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Wedge experiment at MICE

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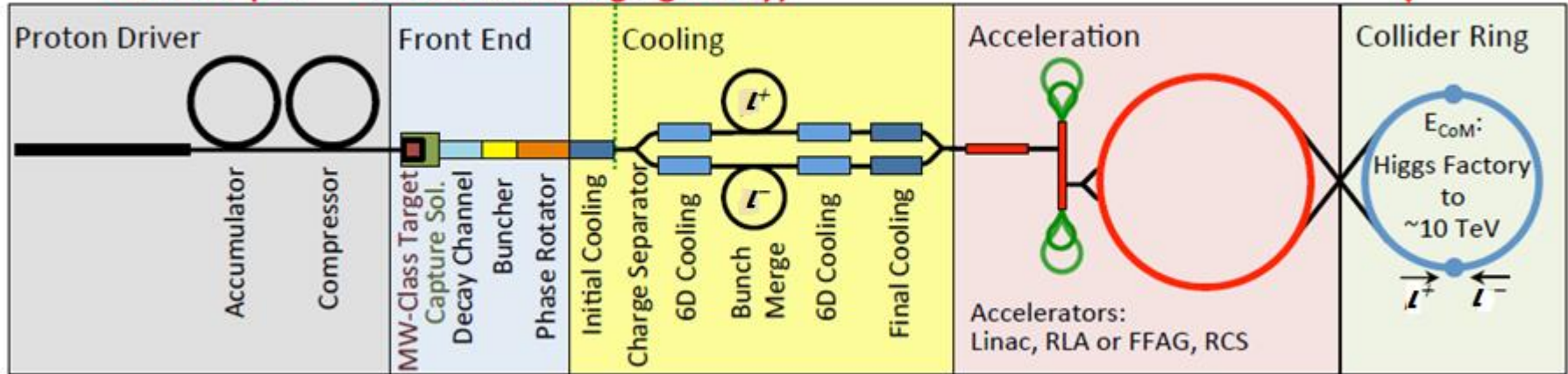
Outline

- Motivation-Final Cooling for a High-Energy Collider
 - small transverse emittances, larger longitudinal emittances
- Final Cooling scenario with wedge exchange
 - w /D. Summers & T. Hart
 - emittance exchange
- MICE – scale model experiment
 - ~10X larger beam, absorber, emittance, ...
 - Similar dynamics ~ factor of 4 emittance exchange

Towards multi-TeV lepton colliders

M. Palmer, FRXC3

Collider (*Lepton* Accelerator Staging Study)



Parameter	Unit	Higgs factory	3 TeV design	6 TeV design
Beam energy	TeV	0.063	1.5	3.0
Number of IPs		1	2	2
Circumference	m	300	2767	6302
β^*	cm	2.5	1	1
Tune x/y		5.16/4.56	20.13/22.22	38.23/40.14
Compaction		0.08	-2.88E-4	-1.22E-3
Emittance (Norm.)	mm·mrad	300	25	25
Momentum spread	%	0.003	0.1	0.1
Bunch length	cm	5	1	1
H. electrons/bunch	10^{12}	2	2	2
Repetition rate	Hz	30	15	15
Average luminosity	$10^{34} \text{ cm}^{-2}\text{s}^{-1}$	0.005	4.5	7.1

D, Neuffer

Final cooling baseline

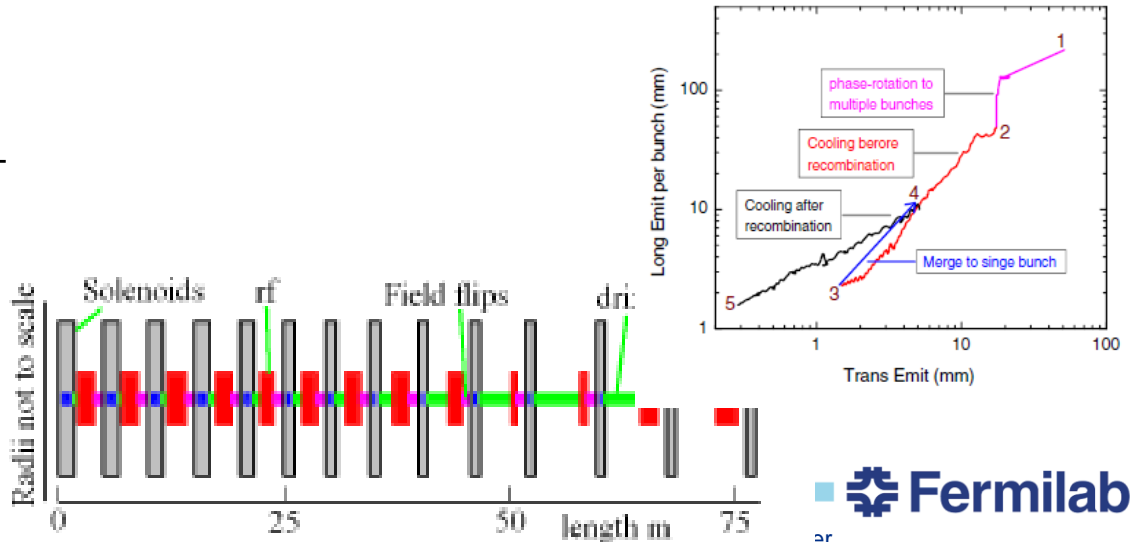
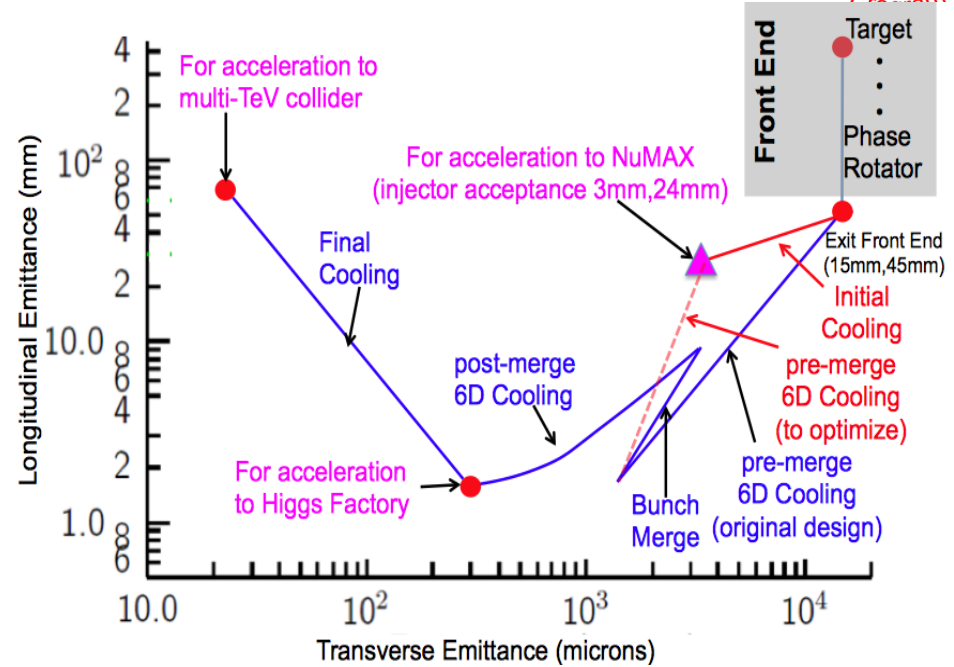
(R. Palmer et al. IPAC12)



- Final Cooling
 - $\epsilon_{t,N} : 3.0 \rightarrow 0.25 \times 10^{-4} \text{ m}$
 - $\epsilon_L : 1.0 \rightarrow 70 \text{ mm}$
 - expensive emittance exchange
- solenoids, $B \rightarrow 30\text{--}50\text{T}$
 - H_2 absorbers,
 - Low momentum

$$\epsilon_{N,eq} \cong \frac{\beta_t E_s^2}{2\beta m c^2 L_R (dE/ds)}$$

$$\beta_t \cong \frac{2P_l (GeV/c)}{0.3B}$$



Detailed simulation of final cooling

(H. Sayed et al. IPAC14, PRSTAB 18)

- **G4Beamline simulation of final cooling scenario**

- System is ~135m long

- $\epsilon_{t,N} : 3.0 \rightarrow 0.5 \cdot 10^{-4} \text{ m}$

- $\epsilon_L : 1.0 \rightarrow 75 \text{ mm}$

- $P_i : 135 \rightarrow 70 \text{ MeV/c}$

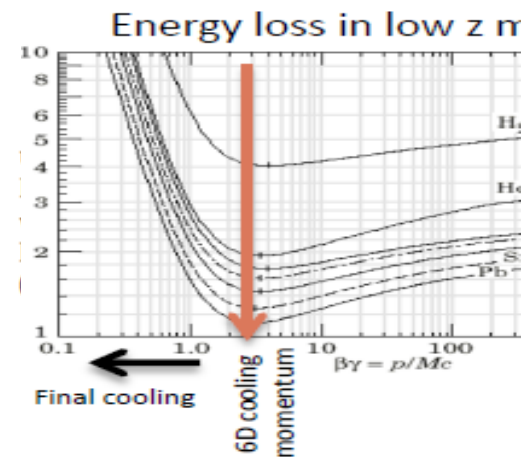
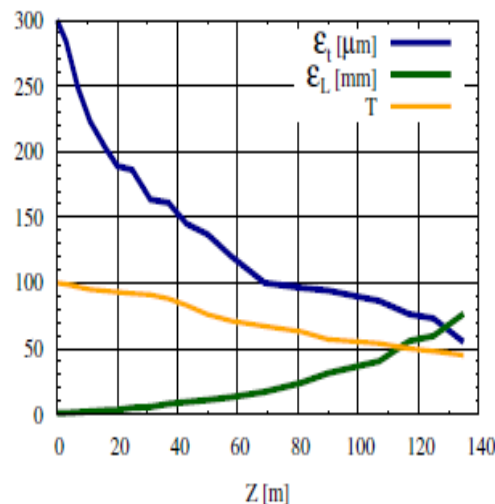
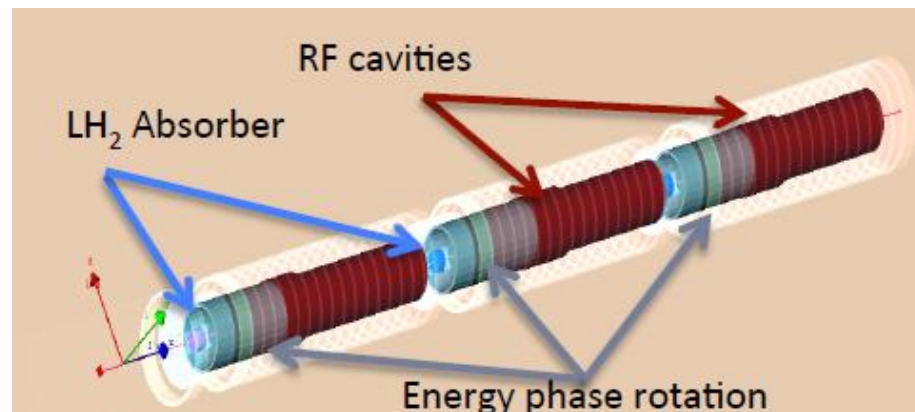
- $B : 25 \rightarrow 32 \text{ T}; 325 \rightarrow 20 \text{ MHz}$

- not quite specs

- Transmission ~ 50%

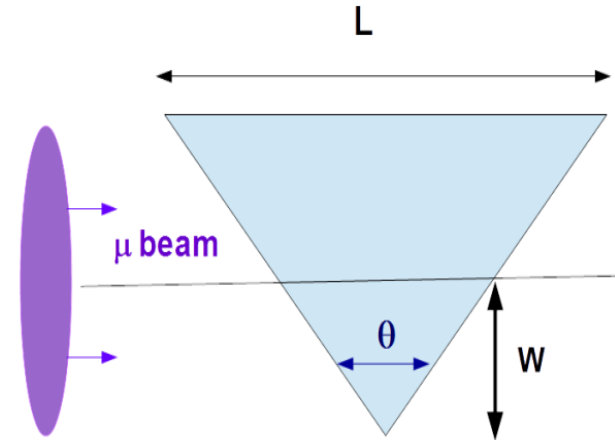
- **Predominantly $\epsilon_{t,N} / \epsilon_L$ emittance exchange**

- **Use explicit emittance exchange instead?**



Variant: “thick” wedge emittance exchange

- Use wedge to decrease ε_x
 - By increasing $\delta p/p$
 - increases ε_L , decrease ε_x
- If $\delta p/p$ introduced by wedge $\gg \delta p/p_{\text{bea}}$
 - can get large emittance exchange
- Example:
 - 100 MeV/c; $\delta p=0.5\text{MeV/c}$
 - $\varepsilon_{\perp}=10^{-4}\text{m}$, $\beta_0=1.2\text{cm}$, Be wedge
0.6cm, 140° wedge
 - obtain factor of ~ 5 exchange
 - $\varepsilon_x \rightarrow 0.2 \times 10^{-4}\text{m}$; $\delta p=2.5\text{ MeV/c}$
- Much simpler than equivalent
final cooling section



“Thick” Wedge theory (MuCool-003)

- Dispersion + wedge is product of two matrices (in $x - dp/p$)

$$\mathbf{M}_\delta = \begin{bmatrix} 1 & 0 \\ -\delta' & 1 \end{bmatrix} \quad \mathbf{M}_\eta = \begin{bmatrix} 1 & \eta_0 \\ 0 & 1 \end{bmatrix}$$

- $\delta' = dp/ds \ 2 \tan[\theta/2]/p$
- variables are $[x, \delta]$, where $\delta = dp/p$
- transport through wedge is transport of $[x, \delta]$ phase space ellipse, initially

$$g_0 x^2 + b_0 \delta^2 = \sigma_0 \delta_0$$

- becoming

$$g_1 x^2 + 2a_1 x\delta + b_1 \delta^2 = \sigma_0 \delta_0$$

Results of wedge analysis

- **new coefficients**

$$b_1 = b_0 + (\eta_0)^2 g_0$$

$$a_1 = \delta' b_0 - \eta_0 (1 - \delta' \eta_0) g_0$$

$$g_1 = \delta'^2 b_0 + (1 - \delta' \eta_0)^2 g_0$$

$$\beta_1 = m_{11}^2 \beta_0 + 2m_{11}m_{12}\alpha_0 + m_{12}^2 \gamma_0$$

$$\alpha_1 = -m_{11}m_{21}\beta_0 + (m_{11}m_{22} + m_{12}m_{21})\alpha_0 - m_{12}m_{22}\gamma_0$$

$$\gamma_1 = m_{21}^2 \beta_0 + 2m_{21}m_{22}\alpha_0 + m_{22}^2 \gamma_0$$

- **new $\delta p/p$ width ($\epsilon_{L,1} = \epsilon_{L,0} (\delta_1/\delta_0)$)**

$$\delta_1 = \sqrt{g_1 \sigma_0 \delta_0} = \delta_0 \left[(1 - \eta_0 \delta')^2 + \frac{\delta'^2 \sigma_0^2}{\delta_0^2} \right]^{1/2}$$

- **new transverse emittance ($\epsilon_{x,1} = \epsilon_{x,0} (\delta_0/\delta_1)$)**

$$\epsilon_{x,1} = \sqrt{g_1 \sigma_0 \delta_0} = \epsilon_{x,0} \left[(1 - \eta_0 \delta')^2 + \frac{\delta'^2 \sigma_0^2}{\delta_0^2} \right]^{-1/2}$$

new β_x, η

$$\eta_1 = -\frac{a_1}{g_1} = \frac{\eta_0 (1 - \eta_0 \delta') - \delta' \frac{\sigma_0^2}{\delta_0^2}}{(1 - \eta_0 \delta')^2 + \delta'^2 \frac{\sigma_0^2}{\delta_0^2}}$$

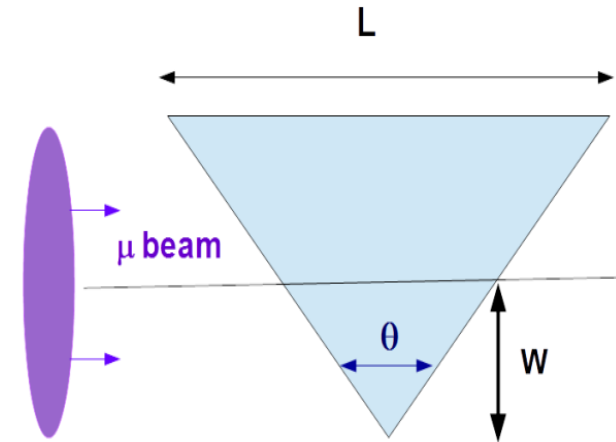
$$\beta_1 = \beta_0 \left[(1 - \eta_0 \delta')^2 + \frac{\delta'^2 \sigma_0^2}{\delta_0^2} \right]^{-1/2}$$

By choosing η_0, δ' (and δ_0, σ_0)

Can decrease ϵ_x and increase ϵ_L or vice versa

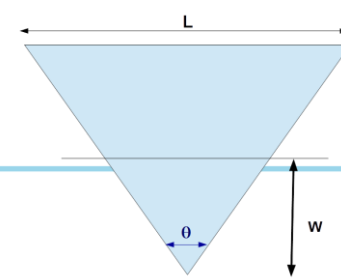
Can get large emittance exchange from single wedges

- Set reference momentum at 100 MeV/c
 - $\epsilon_x, \epsilon_y \rightarrow 10^{-4} \text{ m}$
 - $\beta_x = \beta_y = \sim 1 \text{ cm}$
 - $\sim 1 \text{ mm}$ beam
 - no initial dispersion η
 - could be improved with matched η
- Need small $\delta p/p$
 - $\delta p \sim 0.5 \text{ MeV/c}$
 - obtain by lengthening and flattening bunch ($\epsilon_L = \sim 0.001 \text{ m}$)
- Need dense low-Z wedge
 - Be ($\rho=1.86$) \rightarrow Diamond
 - ($\rho=3.6 \text{ C}$)
 - $dp/ds = 15.1 \text{ MeV/c /cm}$
 - B_4C ($\rho=2.52$) also good



- Evaluate in ICOOL
 - wedge and beam definition, match
 - emitcalc evaluates eigen emittances before/after
 - a few cm transport

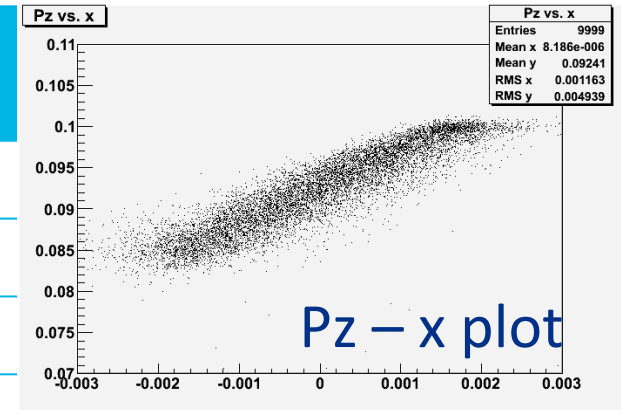
Numerical examples



- **Wedge parameters**

- **Diamond, $w=1.75\text{mm}$, $\theta = 100^\circ$ (4.17mm thick at center)**

$z(\text{cm})$	P_z	$\epsilon_x(\mu)$	ϵ_y	$\epsilon_L(\text{mm})$	σ_E MeV	6-D ϵ increase
0	100	97	95.5	1.27	0.46	1.0
0.4	96.4	33.4	96.3	4.55	1.64	1.24
0.8	92.4	22.7	96.5	8.94	3.22	1.65



- **reduces ϵ_x by factor of 4.3, ϵ_L increases by factor of 7.0**

- **first half of wedge more efficient than second half ...**

- **Second wedge ?**

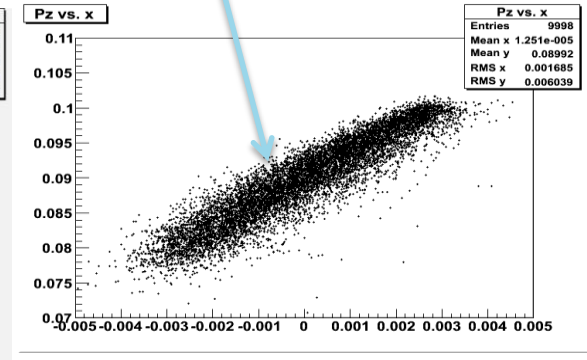
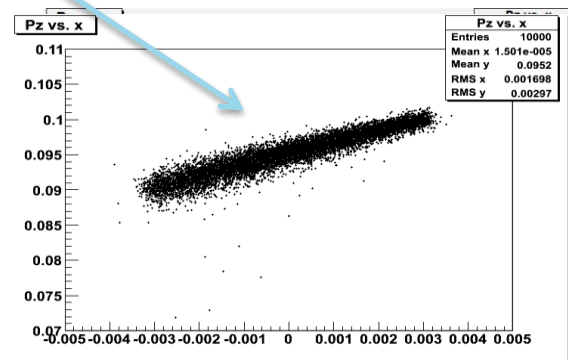
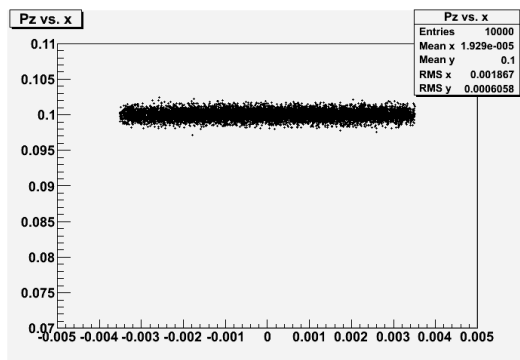
- **if matched to same optics ($P_z \rightarrow 100 \text{ MeV}/c$, $\sigma_E \rightarrow 0.46 \text{ MeV}$)**

- **$\epsilon_x : 23 \rightarrow 27\mu$; $\epsilon_y : 97 \rightarrow 23 \mu$**

Parameter variations...

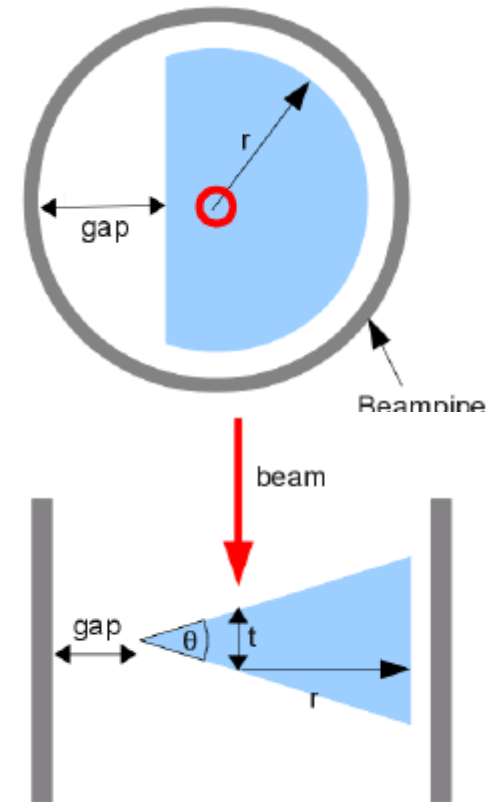
- Go to larger initial beams, larger wedge
 - $\beta_t=2.6\text{cm}$, $\epsilon_t =130\mu$
 - Diamond, $w=3.0\text{mm}$, $\theta = 85^\circ$ (5.6mm thick at center)

Z(cm)	P_z	$\epsilon_x(\mu)$	ϵ_y	ϵ_L (mm)	σ_E MeV	6-D ϵ increase
0	100	129	127	1.0	0.50	1.0
0.6	95.2	40.4	130	4.03	1.95	1.29
1.2	90.0	25.0	127	7.9	3.87	1.54



Scale model of final emittance exchange

- Mice 290 (Rogers, Snopok, Coney, Janssen) considered adding wedges within the focus coil to explore emittance exchange
 - LiH, CH₂, Be wedges, 3 angles
- For final exchange scale demo, only need CH₂, 60° wedge
- Use 200 MeV/c beam
 - standard settings, zero η



material	LiH	LiH	LiH	C ₂ H ₄	C ₂ H ₄	C ₂ H ₄	Be	Be	Be
θ [°]	30	60	90	30	60	90	30	60	90
r [mm]	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0	225.0
t [mm]	75.4	75.4	75.4	60.5	60.5	60.5	40.2	40.2	40.2
h [mm]	365.7	290.3	262.7	337.9	277.4	255.3	300.0	259.8	245.1
l [mm]	98.0	167.6	225.0	90.5	160.2	225.0	80.4	150.0	225.0
d [mm]	0	0	37.7	0	0	25.1	0	0	20.1
mass [kg]	12.16	16.27	17.7	12.4	17.3	19.0	20.5	30.6	34.2

Table 3: Parameters of the wedges described in this note. Dimensions are labelled in Figure 9.

Mice experiment wedges

- Step 4 geometry, fields

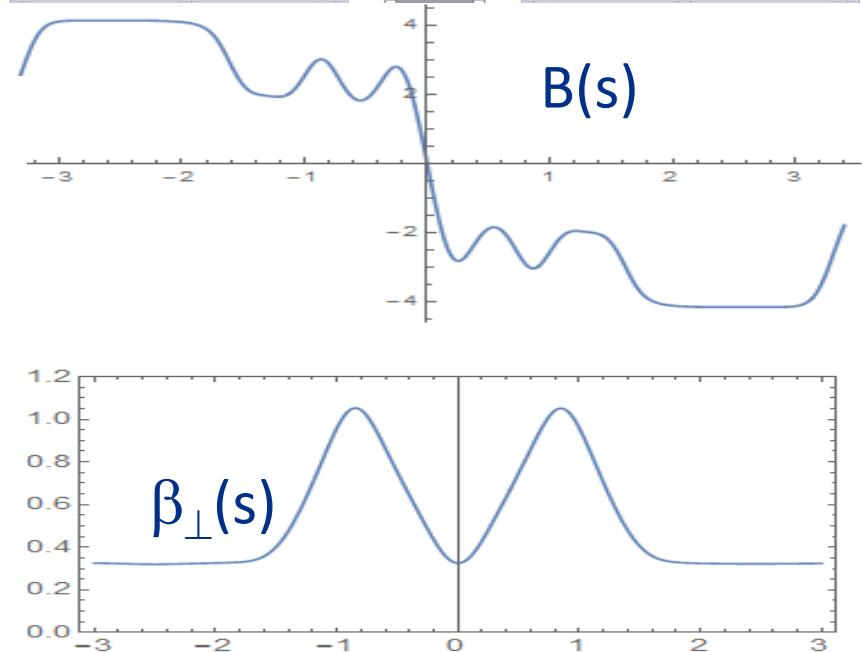
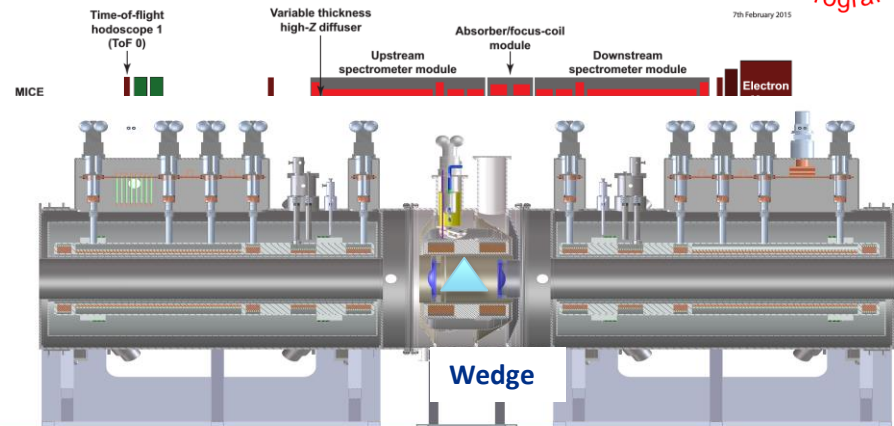
- 4.14T solenoids
- $\beta^* = \sim 0.35\text{m}$

Input Beam:

200MeV/c 0-diffuser setting is best fit:

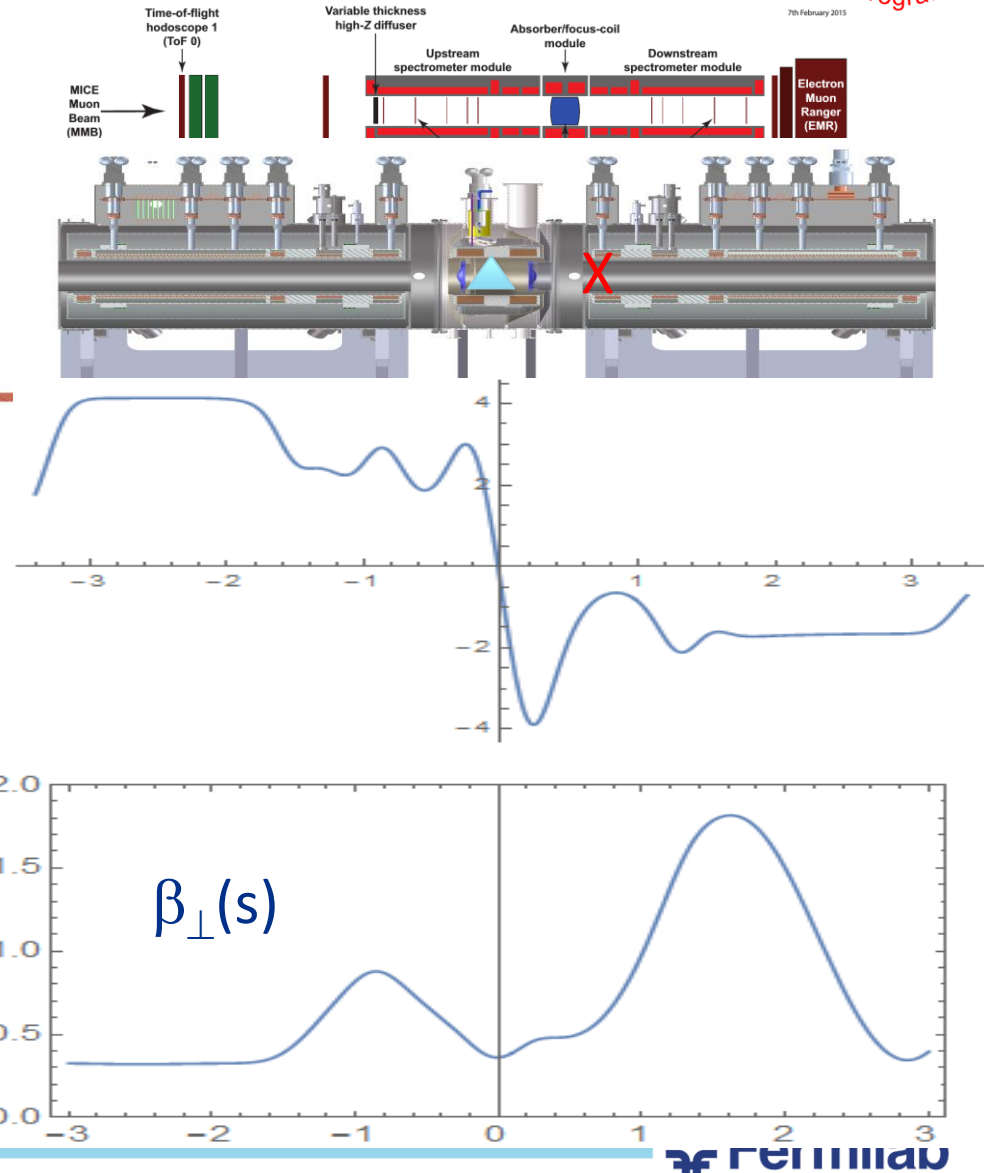
		p_z (MeV/c)		
		140	200	240
ϵ_N (π mm · rad)	3	$d = 0.0$ $P_{diff} = 151$ MeV/c $\alpha = 0.2$ $\beta = 56$ cm	$d = 0.0$ $P_{diff} = 207$ MeV/c $\alpha = 0.1$ $\beta = 36$ cm	$d = 0.0$ $P_{diff} = 245$ MeV/c $\alpha = 0.1$ $\beta = 42$ cm
	6	$d = 0.9X_0$ $P_{diff} = 156$ MeV/c $\alpha = 0.3$ $\beta = 113$ cm	$d = 1.3X_0$ $P_{diff} = 215$ MeV/c $\alpha = 0.2$ $\beta = 78$ cm	$d = 1.3X_0$ $P_{diff} = 256$ MeV/c $\alpha = 0.2$ $\beta = 80$ cm
	10	$d = 1.8X_0$ $P_{diff} = 164$ MeV/c $\alpha = 0.6$ $\beta = 198$ cm	$d = 2.8X_0$ $P_{diff} = 229$ MeV/c $\alpha = 0.4$ $\beta = 131$ cm	$d = 2.8X_0$ $P_{diff} = 267$ MeV/c $\alpha = 0.3$ $\beta = 129$ cm

$\epsilon_{N,rms} = \sim 3$ mm-R, select small energy spread for reverse emittance exchange expt.



Mice experiment wedges (step 4-)

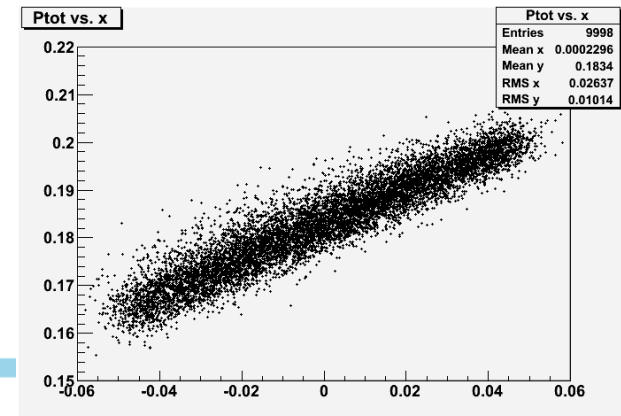
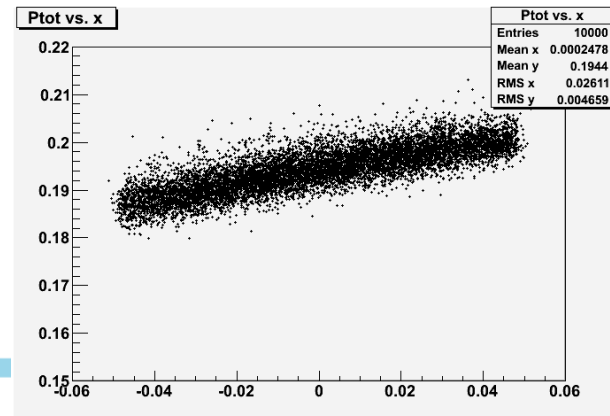
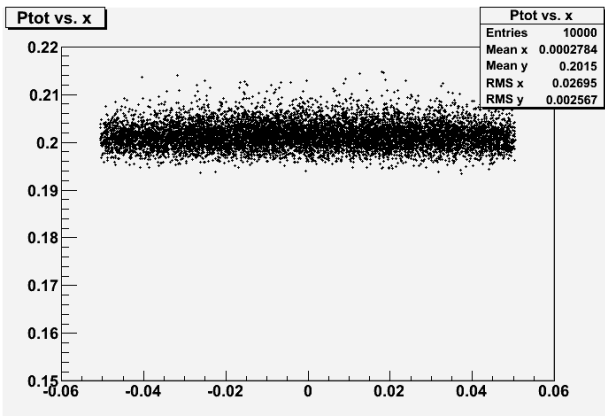
- Step 4 geometry, fields
 - 4.14T solenoids
 - $\beta^* = \sim 0.35\text{m}$
- With SSD M1 coil off
 - mismatched downstream
 - Keep β^* manageable
 - $B \rightarrow 1.7\text{T}$
- same lattice into wedge
 - rematch after wedge
 - $\beta_{\text{max}} \rightarrow 1.8\text{m}$



Parameters closer to MICE

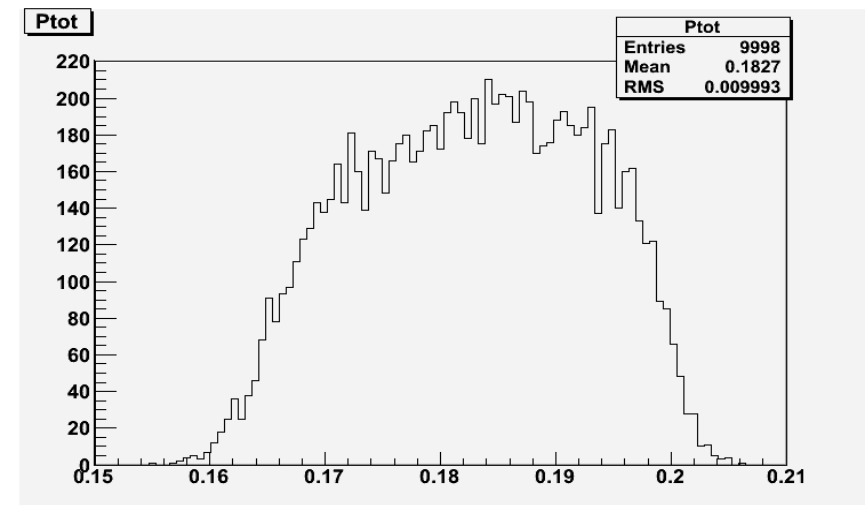
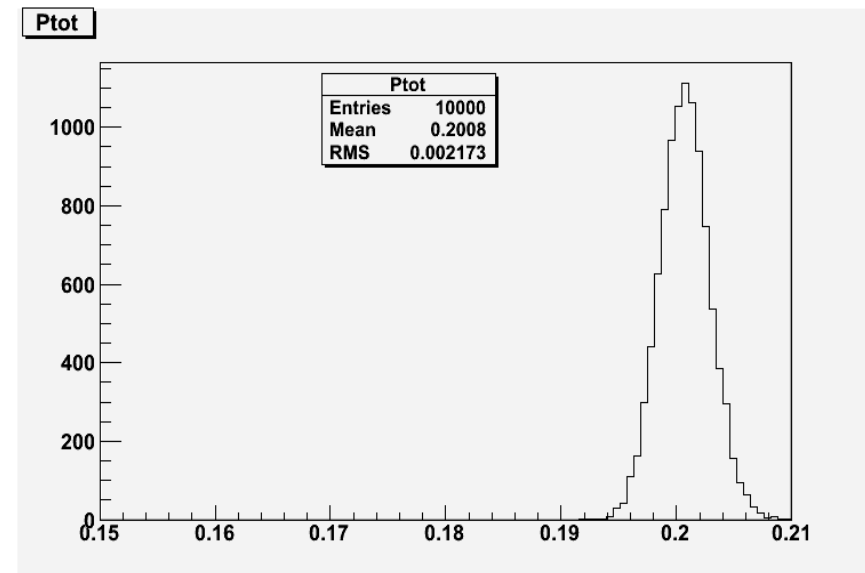
- **Wedge at \sim MICE parameters**
 - simplest case: Polyethylene wedge (C_2H_4)
 - 200 MeV/c beam $\beta_t=36$ cm, $\epsilon_t=3.04$ mm
 - C_2H_4 , $w=5$ cm, $\theta=60^\circ$ (6 cm thick at center)

Z(cm)	P_z MeV/c	ϵ_x (mm)	ϵ_y	ϵ_L (mm)	σ_E MeV	6-D ϵ increase
0	200	3.04	2.99	2.90	1.82	1.0
6	193	1.44	3.02	6.82	3.86	1.13
12	182	0.76	3.00	14.27	8.63	1.23



Proof of Principle wedge test

- Simplest verification
 - Does momentum spread increase match calculation?
- Large effect
 - σ_p : 2.0 \rightarrow 10 MeV/c
- 6-D phase space eigenemittance reconstruction needed to see ϵ_x reduction
 - $x \rightarrow x + \eta(\delta p/p)$
 - more difficult



Advantages of experiment (Step 4+)



- **Simple hardware**
 - plastic wedge (not LiH or Be)
 - Does not need rf
 - step 4 beam configuration
- **Can we do without SSD M1 ?**
 - **define initial beam in SSU**
 - match to wedge input
 - focus to $\beta^* \sim 0.3\text{—}0.4$ m
 - **measure change in δp**
 - agreement establishes **scale model** of final cooling scenario
- **6-D eigen emittance measurement would be betterwith SSD**

Disadvantages

- Beam comes out of wedge with x-y asymmetry
 - $\beta_x = \sim 0.1\text{m}$, Dispersion $\eta_x = \sim 0.5\text{m}$
 - ($\beta_y = \sim 0.4\text{m}$, $\eta_y = \sim 0$)
- MICE optics (solenoids) does not match
 - 6-D emittance measurement of beam may be difficult
 - canonical eigenemittance calculation?
- Optics matching (after wedge) is nontrivial

Summary

- **Thick Wedges could be important part of final cooling scenario**
 - Details depend on optimization and design studies
 - An important part of collider scenario
 - could be part of other particle sources
- **Experiment at MICE could demonstrate principle**
 - **Large** effect could be demonstrated
 - (unlike baseline cooling demo)
 - variation in input beam selection could also show longitudinal cooling
 - other applications ...