

# Effective volume of correlated charm quark pair

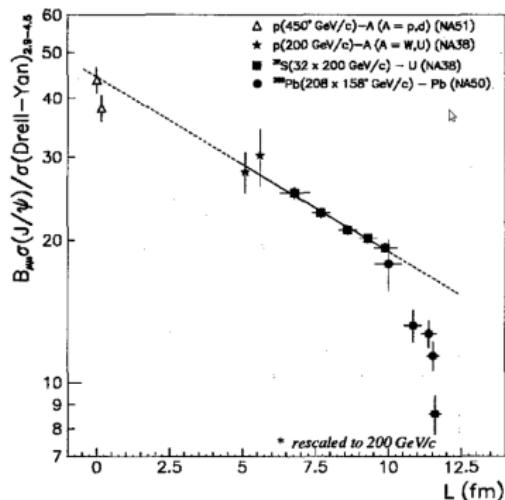
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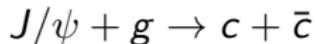
June 9, 2016

In collaboration with  
Che-Ming Ko  
Feng Li

# $J/\psi$ in HIC



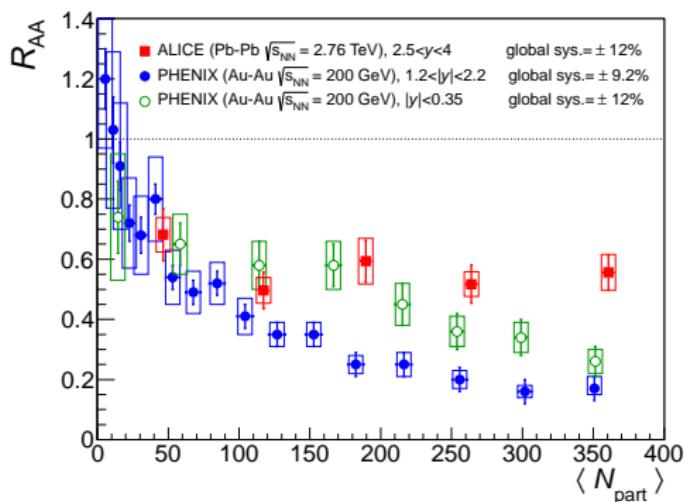
(SPS data, Nucl. Phys. A610,  
404c (1996))



- Sequential dissociation  
(H. Satz, et al)
- Statistical model  
(P. Braun-Munzinger, et al)
- Transport model  
(**P. Zhuang**, R. Rapp,  
T. Song, et al)

# $J/\psi$ in HIC

$$R_{AA} = \frac{N_{J/\psi}^{AA}}{N_{J/\psi}^{pp} N_{coll}}$$

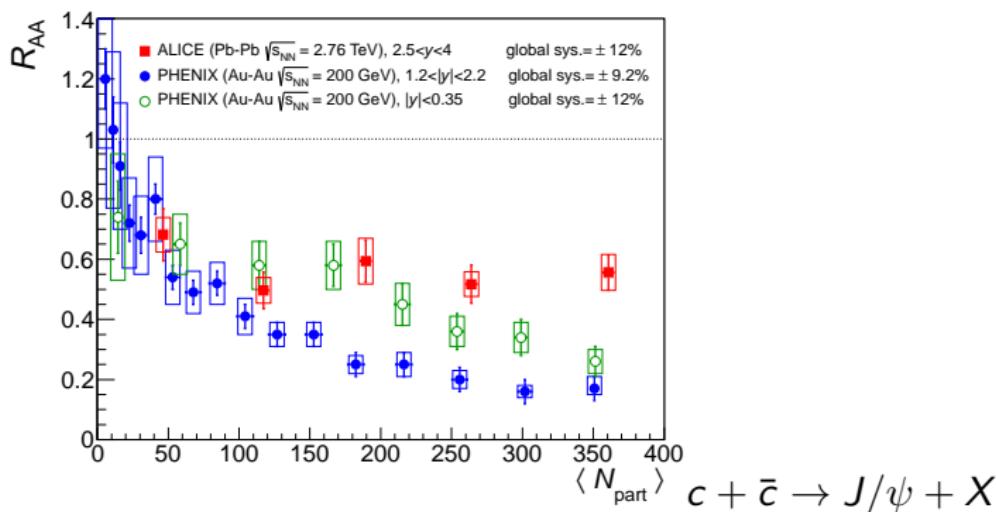


(Alice data, Phys. Rev. Lett. 109, 072301 (2014))

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- Y. Liu, Z. Qu, N. Xu, **P. Zhuang**, J/psi Transverse Momentum Distribution in High Energy Nuclear Collisions at RHIC, Phys. Lett. B678 (2009) 72-76.
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- Y. Liu, B. Chen, N. Xu, **P. Zhuang**, Upsilon Production as a Probe for Early State Dynamics in High Energy Nuclear Collisions at RHIC, Phys. Lett. B697 (2011) 32-36.
- K. Zhou, N. Xu, Z. Xu, **P. Zhuang**, Medium effects on charmonium production at ultrarelativistic energies available at the CERN Large Hadron Collider, Phys. Rev. C89 (2014) no.5, 054911.
- ...

# $J/\psi$ Regeneration

$$R_{AA} = \frac{N_{J/\psi}^{AA}}{N_{J/\psi}^{pp} N_{coll}}$$



(Alice data, Phys. Rev. Lett. 109, 072301 (2014))

# $J/\psi$ Regeneration

Possible heavy quarkonium regeneration from rare heavy quarks:

- $J/\psi$  at SPS
- $J/\psi$  in peripheral collisions at RHIC
- $\Upsilon$  at SPS or RHIC
- $J/\psi$  in  $p+\text{Pb}$  collisions at LHC
- $\Upsilon$  in  $p+\text{Pb}$  collisions at LHC
- ...

# Outline

- 1 Canonical ensemble effect
- 2 Effective volume
- 3 Results from a toy model
- 4 Qualitative conclusions

# Canonical effect of $J/\psi$ production in the statistical model

Statistical model:

$$N_{c\bar{c}}^{dir} = \frac{1}{2} N_{open} + N_{hidden}$$

- Grand canonical ensemble:

$$N_{c\bar{c}}^{dir} = \frac{1}{2} \gamma n_{open}^{th} V + \gamma^2 n_{hidden}^{th} V$$

$$\frac{N_{hidden}}{N_{open}^2} \propto \frac{1}{V}$$

# Canonical ensemble

- Canonical ensemble (rare charm limit):

$$\begin{aligned} N_{c\bar{c}}^{dir} &= \frac{1}{2}\gamma n_{open}^{th} V \frac{I_1(\gamma n_{open}^{th} V)}{I_0(\gamma n_{open}^{th} V)} + \gamma^2 n_{hidden}^{th} V \\ &\approx (\frac{1}{2}\gamma n_{open}^{th} V)^2 + \gamma^2 n_{hidden}^{th} V \end{aligned}$$

$$\frac{N_{hidden}}{N_{open}} \propto \frac{1}{V}$$

# Canonical effect of $J/\psi$ production in the statistical model

Statistical model:

$$N_{c\bar{c}}^{dir} = \frac{1}{2} N_{open} + N_{hidden}$$

- Grand canonical ensemble:

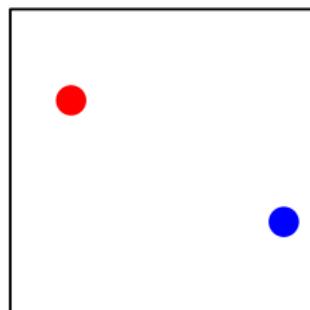
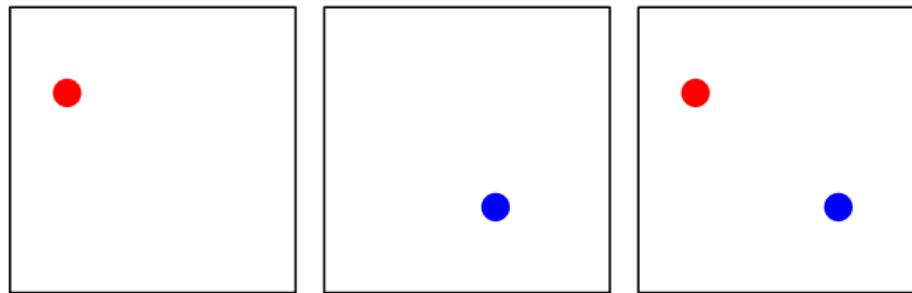
$$\frac{N_{hidden}}{N_{open}^2} \propto \frac{1}{V}$$

- Canonical ensemble (rare charm limit):

$$\frac{N_{hidden}}{N_{open}} \propto \frac{1}{V}$$

Microscopic view:  $V \rightarrow$  collision rate  $\rightarrow N_{hidden}$ .

# Heavy quark correlation and the effective volume



VS



# Approach

## Set up

- gluons: Boltzmann distribution
- charm quarks:

$$\partial_t f_c + \mathbf{v} \cdot \nabla f_c = C_{c+g \rightarrow c+g}$$

- parameters:
  - $\sigma_{cg} = 4 \text{ mb}$ , isotropic
  - $m_c = 1.25 \text{ GeV}$
  - $m_g = 0$
  - constant temperature  $T$
  - initial momentum of charm quarks  $p_0$

# Approach

# of collisions between  $c$  and  $\bar{c}$  in  $\Delta t$

$$\begin{aligned}\Delta N_{\text{coll}} &\propto \sigma \\ &\propto \Delta t \\ &\propto N_c \\ &\propto N_{\bar{c}} \\ &\sim \text{correlation between } c \text{ and } \bar{c}\end{aligned}$$

For thermalized charm quarks in volume  $V$

$$\Delta N_{\text{coll}}^{th} = \frac{N_c N_{\bar{c}} \sigma \Delta t}{V} g(m_c/T),$$

where  $g(z) \equiv \frac{4K_3(2z)}{zK_2^2(z)}$ .

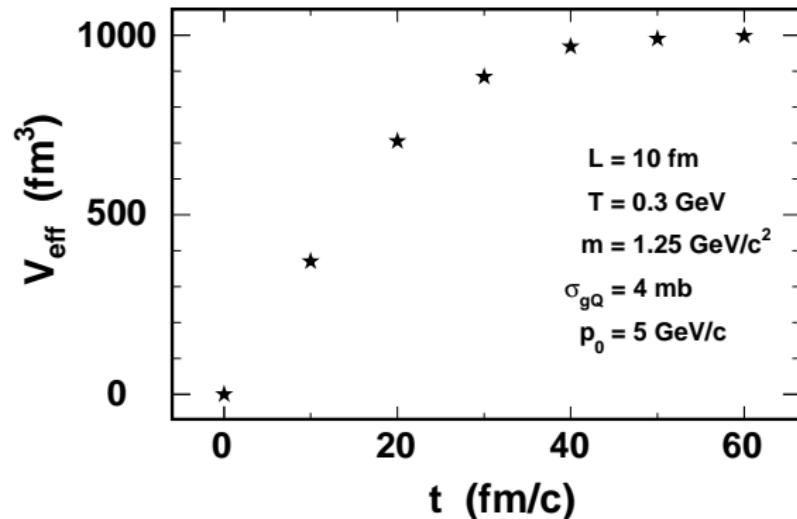
# Approach

$$\Delta N_{\text{coll}}^{th} = \frac{N_c N_{\bar{c}} \sigma \Delta t}{V} g(m_c/T),$$

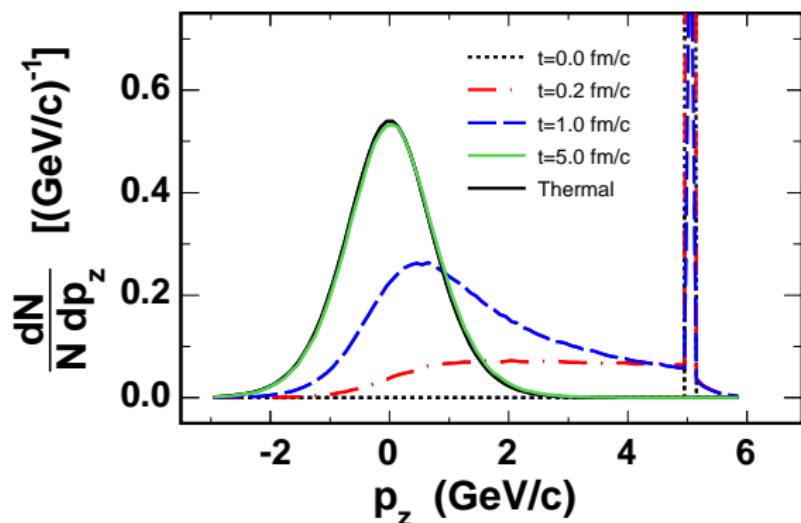
We define the effective volume

$$V_{\text{eff}} = \lim_{\substack{N_c, N_{\bar{c}} \rightarrow \infty \\ \Delta t \rightarrow 0 \\ \sigma \rightarrow 0}} \frac{N_c N_{\bar{c}} \sigma \Delta t}{\Delta N_{\text{coll}}} g(m_c/T).$$

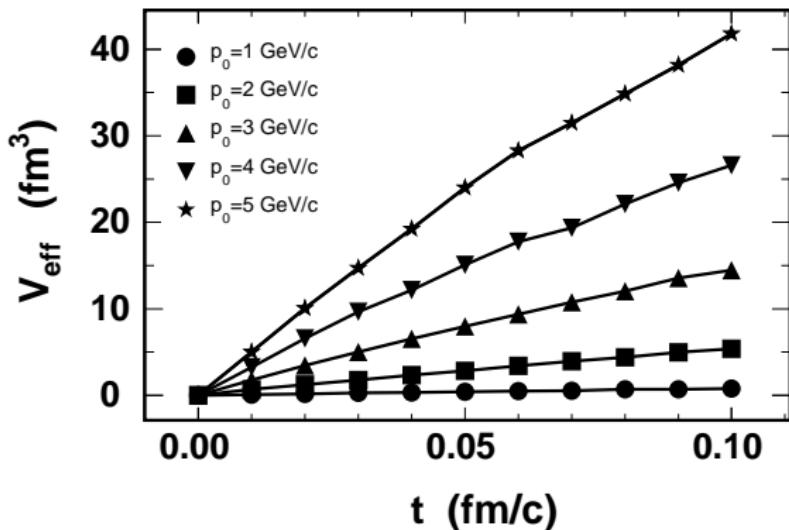
# Box Test



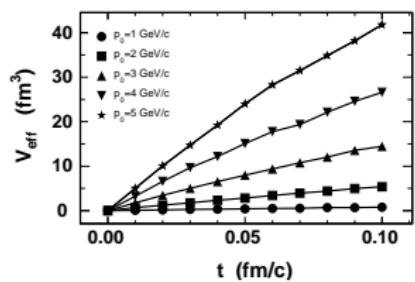
# $p_z$ distribution of the charm quark



## Short Time

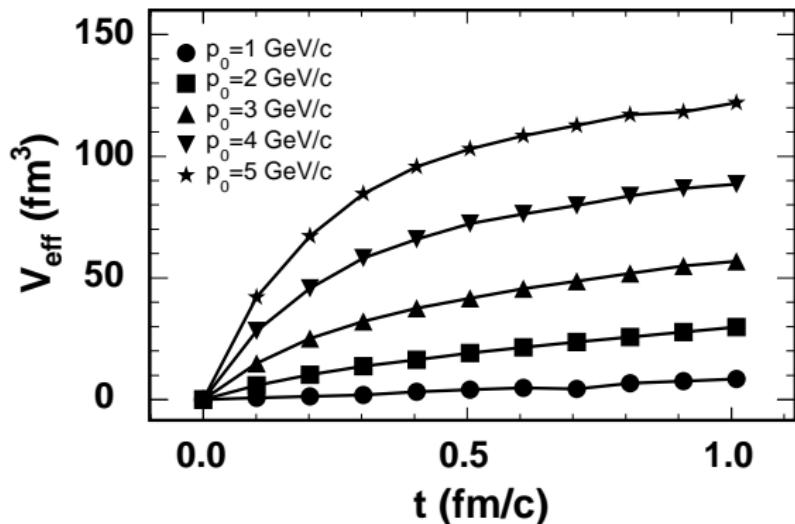


# Short Time

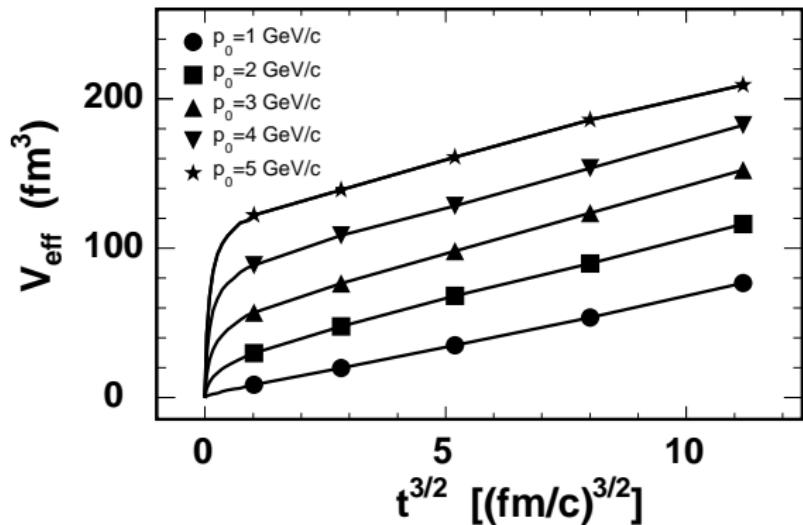


$$\begin{aligned}
 N_{J/\psi} &\propto \int \frac{\Delta N_{Q\bar{Q}}}{\Delta t} dt \\
 &\propto \int \frac{1}{V_{\text{eff}}} dt \\
 &\propto \int \frac{1}{t} dt
 \end{aligned}$$

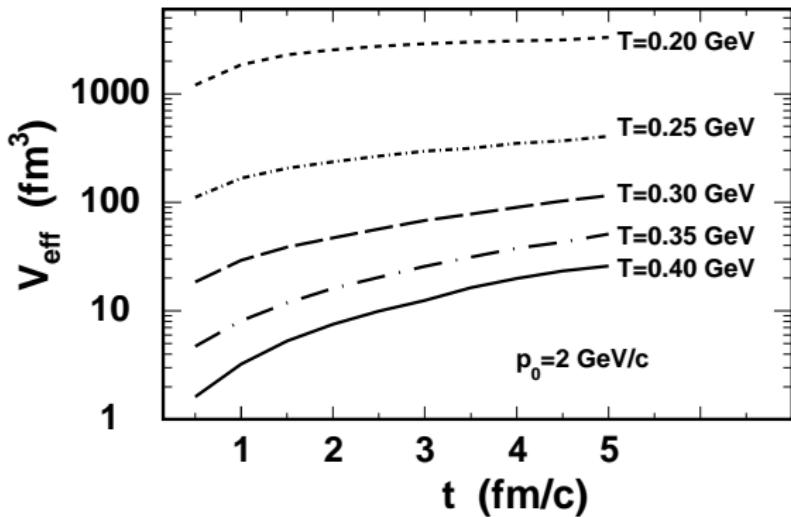
# Intermediate Time

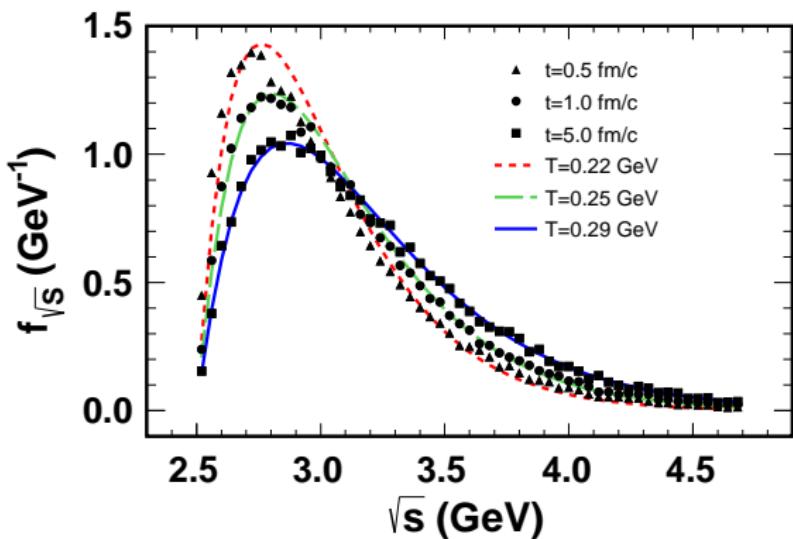


# Long Time



# $T$ dependence



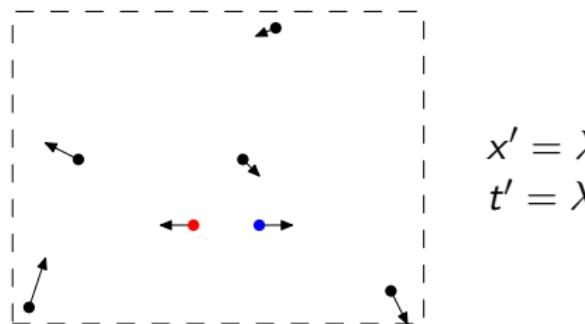
$\sqrt{s_{Q\bar{Q}}}$  distribution of colliding  $Q\bar{Q}$  pair

# Qualitative conclusions

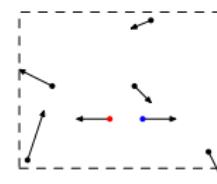
In rare charm events:

- ① Charm quark correlation, instead of the volume of the fireball, plays an important role in charmonium regeneration;
- ② The regeneration of charmonia depends on the temperature and initial momentum sensitively;
- ③ Regenerated charmonia only contribute to the low momentum region even if the charm quark distribution is far from equilibrium.

# Proof of the linear behavior



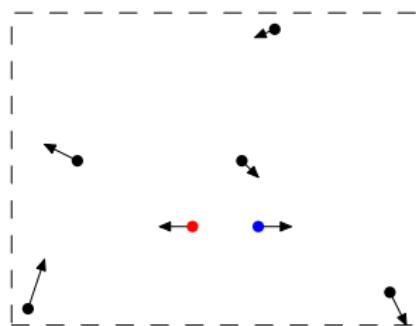
$$x' = \lambda x \\ t' = \lambda t$$



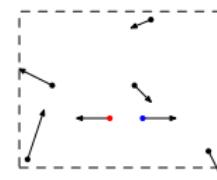
$$\Delta N_{Q\bar{Q}}(t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g) = \Delta N_{Q\bar{Q}}(t', \Delta t', \sigma'_{Q\bar{Q}}, \sigma'_{gQ}, f'_g)$$

$$\begin{aligned} \Delta N_{Q\bar{Q}} &\propto \Delta t, \\ \Delta N_{Q\bar{Q}} &\propto \sigma_{Q\bar{Q}}, \\ \Delta N_{Q\bar{Q}} &\propto \sigma_{gQ} f_g, \\ \Delta N_{Q\bar{Q}} &\propto \sigma_{g\bar{Q}} f_g \end{aligned}$$

# Proof of the linear behavior



$$\begin{aligned}x' &= \lambda x \\t' &= \lambda t\end{aligned}$$



$$\begin{aligned}\Delta N_{Q\bar{Q}}(t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g) &= \Delta N_{Q\bar{Q}}(t', \Delta t', \sigma'_{Q\bar{Q}}, \sigma'_{gQ}, f'_g) \\&= \lambda \Delta N_{Q\bar{Q}}(\lambda t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g)\end{aligned}$$

$$\frac{1}{\lambda} \Delta N_{Q\bar{Q}}(t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g) = \Delta N_{Q\bar{Q}}(\lambda t, \Delta t, \sigma_{Q\bar{Q}}, \sigma_{gQ}, f_g)$$