Higgs Theory

Scott Thomas

Lecture 1

Standard Model in the Phase we Observe

Point-like $J = 1$ Bosons:

$W^+,\ W^-,\ Z$ massive

γ massless

Interactions well described by local $SU(2)_L \times U(1)_Y$

$$\begin{pmatrix} Z \\ A \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W^3 \\ Z \end{pmatrix} \qquad W^{\pm} = \frac{1}{\sqrt{2}}\left(W^1 \mp iW^2\right)$$

Add $J = 1$ mass terms (address gauge invariance later)

$$\frac{1}{2} m_a^2 \, A_\mu^a A^{a\mu}$$

$W^+W^- \to W^+W^-$

Unpolarized $2 \to 2$ scattering - Expand in partial waves

$$\mathcal{A}(\cos\theta) = 16\pi \sum_{\ell} P_{\ell}(\cos\theta)\ a_{\ell}$$

S-Matrix

$$\psi_f = S\,\psi_i$$

Probability Conservation $\Rightarrow S^\dagger S = 1$ Unitarity

Optical Theorem Foward Scattering $S = 1 + i\,T$

$$-i\,(T - T^\dagger) = T^\dagger T$$

$$\sigma(i \to \text{All}) = \frac{1}{2s} \int d\Phi_f\ T^\dagger_{if} T_{fi}$$

$$\mathrm{Im}\,\mathcal{A}(2 \to 2) = s\,\sigma(2 \to \mathrm{All}) \geq s\,\sigma(2 \to 2)$$

$$\sigma(2 \to 2) = \frac{16\pi}{s} \sum_{\ell} (2\ell + 1)\, |a_\ell|^2$$

Optical Theorem - Partial Wave Unitarity

$$|a_\ell|^2 \leq \mathrm{Im}\, a_\ell \qquad\qquad |\mathrm{Re}\, a_\ell| \leq \frac{1}{2}$$

Calculate $\mathcal{A}(W^+W^- \to W^+W^-)$ tree-level $\ell = 0$

limit: $m_W^2 \to 0$, $s =$ fixed

$$a_0 = -\frac{g^2\, s}{128\pi\, m_W^2}$$

Violates unitarity $s \geq 64\pi\, m_W^2/g^2 \sim 1.7$ TeV

Something must happen to save Unitarity!

What about quantum corrections (loops)?

Consider limit: $g^2 \to 0$, $m_W^2 \to 0$, $g^2/m_W^2 =$ fixed

n-loop $J = 1$ corrections $\mathcal{O}(g^2/16\pi^2)^n \to 0$

Look at Polarization Vectors of $W^{\pm}$

$J = 1$, $m \neq 0$, 3 Polarization states, $\quad k^{\mu}\epsilon_{\mu}^{i} = 0 \quad i = 1, 2, 3$

CM frame $\quad s \gg m_W^2 \quad W^{\pm}$ Highly Boosted

Boost along z-axis $\quad k^{\mu} = (E, 0, 0, k)$

$$\epsilon_T^{\mu\,\prime} = \epsilon_T^{\mu}$$

$$\epsilon_L^{\mu\,\prime} = (k/m_W, 0, 0, E/m_W) \simeq k^{\mu}/m_W + \mathcal{O}(m_W/E)$$

Longitudinal components of $W^{\pm}$ are strongly coupled $s \gg m_W^2$
(Culprit)

$W_L^+ W_L^- \to W_L^+ W_L^-$ strongly coupled in $J = 0$, $I_3 = 0$ channel

Intermediate state has quantum numbers of the vacuum

Address Gauge Invariance

$$\frac{1}{2} m_a^2 A_\mu^a A^{a\mu}$$

Add auxiliary fields + Completion to restore gauge invariance

$$\Sigma = v \; \exp(i\, G/V) \qquad G = G^a \tilde{T}^a \quad a = 1,2,3$$

Under $U = \exp(i\,\alpha^a T^a) \in SU(2)_L \times U(1)_Y$

$$\Sigma \to U_L \, \Sigma \, U_Y$$

$$A_\mu \to A_\mu + (\partial_\mu U) U^{-1} + U A_\mu U^{-1}$$

Covariant derivative $D_\mu = \partial_\mu + i A_\mu \qquad A_\mu = A_\mu^a T^a$

$$\mathrm{Tr}\,(D_\mu \Sigma)^\dagger \, D^\mu \Sigma = \frac{v^2}{2} A_\mu^a A^{a\mu} + \cdots$$

Return to $W^+W^- \to W^+W^-$

Formalism now Gauge Invariant

G^a transform inhomogeneously $G^a \to G^a + v\,\tilde{\alpha}^a + \cdots$

Unitarity gauge $G^a = 0$ Calculation reduces to previous

So gauge invariance isn't the problem

Consider limit: $g^2 \to 0$, $m_W^2 \to 0$, $g^2/m_W^2 =$ fixed

$J = 1$ fields have infinite kinetic inertia, don't propagate

$$-\frac{1}{4g^2} F_{\mu\nu}^a F^{a\mu\nu}$$

Alternately, with canonical normalization, $J = 1$ fields decouple

So left with (in general gauge)

$$\mathrm{Tr}\, (\partial_\mu \Sigma)^\dagger \, \partial^\mu \Sigma + \cdots$$

Non-linear σ-Model describing Low Energy interactions of Goldstone bosons of spontaneously broken global symmetry

$\Sigma =$ Order Parameter $\quad G \to H$

G^a Goldstone bosons parameterize G/H Vacuum Manifold

Momentum Expansion ...

Consider $G^+G^- \to G^+G^-$ Scattering

$\mathcal{A}(G^+G^- \to G^+G^-)$ tree-level $\ell = 0$ Leading order in momentum expansion

$$a_0 = -\frac{s}{32\pi v^2}$$

Exactly same result as calculation with $J = 1$ fields in limit
$m_W^2 \to 0$, $s =$ fixed

Longitudinal Modes of Massive $J = 1$ fields $\simeq$ Goldstone Bosons

Equivalence Theorem

$$\mathcal{A}(X + n\, A_L^a) = \mathcal{A}(X + n\, G^a)(1 + \mathcal{O}(m_A^2/p_{AX}^2)\,)$$

Terminology: Massless $J = 1$ gauge bosons "Eat"
Goldstone bosons to become Massive $J = 1$ fields

$$2_T + 1_L = 3 \;\; \text{Polarization States}$$

Gauged Non-linear σ-Model

$$\mathrm{Tr}\,(D_\mu\Sigma)^\dagger\,D^\mu\Sigma + \cdots$$

Growth of $\mathcal{A}(W^+W^- \to W^+W^-)$ tree-level $\ell = 0$ due to leading term in Non-linear σ-Model.

(Only) Theory of gauged motions on G/H. Effective theory with leading term only, necessarily breaks down at some energy scale (to preserve Unitarity). Details depend on UV completion.

Higgs Boson

Linear σ-model

$$\Sigma = (v + h)\ \exp(i\, G/V) \qquad G = G^a \tilde{T}^a \quad a = 1, 2, 3$$

Allow magnitude of Order parameter to fluctuate

Four fields $\quad h, G^+, G^0, G^-$

Linear σ-model

$$\mathrm{Tr}\,(\partial_\mu \Sigma)^\dagger\, \partial^\mu \Sigma + \cdots + V(\Sigma^\dagger \Sigma)$$

Broken phase V minimized for $v \neq 0$

Unbroken phase V minimized for $v = 0$

Gauged Linear σ-Model

$$\mathrm{Tr}\,(D_\mu\Sigma)^\dagger\, D^\mu\Sigma + \cdots + V(\Sigma^\dagger\Sigma)$$

$\mathcal{A}(W^+W^- \to W^+W^-)$ tree-level $\ell = 0$ Leading order in momentum expansion, limit $m_W^2 \to 0$, $\;s, m_h^2 =$ fixed

$$a_0 = -\frac{s}{32\pi v^2} + \frac{s^2}{32\pi v^2(s - m_h^2)}$$

s-channel Higgs exchange cancels leading non-linear σ-model divergence

Higgs boson is an Answer to what can happen in $W^+W^- \to W^+W^-$ scattering in $J = 0$, $I = 0$ channel (quantum numbers of vacuum)

$m_h^2 \ll v^2$ limit: Gauged Linear σ-model is weakly coupled

What Else Does Introducing Higgs Boson Buy

Linear Representation $SU(2)_L \times U(1)_Y$

$$\Sigma = \begin{pmatrix} H^0 & H^+ \\ H^{+*} & H^{0*} \end{pmatrix} \qquad H^0, H^+ \in \mathbb{C}$$

"The" Higgs field

$$H \equiv \begin{pmatrix} H^+ \\ H^0 \end{pmatrix} \quad \in \ \mathbf{2}_1 \ \text{ of } \ SU(2)_L \times U(1)_Y$$

Global limit: Broken Phase $\langle H \rangle = v/\sqrt{2}$

$SU(2)_L \times U(1)_Y \to U(1)_Q$

Broken Phase rectangular coordinates $\quad G^+ \in \mathbb{C},\ v, h, G^0 \in \mathbb{R}$

$$H = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + h + i\, G^0) \end{pmatrix}$$

Gauged Linear σ-model Can be Truncated at Renormalizable Level (Relevant and Marginal Interactions Only)

$$(D_\mu H)^\dagger D^\mu H + m^2 H^\dagger H + \lambda (H^\dagger H)^2$$

Only Two Higgs Sector Parameters: $m^2, \lambda \leftrightarrow v, m_h$

Now both measured!

The (Renormalizable) Standard Model (with large UV scale for irrelevant interactions) is Predicitive

For $m_h \simeq 125$ GeV the self coupling λ is small, $\lambda \simeq 2g^2$

Mixing of Massive $J = 1$ States

$$(D_\mu H)^\dagger D^\mu H = \frac{1}{2} m_{ab}^2 \, A_\mu^a \, A^{b\mu} + \cdots$$

$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ Z_\mu \end{pmatrix}$$

$\sin\theta_W = f(g, g')$ Depends on H $SU(2)_L \times U(1)_Y$ representation

For $H \in \mathbf{2}_1$ of $SU(2)_L \times U(1)_Y$

$$\sin^2\theta_W = \frac{g'}{\sqrt{g^2 + {g'}^2}} \simeq 0.235 \qquad \cos^2\theta_W = \frac{m_W^2}{m_Z^2}$$

For $T \in \mathbf{3}_0$ of $SU(2)_L \times U(1)_Y$

$$\langle T \rangle / \langle H \rangle < \text{few} \times 10^{-2}$$

Standard Model Fermions

Two component Weyl $\psi^\alpha \in (\frac{1}{2}, 0)$ of $SU(2) \times SU(2) \cong SO(4)$
$\alpha = 1, 2$ (See Dreiner, Haber, Martin Review)

	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Q	$\mathbf{3}$	$\mathbf{2}$	1/3
$\bar{u}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	−4/3
$\bar{d}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	2/3
L	$\bar{\mathbf{1}}$	$\mathbf{2}$	−1
$\bar{e}$	$\bar{\mathbf{1}}$	$\mathbf{1}$	2

Chiral Representations: No Gauge + Lorentz Invariant $\psi\psi$
SM Fermions are Massless (without the Higgs field)

Fermion-Higgs Yukawa Couplings

Higgs Field Representation Allows the marginal Yukawa couplings

$$\lambda^u_{ij}\, Q_i H \bar{u}_j \,+\, \lambda^d_{ij}\, Q_i H^c \bar{d}_j \,+\, \lambda^\ell_{ij}\, L_i H^c \bar{\ell}_j \,+\, \text{h.c.} \qquad i,j = 1,2,3$$

where $H^c = i\sigma^2 H^* \in \mathbf{2}_{-1}$ of $SU(2)_L \times U(1)_Y$

$$H = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+h) \end{pmatrix} \qquad Q = \begin{pmatrix} u \\ d \end{pmatrix} \qquad L = \begin{pmatrix} \nu \\ \ell \end{pmatrix}$$

$$\frac{1}{\sqrt{2}}(v+h)\left[\,\lambda^u_{ij}\, Q_i H \bar{u}_j \,+\, \lambda^d_{ij}\, Q_i H^c \bar{d}_j \,+\, \lambda^\ell_{ij}\, L_i H^c \bar{\ell}_j \,+\, \text{h.c.}\right]$$

In mass basis, $h\psi\psi$ couplings are diagonal $\propto\ m_{ij} = \lambda_{ij} v/\sqrt{2}$

No Tree-Level FCNC with Single Higgs field

$\lambda_e \sim 10^{-5}$, . . . , $\lambda_t \sim 1$

Fermion-Higgs Yukawa Couplings - More than One Higgs Field

$$\lambda^u_{ijA}\, Q_i H_A \bar{u}_j \;+\; \lambda^d_{ijA}\, Q_i H^c_A \bar{d}_j \;+\; \lambda^\ell_{ijA}\, L_i H^c_A \bar{\ell}_j \;+\; \text{h.c.} \quad A = 1, \ldots, n_H$$

$$\frac{1}{\sqrt{2}}(v_A + h_A)\left[\,\lambda^u_{ijA}\, Q_i \bar{u}_j \;+\; \lambda^d_{ijA}\, Q_i \bar{d}_j \;+\; \lambda^\ell_{ijA}\, L_i \bar{\ell}_j \;+\; \text{h.c.}\right]$$

Higgs mass eigenstates $h_X = O_{XA} h_A$ In general Tree-Level FCNC

Glashow-Weinberg Condition:

Given type of fermion couples to a given Higgs field

$$\frac{1}{\sqrt{2}}(v_u + h_u)\lambda^u_{ij}\, Q_i \bar{u}_j + \frac{1}{\sqrt{2}}(v_d + h_d)\lambda^d_{ij}\, Q_i \bar{d}_j + \frac{1}{\sqrt{2}}(v_\ell + h_\ell)\lambda^\ell_{ij}\, L_i \bar{\ell}_j + \text{h.c.}$$

Higgs mass eigenstates $h_X = O_{XA} h_A$ No Tree-Level FCNC

Neutrino-Higgs Couplings

Irrelevant Dimension 5 coupling

$$\frac{\lambda^{\nu}_{ij}}{M} L_i H L_j H + \text{h.c.}$$

$$m^{\nu}_{ij} = \lambda^{\nu}_{ij}\, v^2/M$$

If $m_\nu^2 > \Delta m_\nu^2 \qquad M/\lambda^\nu < 10^{14-15}$ Gev

Higgs Boson Couplings - General Considerations

Low Energy (Soft) Higgs Theorem (Applies On/Off Shell Higgs):

$$\lim_{p^2_{Xh}\to 0} \mathcal{A}(X+h) = h \frac{\partial}{\partial v} \mathcal{A}(X)$$

h is just an excitation of Higgs condensate.
Limit $p^2 \to 0$: long wavelength, low frequency, local change in v

$$\frac{\partial}{\partial v} = \sum_i \frac{\partial m_i}{\partial v} \frac{\partial}{\partial m_i} \qquad i = \text{All Particles}$$

For Particles that gain All mass from EWSB
(SM Massive $J = 1/2,\ J = 1$)

$$\frac{\partial m_i}{\partial v} = \frac{m_i}{v}$$

Higgs - SM Fermion Couplings :

$$-m\,\psi\bar{\psi} \quad \rightarrow \quad -\,\frac{h}{v}\,m\,\psi\bar{\psi}$$

Higgs - SM Massive Gauge Boson Couplings :

$$\frac{1}{2}\,m^2\,A^a_\mu A^{a\mu} \quad \rightarrow \quad \frac{h}{v}\,m^2\,A^a_\mu A^{a\mu}$$

Multiple Soft Higgs Theorem:

$$\lim_{p^2_{Xh}\to 0}\,\mathcal{A}(X + h^n) = h^n\,\frac{\partial^n}{\partial v^n}\,\mathcal{A}(X)$$

Higgs - Massless Gauge Boson Couplings:

No Coupling for All $p^2 \to 0$ since $m_A^2 = 0$

Expansion in p_A^2

$$-\frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu} \qquad \text{Quantum Theory} \quad g^2 = g^2(v)$$

Scale Dependence $g^2 = g^2(\mu)$

$$\beta_{1/g^2} \equiv \frac{\partial}{\partial \ln \mu} \frac{1}{g^2} = \frac{b}{8\pi^2} + \cdots \qquad b_{SU(N_c)} = \frac{11}{3} N_c - \frac{1}{3} N_\psi - \frac{1}{6} N_\phi$$

$$\frac{1}{g^2(\mu)} - \frac{1}{g^2(\mu')} = \frac{b}{8\pi^2} \ln(\mu/\mu')$$

Heavy threshold mass m from EWSB - Matching $\quad g^2 = g^2(m, \mu)$

$$\frac{\partial}{\partial m} \frac{1}{g^2} = \frac{\Delta b}{8\pi^2} \frac{\partial}{\partial m} \ln(m/\mu) = \frac{\Delta b}{8\pi^2} \frac{1}{m} \frac{\partial m}{\partial v} = \frac{\Delta b}{8\pi^2\, v}$$

Higgs-Gluon-Gluon Coupling from Top Quark Loop

Low Energy (Soft) Higgs Theorem good for $m_h^2 \ll m_t^2$

$\Delta b_t = -2/3$

$$\frac{1}{48\pi^2}\frac{h}{v}F^a_{\mu\nu}F^{a\mu\nu}$$

Note: independent of λ_t or m_t for $m_h^2 \ll m_t^2$
(IR divergence cutoff by m_t)

For $p_h^2 = m_h^2$ full one-loop calculation gives a form factor

$$f(m_h^2/m_t^2) = 1 + \frac{11}{1155}\frac{m_h^2}{m_t^2} + \cdots$$