Axial charge dynamics: topological transition and quark mass effect

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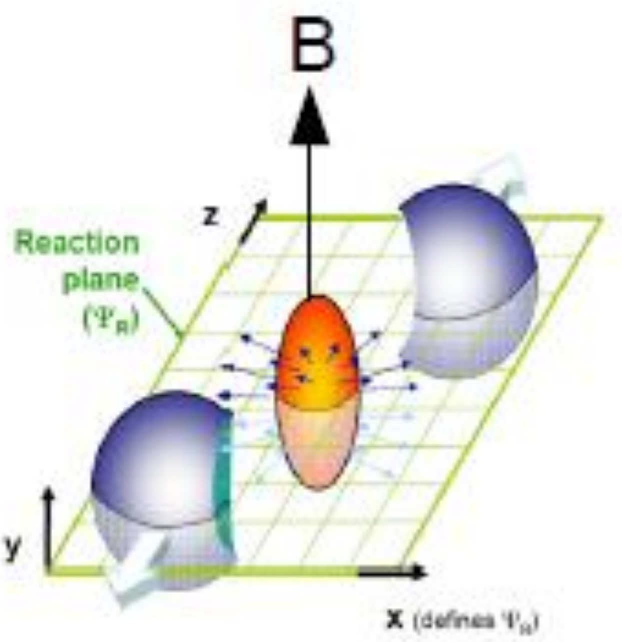
QCD Phase Structure III, CCNU, 2016

Iatrakis, SL, Yin. 1411.2863, PRL 2015
1506.01384, JHEP 2015
Guo, SL. 1602.03952, PRD 2016

Outline

- Motivation
- Basic hydrodynamics in HIC
- Axial charge in hydrodynamics
- Topological transition
- Quark mass effect
- Summary

Local parity violation in heavy ion collisions



Chiral Magnetic Effect (CME)

$$\vec{j}_V = \frac{N_c \mu_A}{2\pi^2} e\vec{B}$$ QED anomaly

μ_A: chiral imbalance in QGP
$eB \sim {m_\pi}^2$: strong magnetic field in heavy ion collisions

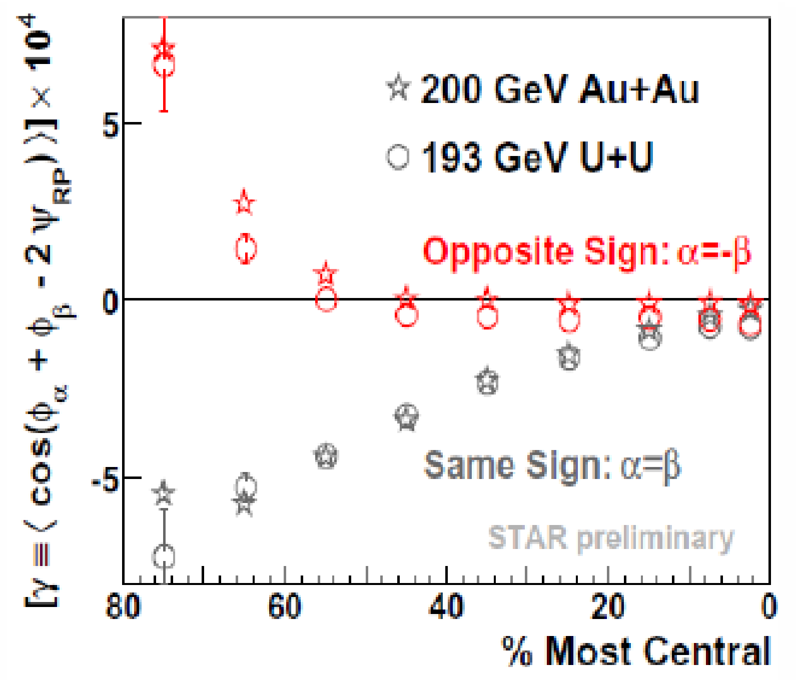
Kharzeev, Zhitnitsky, NPA 2007
Kharzeev, McLerran, Warringa, NPA 2008

Origin of chiral imbalance: $$\partial_\mu j_A^\mu = -\frac{g^2 N_f}{8\pi^2} tr(G\tilde{G})$$ QCD anomaly

$$\Delta N_A = -\frac{g^2 N_f}{8\pi^2} \int dx^4 tr(G\tilde{G})$$ P and CP violation

$$\langle N_A \rangle = 0, \langle {N_A}^2 \rangle \neq 0$$

Experimental signature of CME



$$\vec{j}_V = \frac{N_c \mu_A}{2\pi^2} e\vec{B}$$

$$\langle N_A \rangle = 0, \langle {N_A}^2 \rangle \neq 0$$

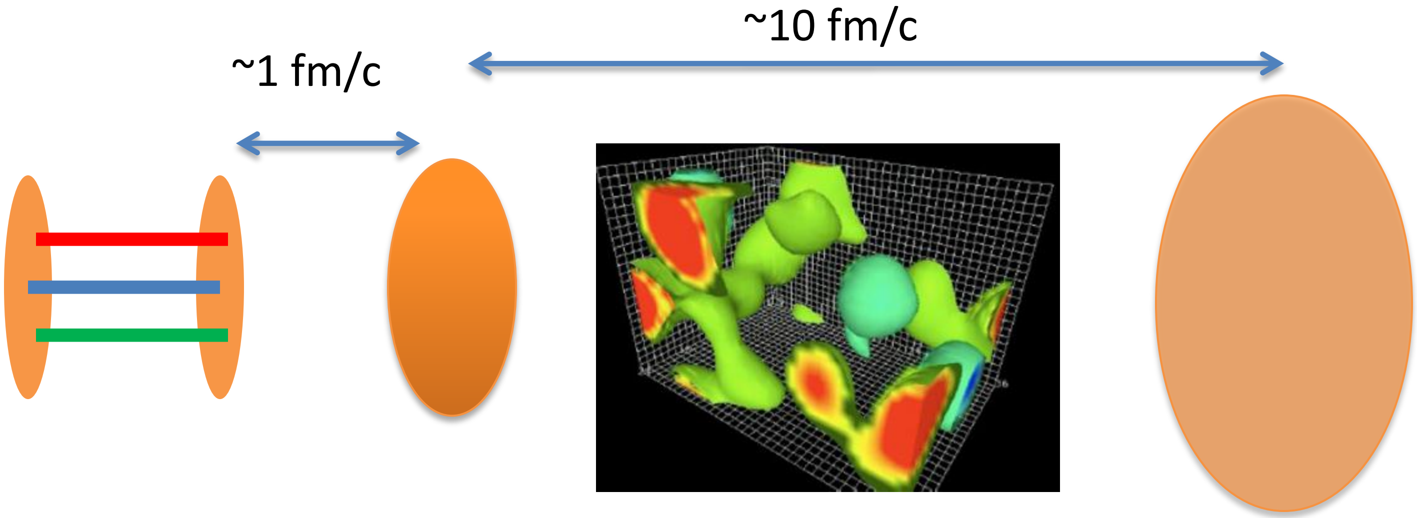
Measurement done on an event-by-event basis

H. Huang's talk

Same charge correlation enhanced than opposite charge correlation due to CME

STAR collaboration, PRL (2014), 1404.1433

Sources of chiral imbalance



Pre-thermal glasma
Parallel chromo E & B

$$\Delta N_A \sim \int dx^4 E_a \cdot B_a$$

Fukushima, Kharzeev, Warringa, PRL (2010)
Hirono, Hirano, Kharzeev, 1412.0311

thermal QGP
Topological transition, e.g. sphaleron decay

Many early works at weak & strong coupling in equilibrium
Arnold, McLerran, Son, Yaffe, 80s-90s
Son, Starinets, 2002

How to integrate in HIC?

Frameworks for axial charge dynamics

chiral kinetic theory (Berry curvature)

Son, Yamamoto, PRL (2012)
Stephanov, Yin, PRL (2012)
Pu, Gao, Wang et al, PRL (2012), (2013), PRD (2014)

Q. Wang’s talk

hydrodynamics (axial charge)

Son, Surowka, PRL (2009)
Neiman, Oz, JHEP (2011)

Relativistic hydrodynamics for HIC

$$\partial_\mu T^{\mu\nu} = F^{\nu\lambda} {j^V}_\lambda$$
$$\partial_\mu {j_V}^\mu = 0$$
$$\partial_\mu {j_A}^\mu = C E^\mu B_\mu$$

Son, Surowka, PRL (2009)

with QED anomaly,
without QCD anomaly

$${j_V}^\mu = n_V u^\mu + {\nu_V}^\mu$$
$${j_A}^\mu = n_A u^\mu + {\nu_A}^\mu$$

talks by Jiang, Huang and Liao

Anomalous part:

$${\nu_V}^\mu = \overset{\text{CVE}}{\overset{\downarrow}{C\mu_V\mu_A\omega^\mu}} + \overset{\text{CME}}{\overset{\downarrow}{C\mu_A B^\mu}}$$
$${\nu_5}^\mu = C/2({\mu_V}^2 + {\mu_A}^2 + \cdots)\omega^\mu + \underset{\text{CSE}}{\underset{\uparrow}{C\mu_V B^\mu}}$$

However, in HIC we need QCD anomaly to generate axial charge!

$$\langle N_A \rangle = 0, \langle {N_A}^2 \rangle \neq 0$$

Axial charge stochastic,
hydrodynamic noise necessary!

How hydro noise is included

Conserved charge as an example

$$\partial_\mu J^\mu = 0,$$

w/o noise

$J^0 = n$ charge density $J_k = -D\partial_k n.$ diffusive current

with noise

$J^0 = n$

$$J_k = -D\partial_k n + r_k$$

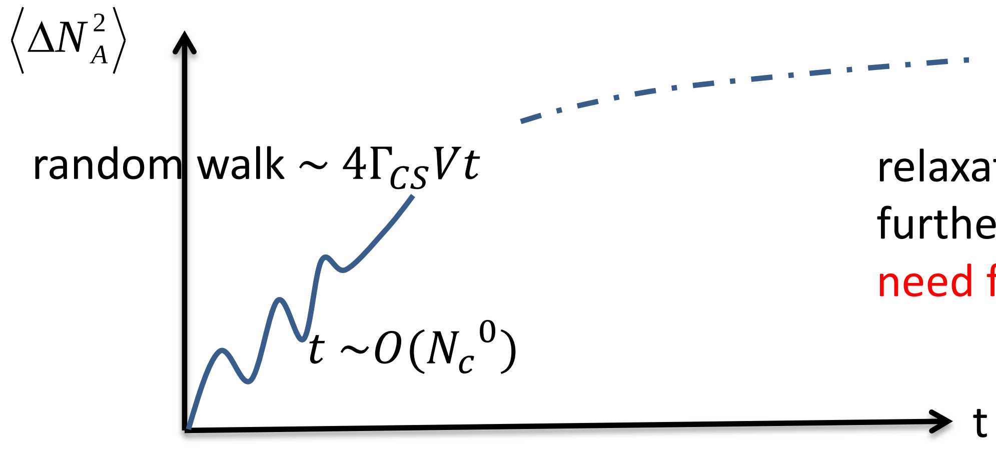
dissipation fluctuation

$$\langle r_i(\mathbf{x},t)\, r_k(\mathbf{x}',t')\rangle = C\delta_{ik}\delta(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Yan's talk

Kovtun, 1205.5040

Axial charge from topological fluctuation



$$t \sim \tau = \frac{\chi T}{2\Gamma_{CS}} \sim O(N_c)$$

relaxation: presence of n_A affects further topological charge fluctuation.
need fermion dynamics

Chern-Simon diffusion rate $\Gamma_{CS} = \int d^4x \langle q(x) q(0) \rangle$

$q \sim trG\tilde{G}$ topological charge density

weak coupling extrapolation: $\Gamma_{CS} \sim 30{\alpha_s}^4 T^4$ Moore, Tassler, JHEP 2011

strong coupling: $\Gamma_{CS} = {\alpha_s}^2 {N_c}^2 T^4/16\pi$ Son, Starinets, JHEP 2002

strong coupling w/B: $\Gamma_{CS} \sim {\alpha_s}^2 {N_c}^2 B T^2$ Basar, Kharzeev, PRD 2012

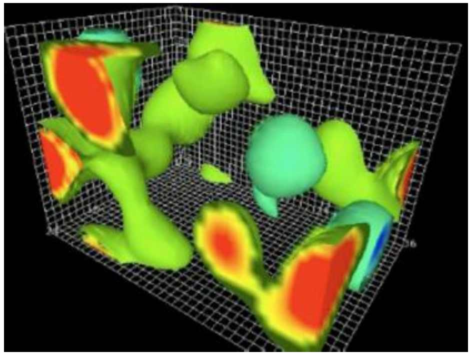
Topological fluctuation as hydro noise

Size of QGP >> fluid cell >> size of topological fluctuation

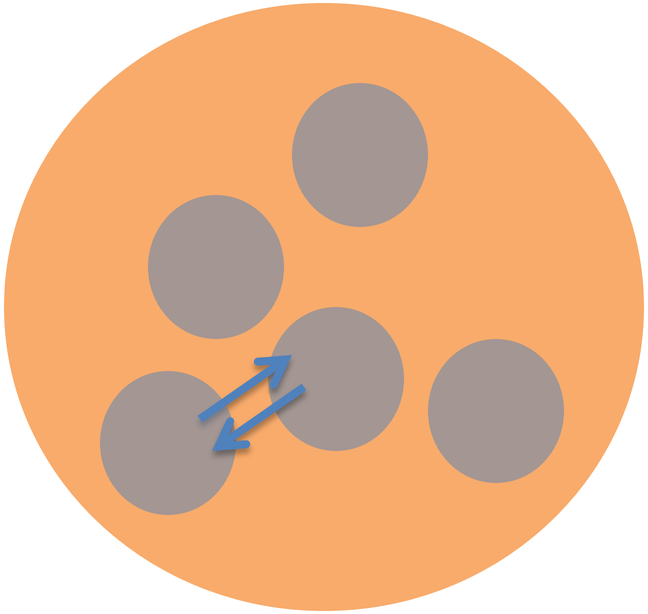
Axial charge fluctuation localized in fluid cell

Topological transition additional source of noise!

within one fluid cell



between fluid cells



The Sakai-Sugimoto model (D4/D8)

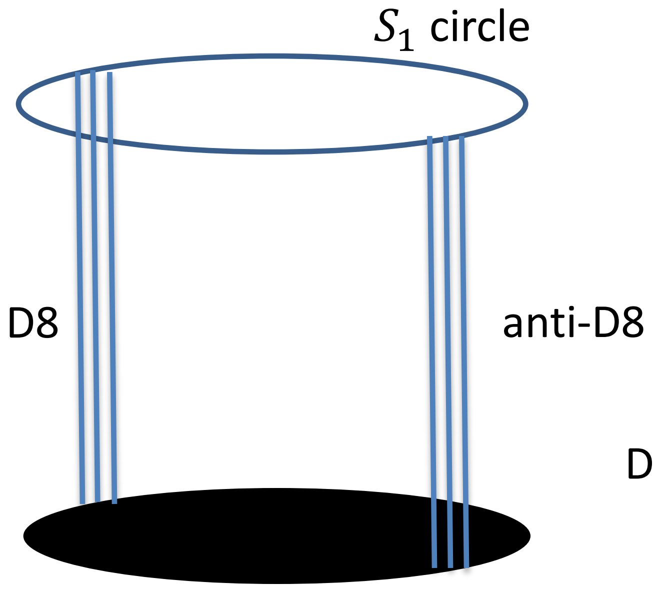
N_c D4 branes wrapped on S^1 circle + N_f D8/anti D8 branes being a point on S^1.

$\Uparrow$ gluons

$\Uparrow$ left/right handed quarks

mass gap $M_{KK} = \dfrac{1}{R_4}$

Sakai, Sugimoto, Prog.Theor.Phys, 2005

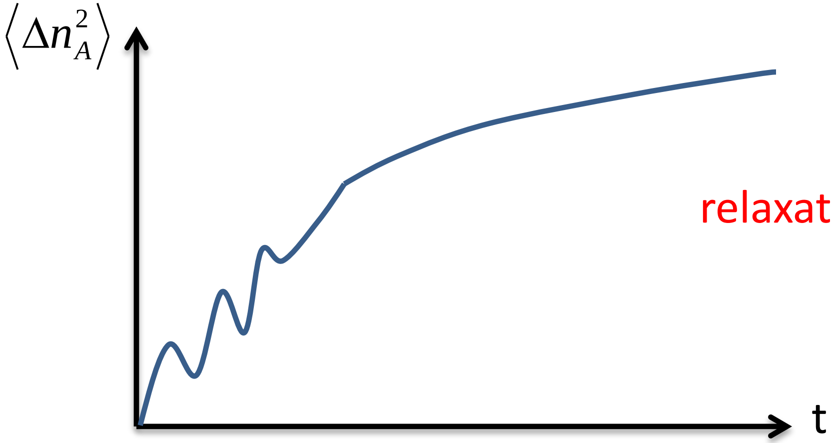


black D4 brane background

Deconfined, chiral symmetry restored

Aharony et al, Annal. Phys. 2006

Axial charge relaxation



relaxation, explicit in our model with fermions

Response of q to n_A

$$q = \frac{\Gamma_{CS}}{\chi T} n_A \Longrightarrow \frac{dn_A}{dt} = -2q = -\frac{2\Gamma_{CS}}{\chi T} n_A = -\frac{n_A}{\tau_{sph}}$$

χ: static susceptibility $\tau_{sph} = \frac{\chi T}{2\Gamma_{CS}}$: relaxation time

consistent with early statistical argument

Also work by Akamatsu, Rothkopf, Yamamoto, JHEP 2016

Iatrakis, SL, Yin, JHEP 2015

Stochastic hydrodynamics for axial charge

Dynamical equation

Constitutive equations

$$\left\{\begin{array}{l} \partial_t n_A(t,x) + \nabla\cdot j_A(t,x) = -2q(t,x) \\ \\ j_A(t,x) = -D\nabla n_A(t,x) + \xi(t,x) \\ q(t,x) = \dfrac{n_A(t,x)}{2\tau_{\text{sph}}} + \xi_q(t,x) \end{array}\right.$$

Non-topological fluctuation $\langle \xi_i(t,x)\xi_j(t,x')\rangle = 2\sigma T\delta_{ij}\delta(t-t')\delta^3(x-x')$

topological fluctuation $\langle \xi_q(t,x)\xi_q(t,x')\rangle = \Gamma_{\text{CS}}\delta(t-t')\delta^3(x-x')$

Iatrakis, SL, Yin, JHEP 2015

Time evolution of axial charge from stochastic hydrodynamics

$$C_{nn}(t,x) \equiv \langle [n_A(t,x) - n_A(0,x)][n_A(t,0) - n_A(0,0)] \rangle$$

$$C_{nn}(t,\boldsymbol{x}) = (\chi T)\left[\delta^3(\boldsymbol{x}) - \frac{1}{(8\pi D t)^{3/2}} e^{-\frac{2t}{\tau_{\rm sph}}} e^{-\frac{|\boldsymbol{x}|^2}{8Dt}}\right]$$

within cell

across cells

Early time $t \ll \tau_{\rm sph}$

$$C_{nn}(t,\boldsymbol{x}) \approx 4\Gamma_{\rm CS}\, t\, \delta^3(\boldsymbol{x})$$

Late time $t \gg \tau_{\rm sph}$

$$C_{nn}(t \to \infty, \boldsymbol{x}) \to (\chi T)\, \delta^3(\boldsymbol{x})$$

thermodynamic limit

Quark mass effect

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma},$$

when $m \ll T$, neglect mass term above

HIC at RHIC, $T \lesssim 350MeV$
Strange quark mass $m{\sim}100MeV$

Mass effect may enhance
fluctuation/dissipation of axial charge.

The D3/D7 model

N_c D3 branes + N_f D7 branes

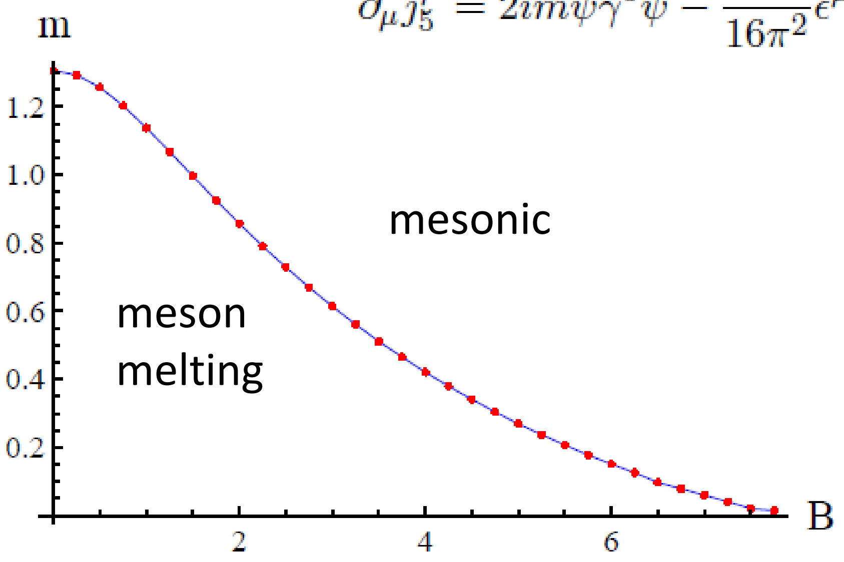
⇑ ⇑

gluons quarks

Pro: quark mass explicit (as compare to D4/D8)

Con: w/o QCD anomaly, but with QED anomaly and mass term

$$\partial_\mu j_5^\mu = 2im\bar{\psi}\gamma^5\psi - \frac{e^2}{16\pi^2}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - \cancel{\frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma}},$$

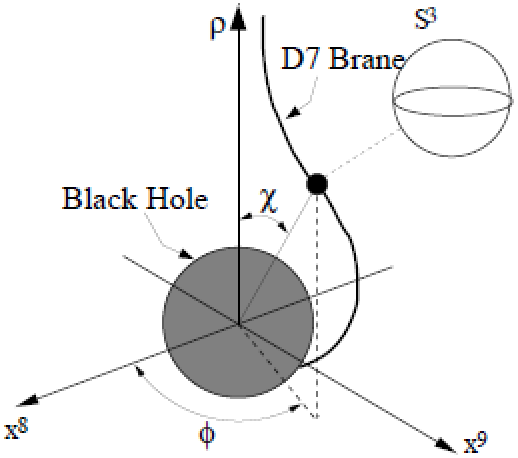


Mateos et al, PRL 2006

Filev et al, JHEP 2007

Erdmenger et al, JHEP 2007

Axial anomaly in D3/D7 model



axial-symmetry realized as rotation
in x8-x9 plane

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
D3	×	×	×	×						
D7	×	×	×	×	×	×	×	×		

$$S = \mathcal{N}\int d^5x\left(-\frac{1}{2}\sqrt{-G}G^{MN}\partial_M\phi\partial_N\phi - \frac{1}{4}\sqrt{-H}F^2\right) - \mathcal{N}\kappa\int d^5x\,\Omega\epsilon^{MNPQR}F_{MN}F_{PQ}\partial_R\phi$$

$$\partial_\mu\left(\frac{\delta S}{\delta\partial_\mu\phi}\right) + \partial_\rho\left(\frac{\delta S}{\delta\partial_\rho\phi}\right) = 0$$

$$J^\mu_R = \int d\rho\tfrac{\delta S}{\delta\partial_\mu\phi} \qquad \partial_\mu J^\mu_R + \underset{\uparrow}{\frac{\delta S}{\delta\partial_\rho\phi}}\Big|^{\infty}_{\rho=\rho_h} = 0$$

ϕ dual to $\quad mi\bar{\psi}\gamma^5\psi + \cdots + \mathcal{N}E\cdot B$

Hoyos et al, JHEP (2011)

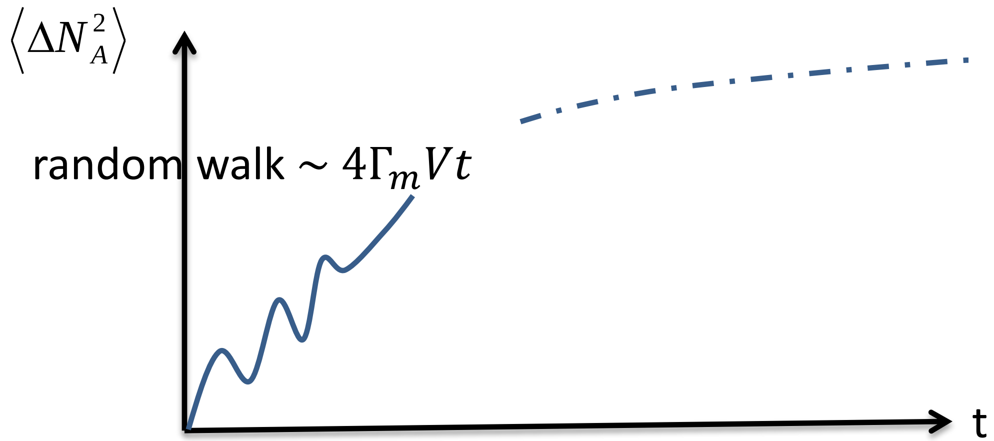
Mass diffusion rate

$$O_\eta = \, mi\bar{\psi}\gamma^5\psi + \cdots$$

$$G_{\eta\eta}(\omega) = \int dt \langle [O_\eta(t), O_\eta(0)] \rangle \Theta(t) e^{i\omega t} \; \sim \frac{-i\omega\Gamma_m}{2T} \qquad \text{as } \omega \to 0$$

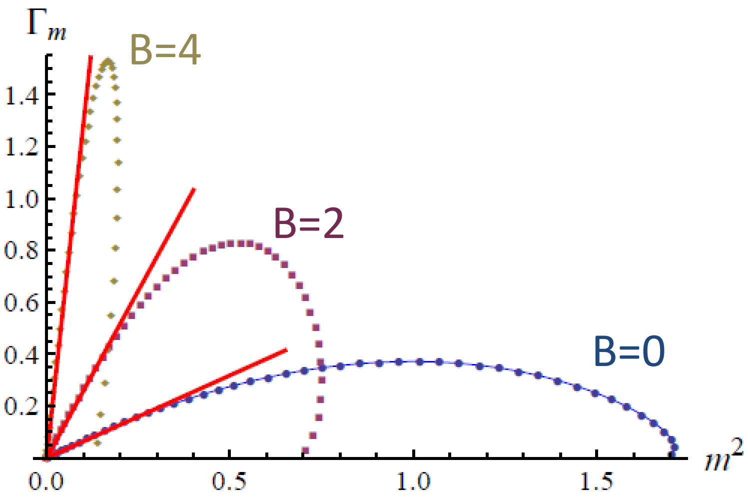
Γ_m analogous to Γ_{CS}

mass diffusion rate   CS diffusion rate (absent in D3/D7)



Guo, SL, PRD (2016)

Mass diffusion rate



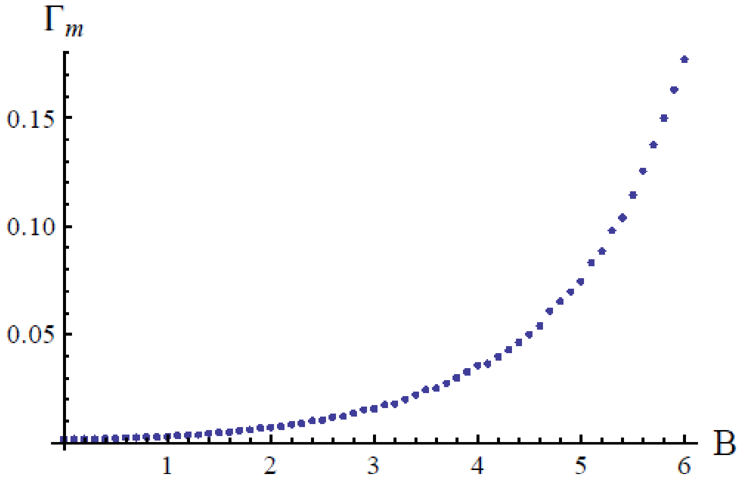
$$\Gamma_m \sim m^2 F(B)$$

Measure of helicity flipping rate

Magnetic field enhances helicity flipping rate

Guo, SL, PRD (2016)

m=1/20



$$B = {m_\pi}^2, T = 300MeV, M = M_s, N_f = 1$$

$$\Gamma_m \sim 6\Gamma_{cs}$$

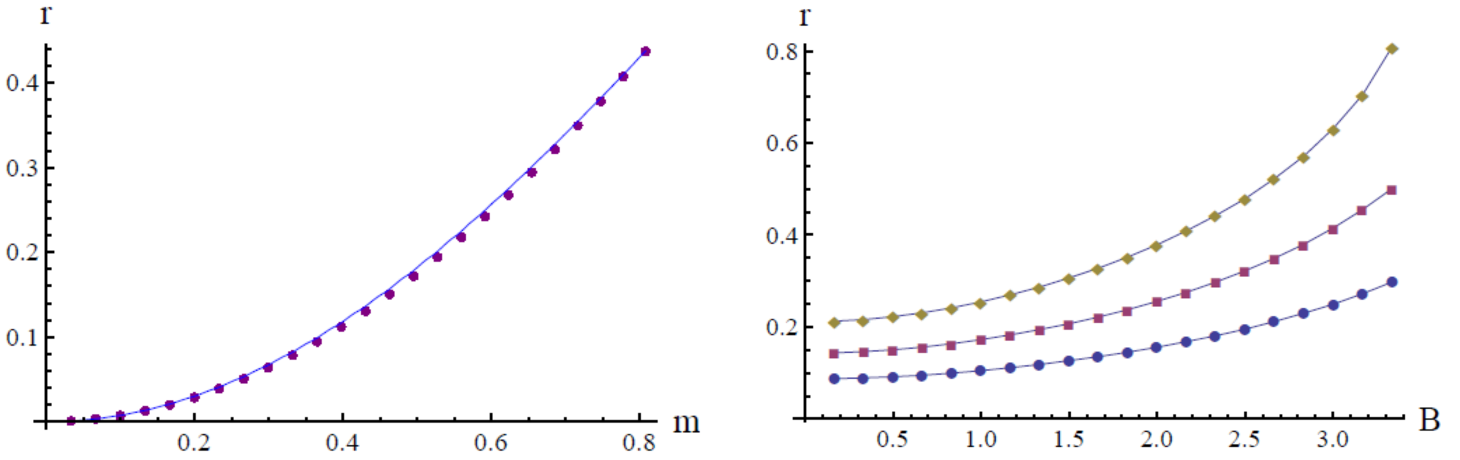
Mass diffusion significant compared to Chern-Simon diffusion

Mass dissipation rate

$$\partial_\mu j_5^\mu = \underset{\underset{O_\eta}{\uparrow}}{2im\bar{\psi}\gamma^5\psi} - \frac{e^2}{16\pi^2}\underset{\underset{E\cdot B}{\uparrow}}{\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}} - \cancel{\frac{g^2}{16\pi^2}\mathrm{tr}\epsilon^{\mu\nu\rho\sigma}G_{\mu\nu}G_{\rho\sigma}},$$

$$r = \frac{O_\eta}{\mathcal{N}E\cdot B}.$$

Produce axial charge by setting up parallel E and B fields for $\omega \to 0$

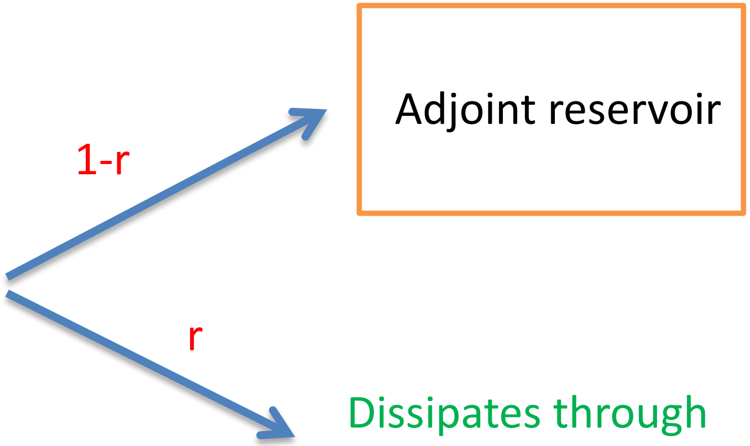


Mass term more effective in dissipating axial charge at large m and B

r<1: axial charge survives the hydro limit for massive quarks?

Guo, SL, PRD (2016)

Mass dissipation rate



One unit of
axial charge
$E \cdot B$ fields

1-r

Adjoint reservoir

$\rho = \rho_h$

$$\partial_\mu J^\mu_R + \frac{\delta S}{\delta \partial_\rho \phi}|^{\infty}_{\rho=\rho_h} = 0$$

r

Dissipates through
mass term

$\rho = \infty$

Effectively only r unit of axial charge is produced, all dissipates through the mass term in hydro limit

Guo, SL, PRD (2016)

Consistent with relaxation time approximation

Landsteiner et al, JHEP 2015

τ_{rel} increases with B, decreases with m

Phenomenology? finite τ_{hydro} versus τ_{rel}

Mass correction to non-dissipative effect

e.g. modified CMW

$$\vec{j}_V = \frac{N_c \mu_A}{2\pi^2} e\vec{B}$$

$$\vec{j}_A = \frac{N_c \mu_V}{2\pi^2} e\vec{B} + O(m^2)$$

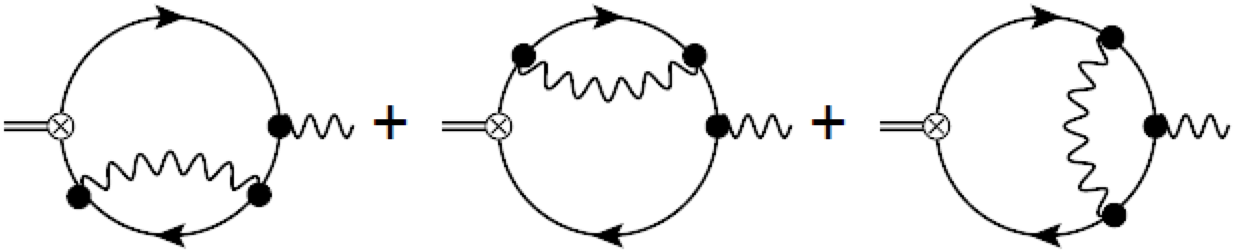
Can be studied reliably in D3/D7 model, no axial charge exchange between quarks and adjoint matter

Quark mass effect on CSE

free theory $\mathbf{j}_5 = e\mathbf{B}\sqrt{\mu^2 - m^2}/(2\pi^2)$

QED at T=0

perturbative correction



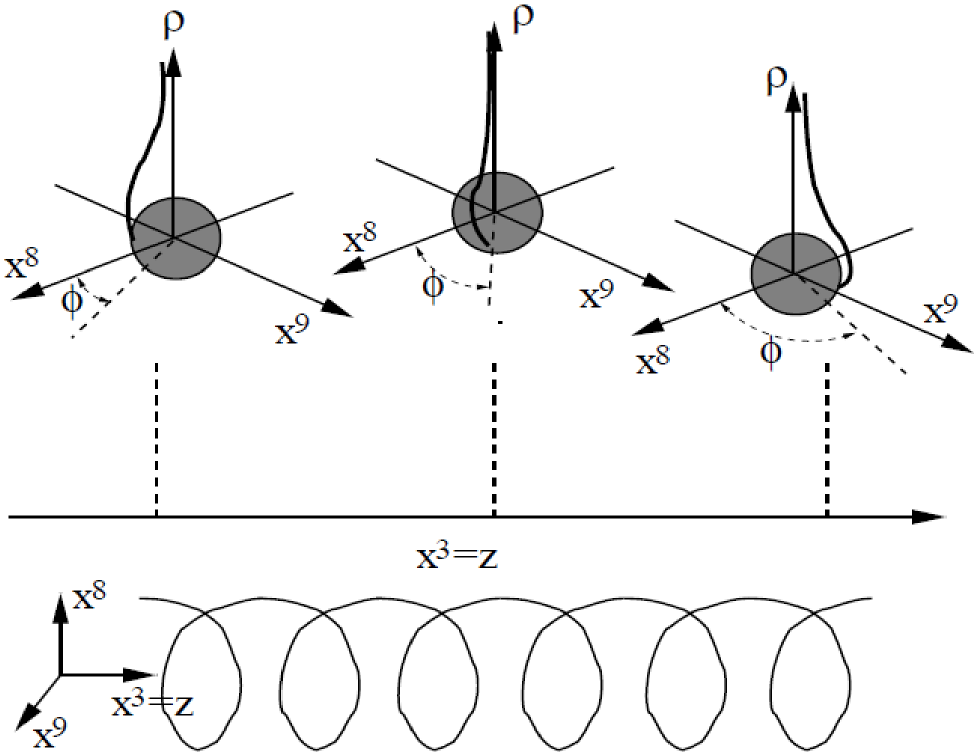
$$\langle j_5^3 \rangle_\alpha = -\frac{\alpha e B \mu}{2\pi^3}\left(\ln\frac{2\mu}{m} + \ln\frac{m_\gamma^2}{m^2} + \frac{4}{3}\right) - \frac{\alpha e B m^2}{2\pi^3 \mu}\left(\ln\frac{2^{3/2}\mu}{m_\gamma} - \frac{11}{12}\right)$$

m_γ IR cutoff

Non-pertubative correction?

Gorbar et al, PRD 2013

Spiral phase and correction to CSE



D7 brane being a point in x8-x9 plane, spiral phase in x3 direction

Kharzeev and Yee, PRD 2011

In spiral phase $i\bar{\psi}\gamma^5\psi \neq 0$ induces correction to CSE in massive case

In progress

Summary

- Axial charge leads to stochastic hydrodynamics: contains two types of noise topological transition in addition to the known (thermal) fluctuation.
- Response of topological charge density to axial charge density gives relaxation of axial charge.
- Quark mass diffusion rate in analogy to CS diffusion rate.
- Quark mass dissipation consistent with relaxation time approximation.
- Quark mass correction to CSE

Thank you!

Dynamical susceptibility from CME

Define dynamical axial chemical potential using CME $J(\omega) = C\mu_A(\omega)B(\omega)$

susceptibility $\chi(\omega) = \dfrac{n_A(\omega)}{\mu_A(\omega)}$

$\chi \sim O(\omega^{-1})$ as $\omega \to 0$ divergent suseptibility Guo, SL, PRD (2016)

- Spontaneous generation of axial charge costs no energy (diffusion)
- Leakage of axial charge from quarks to adjoint reservoir (specific to D3/D7 model)

while m=0 has a finite χ as $\omega \to 0$

Phenomenology? finite m versus ω

Possible contamination of CME

Compare the Chiral Magnetic Current induced by axial charge generated from different sources.

eB constant, and small enough not to affect the dynamics of gluons & quarks

i: n_A from topological fluctuation

QCD X QED anomalies

$$j_V = \frac{N_c eB}{2\pi^2}\frac{n_A}{\chi}$$

Standard CME

ii: n_A from non-topological fluctuation

QED anomaly only

$$j_V = \frac{N_c eB}{2\pi^2}\frac{n_A}{\chi} - D\nabla n_V \xrightarrow{eB \ll \sigma k} \frac{N_c eB}{2\pi^2}\frac{2n_A}{\chi}$$

Twice Standard CME

Experimental CME signal may be contaminated by thermal diffusion!

Iatrakis, SL, Yin, JHEP 2015

Correction due to O_η?

$$\partial_\mu j^\mu_5 = \underset{\substack{\uparrow \\ O_\eta}}{2im\bar{\psi}\gamma^5\psi} - \frac{e^2}{16\pi^2}\underset{\substack{\uparrow \\ E \cdot B}}{\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}}$$

$i\bar{\psi}\gamma^5\psi$ breaks P and T

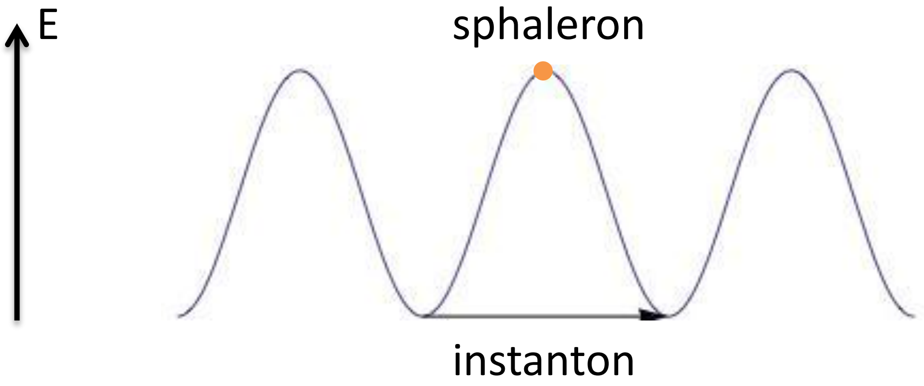
T broken by B, spontaneous breaking of P?

Axial charge generation from topological transition of gluons

We will consider fluctuations in quark gluon plasma
This does not include axial charge generation from glasma field

Hirono’s talk

$$\partial_\mu j^\mu_A = -\frac{g^2 N_f}{8\pi^2} tr(G\tilde{G}) \equiv -2q \qquad \Delta N_A = \int d^3 x n_A(x,t) = -\int d^4 x 2q$$



$$\langle q \rangle = 0$$

$$\langle q^2 \rangle \neq 0$$

Nonperturbative contributions exponentially suppressed $e^{-\frac{\#}{g^2}}$

However, at finite temperature, there are fluctuations of arbitrary size, not restricted to instanton and sphaleron, turning the exponential suppression into a power law suppression.

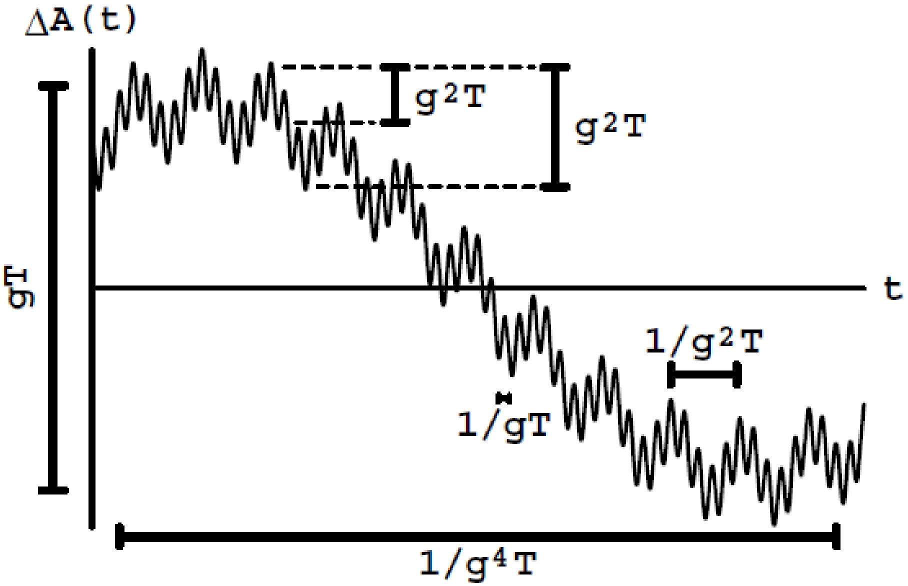
Arnold, McLerran PRD 1987

Chern-Simon rate at weak coupling

$$\left\langle (N_A(t) - N_A(0))^2 \right\rangle = 4Vt\Gamma_{CS} + \dots$$

$$\Gamma_{CS} = \int d^4x \langle q(x)q(0) \rangle \sim \frac{1}{\text{time} * \text{vol}} \sim \# \, g^{10} T^4 \ln g^{-1}$$

volume ~ $1/(g^2T)^3$, time ~ $1/(g^4T)$ $\xrightarrow{\text{color conductivity}}$ time ~ $1/(g^4 T ln(1/g))$



Arnold, Son, Yaffe, PRD 1997
Arnold, Son, Yaffe, PRD 1999

Bodeker's effective theory

Effective theory at non-perturbative scale g^{2}T and below

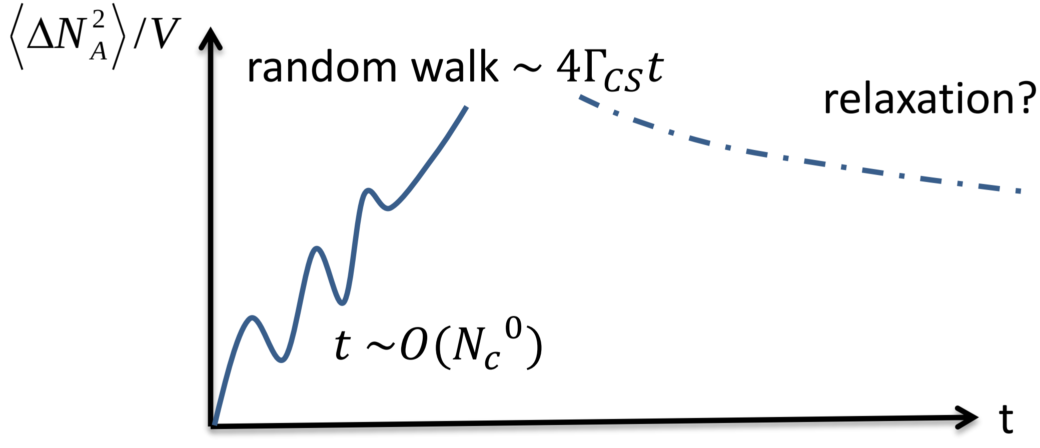
$$D_j F_{ji} + D_t E_i = -J_i \quad = -\sigma E_i + \zeta_i$$

Need real time lattice simulation

σ: conductivity

ζ: noise due to interaction with field above scale g^{2}T

Bodeker, PLB 1998



relaxation? need fermion dynamics

$$t \sim \tau = \frac{\chi T}{2\Gamma_{CS}} \sim O(N_c)$$

Model independent derivation

Gradient expansion:

$$G_R^{qq}(\omega, k) = \frac{1}{2}\left[-i\frac{\Gamma_{\rm CS}}{T}\omega + \tau_{CS}\omega^2 - \kappa_{\rm CS}k^2 + \mathcal{O}(\omega^3, k^3)\right]$$

$$\Longrightarrow$$

$$\partial_\mu j_A^\mu = -2q(t, \vec{x}) = -\left[\frac{\Gamma_{\rm CS}}{T}\partial_t - \tau_{\rm CS}\partial_t^2 + \kappa_{\rm CS}\partial_x^2\right]\theta(\tau, \boldsymbol{x})$$

$$\boldsymbol{j}_{A,\rm anom} = -\kappa_{\rm CS}\nabla\theta + \mathcal{O}(\partial^2) \qquad (1)$$

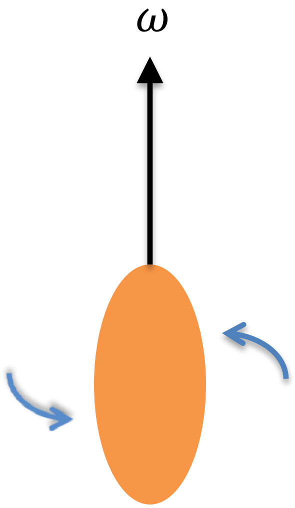
$$n_{A,\rm anom} \equiv j^t_{A,\rm anom} = -\frac{\Gamma_{\rm CS}}{T}\theta + \mathcal{O}(\partial)$$

Diffusion contribution

$$\boldsymbol{j}_{A,\rm norm} = D\nabla(\frac{\Gamma_{\rm CS}}{T}\theta) \qquad (2)$$

Sum of (1) and (2) gives total response to θ

Local parity violation in heavy ion collisions



vorticity: $\vec{\omega} = \frac{1}{2} \nabla \times \vec{v}$

Present in spinning quark gluon plasma

$$\vec{j}_V = \frac{N_c}{2\pi^2} \mu_V \mu_A \vec{\omega}$$

Chiral Vortical Effect (CVE)

Kharzeev, Son, PRL 2011