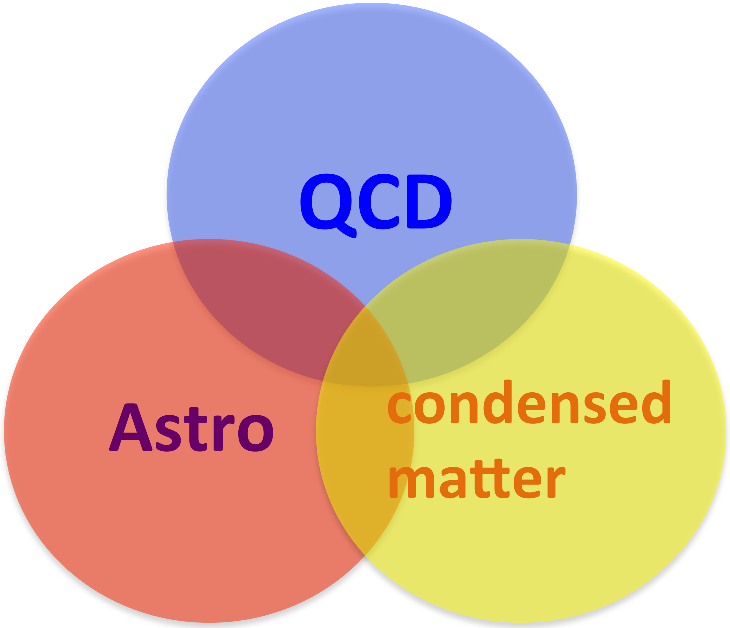
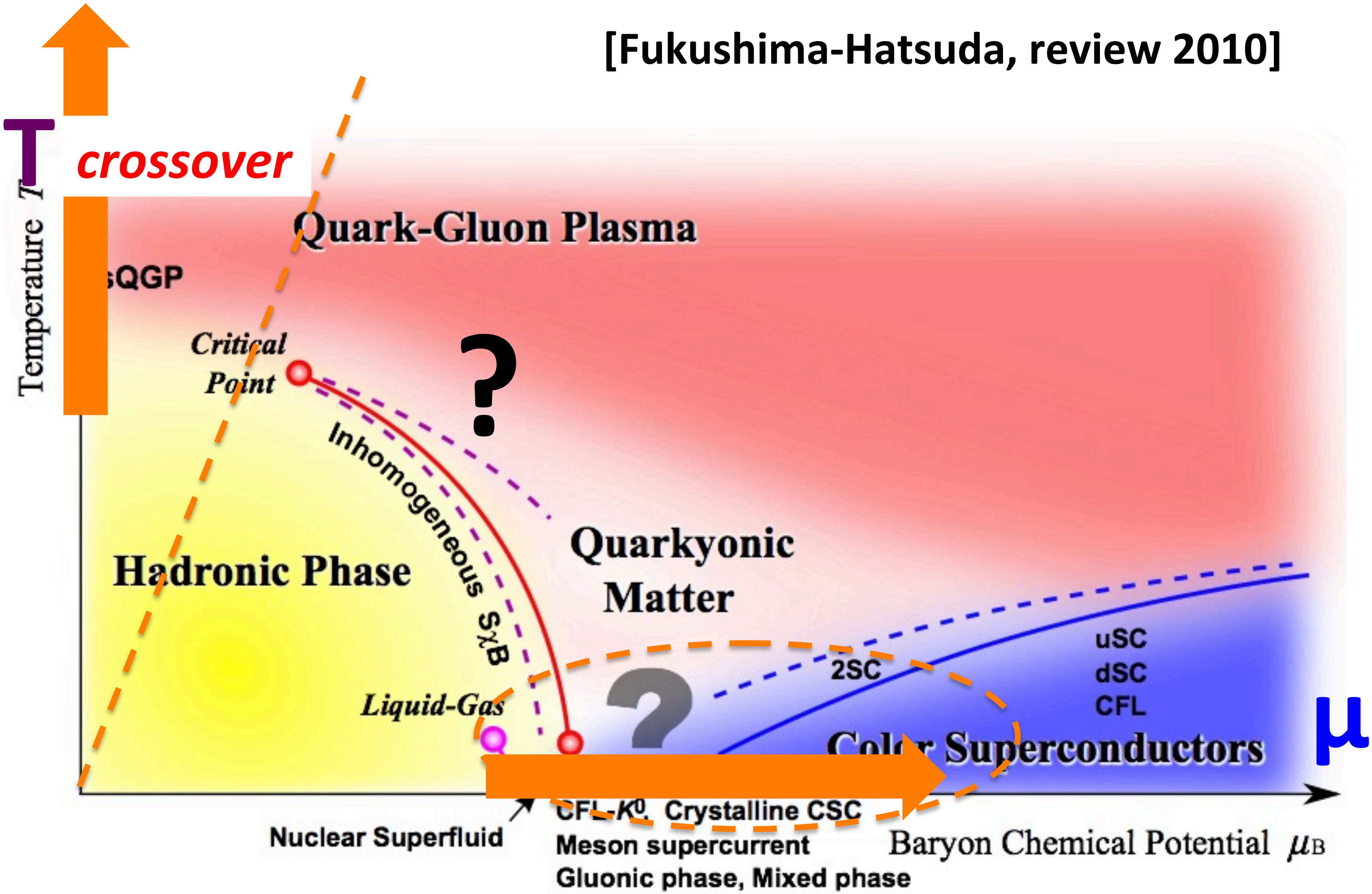
Quarkyonic matter & neutron star equations of state

Toru Kojo (CCNU)



- TK, G. Baym
  1404.1346 [hep-ph] , PRD89, 125008 (2014)
- TK, P.D. Powell, Y. Song, G. Baym
  1412.1108 [hep-ph] , PRD91, 045043 (2015)
- TK, 1508.1108 [hep-ph], review in EPJA
- K. Fukushima & TK, 1509.1108, APJ817(2016)2

Cold, dense QCD: Questions

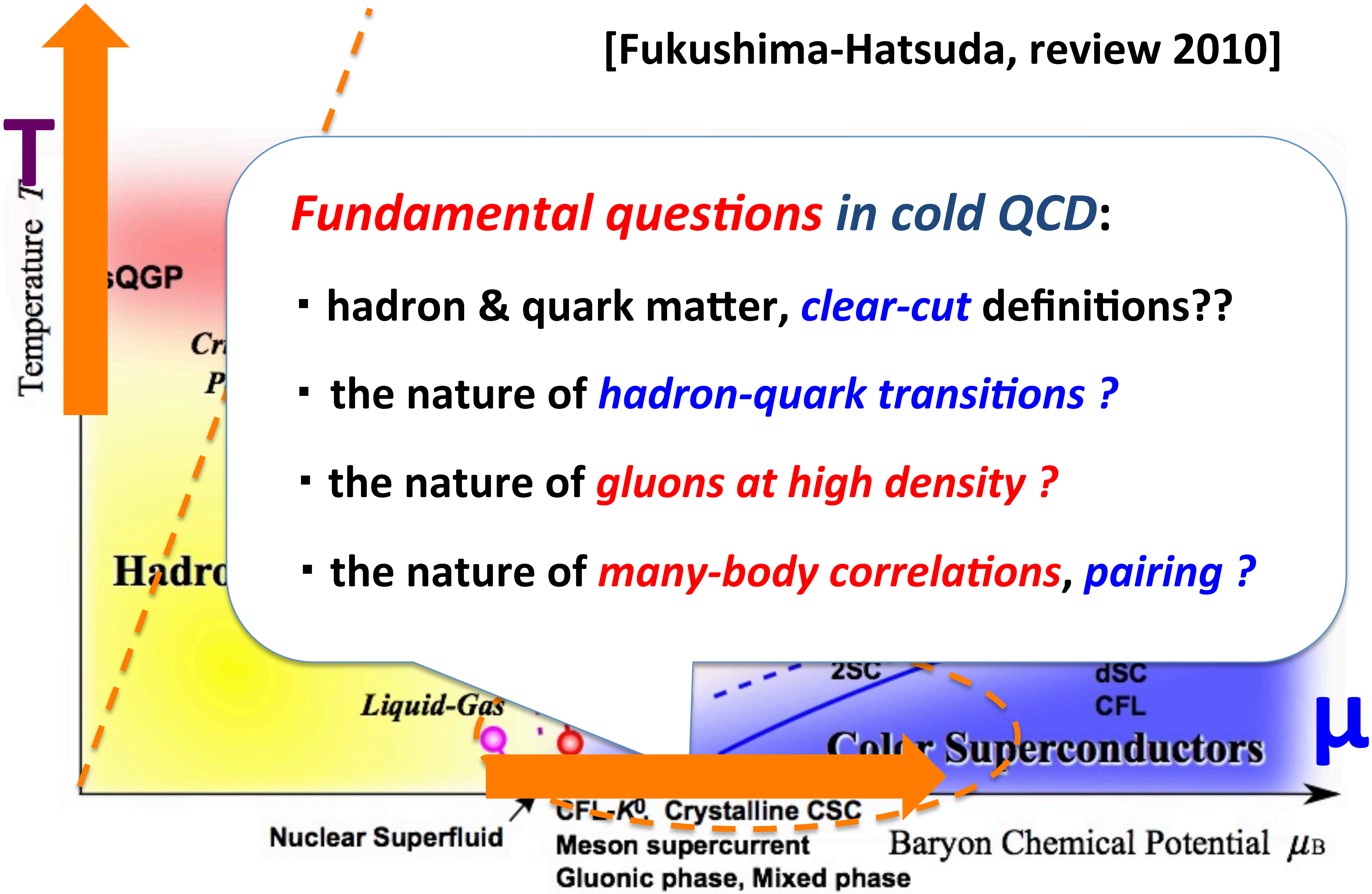


Cold, dense QCD: Questions

[Fukushima-Hatsuda, review 2010]

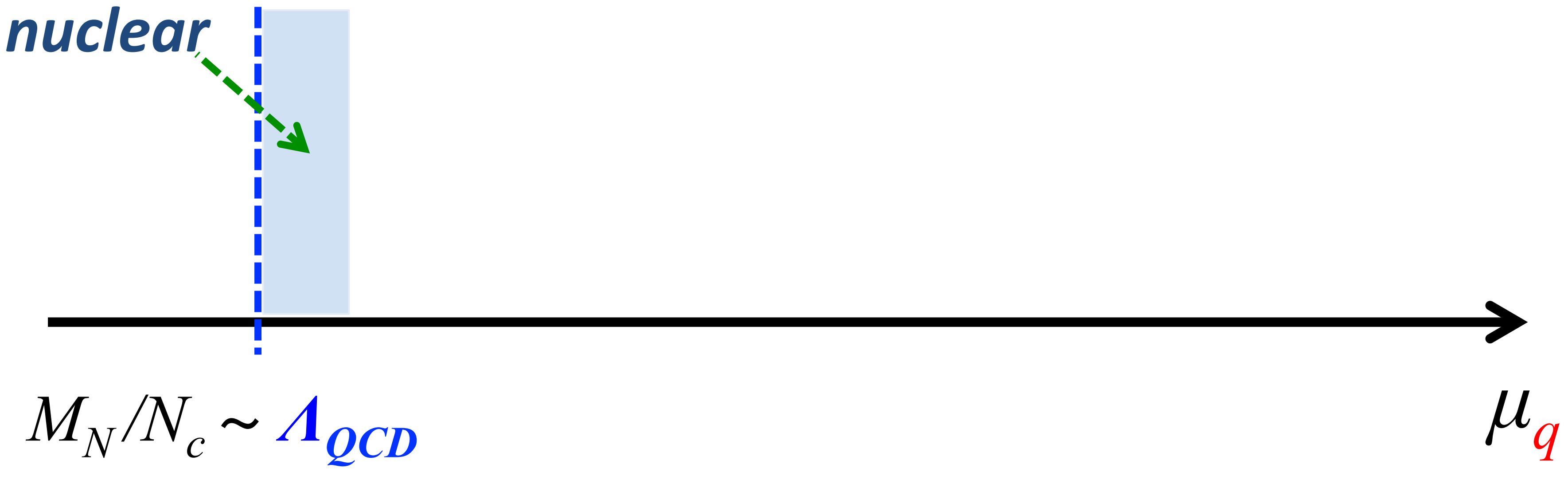
Fundamental questions in cold QCD:

- hadron & quark matter, *clear-cut* definitions??
- the nature of *hadron-quark transitions ?*
- the nature of *gluons at high density ?*
- the nature of *many-body correlations*, *pairing ?*



McLerran-Pisarski's picture

[McLerran-Pisarski '07]



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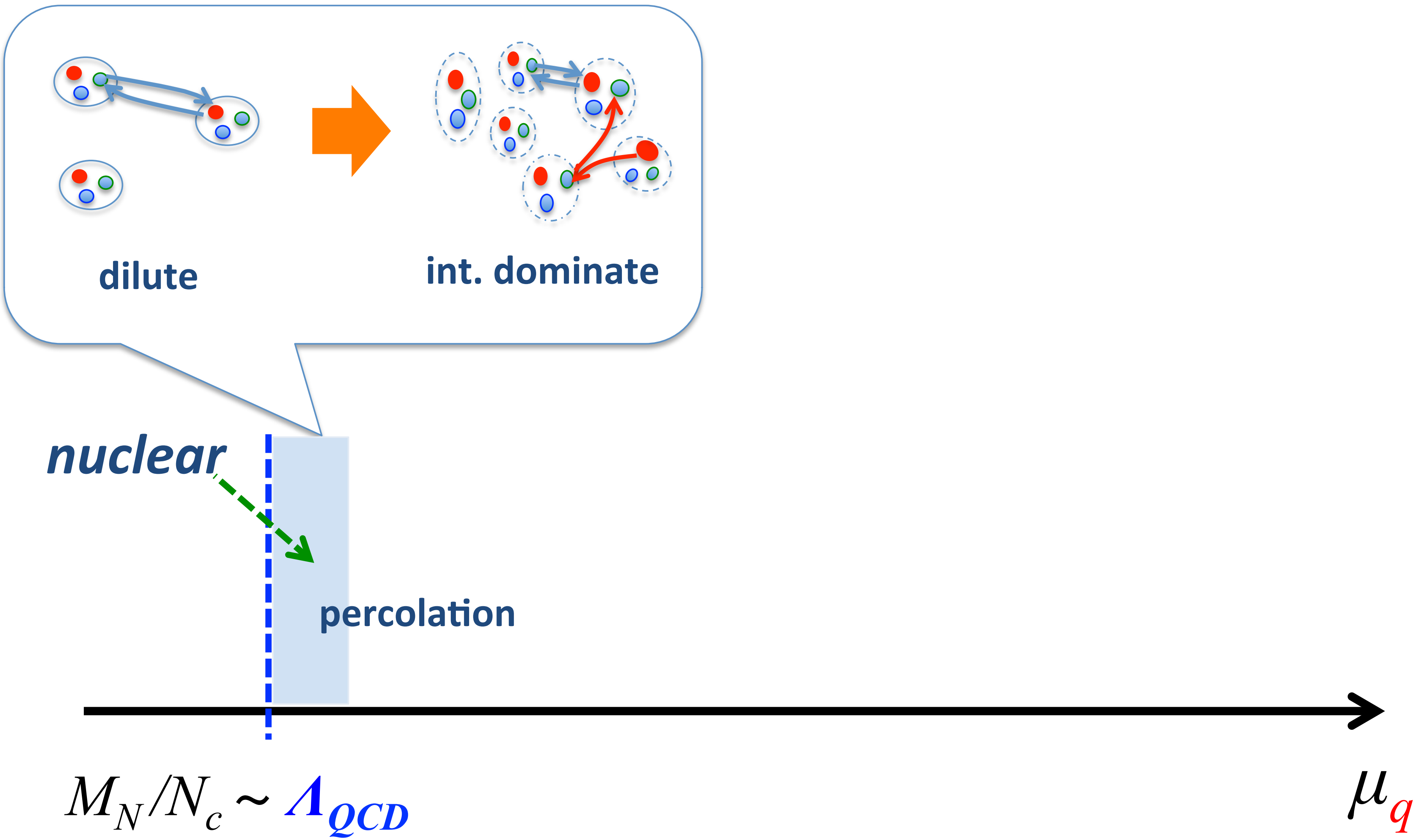

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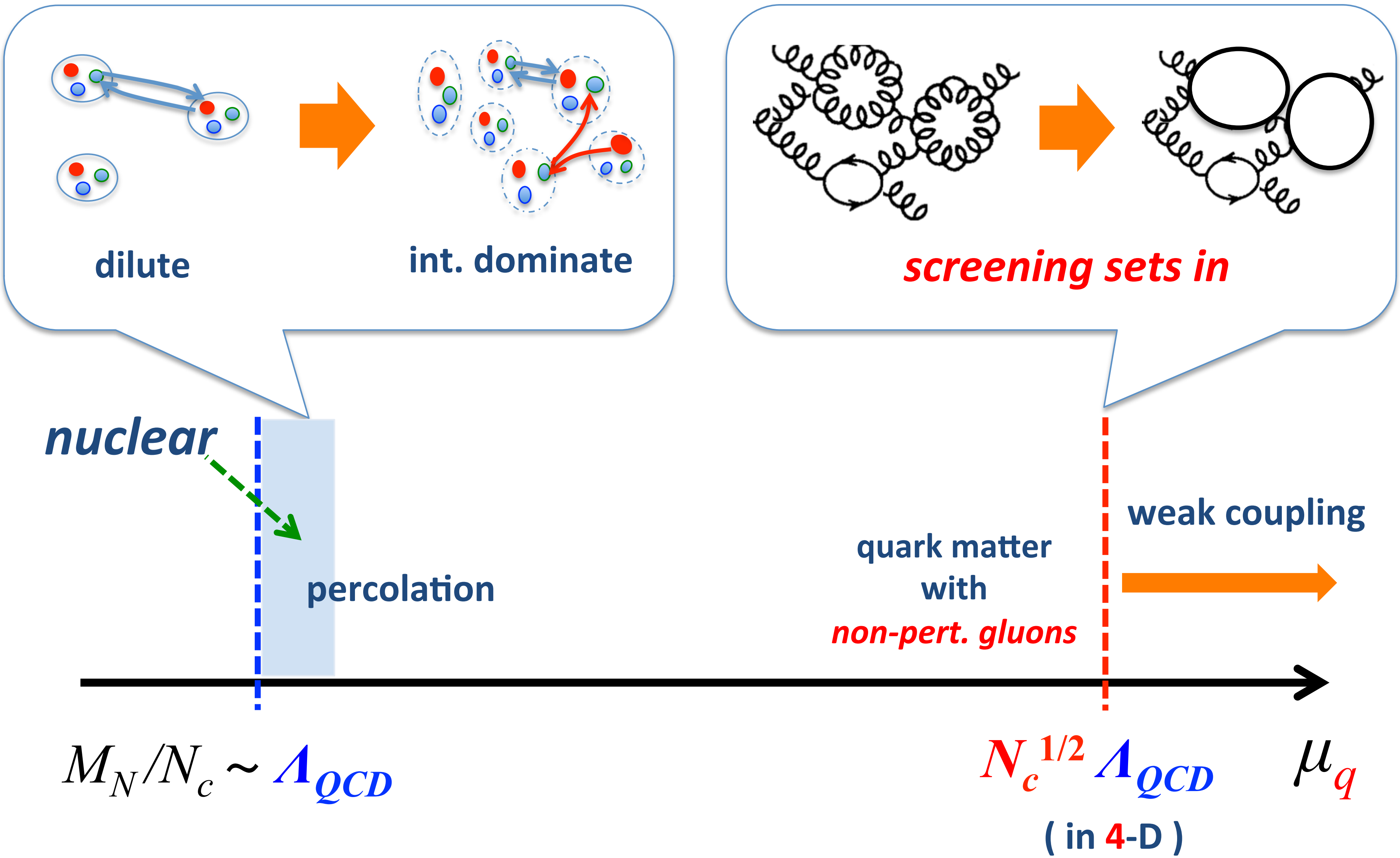

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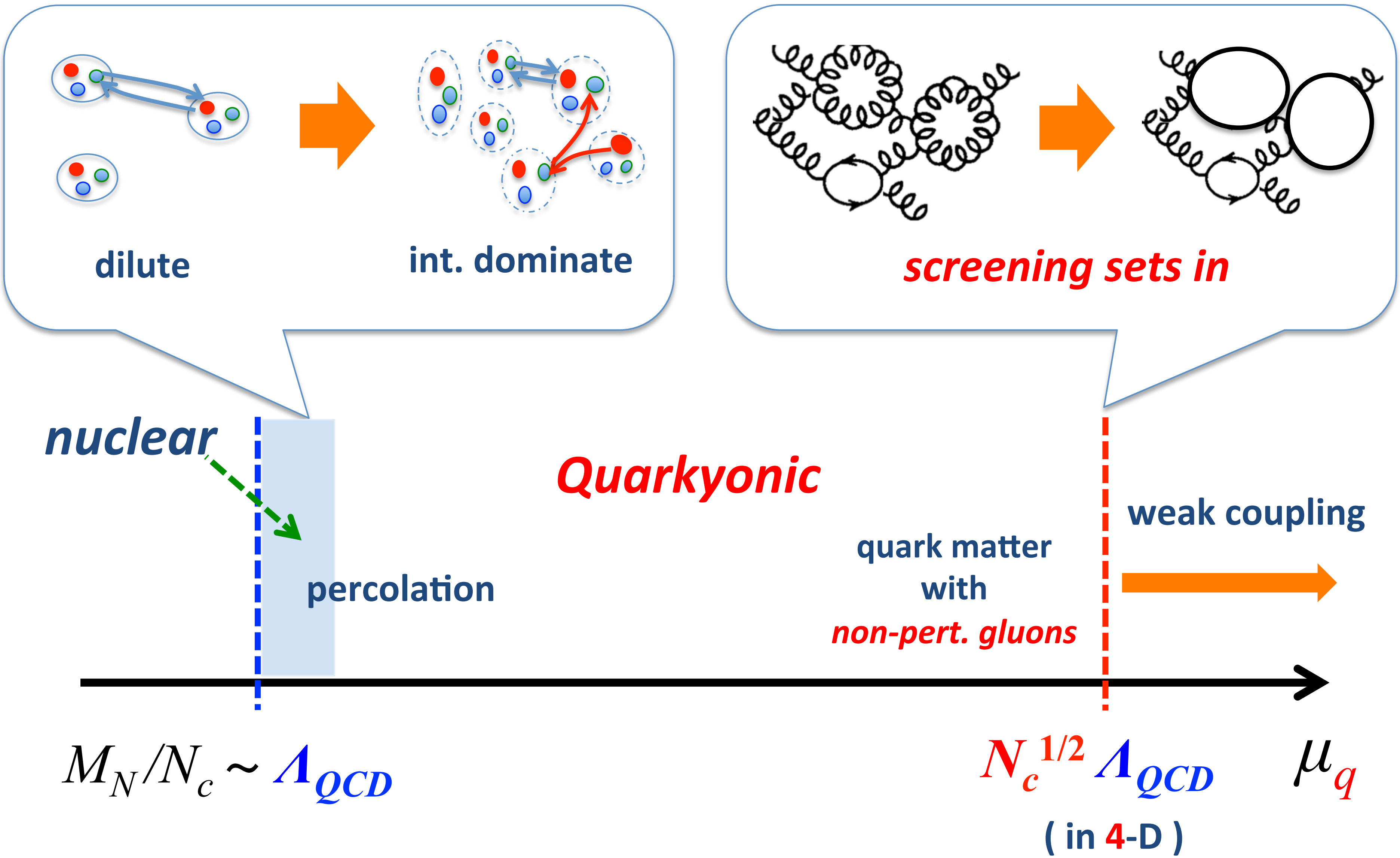


Several branches

- **Confined, *but chiral symmetric* matter** (many papers ...)
  - **have been *challenged* by many model calculations** [Glozman et al. 2007, ...]

- **Confined, *inhomogeneous* chiral *SSB*** (still ongoing ...)

  (*chiral sym. broken only locally*)
  - **Skyrme crystals, ...**
  - **Chiral density wave (1-D periodic structure)** [Carignano-Nickel-Bubbala]
  - **Quarkyonic Chiral Spirals**
  - **Interweaving Chiral Spirals**

  } [TK-Hidaka-Fukushima -McLerran-Pisarski-Tsvelik 09-11]

- **Reinterpretation of *Hadron-Quark Continuity***
  - **Original proposal : Schafer-Wilczek**
  - **CSC in quarkyonic matter & NS context** [Fukushima-TK ’15]

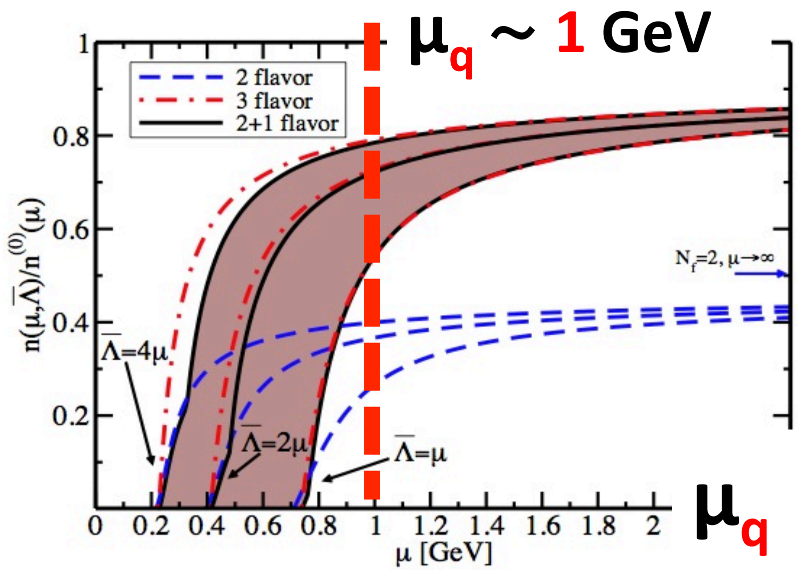
Theoretical guides at N_c=3

- **3-loop *pQCD at large μ_q***

  [Freedman-McLerran 78; Baluni 78
  Kurkela-Romatschke-Vuorinen 09, ...]

  - **large α_s corrections at μ_q < 1 GeV**

    → soft gluons important at n_B < 100 n_0



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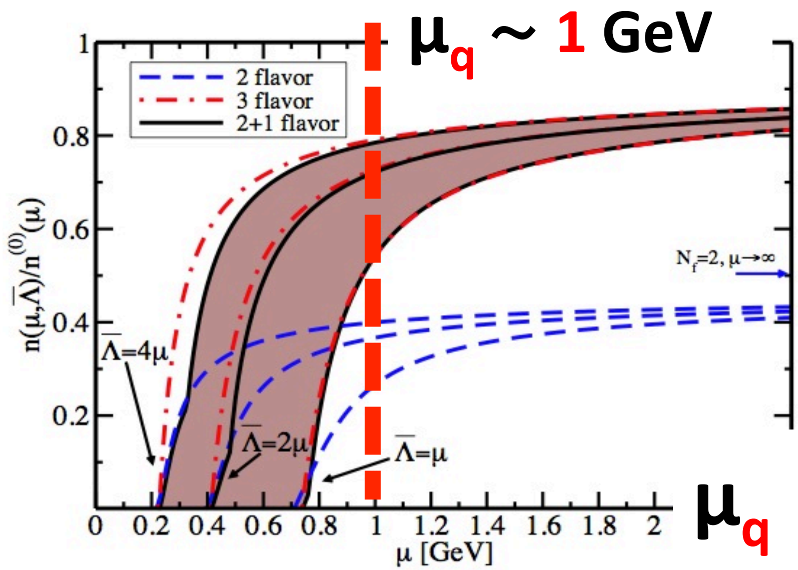


- ***Nuclear calculations* (ChEFT+many-body) *at small* μ_q**

  - **reliable at $n_B \sim n_0$**

    [Akmal et al. (APR) 98; Gandolfi et al. 12, ...]

  - **At n_B > 2n_0**
    - ***convergence* problems : < $V_{2\text{-body}}$ > ~ < $V_{3\text{-body}}$ > ~ ...**
    - ***hyperon softening*, unless introducing ad hoc repulsion**
    - ***changes in hadron w.f. & Dirac sea negligible?***

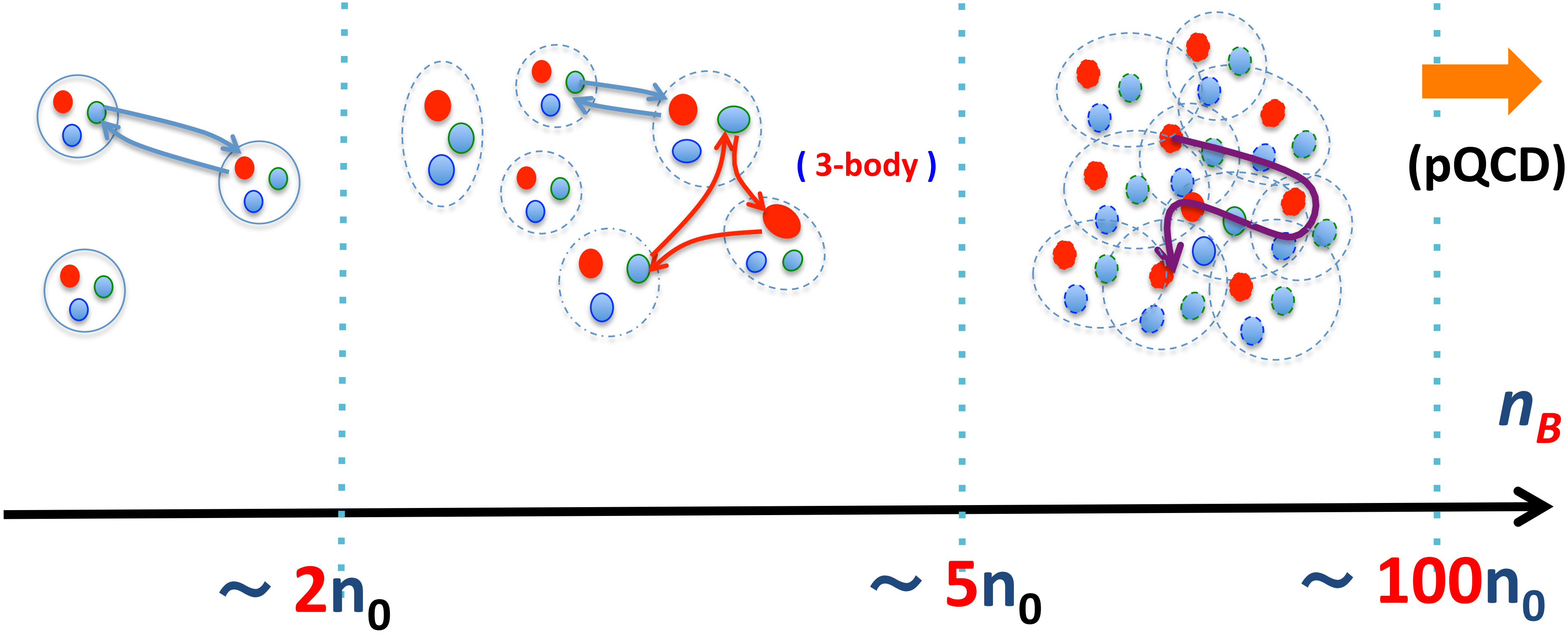
3-window modeling

(Masuda-Hatsuda-Takatsuka 12)

- **few meson exchange**
- **nucleons only**

- **many-meson exchange**
  (mobility --cf: Karsch-Satz '80)
- **structural change of hadrons**

- **Baryons overlap**
- **Quark Fermi sea**



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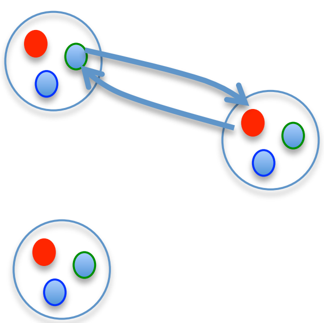

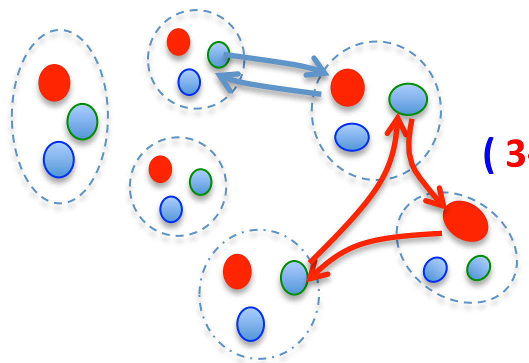

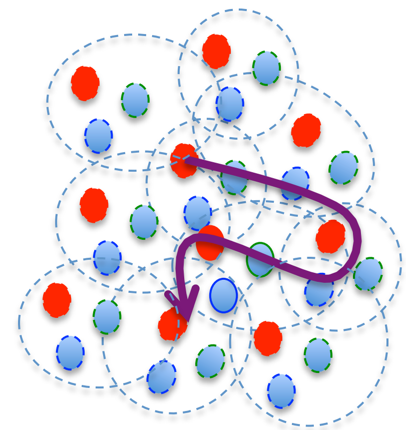

(pQCD)

APR → ***Interpolated EoS*** ← ***Quark models*** n_B

$\sim 2n_0$ $\sim 5n_0$ $\sim 100n_0$

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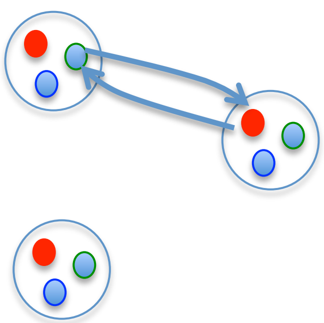

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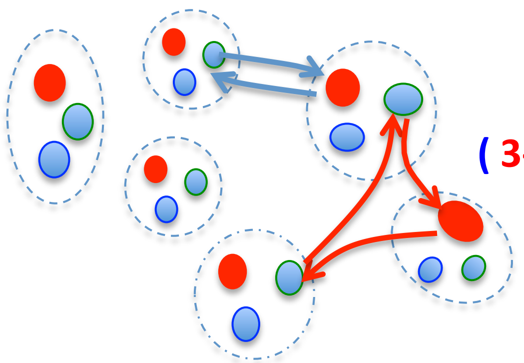

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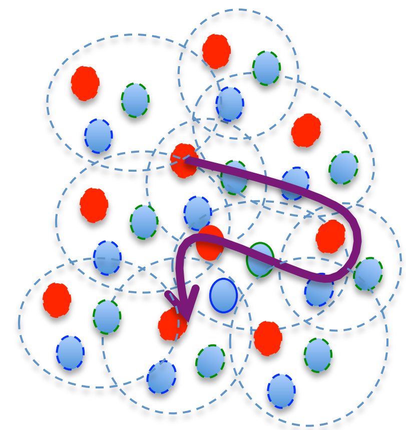


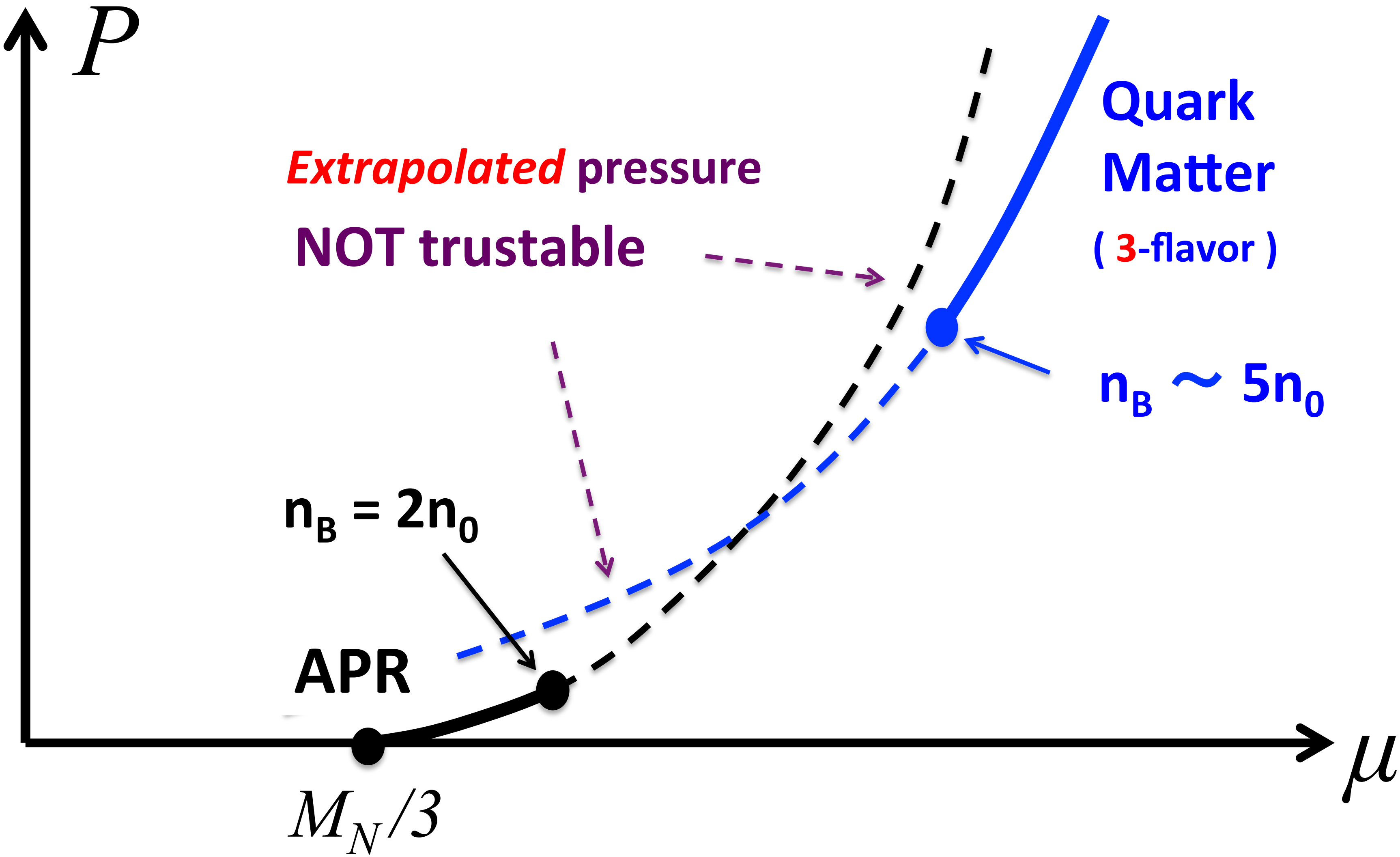
(pQCD)

APR → *Interpolated EoS* ← *Quark models* n_B

~ $2n_0$ ~ $5n_0$ ~ $100n_0$

will be studied through NS observations

3-window modeling : P vs μ



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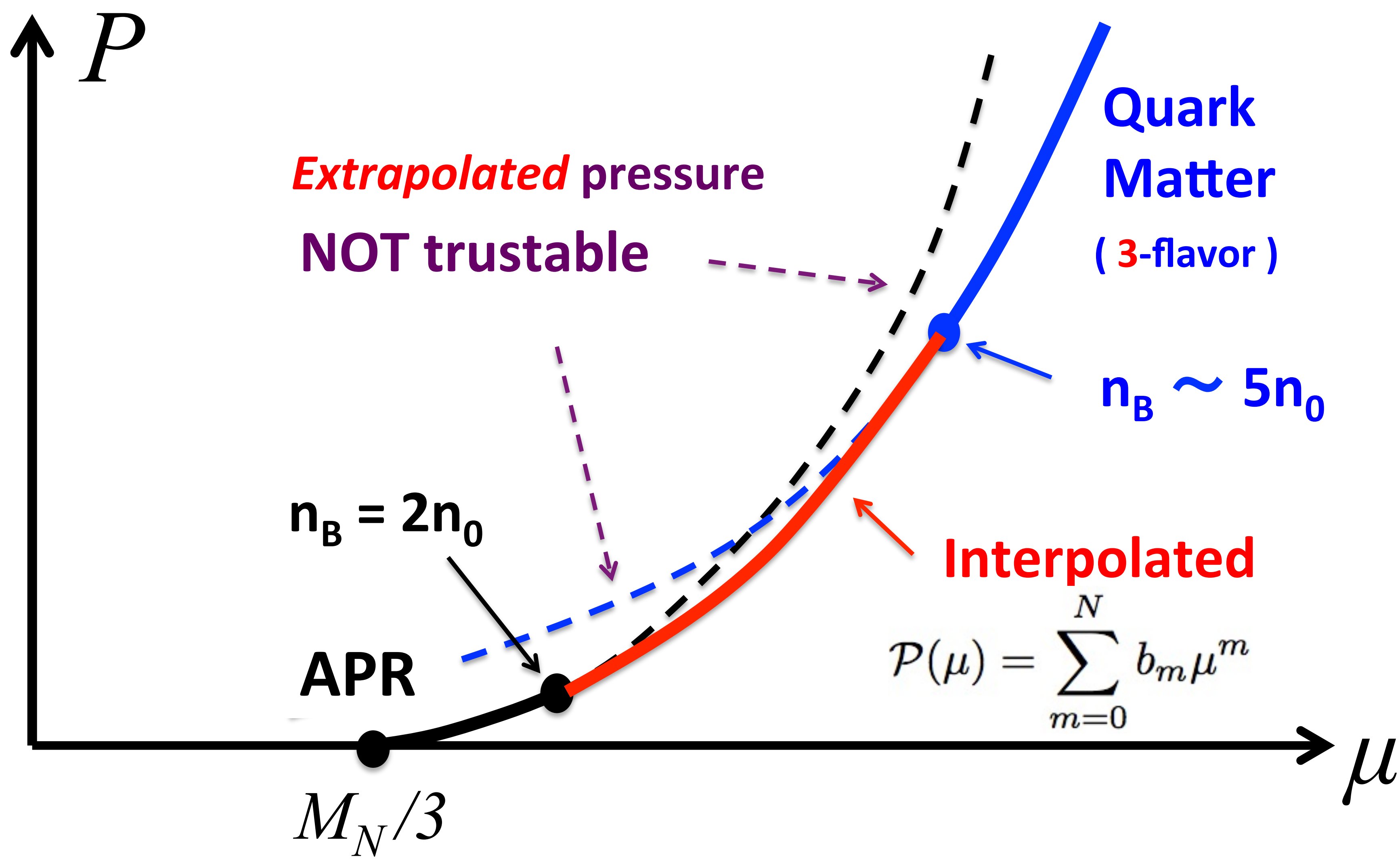
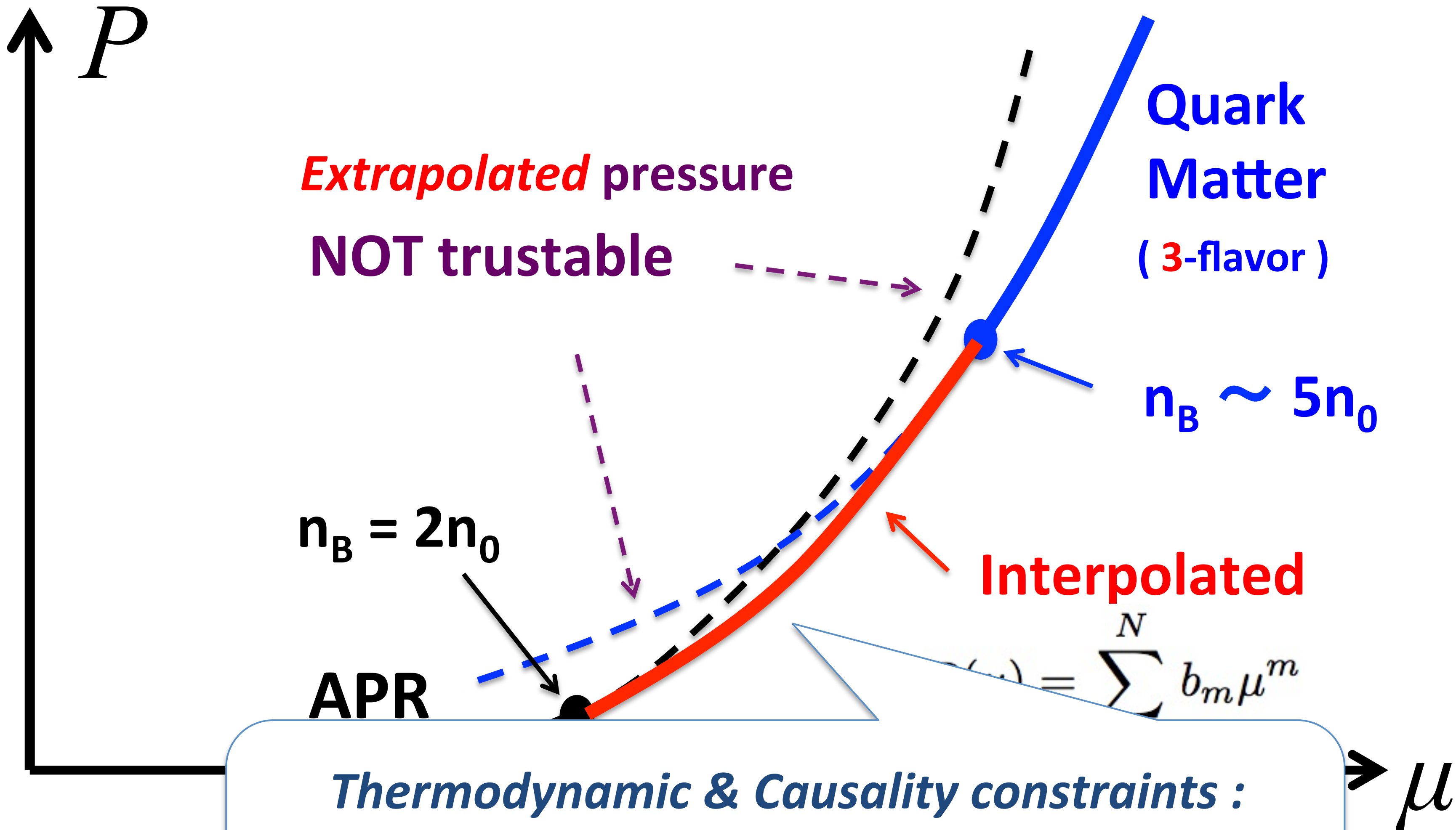


Matching : up to 2nd order of derivatives at $n_B = 2n_0$ & $5n_0$

3-window modeling : P vs μ



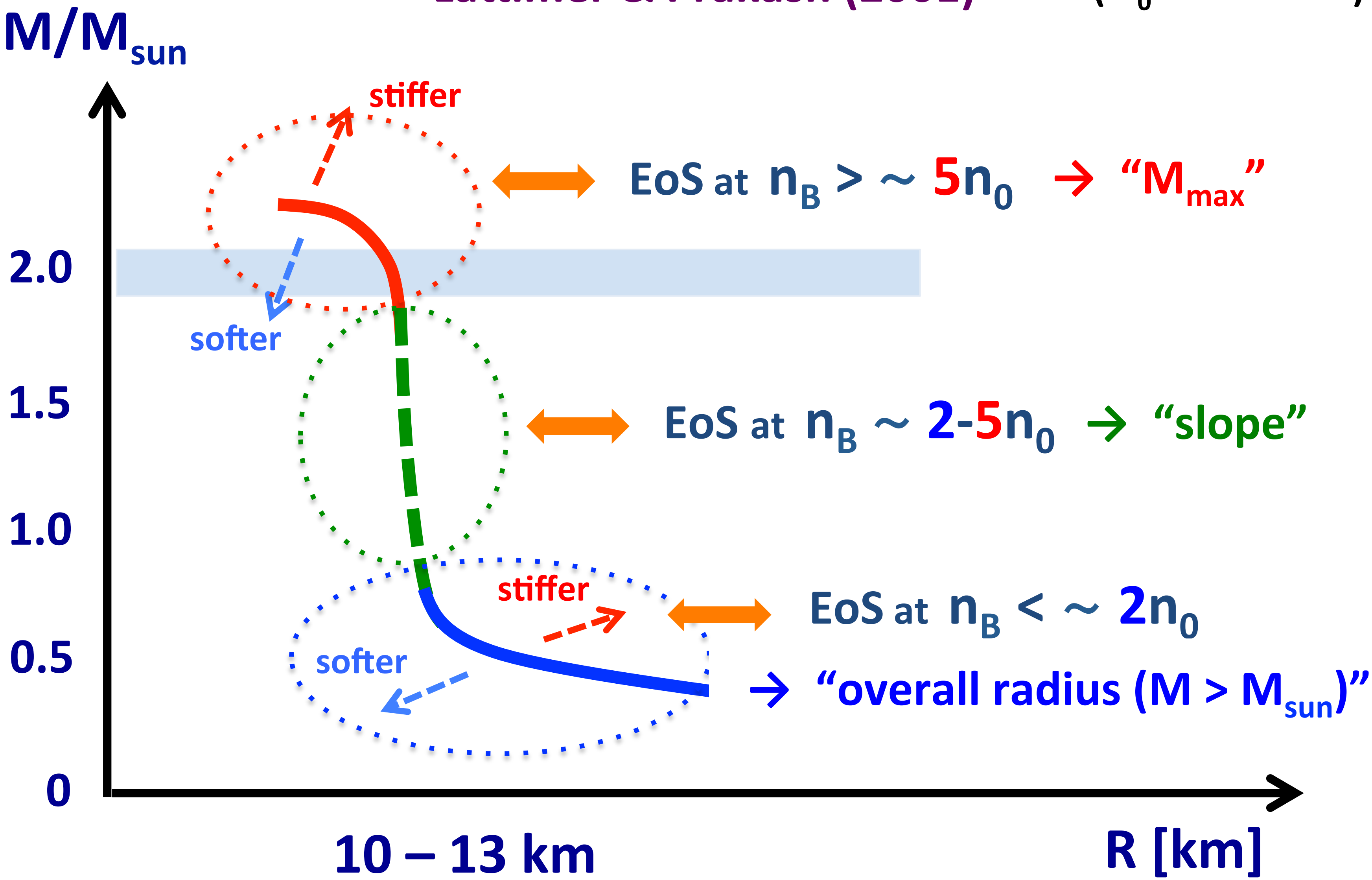
Thermodynamic & Causality constraints :

$$0 < c_s^2 = dP/d\varepsilon < 1$$

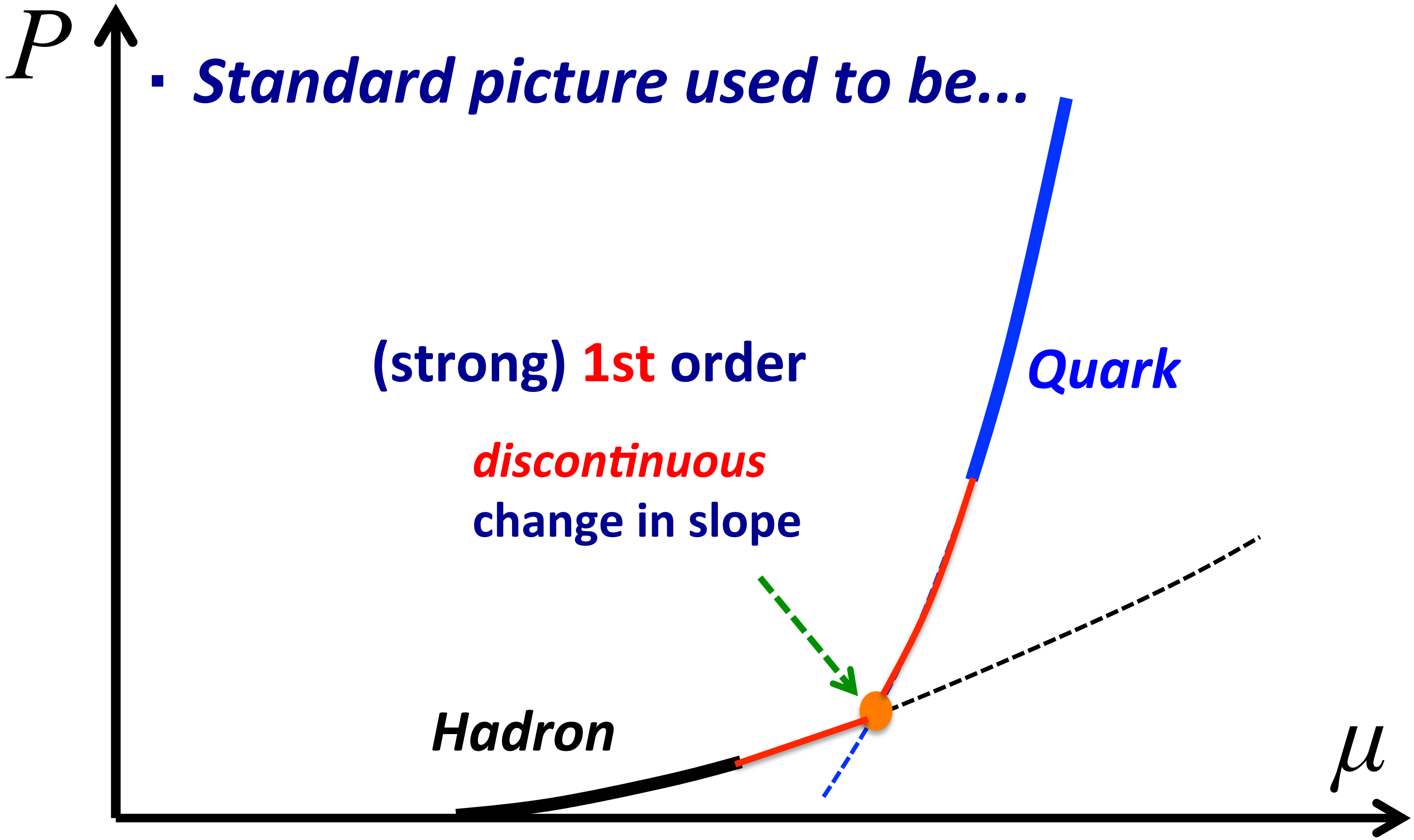
*" **Supposed** " EoS from NS observations*

M-R relation & EoS

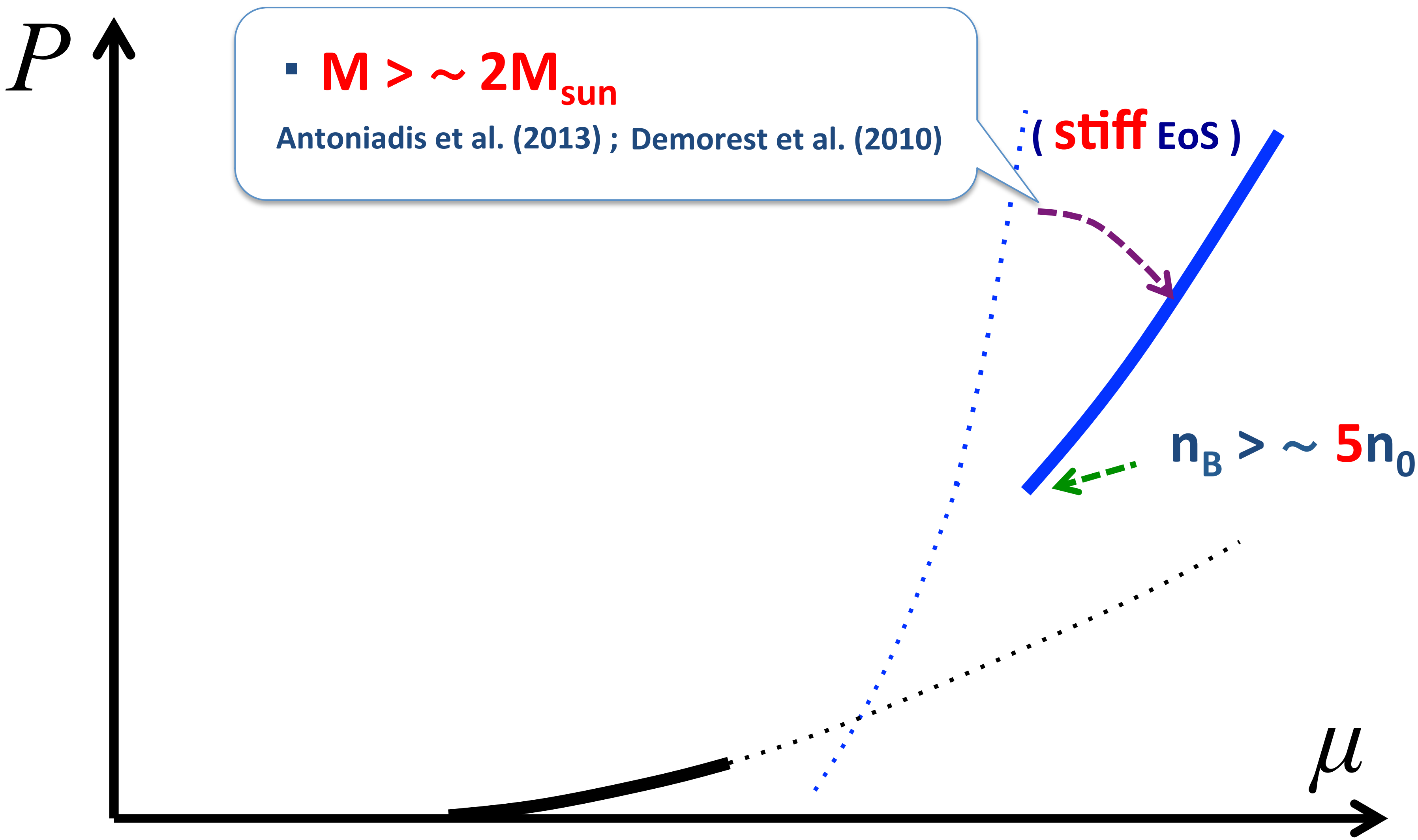
Lattimer & Prakash (2001) ($n_0 = 0.16\ fm^{-3}$)



Observational constraints on P(μ)



Observational constraints on $P(\mu)$



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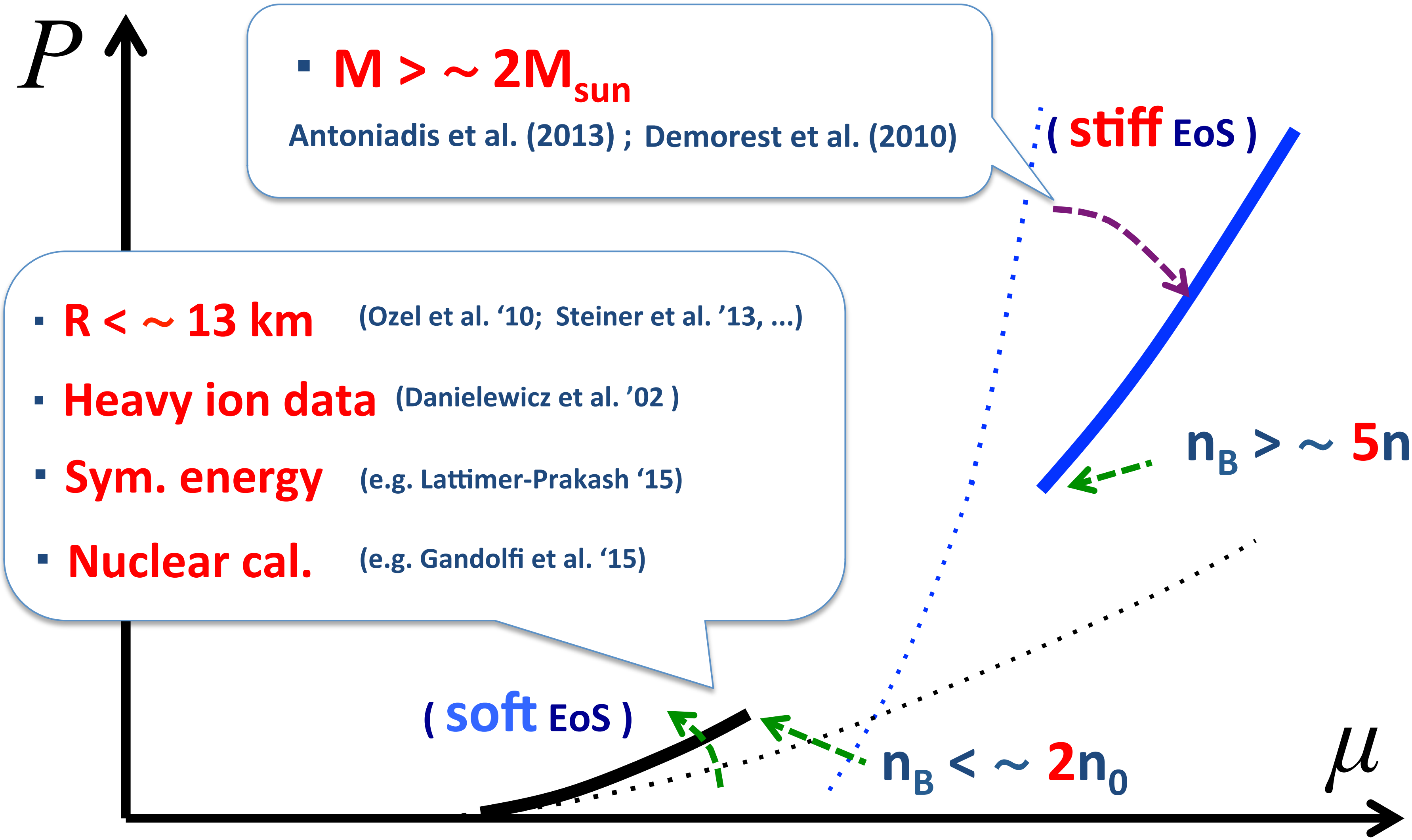
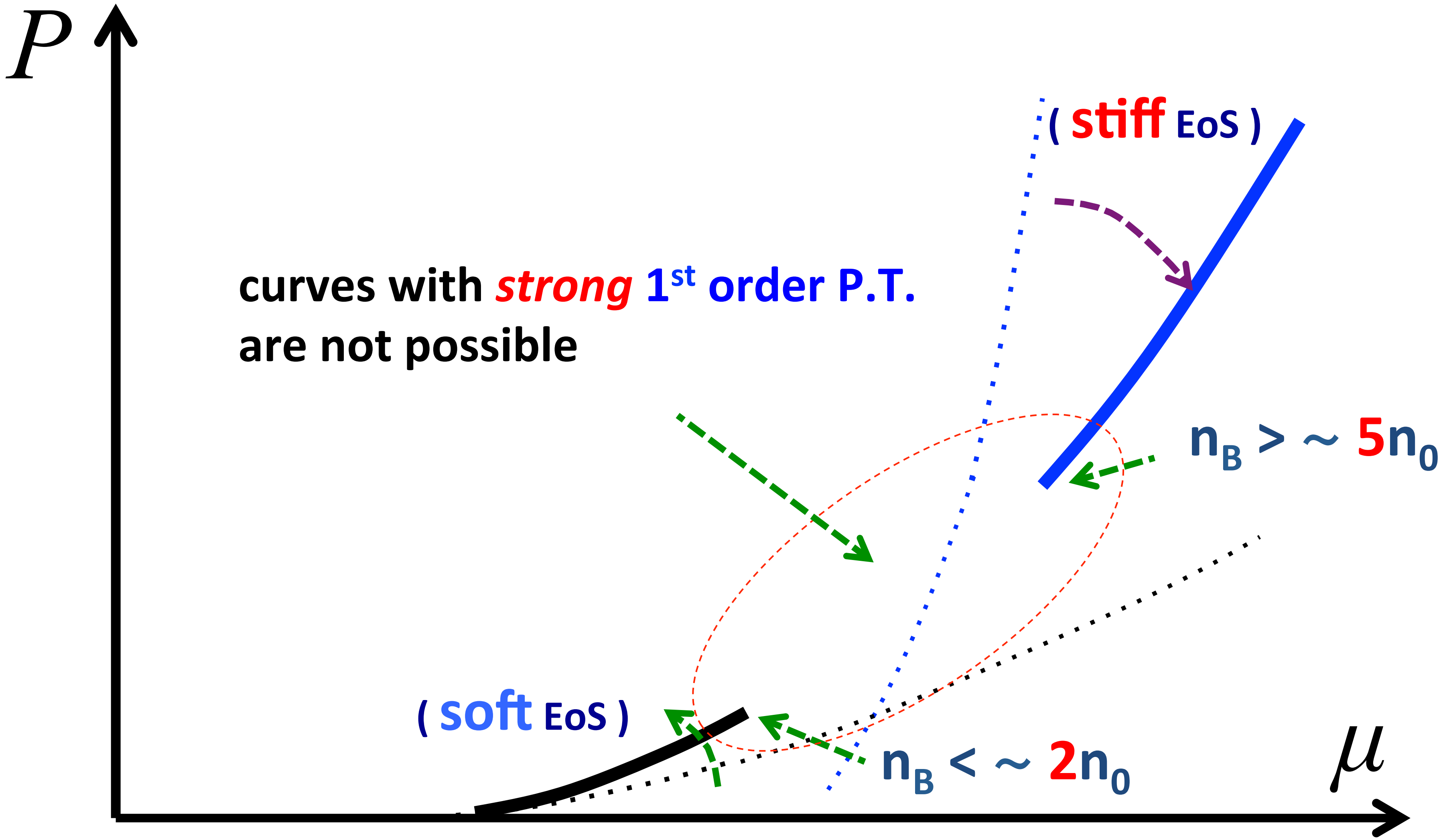


Observational constraints on *P(μ)*



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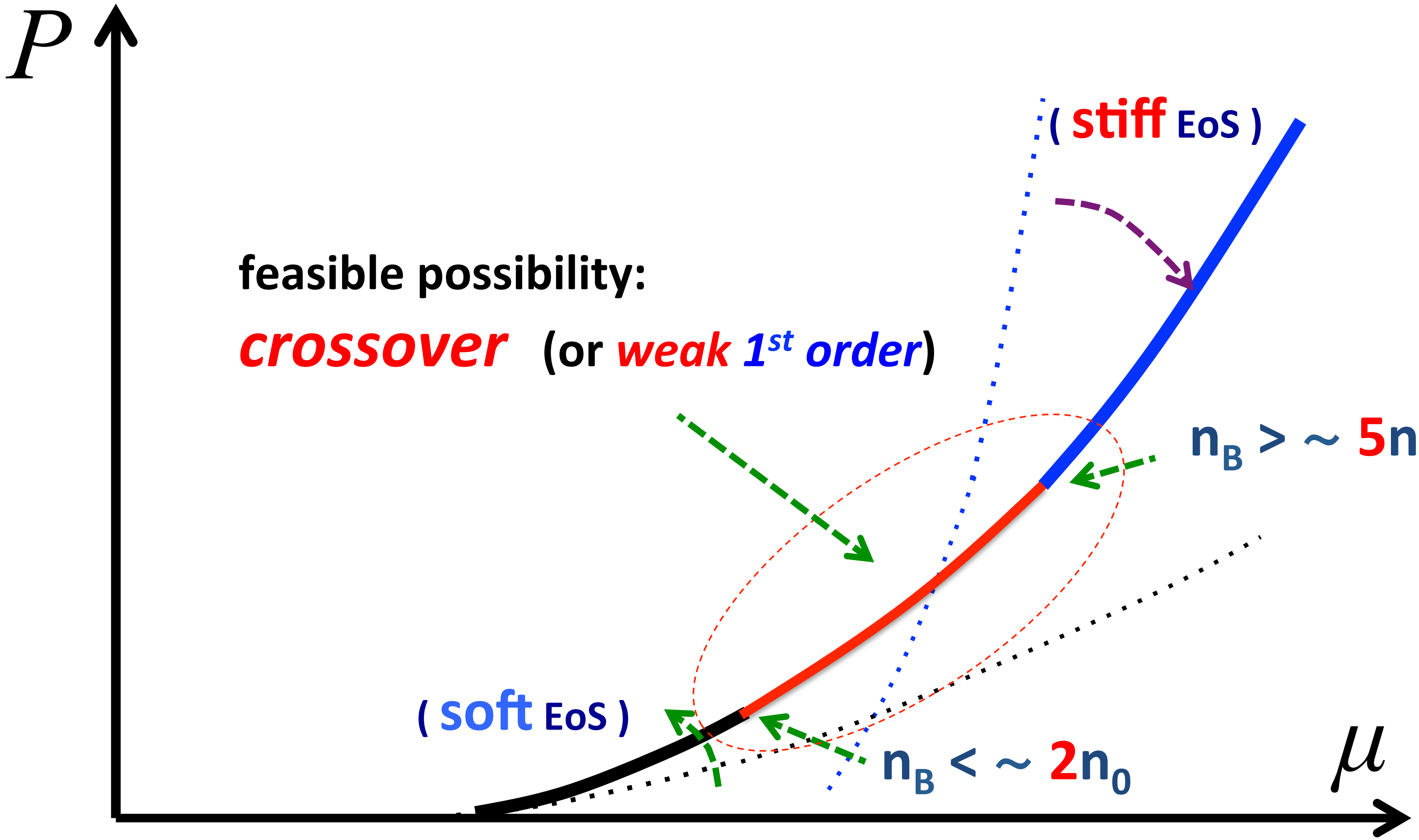


Quark models at $n_B > \sim 5n_0$

3-flavor quark model : template

Effective Hamiltonian (inspired by hadron & nuclear physics):

$$\mathcal{H}_{\rm eff} \sim \bar{\psi}\left[-{\rm i}\vec{\alpha}\cdot\vec{\partial}+m\right]\psi + \mathcal{H}_{\rm NJL}^{\rm 4Fermi+KMT}$$

→ structural change of *Dirac sea* & *quark bases*

$+ \quad \mathcal{H}_{\rm conf}^{3q\to B}$ ➡ will be *ignored* in the *percolated* domain

$+ \quad \mathcal{H}_{\rm OGE}$ ➡ (mag. part) $-H\sum_{A,A'=2,5,7}\left(\bar{\psi}{\rm i}\gamma_5\lambda_A\tau_{A'}\psi_c\right)^2$

(cf: *N-Δ splitting*)

$+ \quad \mathcal{H}_{\rm nucl}$ ➡ $+\, g_V\left(\bar{\psi}\gamma_0\psi\right)^2$ ~ *ω-meson exchange* (*repulsive*)

+ constraints (charge neutrality, β- equilibrium, color-neutrality)

What we will do

“ *Supposed* ” EoS at $n_B \sim 5n_0$

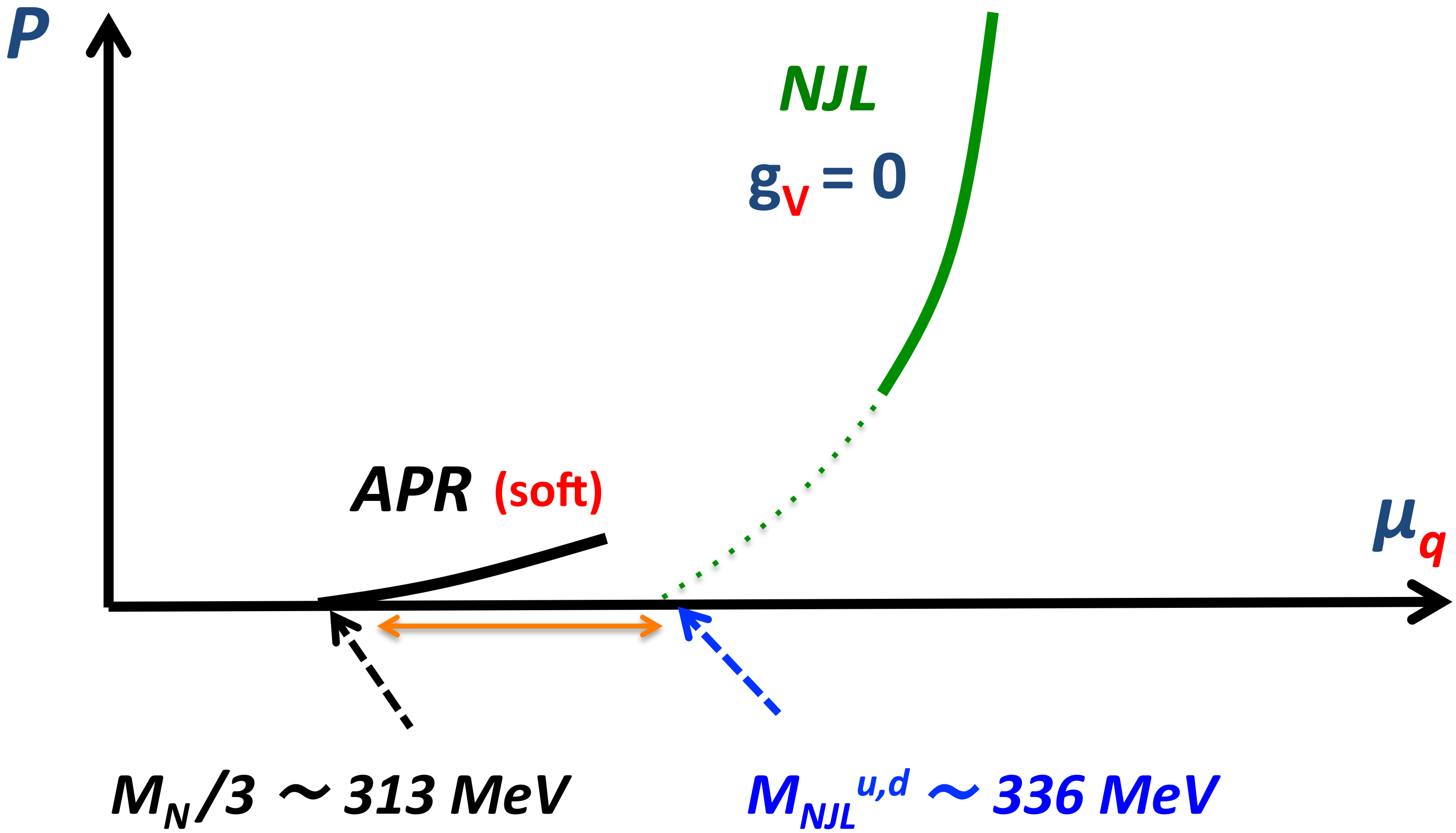
(NS constraints)

Quark models with (*G, H, G_v, ...*) at $n_B \sim 5n_0$

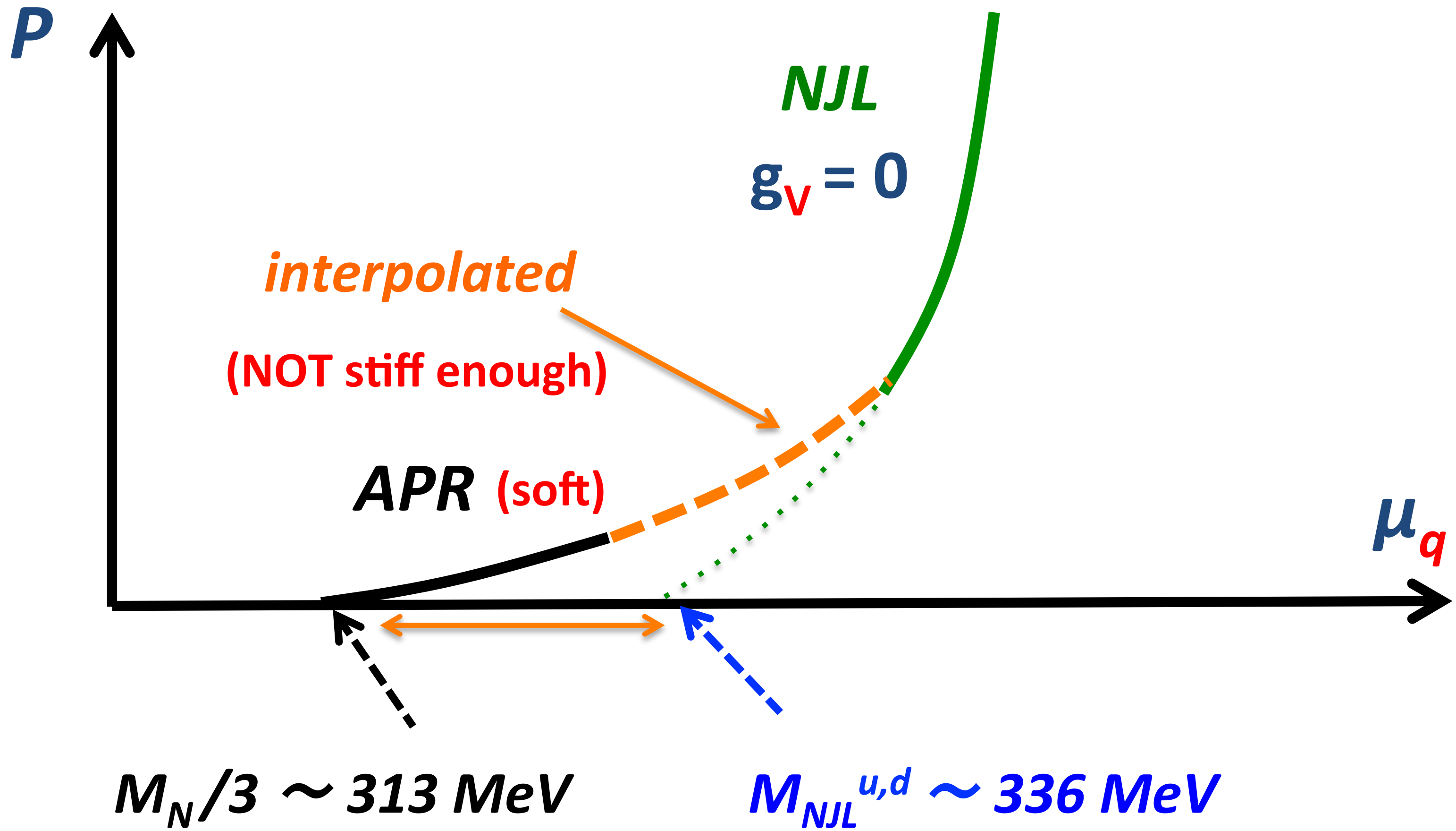
(→ output)

***Delineate* properties of matter at $n_B \sim 5n_0$**

minimal

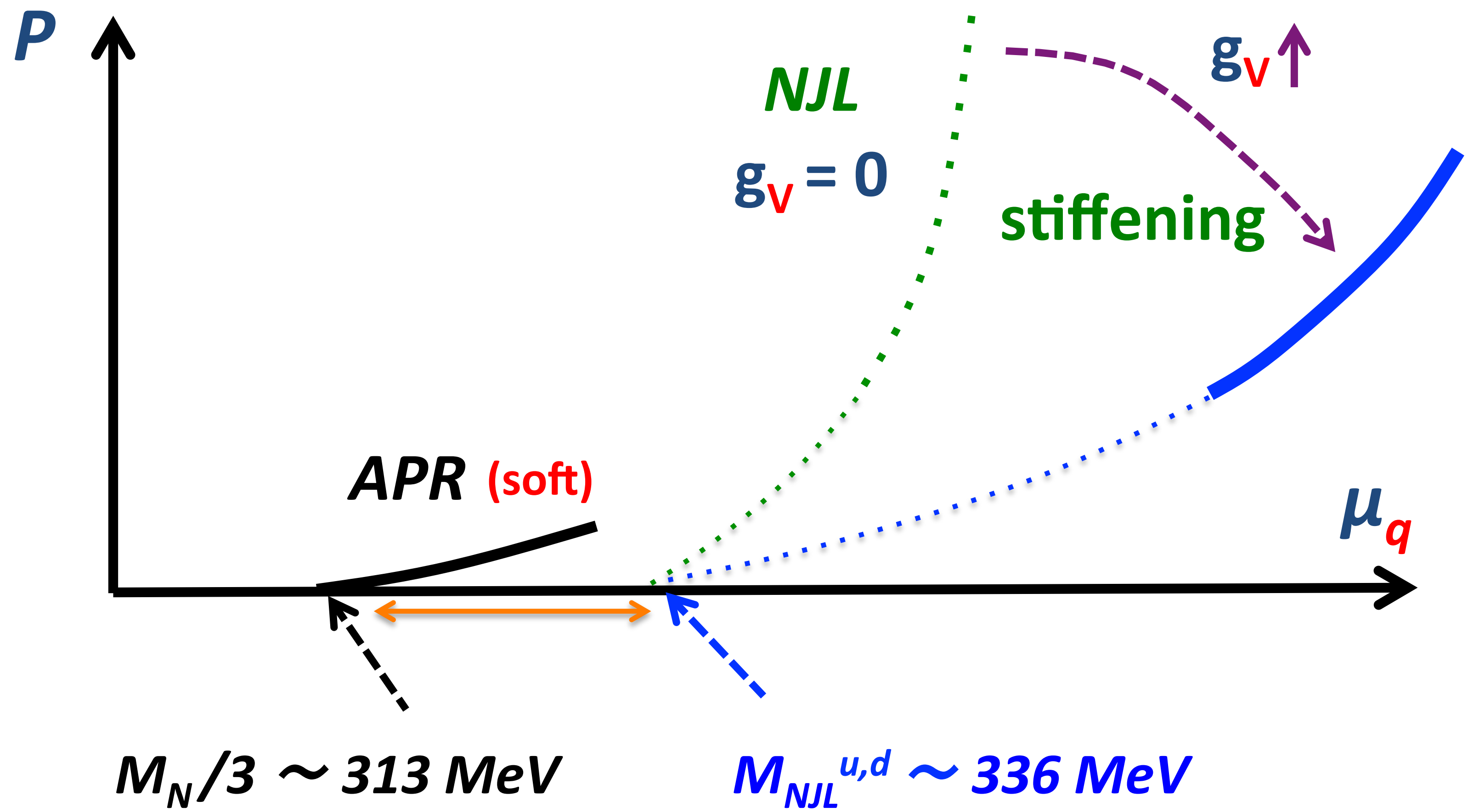


minimal



P
NJL
$g_V = 0$
interpolated
(NOT stiff enough)
APR (soft)
μ_q
$M_N/3 \sim 313$ MeV
$M_{NJL}^{u,d} \sim 336$ MeV

minimal + vector int.



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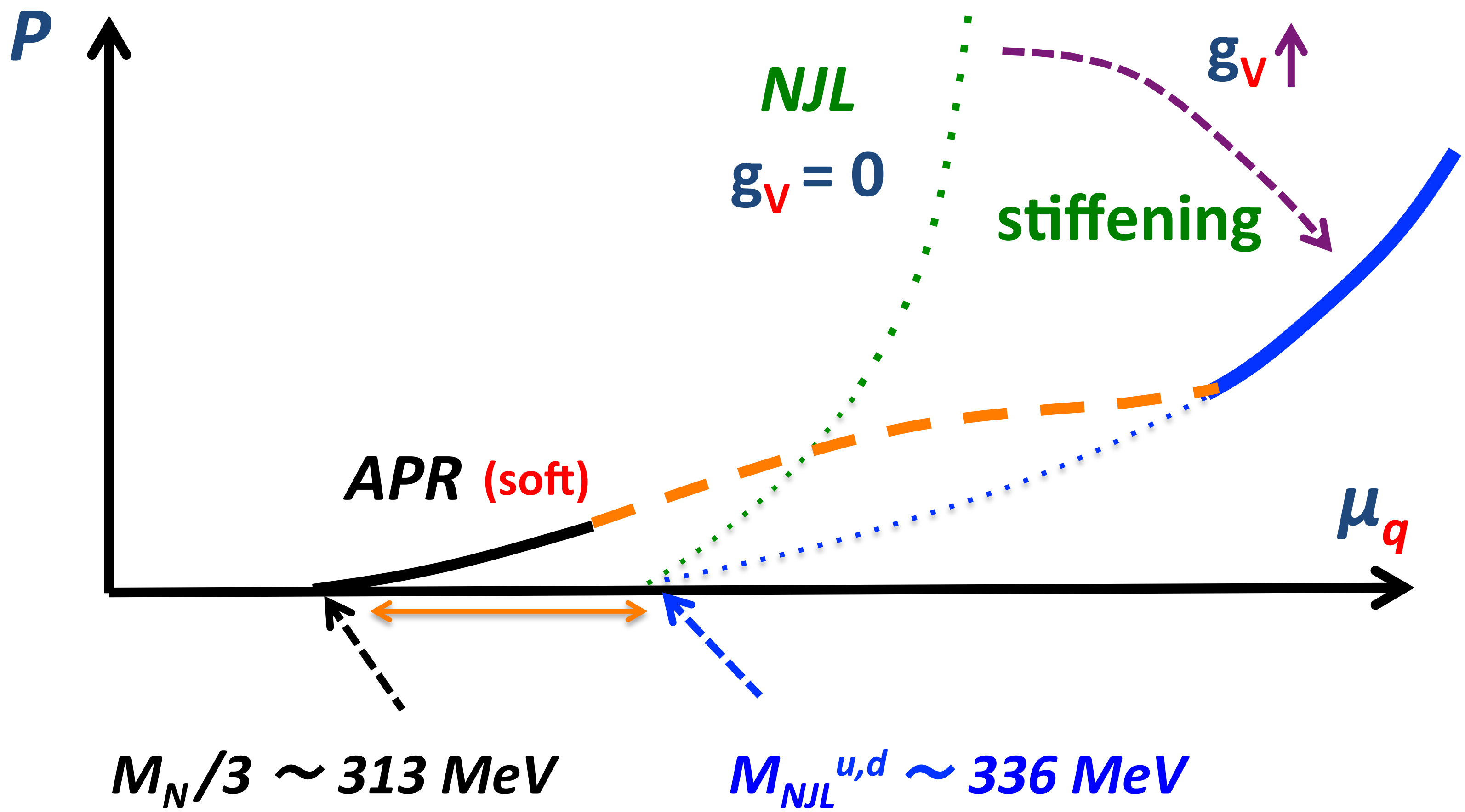

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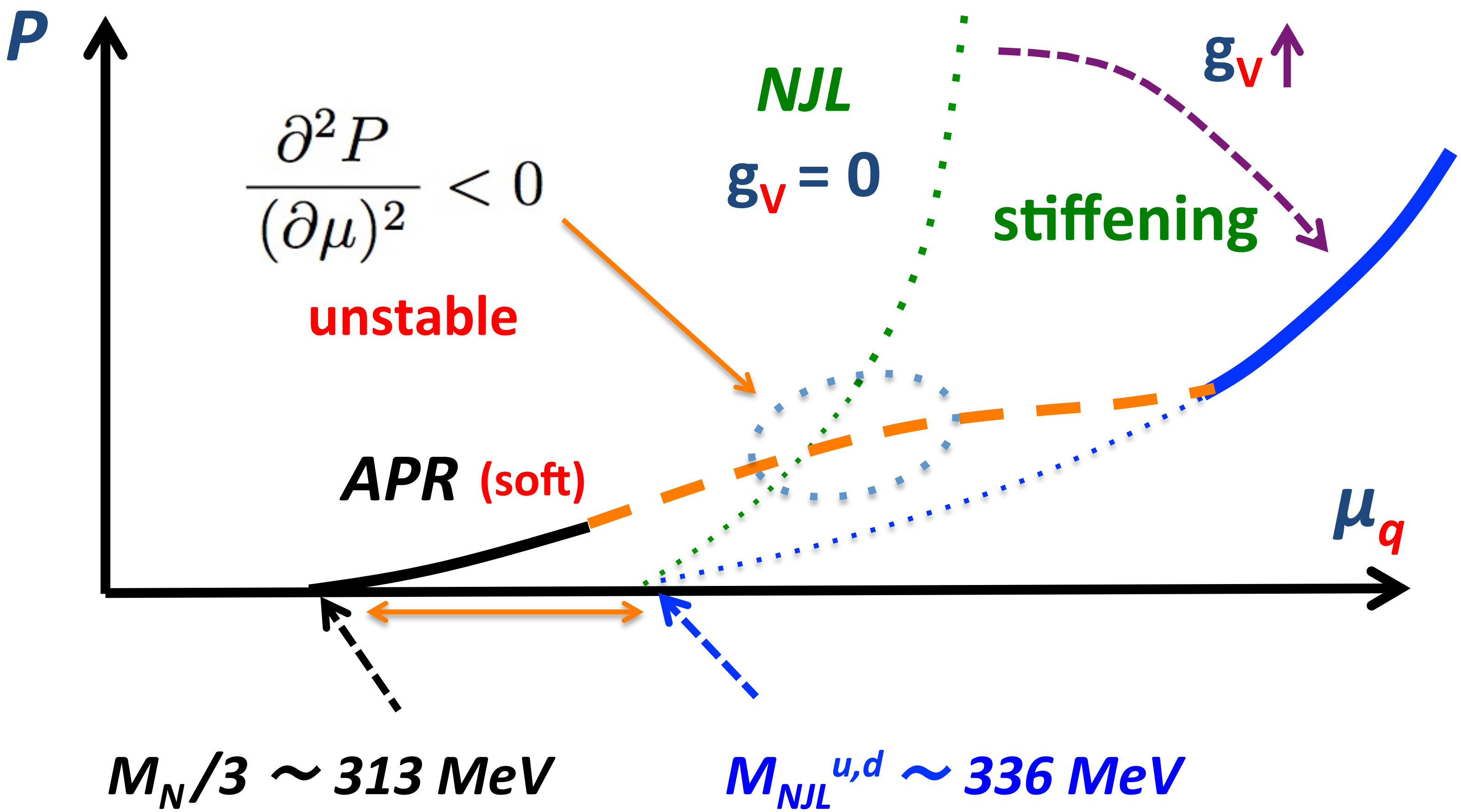
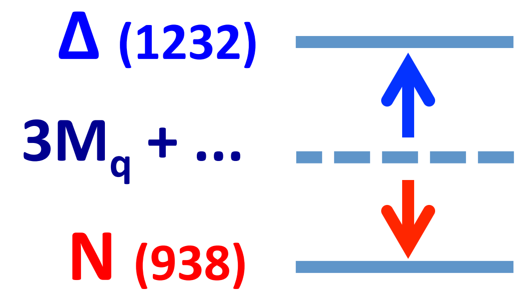
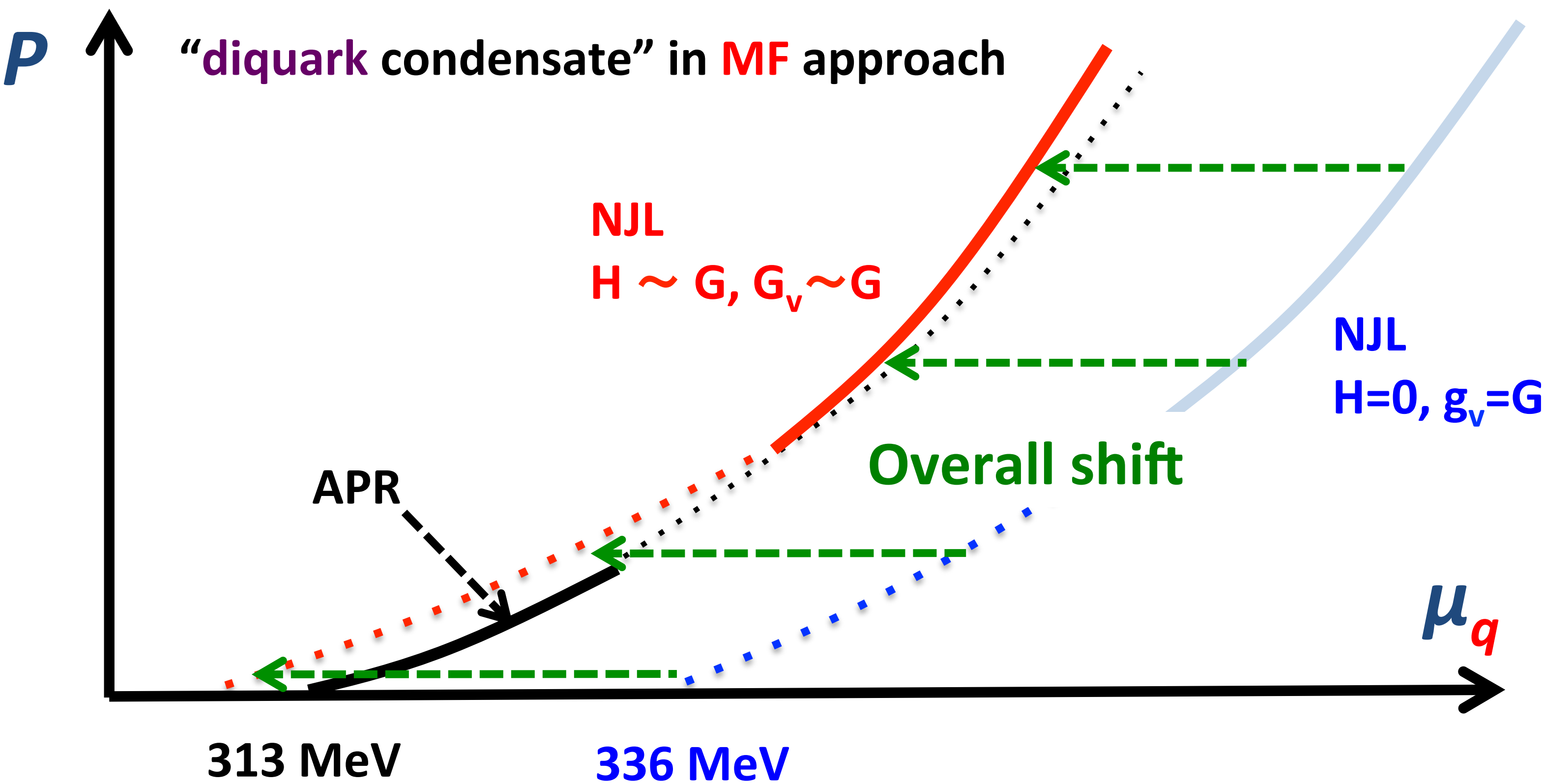


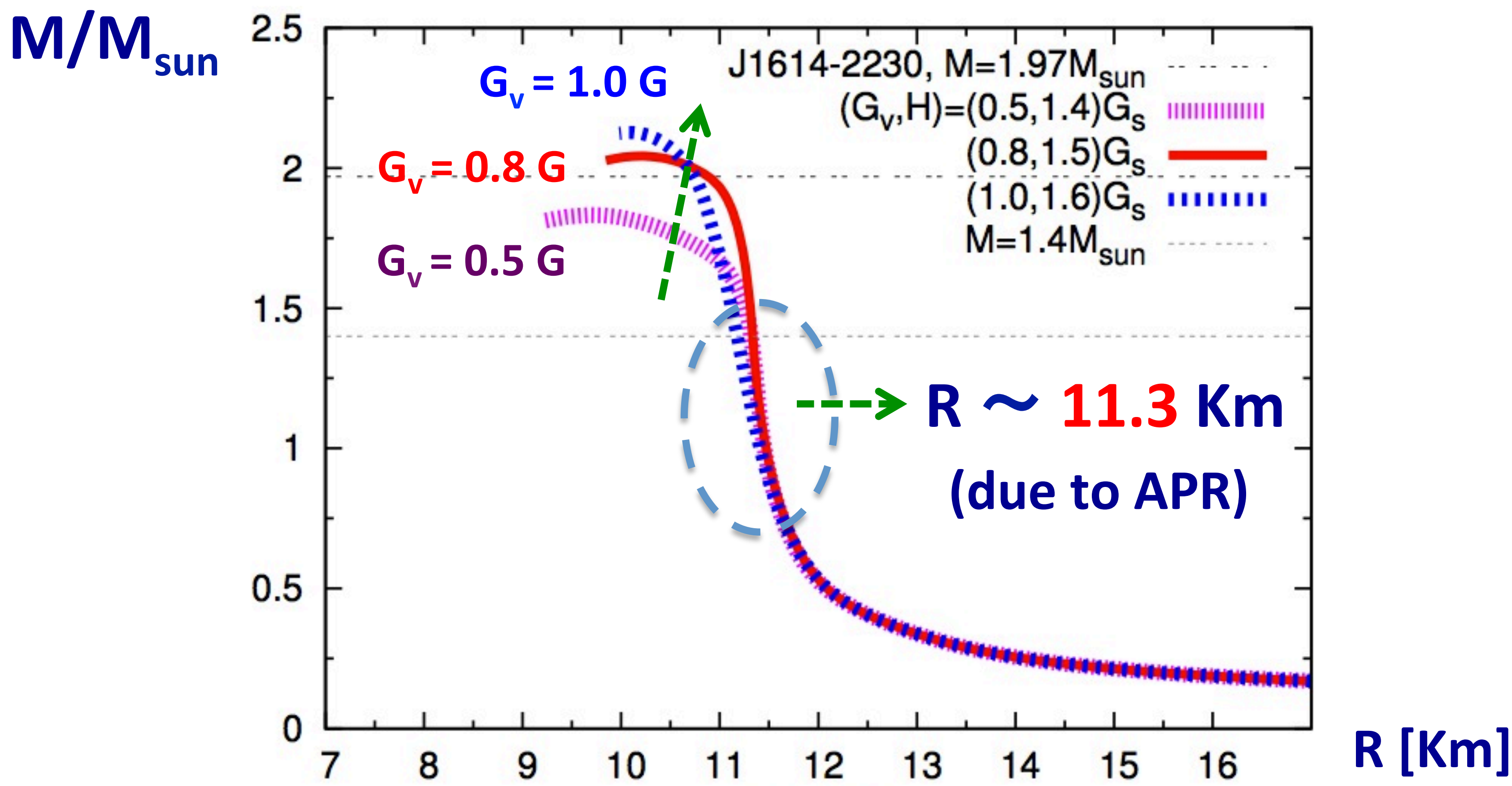
+ *attractive* *color-magnetic int.*

c.f. N – Δ splitting





M-R curves



we need :

$$G_s \sim G_v \sim H \;\; @ \;\; n_B = 5\text{-}10\, n_0 \;\rightarrow\; O(G_s^{vac})$$

Discussions

Discussion : Bag constant ?

Def: $\mathcal{B} \equiv \epsilon^{vac}_{pert} - \epsilon^{vac}_{full} \sim \Lambda^4_{\mathrm{QCD}}$ **> 0**

- **Energy *gain* by *non-pert. effects* ;**

  e.g.) ChSB in Dirac sea, gluon condensation, ...

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If μ is large enough :

(softening)

- **Loss of *non-pert. effects* →**

$$\begin{cases} \epsilon_{\rm matter} \rightarrow \epsilon_{\rm matter} + \mathcal{B} \\ P_{\rm matter} \rightarrow P_{\rm matter} - \mathcal{B} \end{cases}$$

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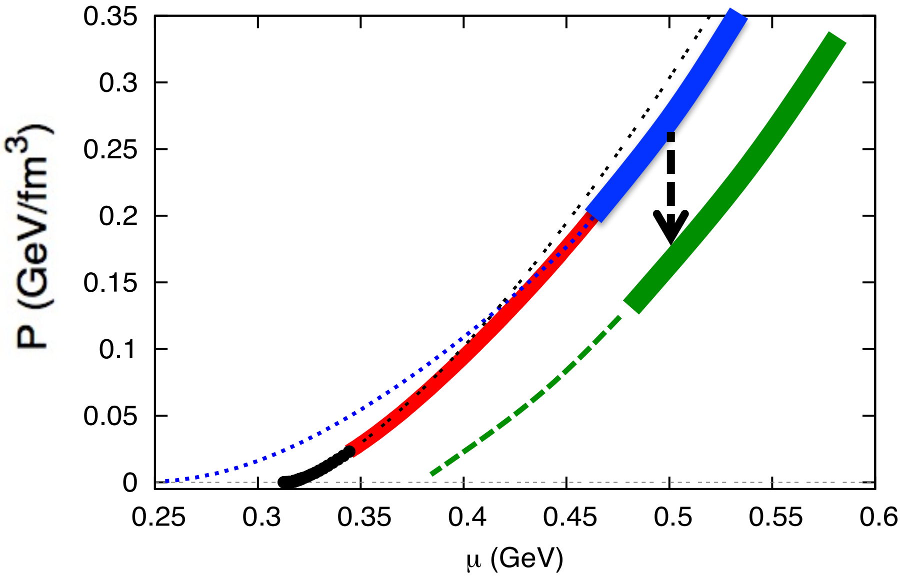
$$\begin{cases} \epsilon_{\rm matter} \to \epsilon_{\rm matter} + \mathcal{B} \\ P_{\rm matter} \to P_{\rm matter} - \mathcal{B} \end{cases}$$

- **NJL takes into account the *vac. contributions* *only partially* ;**

  it *misses* contributions from *gluonic* one, B_g

Discussion : Bag constant ?

P_{NJL} @ 5 n_0 → only 200 - 400 MeV fm^{-3}



If $B_g \sim \Lambda_{QCD}^4$ appears @ $5n_0$

EoS → impossible to pass any constraints

Together with $G_V \sim H \sim G_s^{vac}$, we claim :

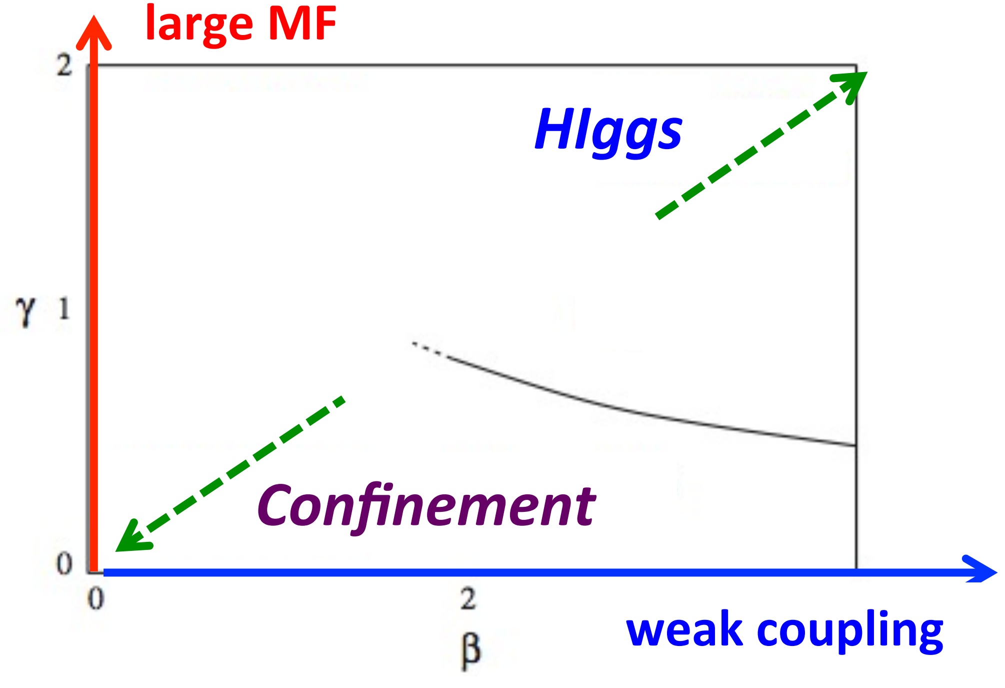
Gluons *should remain non-perturbative* to $n_B \sim$ 5-10 n_0

A question : Conf. vs Higgs ?

e.g.) Higgs model ***with const. amplitude*** *[Fradkin-Shenkar 79]*

1/g² — amp. of Higgs |ϕ| — phase of Higgs

$$S = \beta \sum_{\text{plaq}} \frac{1}{2} \mathrm{Tr}[UUU^{\dagger}U^{\dagger}] + \gamma \sum_{x,\mu} \frac{1}{2} \mathrm{Tr}[\phi^{\dagger}(x) U_{\mu}(x) \phi(x+\hat{\mu})]$$



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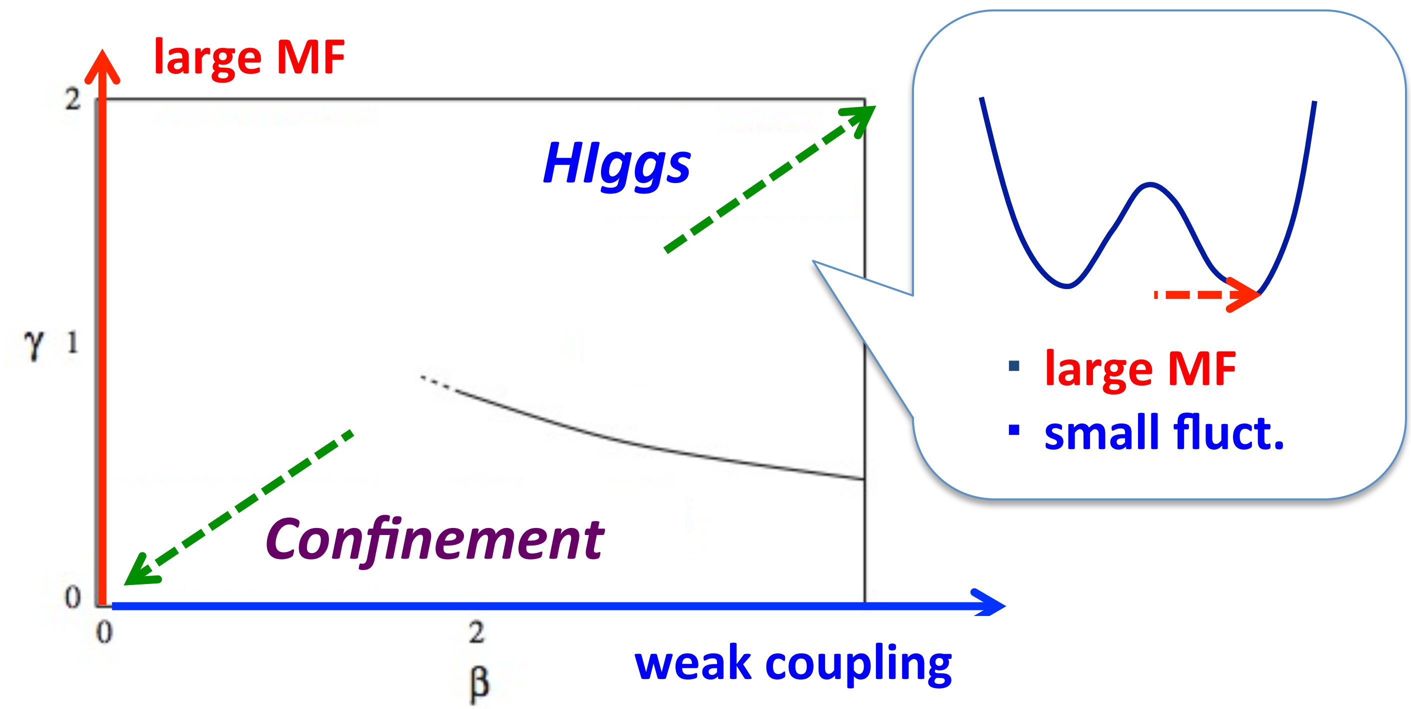

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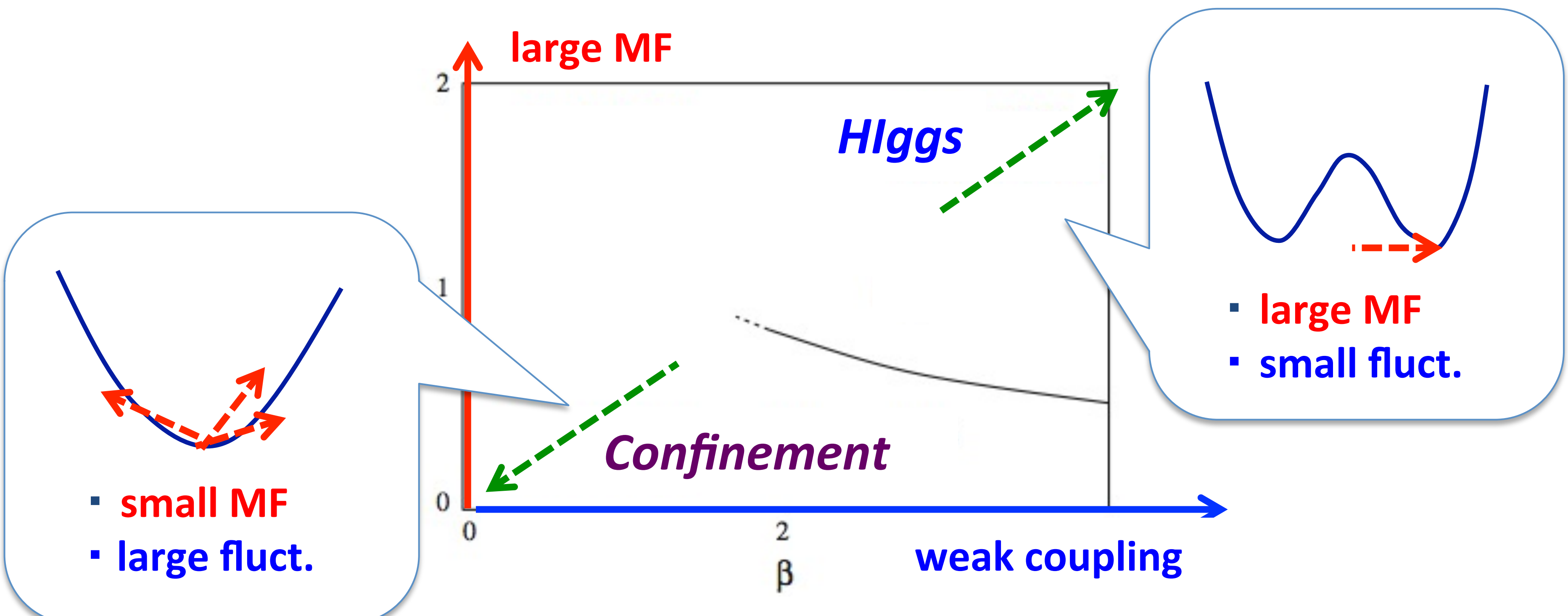

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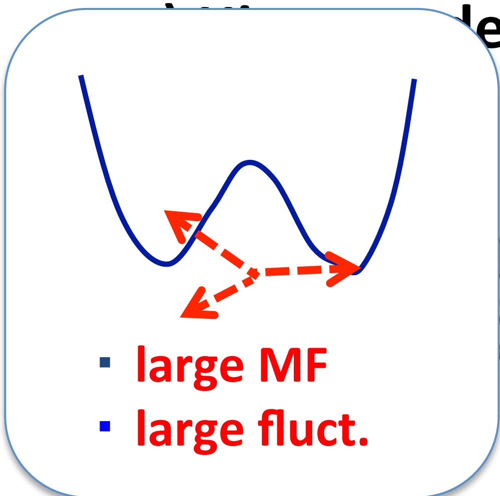
...del *with const. amplitude* *[Fradkin-Shenkar 79]*



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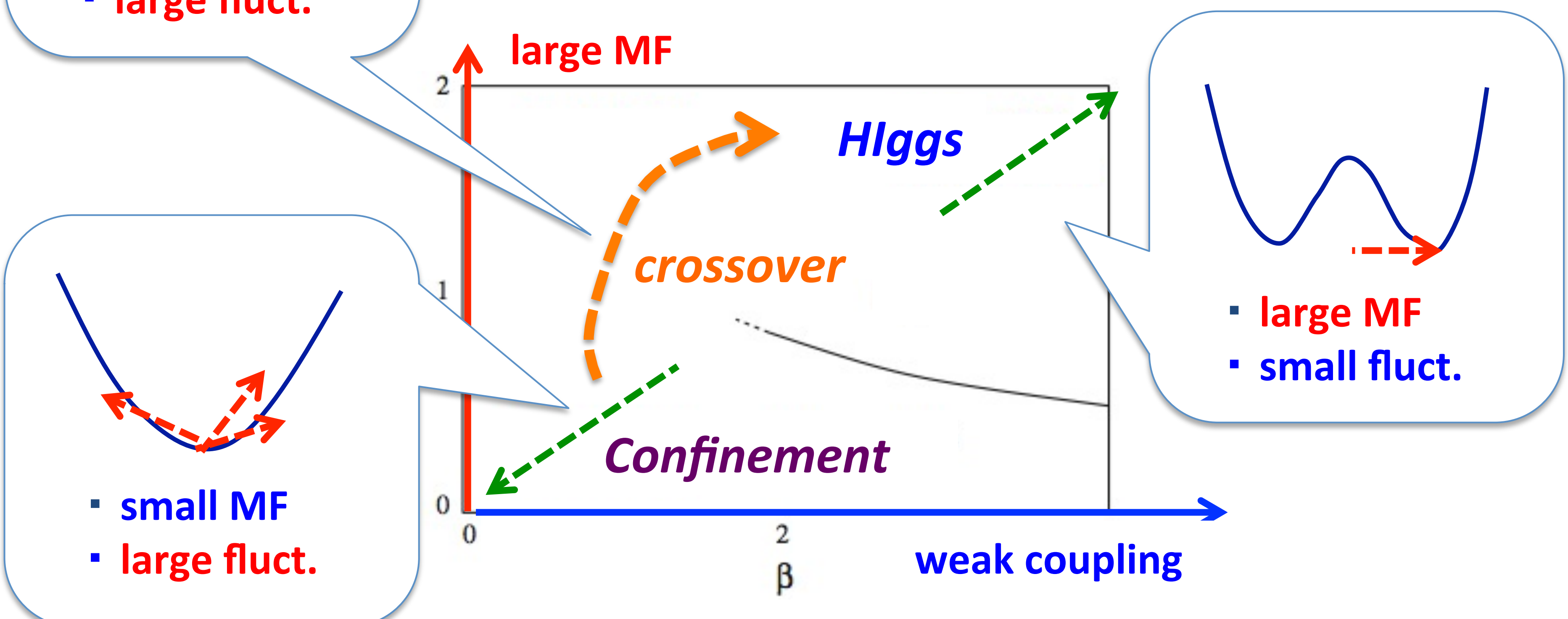


Summary

- *Soft* EoS at *small* n_B & *Stiff* EoS at *large* n_B

  → **crossover** or **weak 1st order** *from H to Q*

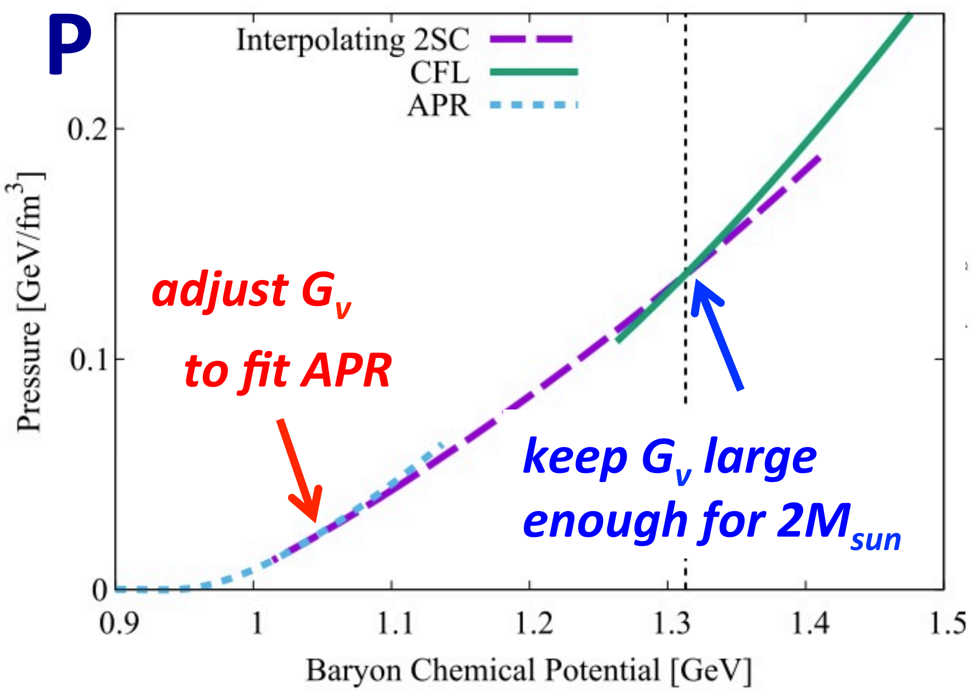
- **Quark matter with** ***non-pert. gluons*** (*Quarkyonic matter*)
  - [G_V, H] @ $5n_0$ is comparable to G_s^{vac}
  - *No* gluonic bag constant at $n_B \sim 5n_0$

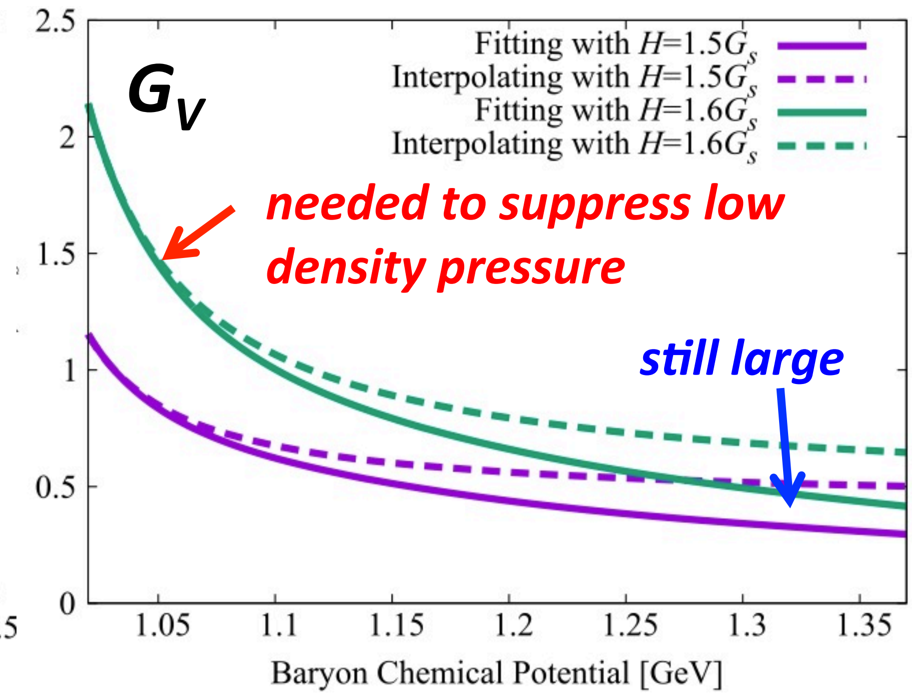
- *Need to confirm these **phenomenological** findings from **microscopic calculations at strong coupling***

Backup

Discussion 2: value of G_V ?

APR constrained NJL with running $G_V(n_B)$ [Fukushima-TK '15]





would offer more concrete modeling for "unified" EoS

than 3-window descriptions

Discussion 3: Hyperon problems ?

How did we avoid hyperon softening ?

- μ_B^{th} for strangeness :
  - $\mu_B \sim 3M_s \sim$ 1.5 GeV (quark picture)
  - $\mu_B \sim \mu_\Lambda, \mu_\Sigma \sim$ 1.1-1.2 GeV (hadron picture)
    (uds, uus,...)

How to interpolate these two pictures ?

Discussion 3: Hyperon problems ?

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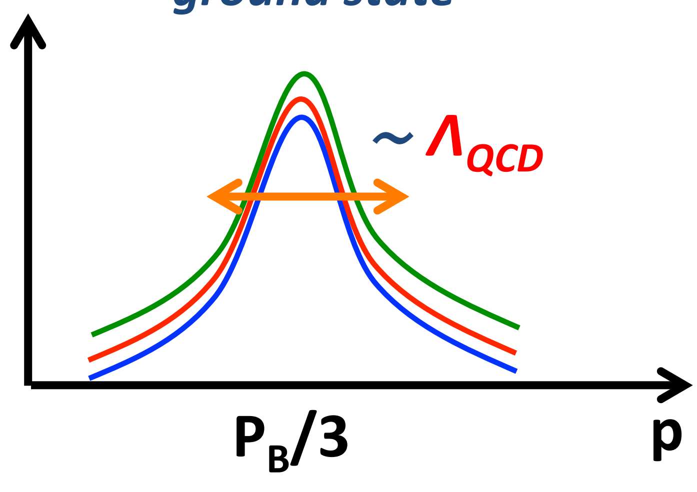
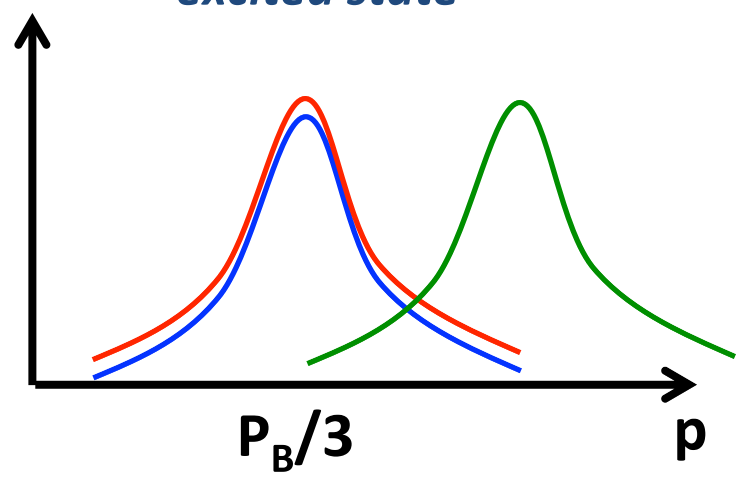
    (uds, uus,...)

- ***A quark w.f. for a baryon*** (e.g. Isgur-Kahl)

ground state

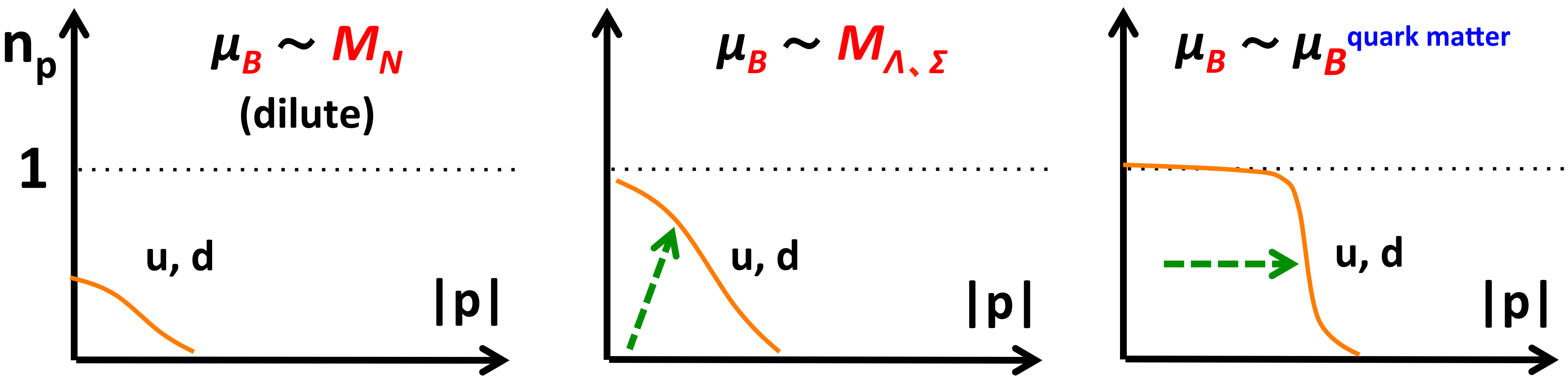


excited state



Discussion 3: Hyperon problems ?

▪ *Quark descriptions of hadronic matter* :



How to put hyperons ??

- $M_{\Lambda、\Sigma}$ at *low P* is *rejected* by quark Pauli blocking on (u,d)
- $M_{\Lambda、\Sigma}$ at *high P* avoid the blocking, but *is energetic*

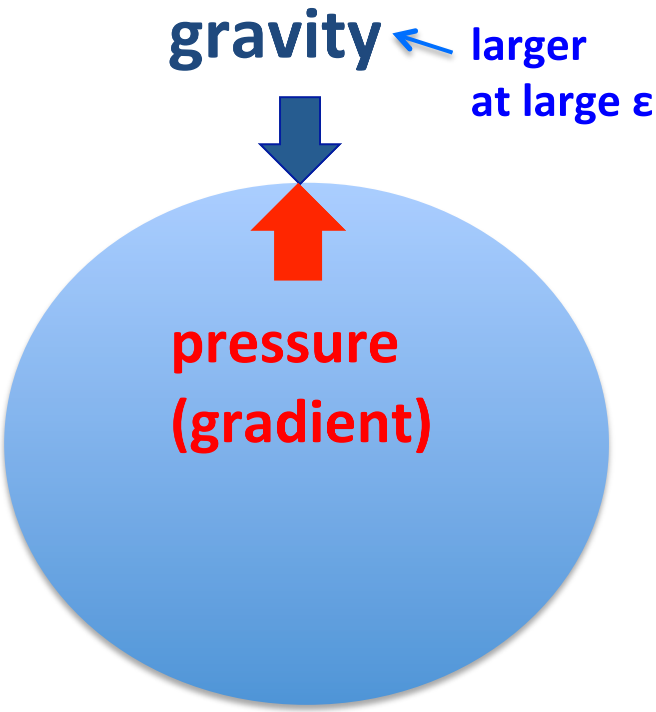
[Note: this argument becomes *more powerful* at *higher* n_B]

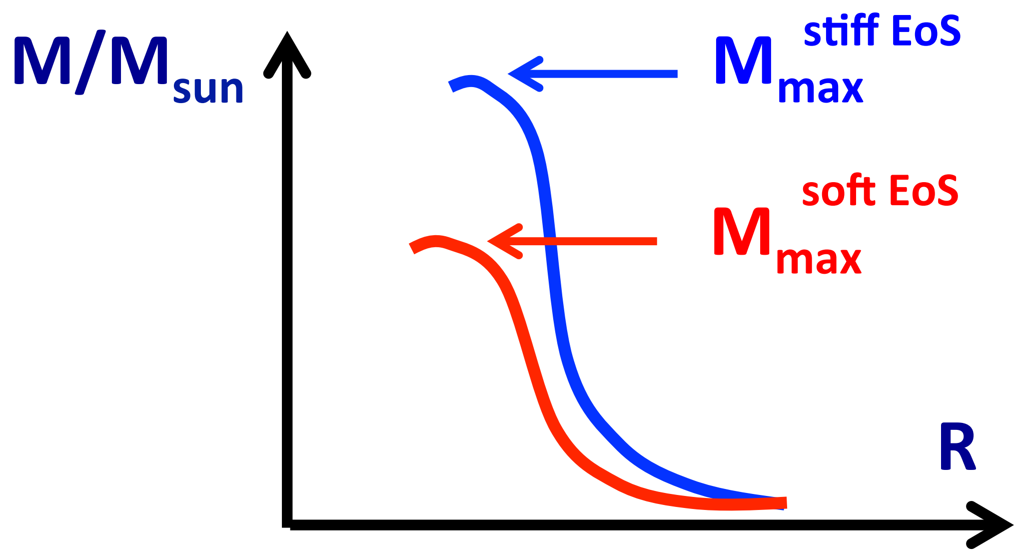
“Stiff” EoS

P* is large *at given ε

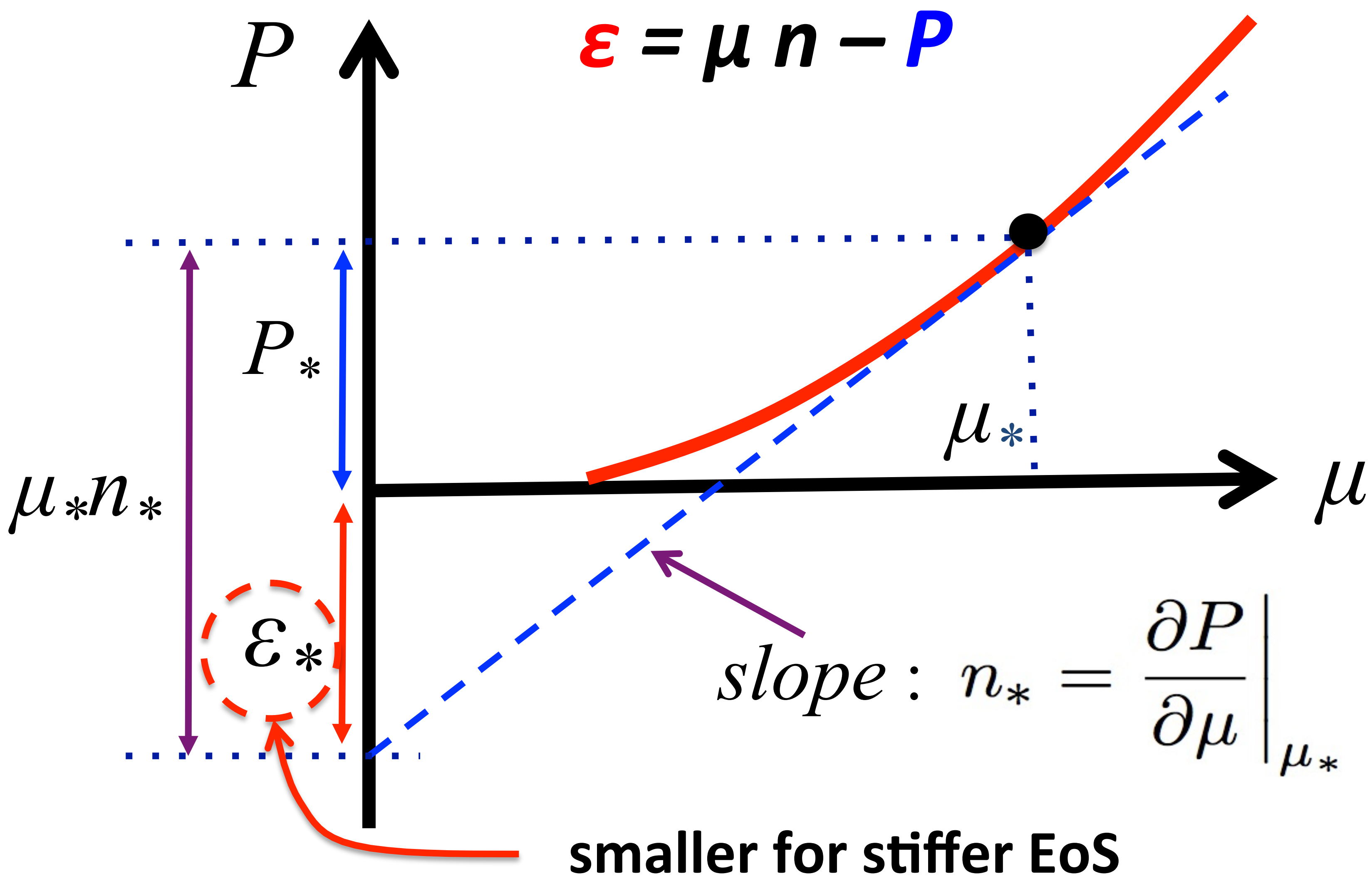
or equivalently,

ε* is small *at given P

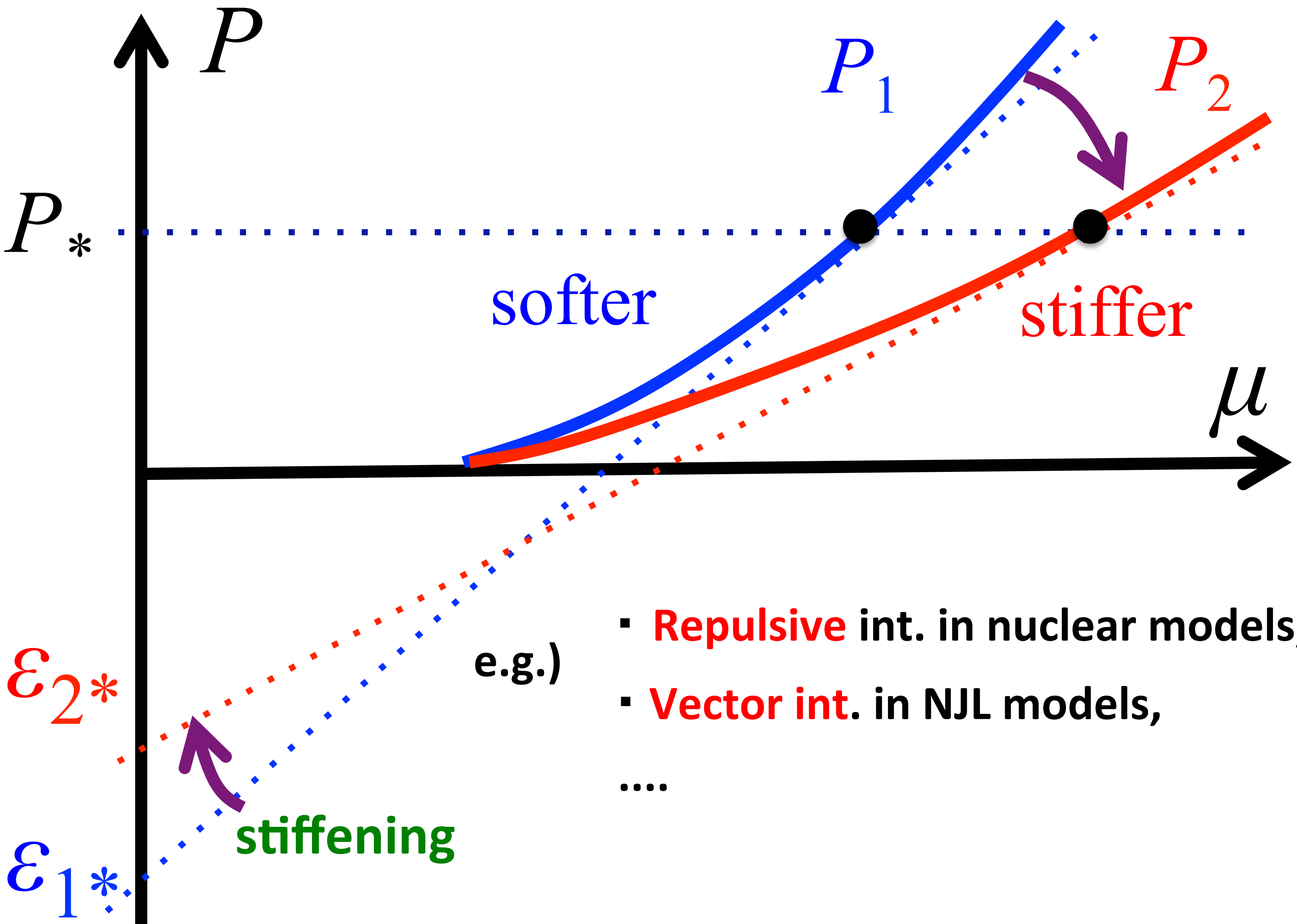




How stiff EoS looks like in $P(\mu)$ curves



Example of stiffening 1

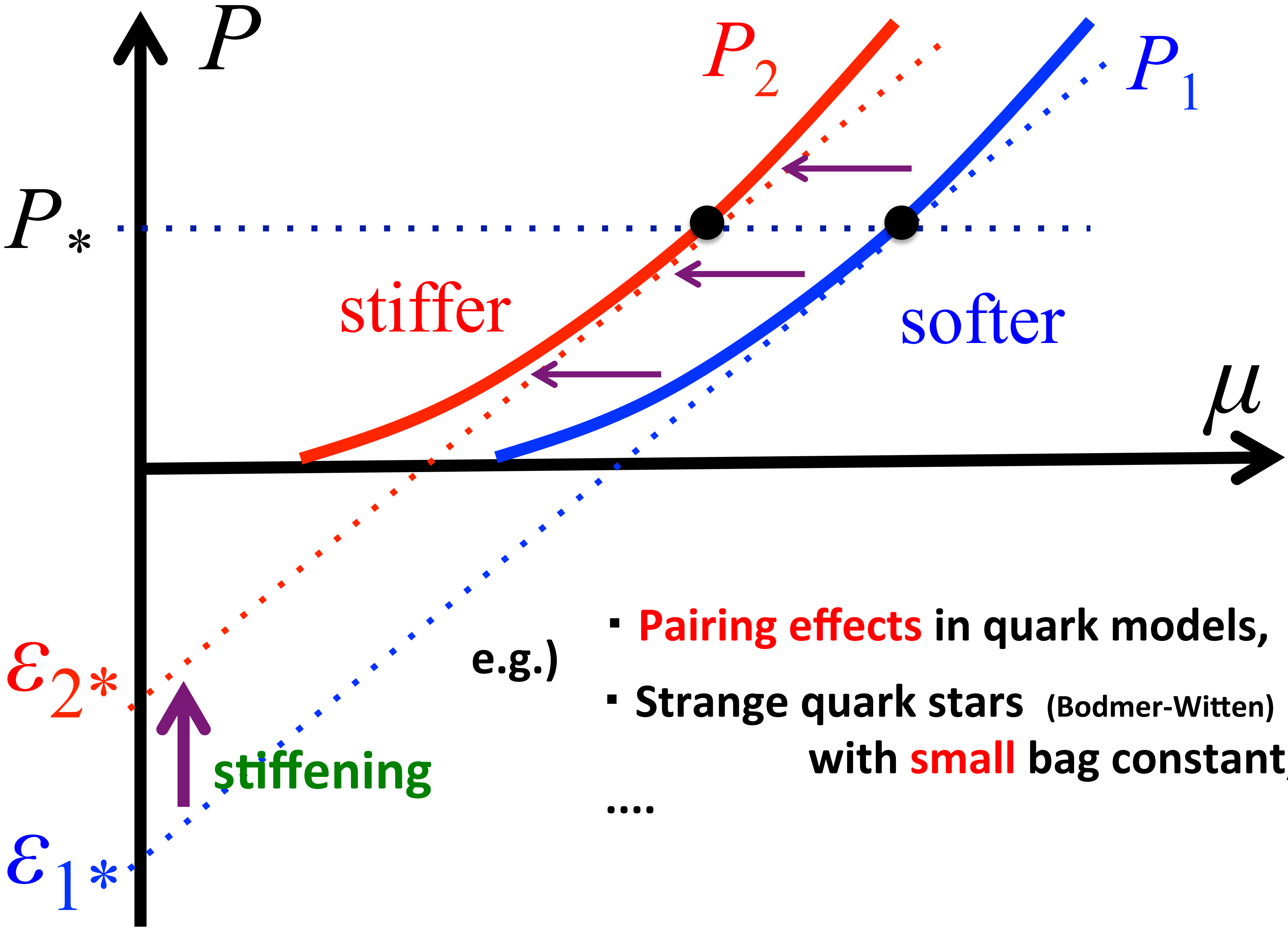


e.g.)

- **Repulsive int. in nuclear models,**
- **Vector int. in NJL models,**

....

Example of stiffening 2



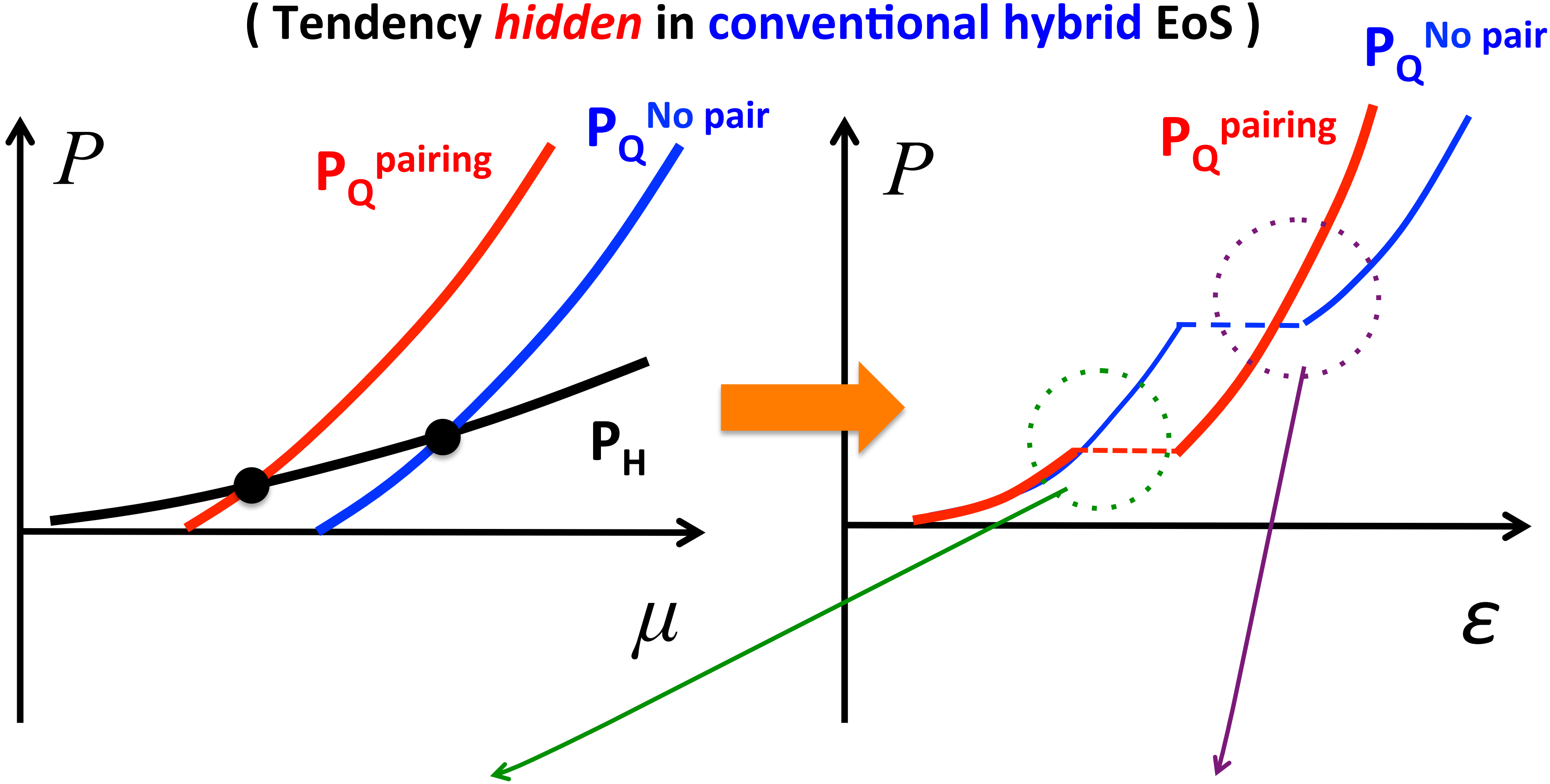
e.g.)

- Pairing effects in quark models,
- Strange quark stars (Bodmer-Witten) with small bag constant,

....

"Pairing" can stiffen EoS

(Tendency *hidden* in conventional hybrid EoS)



→ ***Softening*** **at *low* n_B & *stiffening* at *high* n_B**

Constraints on EoS

- ***Constraint 1*** (very solid)

$M > 2\ M_{sun}$ ➡ ***Stiff*** EoS at $n_B > \sim 5n_0$

PSR J1614 - 2230 Demorest et al. (2010) ; J0348+0432 Antoniadis et al. (2013)

- ***Constraint 2*** (less solid, but likely)

R = 10-13 km ➡ ***Soft*** EoS at $n_B < \sim 2n_0$

- X-ray analyses (Ozel et al. '10; Steiner et al. '13; Guillot et al '14)
- Heavy Ion data (Danielewicz et al. 2002)
- QMC cal. (Gandolfi et al. 2012)

- ***Constraint 3*** $0 < c_s^2 = \frac{\partial P}{\partial \varepsilon} < 1$

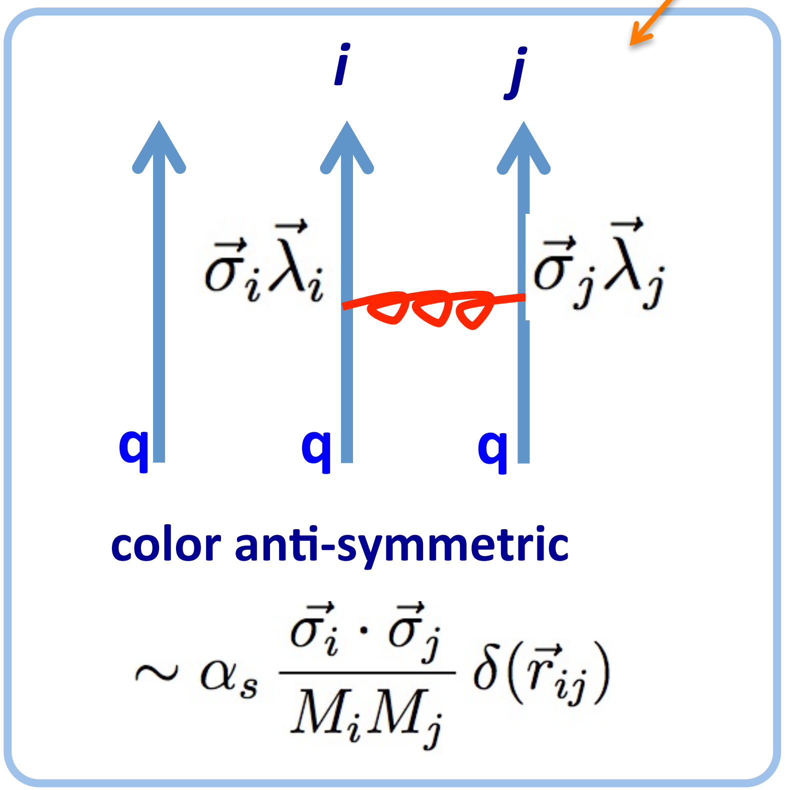
Thermodynamics & causality ➡ "Physical" connection b.t.w. Low & High density EoS

+ *attractive color-magnetic interaction*

$M_q > M_N/3$: This is **NOT** quite unusual in quark models

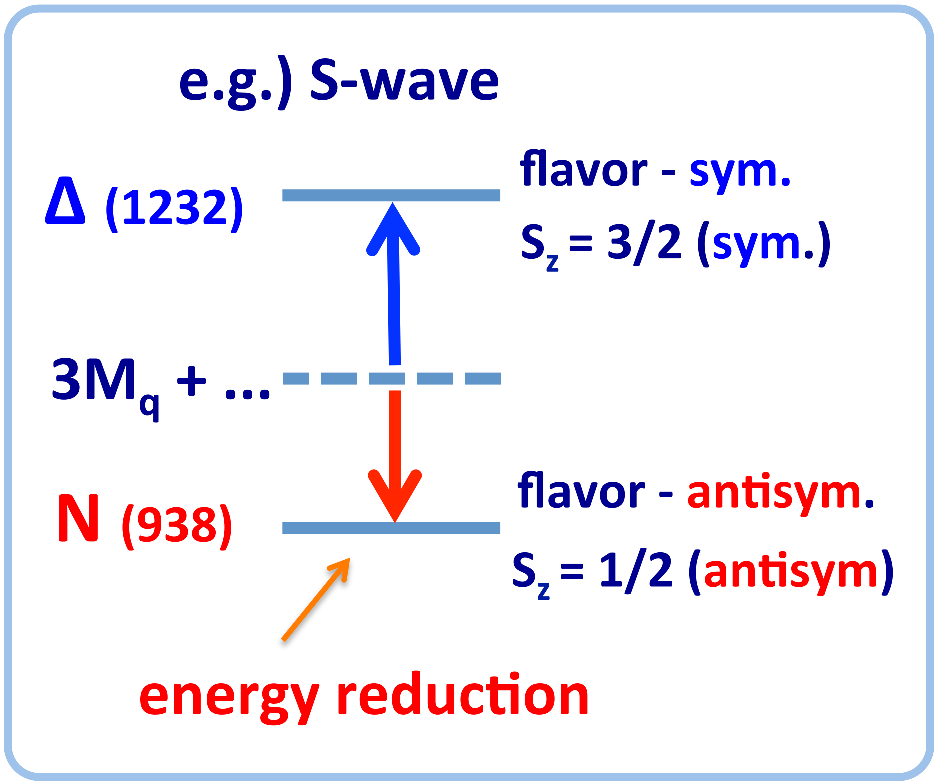
e.g.) *Non-relativistic constituent quark model*

$$M_B = 3\,M_q + \Delta E_{\text{color-mag}} + \Delta E_{\text{conf}} + \ldots$$



$$\vec{\sigma}_i \vec{\lambda}_i \qquad \vec{\sigma}_j \vec{\lambda}_j$$

$$\sim \alpha_s \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{M_i M_j} \delta(\vec{r}_{ij})$$



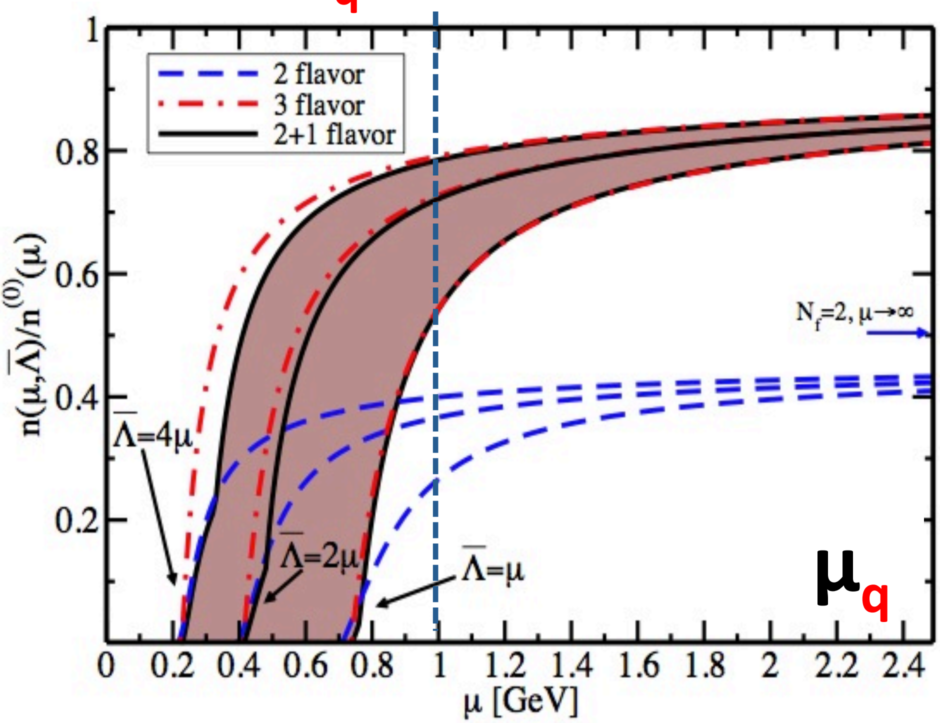
Theory 1 : pQCD for large μ_q

$O(\alpha_s^2)$ & m_s corrections: **Freedman-McLerran 78; Baluni 78**
Kurkela-Romatschke-Vuorinen 09

- ***Number density***

Kurkela-Romatschke-Vuorinen 09

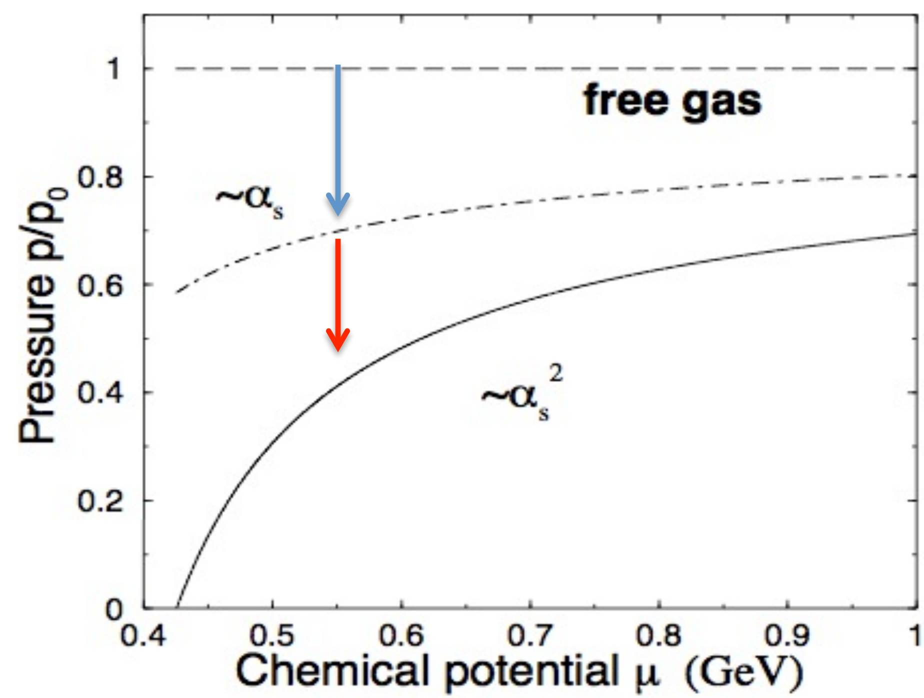
$\mu_q \sim 1$ GeV



- ***Pressure***

Fraga-Pisarski-Schaffner-Bielich 01

check of convergence



Theory 2 : Nuclear physics for small μ_q

e.g.) *Akmal-Pandharipande-Ravenhall (APR) 1998*

[or Peiper et al. 2001; Gandolfi et al. 2012]

- nucleonic EoS
- realistic potentials & sophisticated ab-initio many-body calculations
- reproduce physics at low E & at $n_B \sim n_0$ very well

At $n_B > \sim 2\, n_0$ ($\mu_q \sim 350$ MeV), problems may arise :

1, Convergence problem of many-body interactions

When $< V_{2\text{-body}} > \sim < V_{3\text{-body}} >$ → no reason to truncate 4, 5,.. -body int.

2, Baryon d.o.f. other than nucleons : Hyperons, Δ, ...,

3, Modifications of QCD vac. may come into play.

APR : convergence ?

Many-body interaction (APR-A18+UIX case)

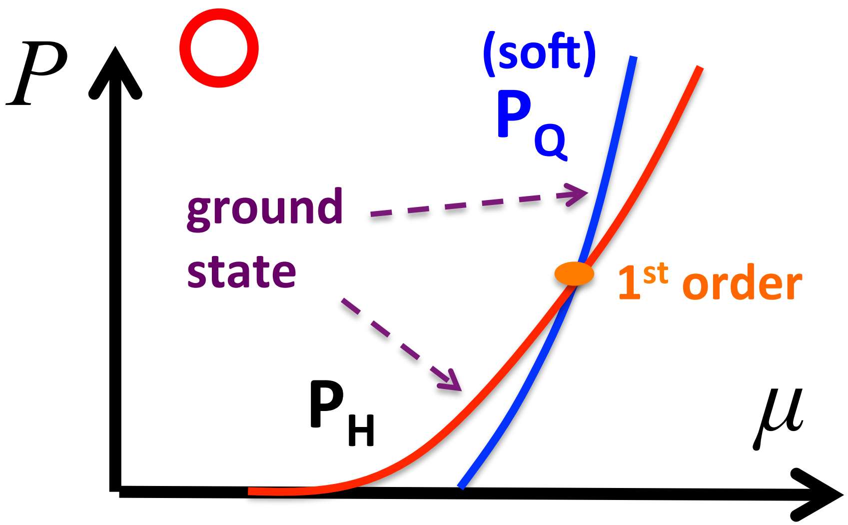
n_B	2 –body int. $\langle v_{ij}^{\pi} \rangle$	2 –body int. $\langle v_{ij}^{R} \rangle$	3 –body int. $\langle V_{ijk}^{2\pi} \rangle$	3 –body int. $\langle V_{ijk}^{R} \rangle$	4 –body int. (our guess)
n_0	−4.1	−29.9	1.2	4.5	small
$2\ n_0$	−25.1	−36.4	−17.4	30.6	marginal
$3\ n_0$	−35.7	−44.7	−34.1	78.0	large
$4\ n_0$	−52.2	−41.1	−76.9	160.3	large

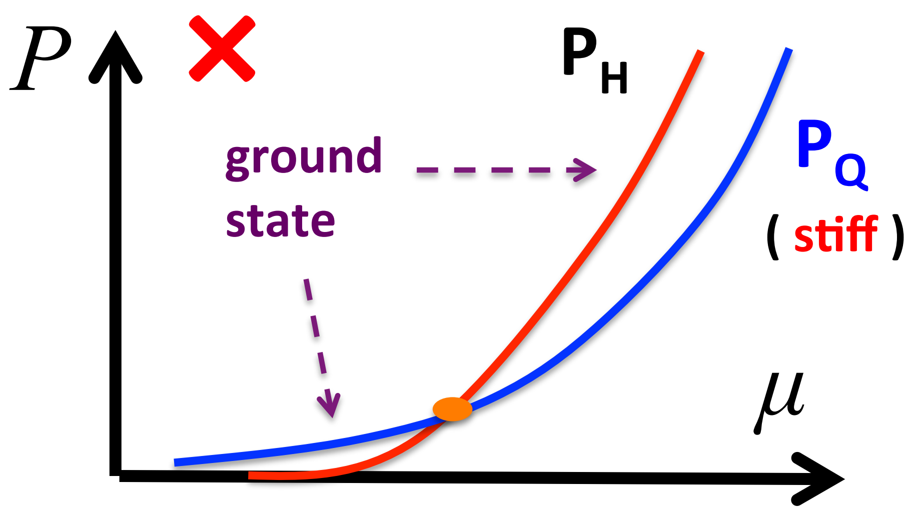
grow rapidly !!

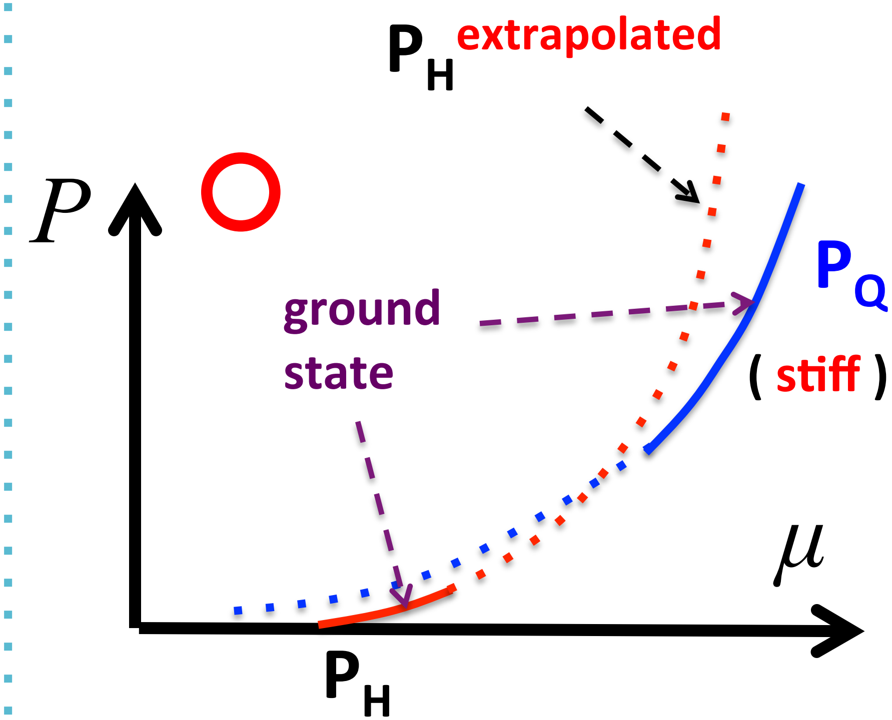
$$< V_{N\text{-body}} > \sim c_N \, (n_B / n_0)^N$$

Conventional ***vs*** *3 - window*

(hybrid, maxwell)

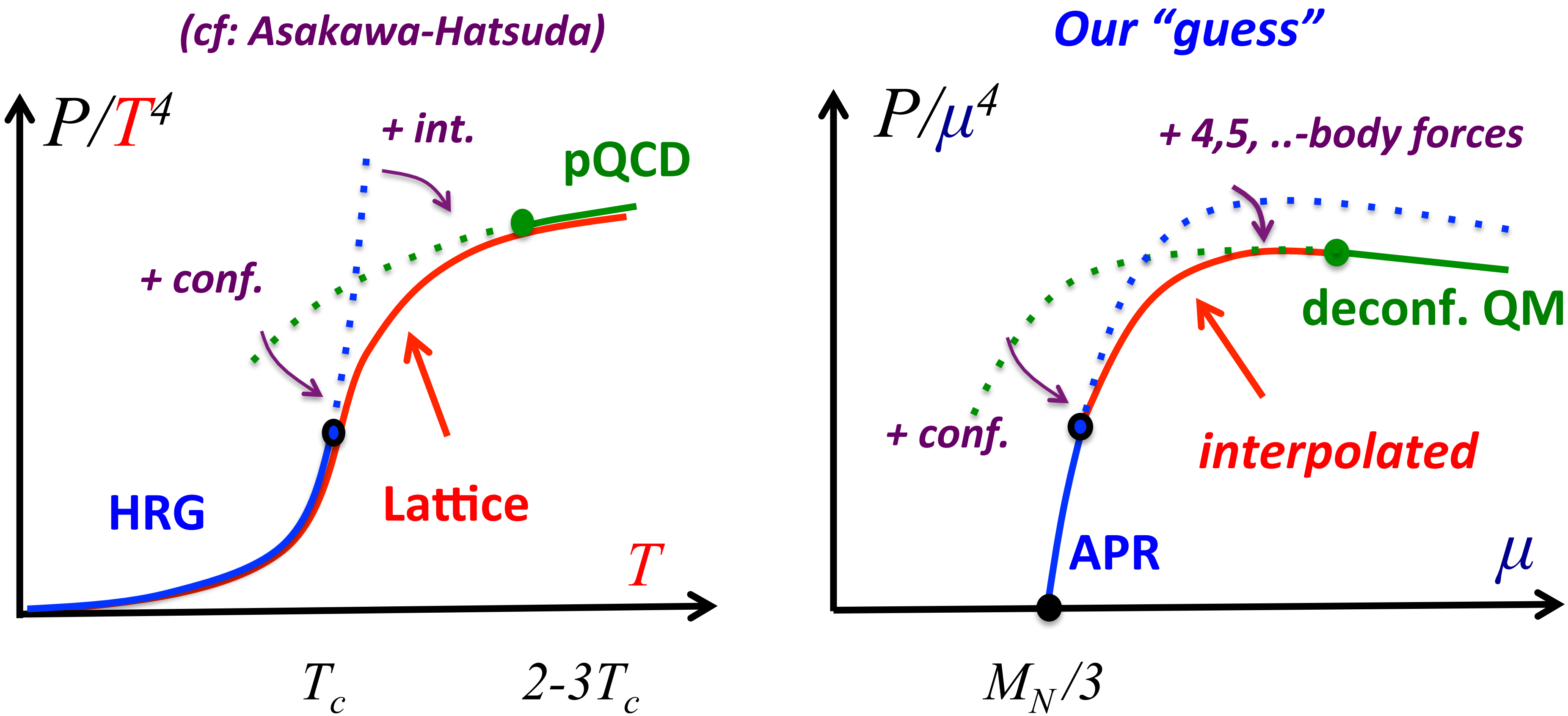




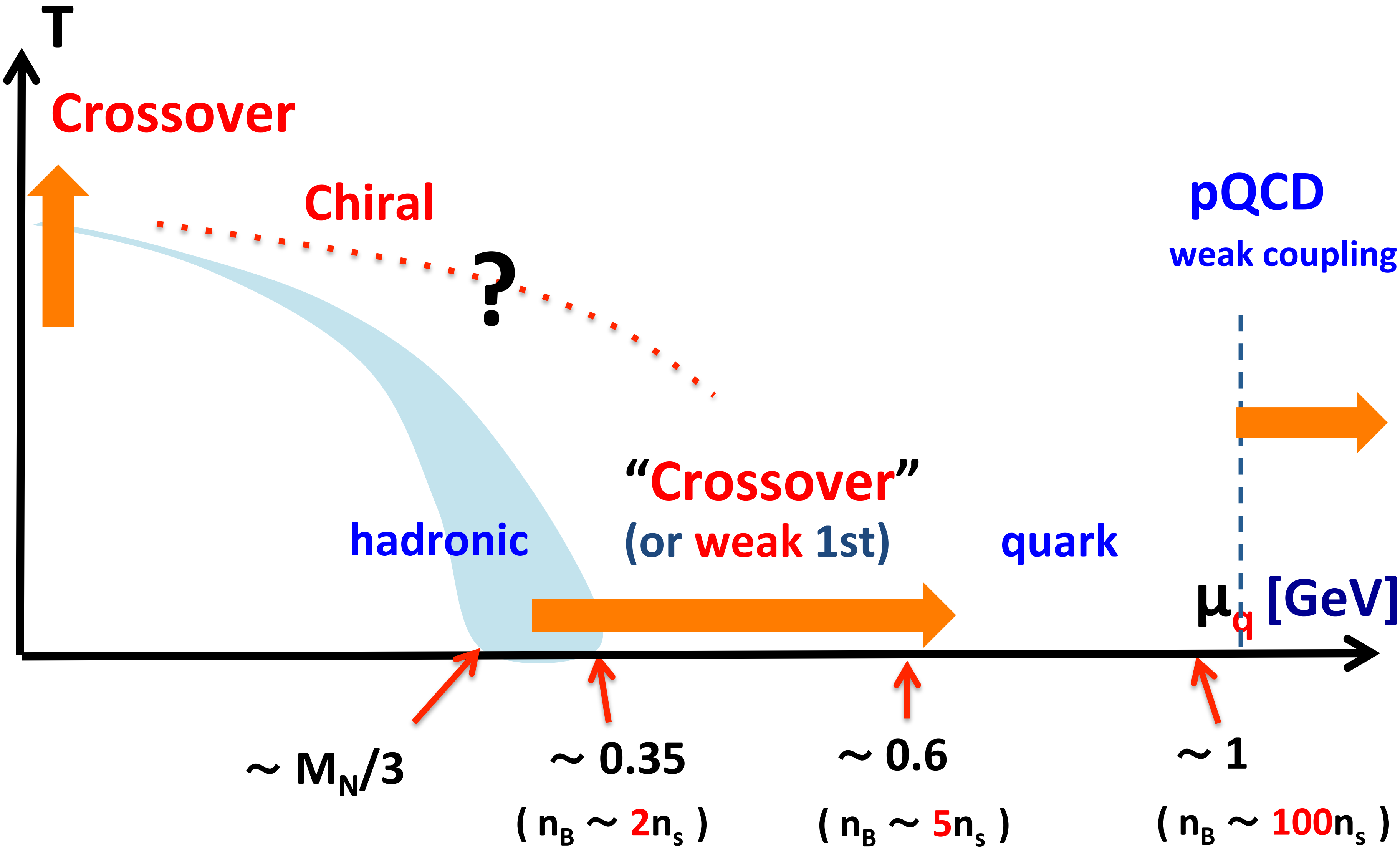


stiff **quark EoS** ***possible***

3-window : Lessons from finite T QCD



QCD phase diagram



To Do List

- Improvement of **radius determination**

  → NS-NS mergers (advanced LIGO, ...)

- Improvement of **theoretical EoS** at $n_B < 2n_0$

  → many-body calculations

- Better understanding of **hyperon-nucleon int.**

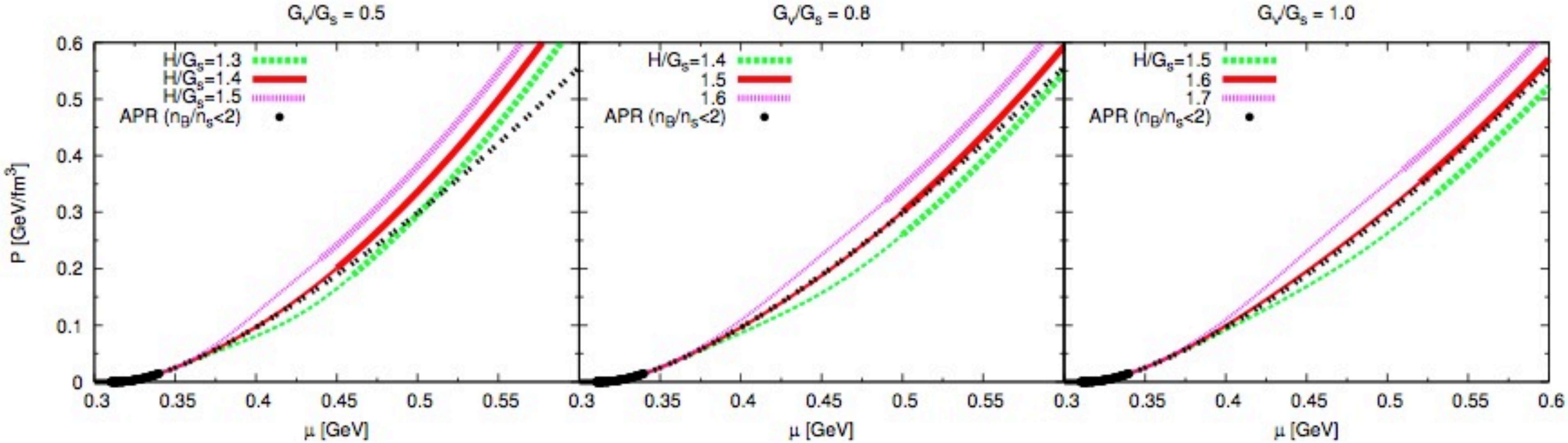
  → lattice QCD, ChEFT

- **Beyond** schematic NJL analyses at $n_B > 5n_0$

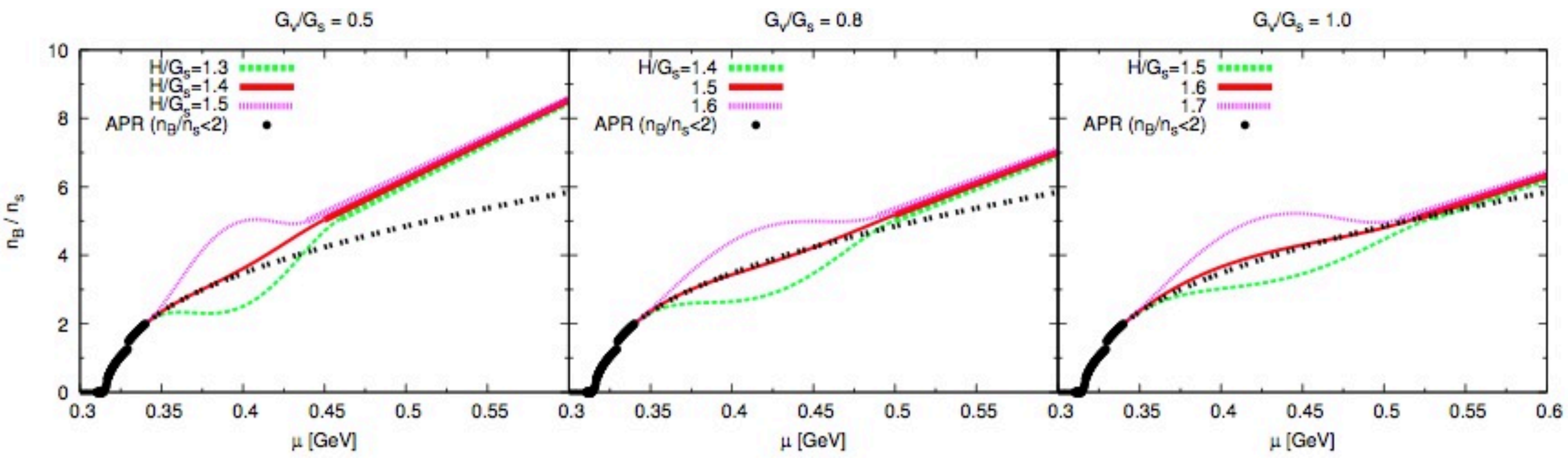
- **HOT EoS** for NS-NS mergers and supernovae

  → beyond ideal gas and purely hadronic EoS

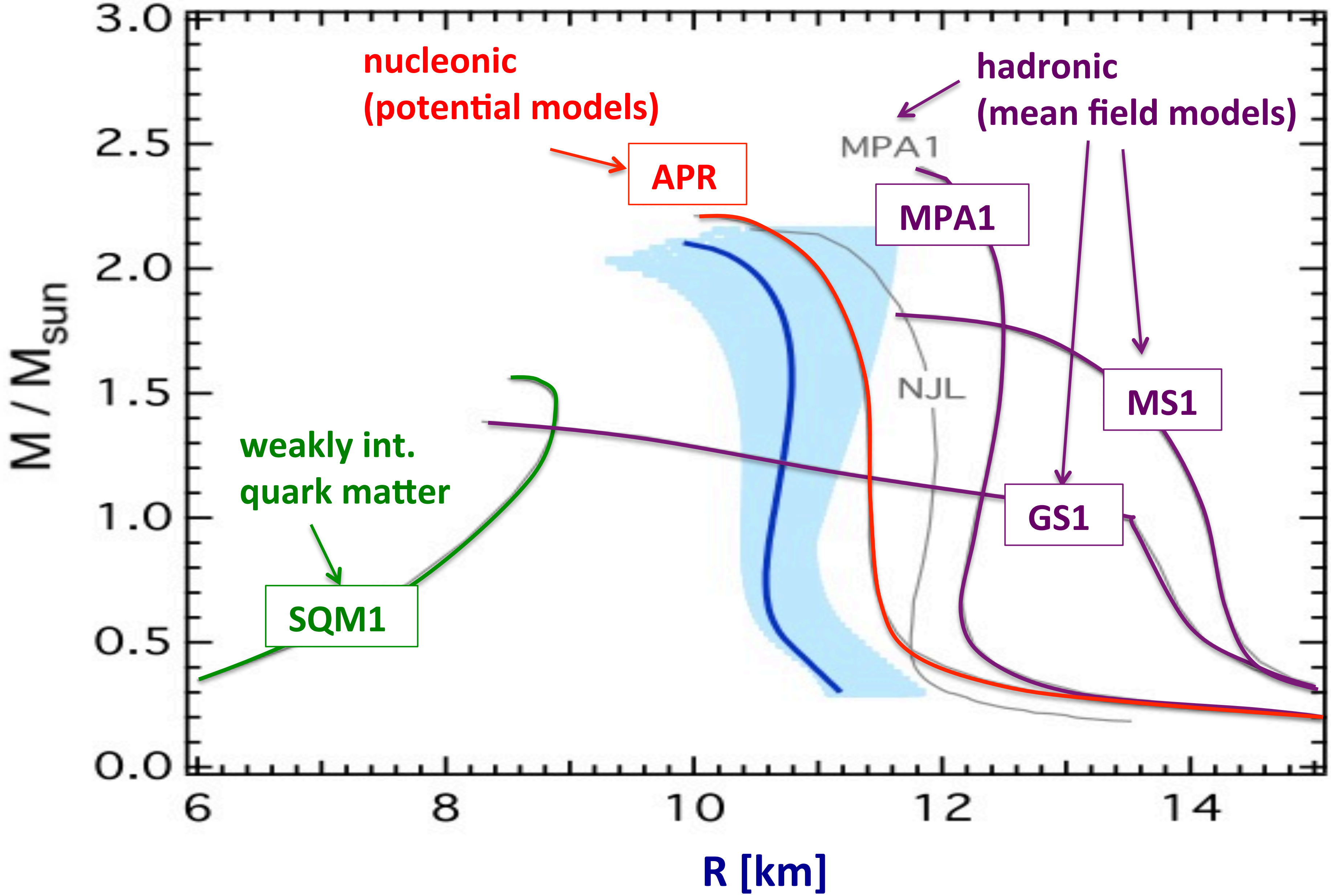
P v.s. μ



n_B/n_0 v.s. μ



M-R relation for various EoS

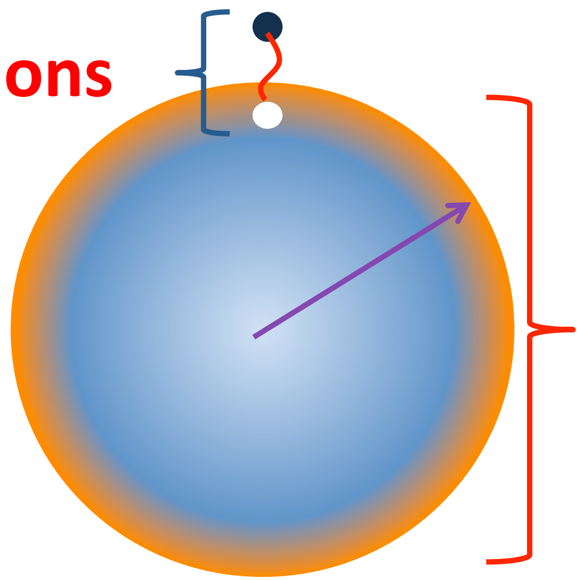


Quarkyonic matter

- *Dense quark matter with confining gluons*

hadronic excitations

- Transport properties
- Phase structures (condensation)
- etc.



Bulk properties:
(EOS, Pressure, etc.)

Weakly int. quarks

- Pauli-blocking.
- forming color singlet B.G.

New state of matter :

Quark Fermi sea + baryonic Fermi surface → Quark-yonic

With MF type H-EoS

[Bonnano & Sedrakian et al, 12]

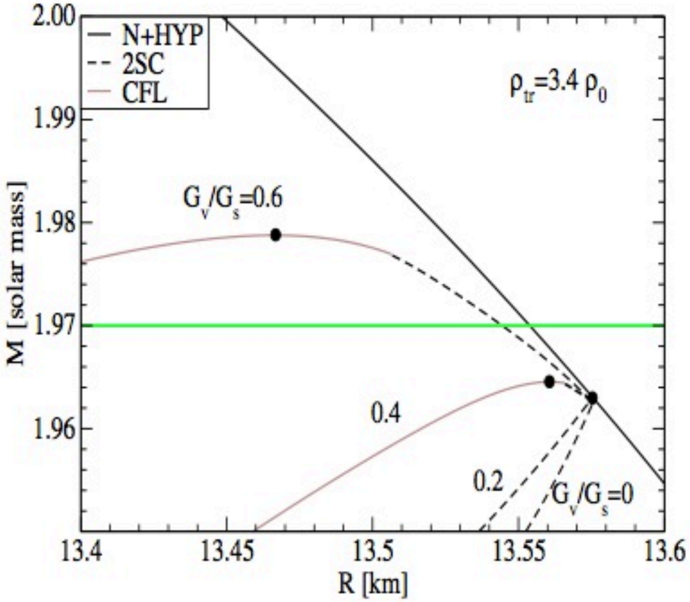
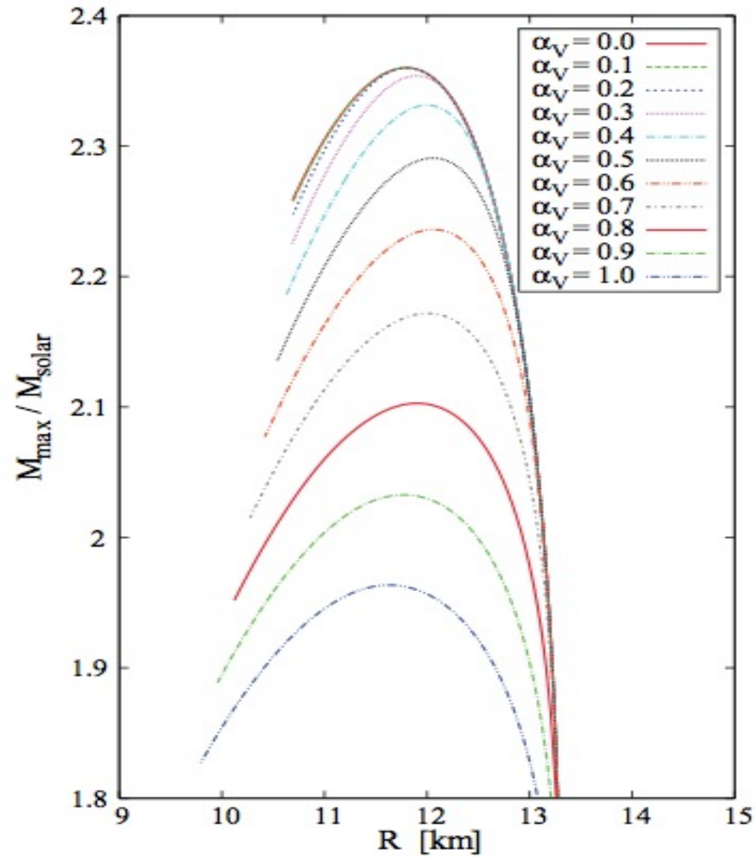


Fig. 2. Mass vs. radius for configurations with quark-hadron transition density $\rho_{tr} = 3.4\rho_0$ for four values of vector coupling G_V/G_S = 0, 0.2, 0.4, 0.6. The purely hadronic sequence (i.e. the sequence that includes nucleons and hyperons) is shown by black solid line. The dashed lines and the gray solid lines show the branches where the 2SC and CFL quark phases are present. The filled circles mark the maximum masses of the sequences. The horizontal line shows the largest mass measurement to date (Demorest et al. 2010).

[Weissenborn et al, 12]



stiff H-EoS at low density -> R > ~ 13 Km

With MF type H-EoS

[Klahn et al, 07]

[Weissenborn et al, 12]

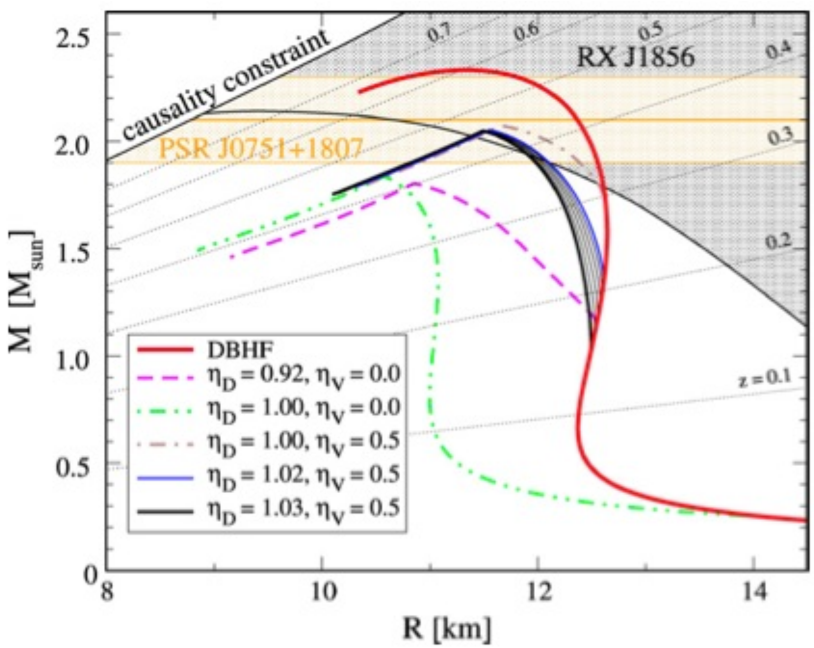


Fig. 2. Mass–radius relationship for CS sequences corresponding to a nuclear matter EoS (DBHF) and different hybrid star EsoS (DBHF + NJL), see text. Indicated are also the constraint on the mass from the pulsar J0751 + 1807 [2] and on the mass–radius relationship from the isolated neutron star RX J1856 [3]. Present constraints on the mass–radius relation of CSs do not rule out hybrid stars. The dotted lines indicate the gravitational redshift, $z = (1 - 2GM/R)^{-1/2} - 1$, of photons emitted from the compact star surface.

When do EoSs become *stiff* ?

simple parameterization at large density :

bag constant.

$$\varepsilon(n) = c_1 n^{4/3} + c_2 n^{2/3} + c_{-2} n^2 + B$$

$\sim p_F^4$ — *kinetic* energy

$\sim p_F^2$ — *pairing effects*

$\sim p_F^6$ — density-density int.

$$P = \frac{\varepsilon}{3} - \frac{2}{3} c_2 n^{2/3} + \frac{2}{3} c_{-2} n^2 - \frac{4}{3} B$$

corrections to the conformal relation

P at given ε becomes large when :

- $c_2 < 0$ → attractive pairing effects (cf. diquark correlation)
- $c_{-2} > 0$ → repulsive density-density interactions (cf. ω-meson exchange or hard-core repuslsion)
- small bag constant

Impact of the bag constant

For chiral sym. restored, deconfined free quark gas (3-flavor) :

$$P(\mu) = c_0\mu^4 - B \qquad \varepsilon(\mu) = 3c_0\mu^4 + B$$

We can find the scaling :

$$M_{\max} \simeq 1.78\, M_\odot \left(\frac{155\,\mathrm{MeV}}{B^{1/4}}\right)^2 \qquad R \simeq 9.5 \left(\frac{155\,\mathrm{MeV}}{B^{1/4}}\right)^2 \mathrm{km}$$

If B were very small, even free gas could give a very large mass !

NJL model : **"quark" bag const. appears through the chiral restoration**

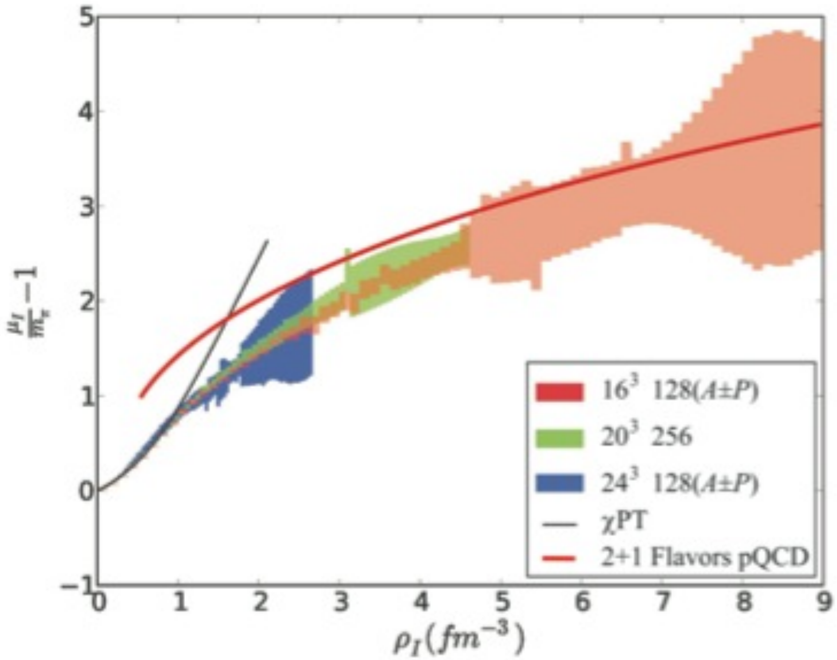
$$B_q^{\mathrm{NJL}} \simeq 284\,\mathrm{MeV/fm^3} = (219\,\mathrm{MeV})^4$$ → $M_{max} \sim$ **0.9** M_{sun}

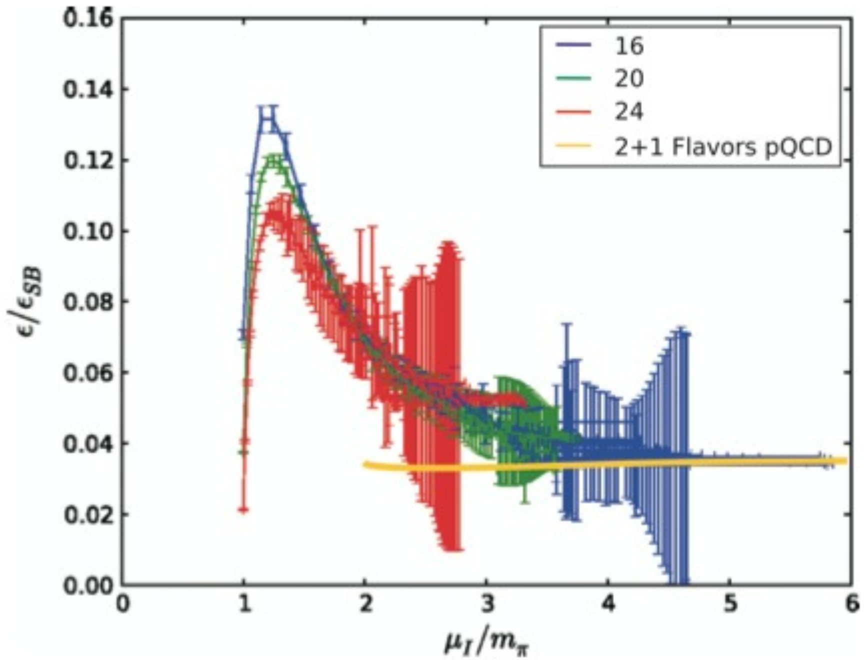
Our question is : ***provided*** **this order of magnitude for B,**

how can we achieve large star masses ?

Outcome

- **Chiral symmetric but confined matter**





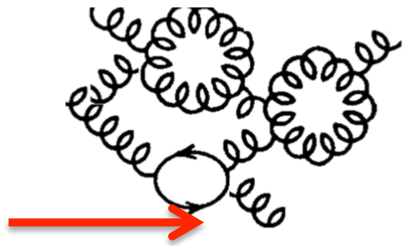
$n_I \sim 50n_0$

$m_\pi \sim 390$ MeV

Outcome

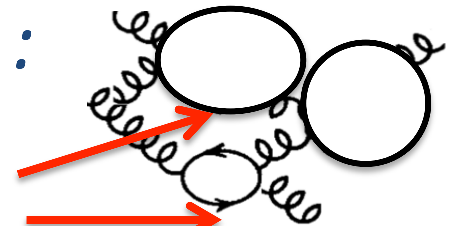
- **Main idea :**

Vacuum:

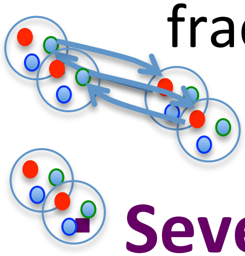


small
fraction

Large μ :



large
fraction



- **Several branches :**