Optimization of Telescope planes spacing

to achieve best sensor spatial resolution

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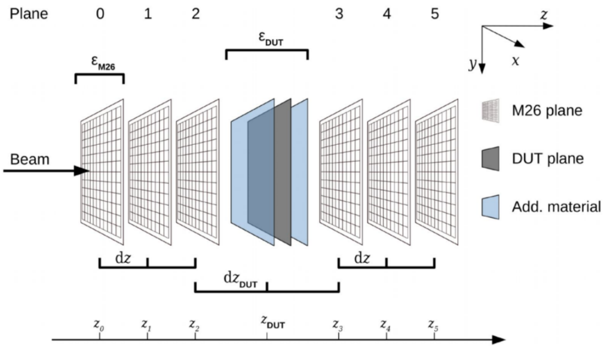
Introduction

Beam telescopes are crucial tools for probing the spatial resolution of new tracking devices by providing a precision reference track trajectory at testbeams. The optimization of telescope spacing is a key feature to achieve the best possible sensor resolution. It depends on several parameters:

- Energy of the incoming particles,
- Type of particles: mass (negligible effect at high energies) and charge
- Thickness of DUT
- Spatial resolution of telescope planes
- Distance to nearest telescope planes

EUDET telescope



Telescope:

- 2 arms
- 6 planes
- MIMOSA-26 (CMOS sensors)
- 3.5 μm resolution
- $0.1\% X_0$ ($X_0 =$ 9.36 cm for Si)

Device under Test (DUT)

Testbeam Conditions

Find optimal dz of telescope planes for several testbeam conditions:

1. CERN: 200 GeV pions;
2. SLAC: 11 GeV electrons;
3. DESY: 5 GeV electrons

And two different distances of DUT to nearest telescope planes:

- 2 cm : case of simple unirradiated DUT inserted between telescope arms
- 15 cm : case of cooling box, mandatory to cool irradiated DUT

Resolution and Multiple Scattering

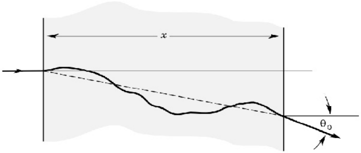
Resolution:

- $\sigma^2_{meas} = \sigma^2_{intr} + \sigma^2_{tel} + \sigma^2_{MS}$
- Lever arm dz and MS angle control the σ_{MS}
- The unbiased track resolution worsens with larger lever arms dz

Multiple scattering:

- For small MS angles:

$$\theta_0 = \frac{13.6}{\beta c p} z \sqrt{x/X_0}\,[1 + 0.038 \ln(x/X_0)]$$



- According to Bethe Bloch formula, low energy particles have higher probability to scatter in matter

Methods

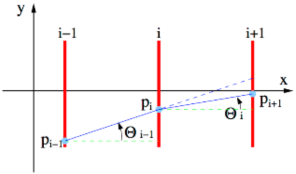
- **Analytical Method**: Minimum least squares method is used to analytically calculate the DUT track resolution. The function depends on the beam type and energy, the detector properties and the distance between the detector planes.

- **Numerical Simulation Method**: Toy MC used to simulate tracks by incoming particles. Then a minimization procedure is performed to fit the tracks and find the telescope configuration which gives best DUT spatial resolution.

Analytical Method

$$\Delta\chi_i^2 = \left(\frac{y_i - p_i}{\sigma_i}\right)^2 + \left(\frac{\theta_i - \theta_{i-1}}{\Delta\theta_i}\right)^2$$

$$\Delta\theta = \frac{13.6 \text{ MeV}}{\beta c p} \cdot z\sqrt{\frac{dx}{X_0}}\left[1 + 0.038 \cdot \ln\left(\frac{dx}{X_0}\right)\right]$$



$$\Theta_i = \frac{p_{i+1} - p_i}{x_{i+1} - x_i}$$

- i = 1, 2, 3, DUT, 4, 5, 6
- X_0: radiation length
- y_i: measured position
- p_i: predicted
- x_i: detector position along x axis
- p: momentum

Assume small angle based on the thinness of detectors and perpendicular beam

Chi square minimization $\Longrightarrow$ beam path

1. Generate rank 7 matrix $A_{ij} = \Delta_{ij}\chi^2$
2. Calculate the track resolution on DUT: $\tilde{\sigma}_{\text{DUT}} = (A^{-1}_{\text{DUT,DUT}})^{1/2}$
3. Plot $\tilde{\sigma}_{\text{DUT}}$ as a function of dz in each case to find the optimal dz where $\tilde{\sigma}_{\text{DUT}}$ is minimized.

Analytical Results

The following result is calculated using Mathematica:

$$\tilde{\sigma}_{\text{DUT}} = \left[\frac{f^6 + f^4(8u^2 + 2uv + 3v^2) + f^2u^2(3u^2 + 6uv + 17v^2) + 2u^4v^2}{f^2(6f^4v^2 + 44f^2u^2v^2 + 12u^4v^2)} \right]^{1/2}$$

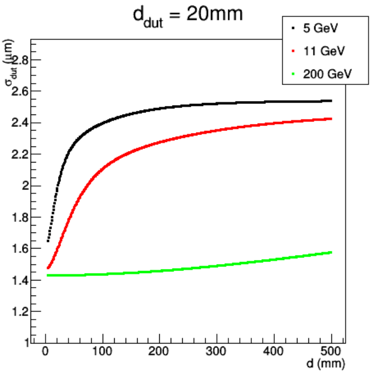
* f$=1/\sigma_{\text{Mimosa}}$
* $u = 1/(dz_{\text{Mimosa}} \times \Delta\theta_{\text{Mimosa}})$
* $v = 1/(dz_{\text{DUT}} \times \Delta\theta_{\text{DUT}})$

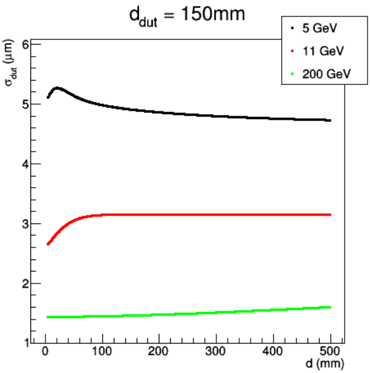
Each $\Delta\theta$ is calculated using the Multiple Scattering formula. σ_{Mimosa} is the resolution of the MIMOSA-26 detectors used in the system. ($\sigma_{\text{Mimosa}} = 3.5\,\mu$m)

Analytical Results Plots

Different trends for $d_{\text{DUT}} = 20\,\text{mm}$ & $d_{\text{DUT}} = 150\,\text{mm}$

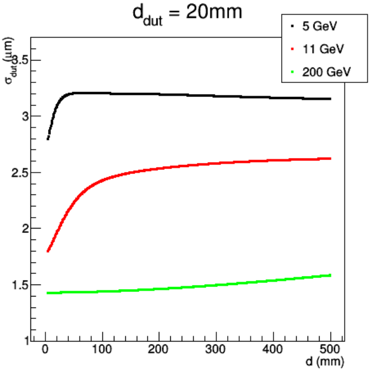
Assume same DUT as telescope plane: $dx_{\text{DUT}}/X_0 = 0.001$

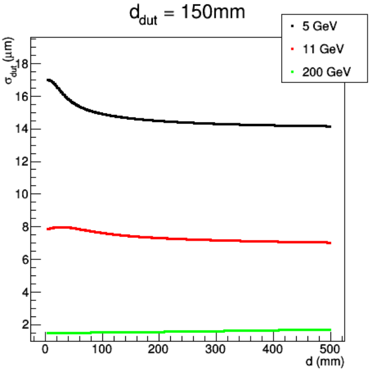




Analytical Results Plots II

Assume $dx_{\mathrm{DUT}}/X_0 = 0.01$:



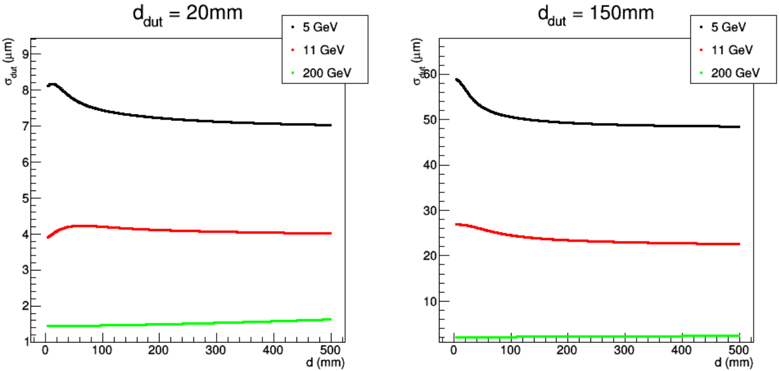


- Large spacing between DUT and nearest Tel planes has a huge impact at low energy:

$$\tilde{\sigma}_{20}^{\mathrm{min}}(5\,\mathrm{GeV}) = 3.1\,\mu\mathrm{m} \qquad \tilde{\sigma}_{150}^{\mathrm{min}}(5\,\mathrm{GeV}) = 14\,\mu\mathrm{m}$$

Analytical Results Plots III

Assume $dx_{\mathrm{DUT}}/X_0 = 0.1$:

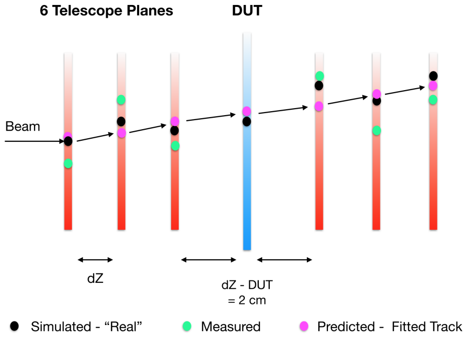


- Plots show that $200 GeV$ beam resolution graph is generally flat and slightly smaller than the resolution of the MIMOSA-26 detectors. As expected from theory, MS effects on the extremely high energy beam is minimal and track resolution is dominated by MIMOSA resolution.

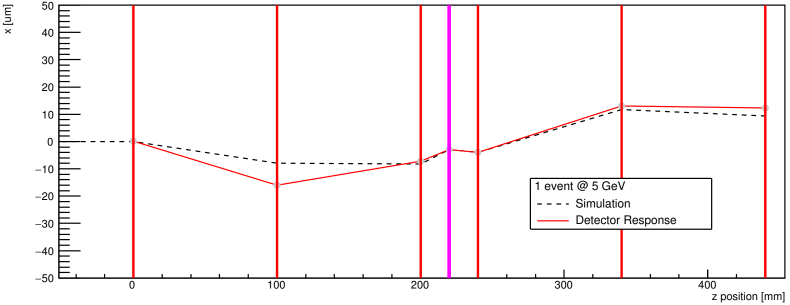
The Experimentalist Approach: Toy MC + Track Fitting

Idea:

- Toy MC to generate data ⇒ real beam position + measured position (detector response)
- 2 dimensions $\widehat{=}$ "real world simulation"
- Fitting via minimization ⇒ reconstructed particle track
- Real position (MC) - predicted position ⇒ resolution at DUT
- Code available on GitHub: `https://github.com/ebrianne/SSI_Project`

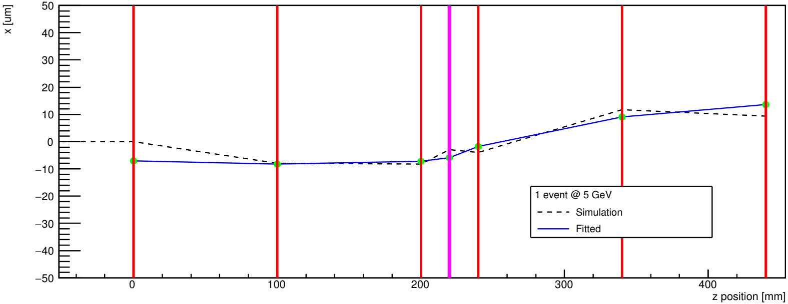


Toy MC Event Generation



- Simulation $\hat{=}$ real position beam passing layer:
  - No beam spread - x_{1-sim}, $y_{1-sim} = 0$
  - $x_{i-sim} = x_{i-1\ -sim}$ + dz * TRandom→Gaus($\Delta\theta_{i-1}$) $\hat{=}$ MS contribution
- Detector response:
  - $x_{i_measured}$ = TRandom→Gaus(x_{i-sim}, σ_i)

Track Fitting & Resolution Extraction

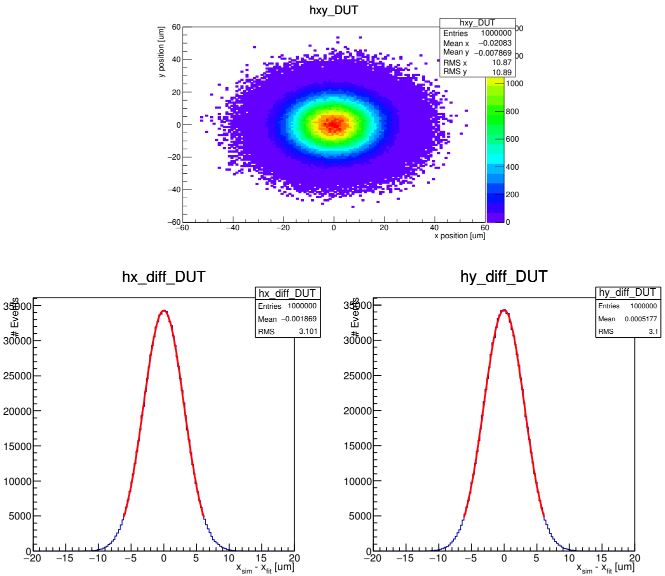


- Calculation of χ^2 used for track fitting:

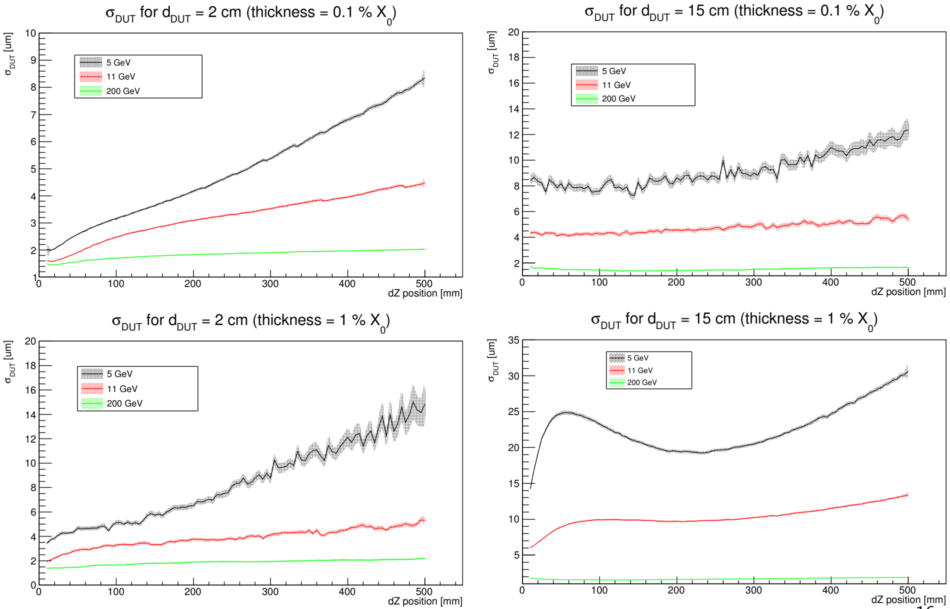
$$\Delta\chi_i^2 = \left(\frac{x_{i-measured} - x_{i-fit}}{\sigma_{i-sensor}}\right)^2\Big|_{i\neq DUT} + \left(\frac{\theta_i - \theta_{i-1}}{\Delta\theta_i}\right)^2\Big|_{i\neq 1,N} \quad (1)$$

$$\theta_i = \frac{x_{i-fit+1} - x_{i-fit}}{dz_{i\ \ to\ \ i+1}} \quad (2)$$

Simulation & Fitting Results



Simulation & Fitting Results



Summary

- From the analytical calculation, for a reasonable dut thickness of $0.1\% X_0$, three different behaviours emerge in terms of telescope plane spacing
  - for high momentum (200 GeV pions), a compact setup with planes placed as close as possible to each other leads to better dut resolution
  - for low momentum (5 and 11 GeV pions) and $d_{DUT} = 2\,\text{cm}$, a compact setup gives better DUT resolution
  - for low momentum (5 and 11 GeV pions) and $d_{DUT} = 15\,\text{cm}$, a large spacing between telescope planes gives better dut resolution

- From the simulation, whatever the incoming particles energy and the spacing between dut and nearest telescope planes are, the compact setup is favored.

Outlook

- Implement cut on χ^2-distribution to improve simulation results

- Further study of the theoretical model and Toy MC using ROOT libraries is needed to account for discrepancies between the results of the two methods.

References

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