# Phase Diagram of QCD 



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— QCD Phase Structure III in CCNU Wuhan -

## A bit about the History

## as an introduction to diquarks

Birth of the QCD Phase Diagram



## Forgotten Candidate




## Forgotten Candidate

## 



## Prototype of Phase Diagram

PHASE DIAGRAM OF NUCLEAR MATTER.
Baym (1983)


June 9 @ QCDPSIII (Wuhan)

## Chiral Phase Diagram




June 9 @ QCDPSIII (Wuhan)

## QCD Critical Point




June 9 @ QCDPSIII (Wuhan)

## Color Superconductivity




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## Chromomagnetic Instabilities




## Some of Highlights

"Charge Neutrality Effects on 2-flavor Color Superconductivity" Mei Huang, Pengfei Zhuang, Weiqin Chao: hep-ph/0207008
"Breached Pairing Superfluidity at Finite Temperature and Density" Jinfeng Liao, Pengfei Zhuang: cond-mat/0307516
"Pion Condensation in Baryonic Matter: from Sarma Phase to Larkin-Ovchnnikov-Fudde (Fulde)-Ferrell Phase" Lianyi He, Meng Jin, Pengfei Zhuang: hep-ph/0604224
"Neutral Color Superconductivity Including Inhomogeneous
Phases at Finite Temperature"
Lianyi He, Meng Jin, Pengfei Zhuang: hep-ph/0610121

## My First? Contact

## New Frontiers in QCD 2008

Fundamental Problems in Hot and/or Dense Matter


## Many QCD Critical Points?

 Zhang-Fukushima-Kunihiro (2008)

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## Polyakov Loop


Fukushima (2003) (2008)


## Revival of Interest




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## BNL-Kyoto-... Diagram




## Inhomogeneity




## Quarkyonic Chiral Spirals

 Kojo-Hidaka-Fukushima-McLerran-Pisarsky (2010)

## Summary




## Alternative Summary


Temperature $T$


Fukushima-Sasaki (2013)
Quark-Gluon Plasma


Hadronic Phase


Nuclear Superfluid

Chemical Potential $\mu_{\mathrm{B}}$

## Interesting Topics Ongoing

Finite isospin / chiral / chemical potentials
$\square$ First-order phase transition? Confirmable with lattice
$\square$ Pion superfluid / LOFF (Ask Pengfei for details)
$\square$ Finite $B(\operatorname{not} E)$ with/without $T$ and density
$\square$ (Inverse) Magnetic catalysis
$\square$ CP in anisotropic pure Yang-Mills theory
Finite rotation (angular momentum)
Xu-guang
Jinfeng...
$\square$ Similar to $B /$ similar to finite density
$\square$ Topological currents
Finite curvature (curved space-time)
$\square$ Chiral symmetric mass gap / Early Universe

## Magnetic Shift of Chemical Freezeout



## Slanting lines

With conservation of $S$ and $Q$

## Shaded lines

Without conserv.
of $S$ and $Q$

KF-Hidaka (2016)

# More about the Diquark 

Never ending project with Jan...

## Biggest Question Mark???



## Nuclear Matter



## Quark Matter

## Diquarks

## Bare vs Constituent

Meson $\sim q \bar{q}+q \bar{q} q \bar{q}+q \bar{q} q \bar{q} q \bar{q}+\cdots$ (Vacuum Re-organized) $\sim q_{\text {con }} \bar{q}_{\text {con }}+($ Bag Constant $)$

How can we be so sure about $M \sim q \bar{q}$

$$
B \sim q q q
$$

beyond quantum num of Quark Model?

A SCHEMATIC MODEL OF BARYONS AND MESONS *
M. GELL-MANN

California Institute of Technology, Pasadena, California
Received 4 January 1964

A simpler and more elegant scheme can be constructed if we allow non-integral values for the
6) James Joyce, Finnegan's Wake (Viking Press, New York, 1939) p. 383. charges. We can dispense entirely with the basic baryon $b$ if we assign to the triplet $t$ the following properties: $\operatorname{spin} \frac{1}{2}, z=-\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer te the members $\mathrm{u}^{\frac{2}{3}}, \mathrm{~d}^{-\frac{1}{3}}$, and $\mathrm{s}^{-\frac{1^{3}}{3}}$ of the triplet as "quarks" 6) $q$ and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be
Primeval expression of diquarks constructed from quarks by using the combinations ( $q q q$ ), ( $q q q q \bar{q}$ ), etc., while mesons are made out of $(q \bar{q})$, ( $q q \bar{q} \bar{q})$, etc. It is assuming that the lowest baryon configuration ( $q$ qq) gives just the representations 1, 8, and 10 that have been observed, while the lowest meson configuration ( $q \bar{q}$ ) similarly gives just 1 and 8.
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## Who was the First?

Perhaps these missing states really do not exist. If baryons were diquark-quark systems, Fig. 2, as Lichtenberg and Tassie noted more than 40 years ago [4], the number of states would be restricted and in fact be very like that currently observed.

Pennington (2011)
Prog. Theor. Phys. Vol. 36 (1966), No. 4

## Baryon Resonances in a Quark Model

Masakuni Ida and Reido Kobayashi
We suppose that baryons consist of a $q q$ pair (or a diquark) and another quark moving around it with orbital angular momentum $L$. In order that for $L=0$ our model can produce the $1 / 2^{+}$octet and the
 $3 / 2^{+}$decuplet, which belong to the " 56 " of $S U(6)$, the $q q$ pair must be in a ${ }^{3} S_{1}$ state and form an $S U(3)$ sextet. Unwanted

# Exotica (scalar nonet) 

# UNCONVENTIONAL STATES OF CONFINED QUARKS AND GLUONS ${ }^{\text {TH }}$ 

R.L. JAFFE* and K. JOHNSON<br>Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Mass. 02139, USA

mentally observed "resonances". In particular we are led to classify the $0^{++}$enhancements known as the $\epsilon$, $\mathrm{S}^{*}$ and $\delta$ as $\mathrm{QQ} \overline{\mathrm{Q}} \overline{\mathrm{Q}}$ states. If correct, this assignment disrupts further the already uneasy state of the P wave mesons in the quark model.
broad exotic $Q Q \bar{Q} \bar{Q}$ states and the $P$-wave baryons states overlap broad $4 Q \bar{Q}$ states. In such cases one might expect that mixing effects will play an essential role in an unravelling of partial widths. This may provide a clue to an understanding of some of the elusive P-wave states such as the $\mathrm{A}_{1}$.

## Can explain why $a_{0}(980)$ heaviest without strangeness Exotic component is to be mixed (via instanton int.)

## Classification



$$
3 \otimes 3=\overline{3} \otimes 6
$$

Quantum numbers and operators
$J^{P} \quad$ Color
Flavor
Operator
$0^{+}$
$1^{+}$
$\overline{\mathbf{3}}$
$\overline{3}$

$$
\bar{\psi}_{\mathcal{C}} \gamma_{5} \psi, \quad \bar{\psi}_{\mathcal{C}} \gamma_{0} \gamma_{5} \psi
$$

6 $\bar{\psi}_{C} \gamma_{i} \psi, \bar{\psi}_{C} \sigma_{0 i} \psi$
$0^{-}$
$1^{-}$
$\overline{3}$
$\frac{6}{3}$
$\bar{\psi}_{C} \psi, \bar{\psi}_{C} \gamma_{0} \psi$
$\bar{\psi}_{C} \gamma_{i} \gamma_{5} \psi, \quad \bar{\psi}_{C} \sigma_{i j} \psi$

## Diquark Phenomenology

## $\Delta I=1 / 2$ rule in non-leptonic weak decay

$$
\begin{aligned}
\Delta I=1 / 2 & \gg \Delta I=3 / 2 \\
& \times \sim 20
\end{aligned}
$$

Stech, Neubert, Xu, Dosch (1987~)
Fierz transformed interaction:

$$
\begin{array}{r}
V_{\mathrm{eff}}=\frac{G_{F}}{\sqrt{2}} V_{u d} V_{u s}\left[c_{-}(u d)_{3}^{\dagger}(s u)_{\overline{3}}+c_{+}(u d)_{6}^{\dagger}(s u)_{6}+\cdots\right] \\
\Delta I=1 / 2 \quad \Delta I=1 / 2,3 / 2
\end{array}
$$

Enhanced by diquarks

## Diquark on Quenched Lattice

 Lattice in Landau gauge (Hess-Karsch-Laermann-Wetzorke 1998)


$$
\begin{gathered}
m_{\text {good }}=694(22) \mathrm{MeV} \\
\left(m_{\pi} \simeq 350 \mathrm{MeV}\right)
\end{gathered}
$$

Mass splitting from $S=1$ and $S=0$

$$
m_{\text {bad }}-m_{\text {good }} \approx \frac{1}{2}\left(m_{\Delta}-m_{N}\right)
$$



## Diquark on Lattice

 Density-density correlator (Alexandrou-de Forcrand-Lucini 2005)

Idealized : static-light-light baryon system

$$
C\left(\boldsymbol{r}_{u}, \boldsymbol{r}_{d}\right)=\langle N| \rho^{u}\left(\boldsymbol{r}_{u}\right) \rho^{d}\left(\boldsymbol{r}_{d}\right)|N\rangle
$$



Lucini et al. (2006)

Correlation $0^{+}>1^{+} \gg 0-$
Characteristic diquark size
$\sim 1.1 \pm 0.2 \mathrm{fm}$ Leinweber (1993)
Larger than meson size


## Diquark and Deconfinement

## Deconfinement in $p Q C D$

 pQCD justifies itself at high $T$ All gluons are screened by $g T$ or $g^{2} T$
## pQCD does not justify itself at high $\mu$

 Magnetic gluons never screenedSuperdense Matter: Neutrons or Asymptotically Free Quarks?
J. C. Collins and M. J. Perry

Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Cambridge CB3 9EW, England (Received 6 January 1975)


Insufficient justification

Color super justifies pQCD at high $\mu$ All gluons are screened by $\boldsymbol{g} \mu$

## No Deconfinement?

## Large- $N_{\mathbf{c}}$ Baryonic (Quark) Matter

$$
\mathcal{O}(1) \quad \longrightarrow \mathcal{O}\left(N_{\mathrm{c}}\right)
$$



## Quarkyonic Matter

McLerran, Pisarski Hidaka, Kojo
Fukushima, Sasaki

## Deconfinement Revisited

## CFL (3-flavor CSC) reads:

SU(3)c broken completely
All 8 gluons get massive (Meissner effect)

## No confinement remains

## Can this be a "definition" of deconfinement?

Private communications with Gordon Baym
If so, quark matter is realized only through diquarks

## Continuity from NM to QM

## $\mathrm{U}(1)_{\mathrm{A}}$ Symmetric



$$
\pi^{+} \sim(u)(\bar{d}) \sim(\bar{d} \bar{s})(s u)
$$


a)

U(1)A Broken

b)
('t Hooft-Isidori-Maiani-Polosa-Riquer 2008)

## Diquark Continuity



## EoS of Quark Matter



## Diquark Continuity



## EoS of Quark Matter

# P <br>  <br> (Quarkyonic) Diquarks 

## EoS of Nuclear Matter

## Implication to the $M-R$ Relation


$M / M_{\text {sun }}$


10~13km
R

## Implication to the $M-R$ Relation

 reannand$M / M_{\text {sun }}$


Pressure at the center
$10 \sim 13 \mathrm{~km}$
R

## Cannot be right!?




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## Duality Hypothesis



## EoS of Quark Matter

P

Dual region describable with APR and NJL Strong constraint from APR to NJL

EoS of Nuclear Matter
$\mu$

## APR-constrained NJL

## All non-perturbative effects renormalized in $G_{V}$

## APR can be reproduced with "running" vector interaction



Best fit function $\sim$ inverse $\log$

$$
\begin{aligned}
& \text { cf. } \quad \alpha_{\mathrm{s}}(\mu)=\frac{1}{b \log (\mu / \Lambda)} \\
& \text { Suggestive!!! }
\end{aligned}
$$

Nuclear matter knows the running coupling?

## APR-constrained NJL

## Fukushima-Kojo (2015)




Weak 1st-order Phase Transition (2SC-CFL)
Single unified theory covering all the densities!

## M-R Relation

## 

 Fukushima-Kojo (2015)

June 9 @ QCDPSIII (Wuhan)

## Summary and Speculation

## Summary

Nuclear Matter = 2SC + Chiral Cond. + 6-diquark
$\square$ More reasonable than CFL-NM continuity
$\square$ Chance to access the diquark superfluid phase in HIC?
$\square$ Enhanced fluctuations from (critical) diquarks
$\square$ Refined HRG with diquarks?
New Model = APR-constrained NJL
$\square$ Microscopic information superseding parametrization
$\square$ Less crazier than using APR to $\sim 5 n_{0}$ ! (Ask Toru!)
Spectroscopy of Qqq baryons?


## Provocative Speculation

Quantum percolation at $p=p_{q}$

(Nothing happens) (Anderson Metal-Insulator Transition)

| (Confined) | $\begin{array}{l}\text { Quarkyonic } \\ \text { Nuclear Matter }\end{array}$ | $\begin{array}{l}\text { (Deconfined) } \\ \text { Regime }\end{array}$ |
| :--- | :--- | :--- |
| Quark Matter |  |  |



Chemical potential $\mu$
Concentration $p$

