Phase Space Tessellation to Estimate Non-Linearities in the MICE Lattice

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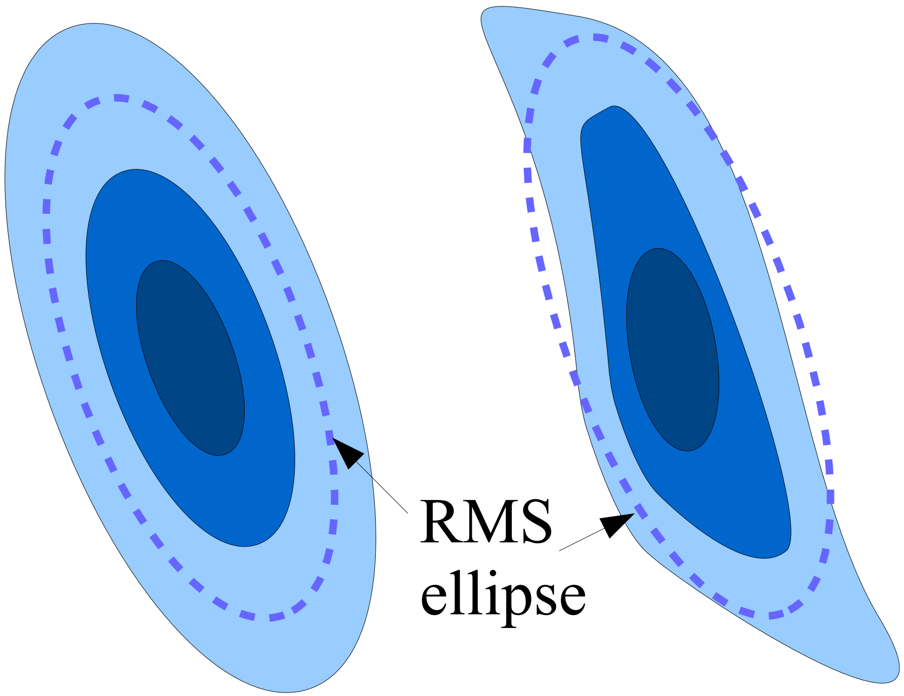
Emittance Growth picture



- Emittance growth is caused by morphing of tails of distribution
  - "Non-linearities"; "filamentation"
  - Note centre of distribution stays more or less elliptical
    - "linear approximation"; "paraxial approximation"
  - Growth due to different focussing vs energy "chromatic"
  - Growth due to different focussing vs x/px/y/py "spherical"
- Area inside the contours is conserved "Liouville's theorem"
- RMS emittance is sensitive to distribution tails

Plan

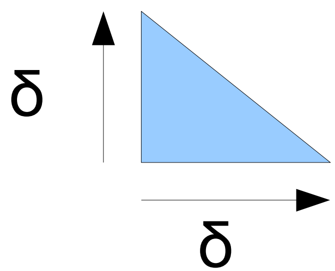
- Develop a story of why we see non-linear emittance growth
  - What makes non-linearities in the beam? (suspect particles at high radius i.e. near the coils)
  - Can we predict how strong the spherical aberrations will be?
  - Can we predict how strong the chromatic aberrations will be?
- Develop a tool set to obviate the emittance growth
  - Fractional emittance (ellipse fitting neglecting the tail)
  - Area inside contours in phase space – kernel density estimator
  - Phase space density – tesselation
  - Track extrapolation to get upstream of M1/SSD
- Need to function in 2D, 4D and 6D phase space

Tesselating phase space

- One way to get around filamentation is to calculate the actual phase space volume occupied by the beam
- Consider dividing the beam into simplices (ND triangles)
- The content (ND area) of these simplices should be fairly well conserved
  - Assuming a reasonable density of particles, it should be possible to calculate phase space volume neglecting filamentation
- Let's test the hypothesis – in MC
- Analysis code is pushed to:
  - https://github.com/chrisrogers1234/simplex_analysis

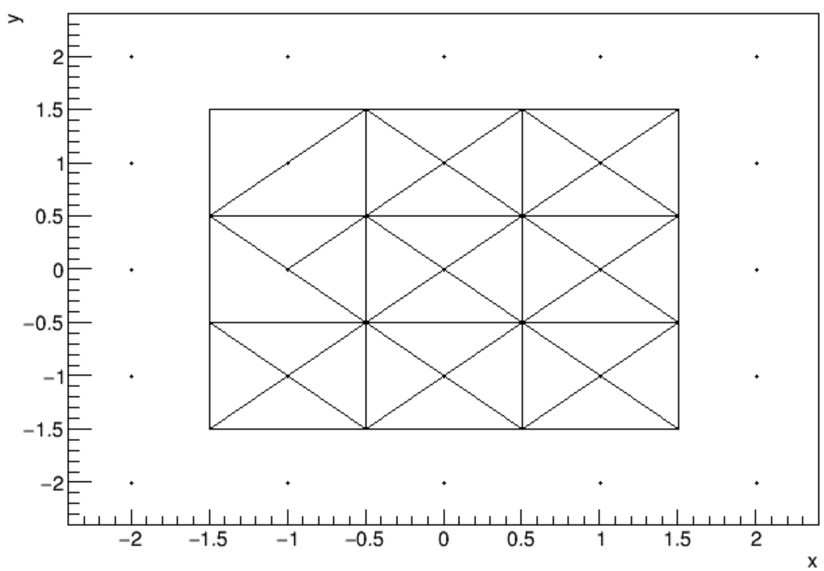
Testing the idea

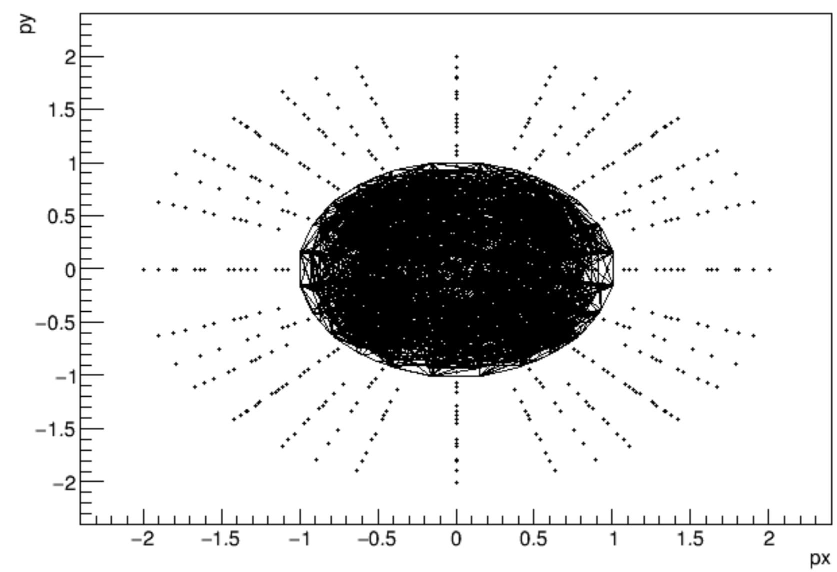
- Track a set of particles through e.g. 140 MeV/c cooling demo lattice and calculate evolution of simplex volume
  - Particles are initially on a right-angled simplex
  - dt and dE are 0 – I assume this is okay
    - I work in 4D phase space x, p_x, y, p_y
  - Phase space volume should be conserved…
- Parameter δ is size of simplex – 2D slice:



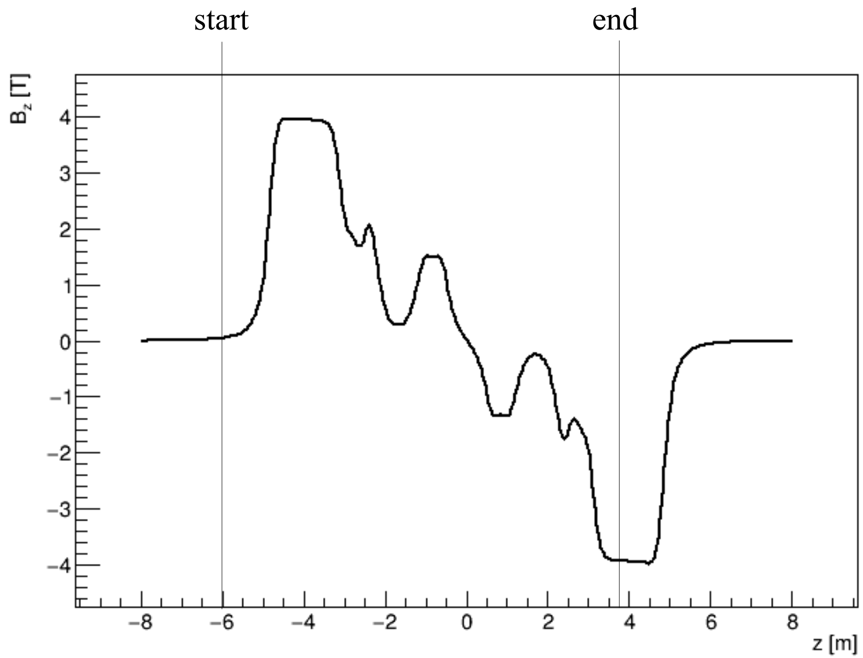
Simplex volume calculation

- Use Cayley-Menger determinant (look it up)
- Test by meshing a (4D) hypercube and calculating volume
- Test by meshing a (4D) hypersphere and calculating volume
- Compare with analytical formulae
  - Approximate hypersphere by 7x7x7x7 sided polygon (3 % error)
  - Check that hypersphere volume does not vary when moving off axis
    - Constant to 9th significant figure(!)



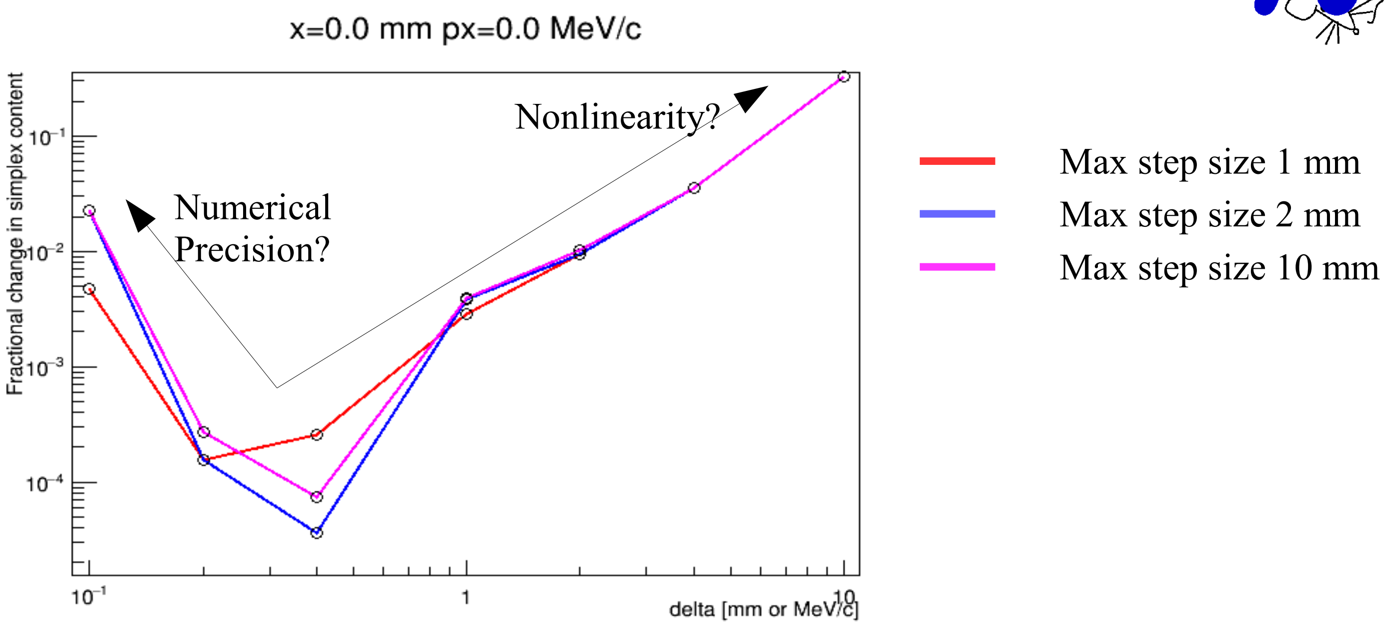


Lattice



- Lattice is Demo 140 MeV/c flip lattice
- Magnets only
  - no physical apertures or scattering
- Start outside the 4 T region
  - worried about angular momentum issues
- End at downstream TRP

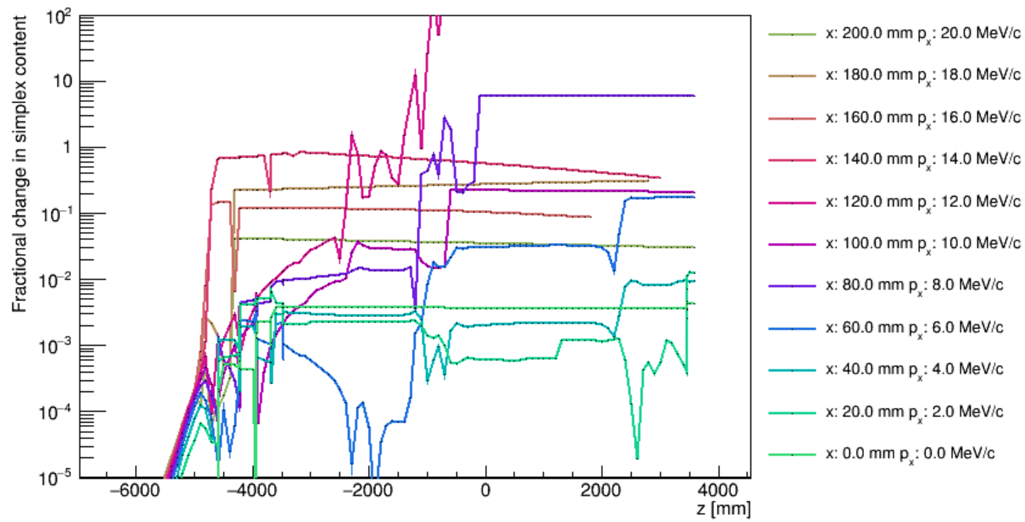
On axis

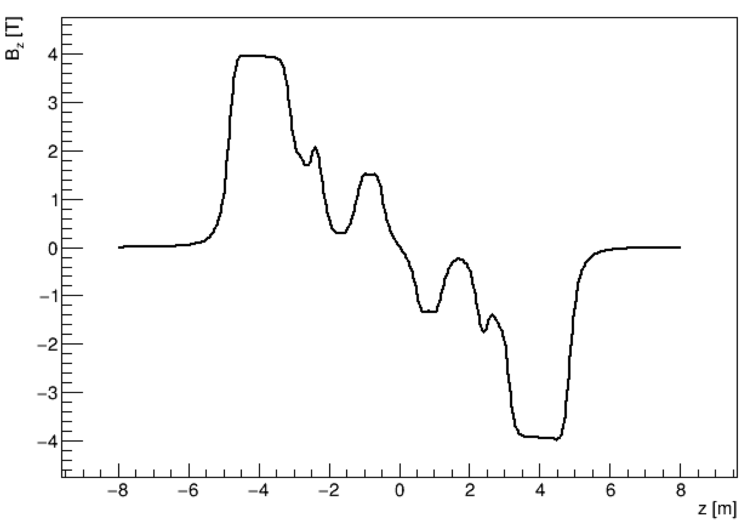


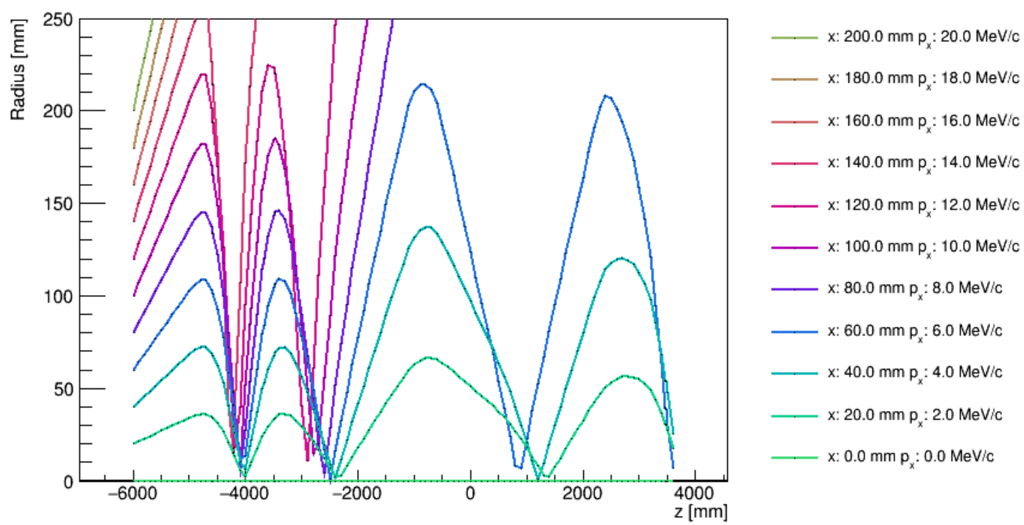
- Near to the axis
  - For delta < ~ 0.5 mm numerical precision issues maybe dominate
  - For delta > ~ 0.5 mm non-linearity (or something) dominates
- Step size is G4 “Max step size” parameter
- Delta is the initial size of the triangle edges

Moving off-axis

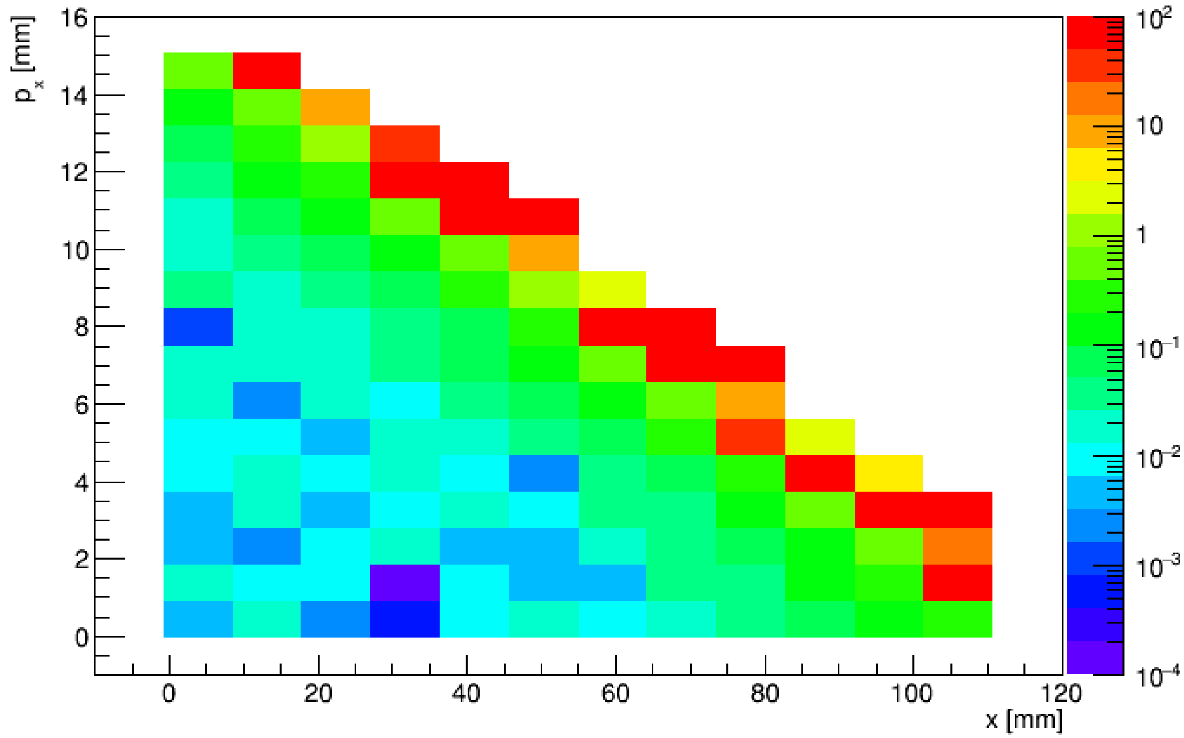
- Content growth as a function of (initial) distance from axis
  - For delta = 1 mm/1 MeV/c







Dependence on distance from axis



- Heating map
- Clear sign of dynamic aperture

Dependence on distance from axis

- Movie

What has been achieved

- Algorithm to understand phase space volume growth
- Independent of the behaviour of some (arbitrary) beam centroid
- Clearly expose the dynamic aperture issues
- Have not demonstrated phase space volume conservation
- Questions:
  - Can we access this experimentally?
    - Measurement error
    - Beam selection
  - Can we excite Dynamic Aperture and measure it?