Gaussian Kernel Density Estimation (KDE) in MICE

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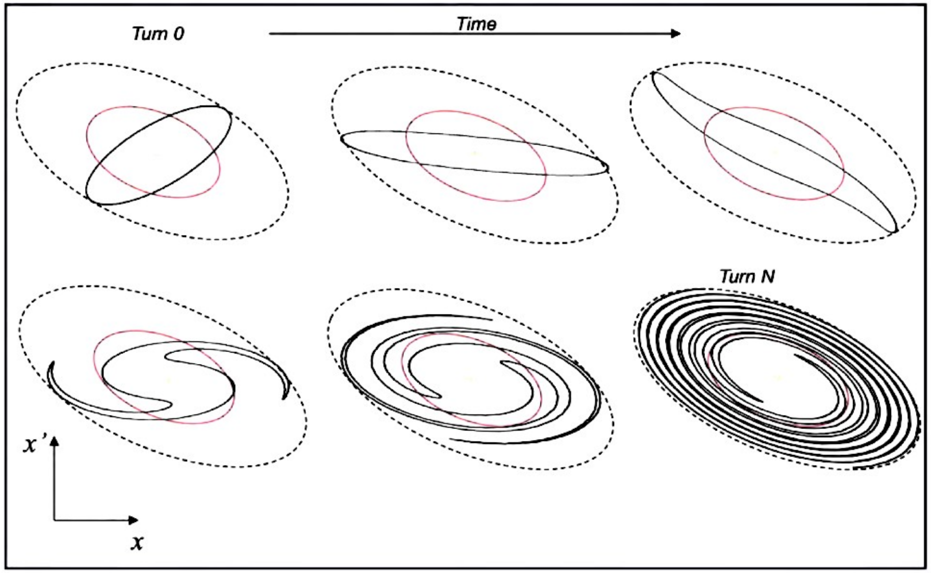
MICE

Motivation

- Solenoid beam optics prone to filamentation and other non-linear effects.

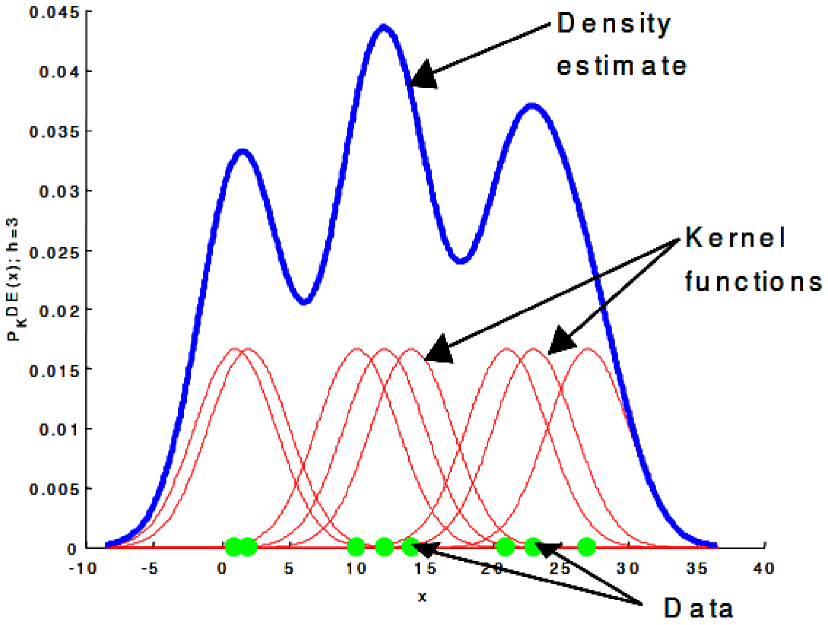


V. Kain, “Beam Transfer and Machine Protection”, CERN

- Non-linearities cause the beam's distorted shape to fill larger ellipse → “apparent” emittance growth.
- Need to study alternative measures of estimating the true phase space volume occupied by the beam as opposed to RMS emittance → Kernel Density Estimation (KDE) can be used.

Background

- KDE → estimates PDF of the particle distribution in phase space using pre-defined kernel functions.
- KDE is a non-parametric DE method, defined as below (n number of points and h smoothing parameter),



$$D = \frac{1}{nh}\sum_{i=1}^{n} K\left(\frac{x - X_i}{h}\right)$$

R. Gutierrez Osuna, "Kernel density estimation", CSCE 666 Pattern Analysis, Texas A&M University.

- MICE has ~gaussian beam → PDF estimation using guassian kernel,

$$K(\frac{x - X_i}{h}) = \frac{1}{\sqrt{(2\pi)^d}} e^{\frac{-1}{2}(\frac{x-X_i}{h})^2}$$

Approach

- 2D KDE algorithm routine:

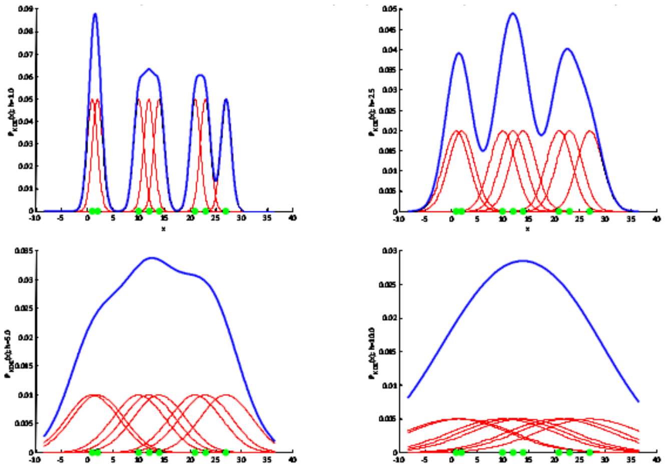
➔ Set up a grid by separately meshing (x, x') and (y, y').

➔ Reshape the grid to (# dimensions, d, # points, n) for KDE evaluation. Stats.gaussian_kde() module in scipy used.

➔ Estimate the probability density functions of reshaped (x, x') and (y, y') grid using gaussian kernels.

➔ Define bandwidth method (smoothing parameter)

→ used scott's factor, $h = n^{\frac{-1}{d+4}}$



➔ Make a contour plot where contour lines around different levels of the distribution represent the estimated density.

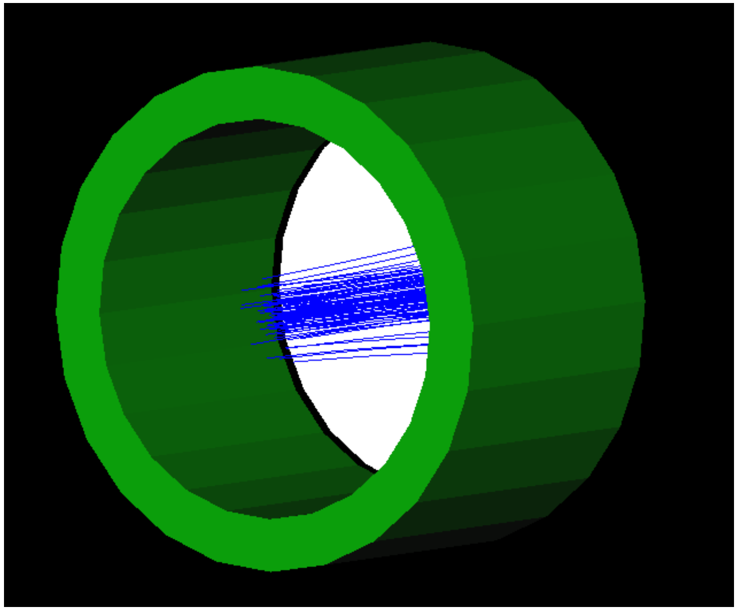
➔ Calculate the area within the individual contour lines using Green's theorem,

$$A = \frac{1}{2} \int_c x dy - y dx$$

Algorithm Validation

- Generated a g4beamline gaussian beam and passed it through a vertically defocusing quadrupole magnet. Simulation parameters below,

Simulation Parameters	Values
Number of events	100,000
Quad size (iron length X radius)	457 X 381 mm
Field gradient	1.15 T/m
Field size (field length X aperture)	396 X 301.5 mm
Beam type	Gaussian
Sigma x	30 mm
Sigma y	30 mm
Sigma px	3 MeV/c
Sigma py	3 MeV/c
Reference Momentum	200 MeV/c

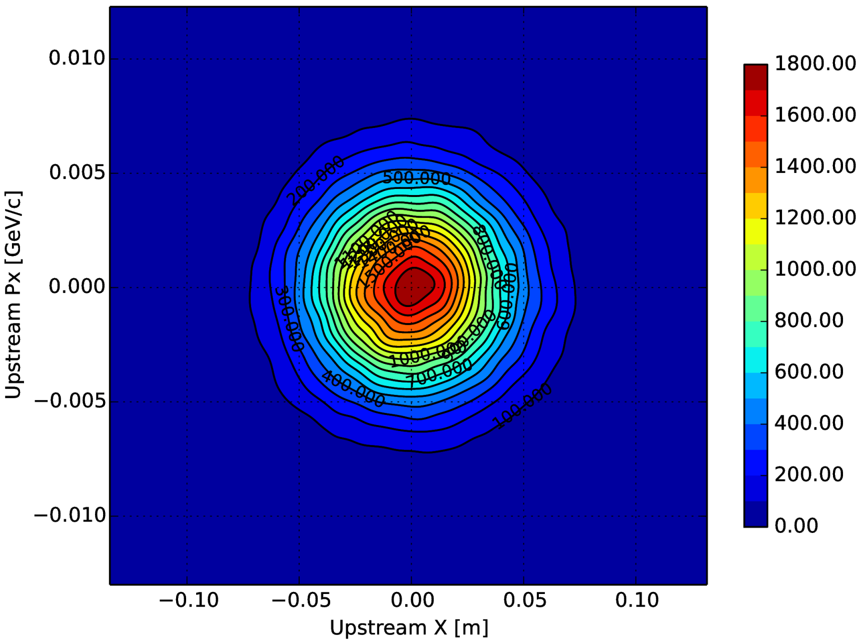


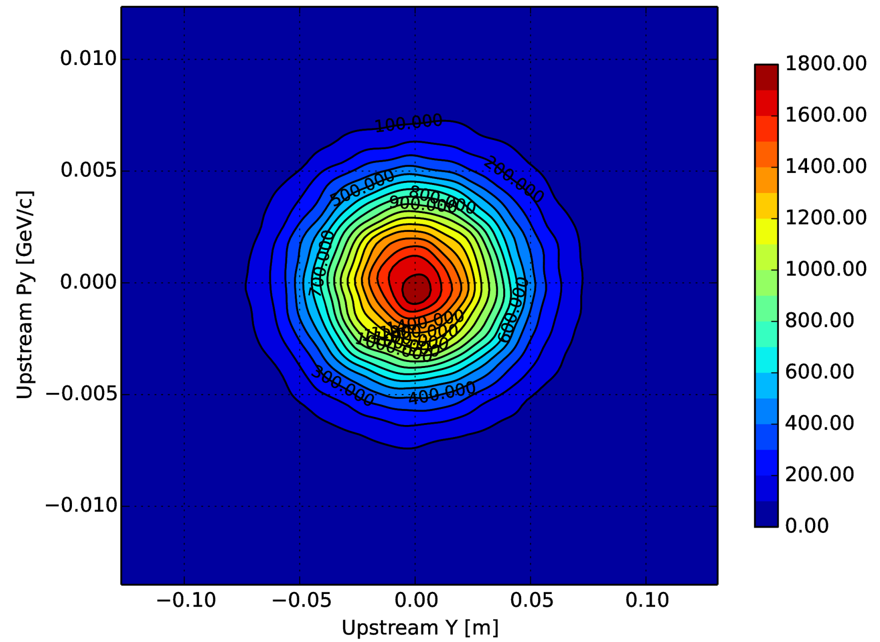
- Placed virtual detectors at the upstream and downstream boundaries of the field length.

Algorithm Validation cont.

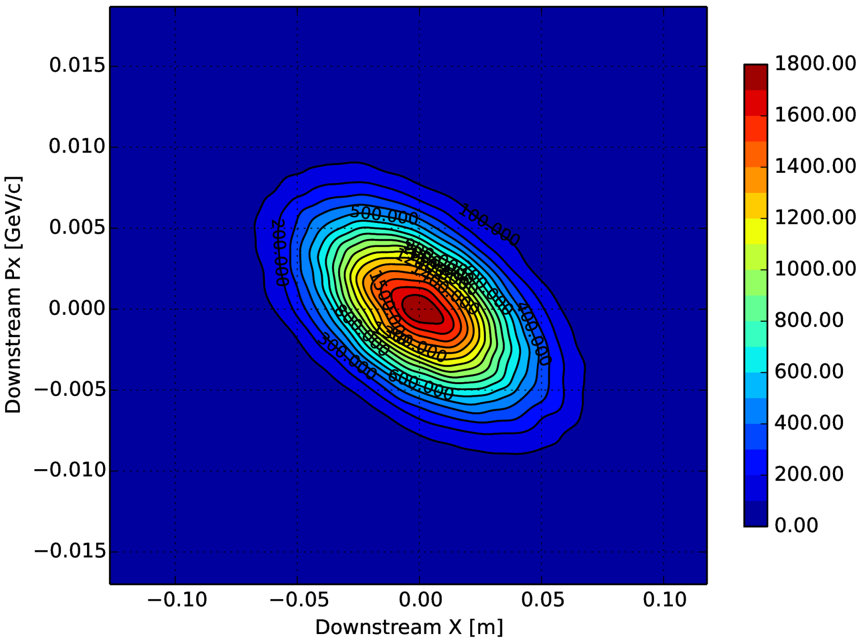
(x,x')

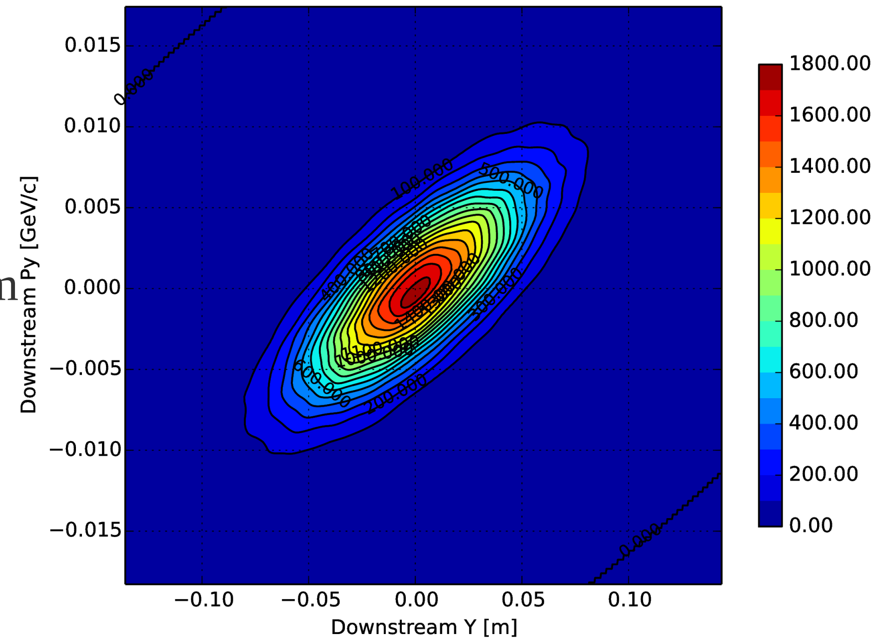
(y,y')





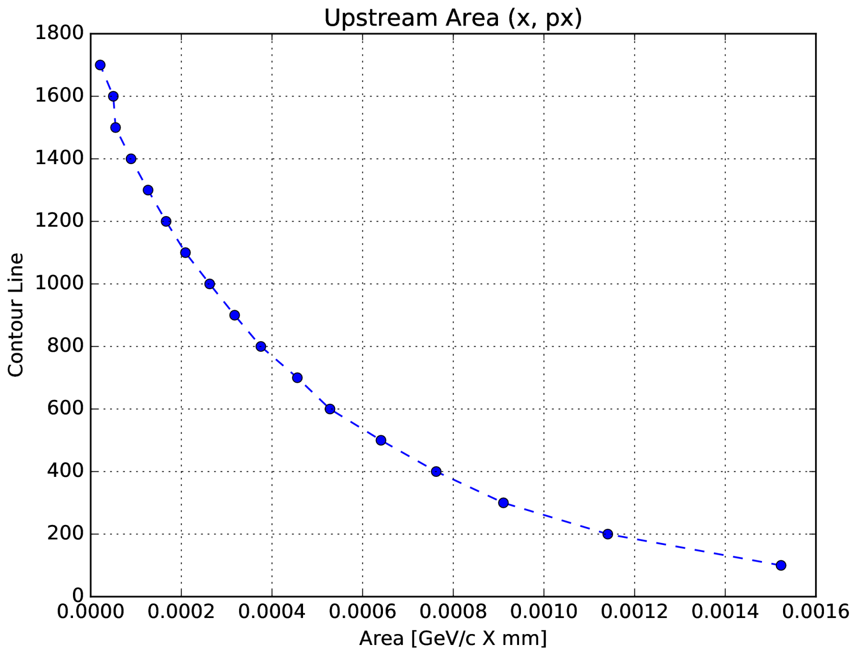
Upstream

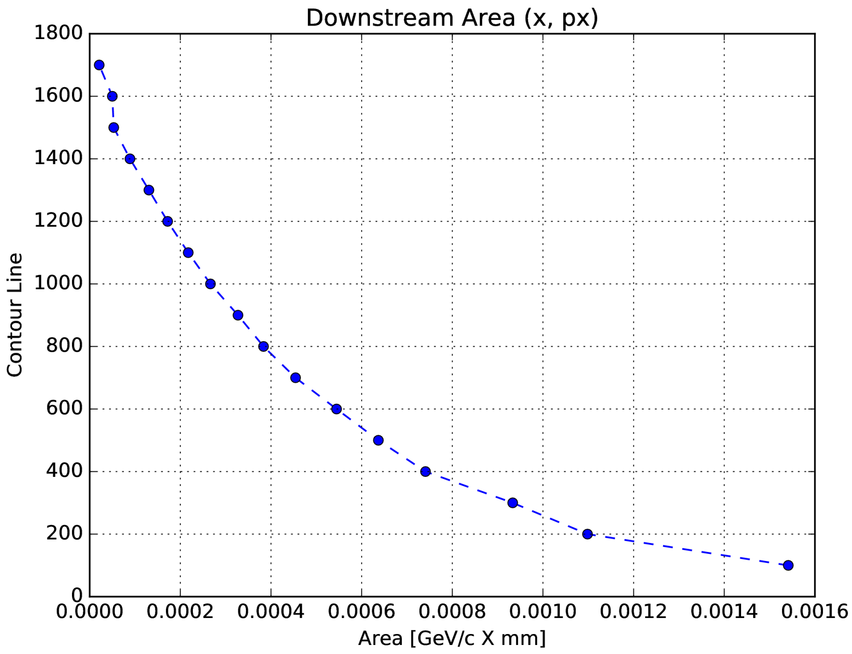


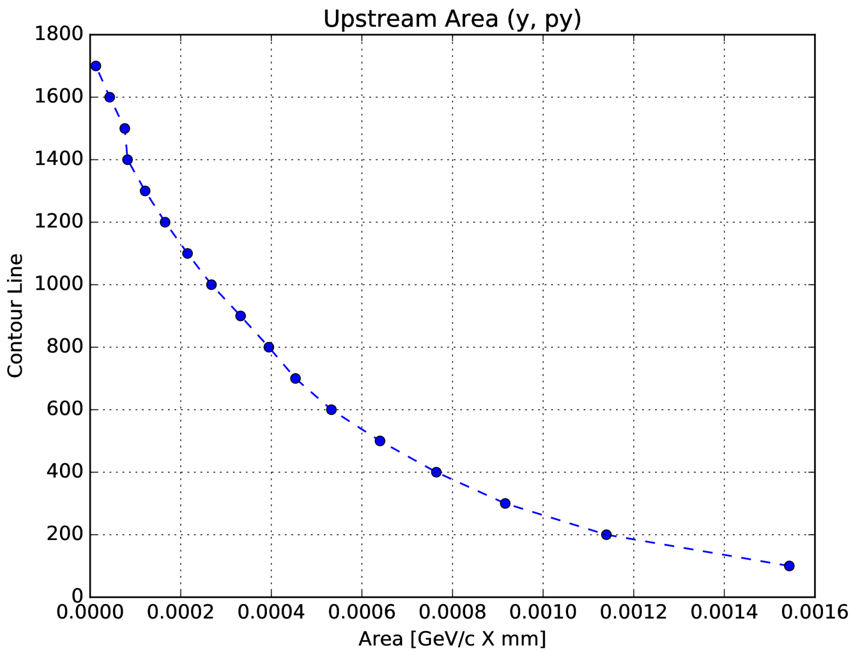


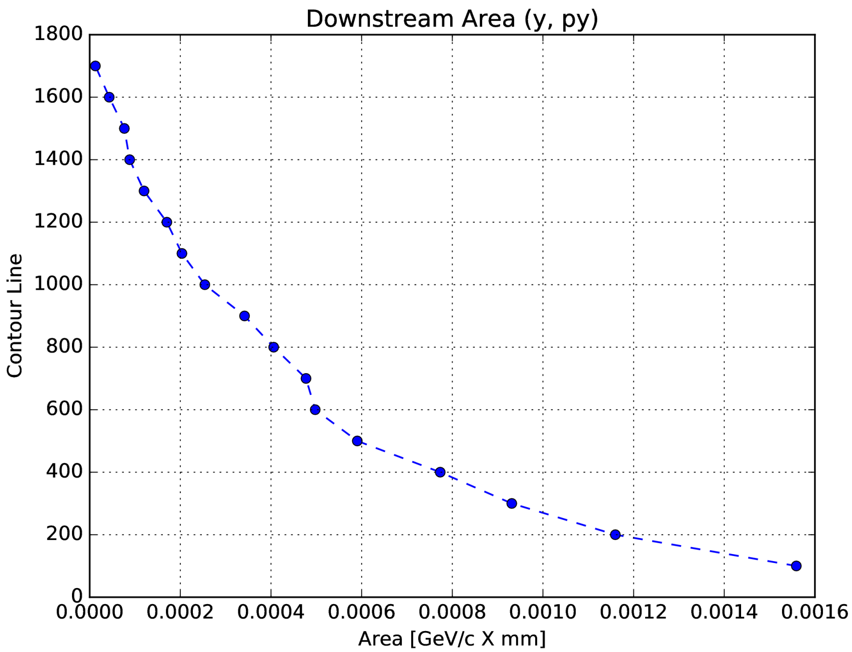
Downstream

Algorithm Validation cont.









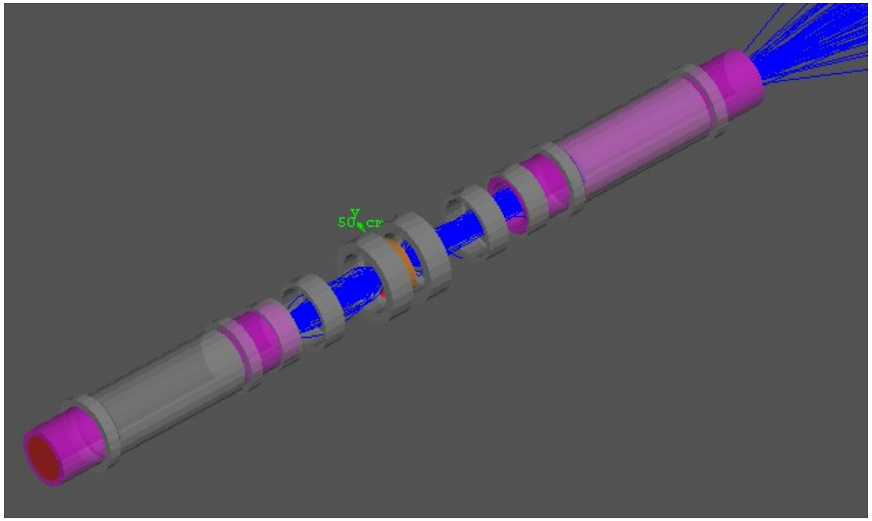
Performance of MICE Step IV with KDE

- Simulation routine:

→ Beam generation script uses MAUS and xboa routines to produce a beam input file. Script and config files provided by Chris Rogers.

→ Beam input file is read by Pavel and Ao's MICE Step IV lattice without downstream Match coils.

→ Beam parameters as shown in table.

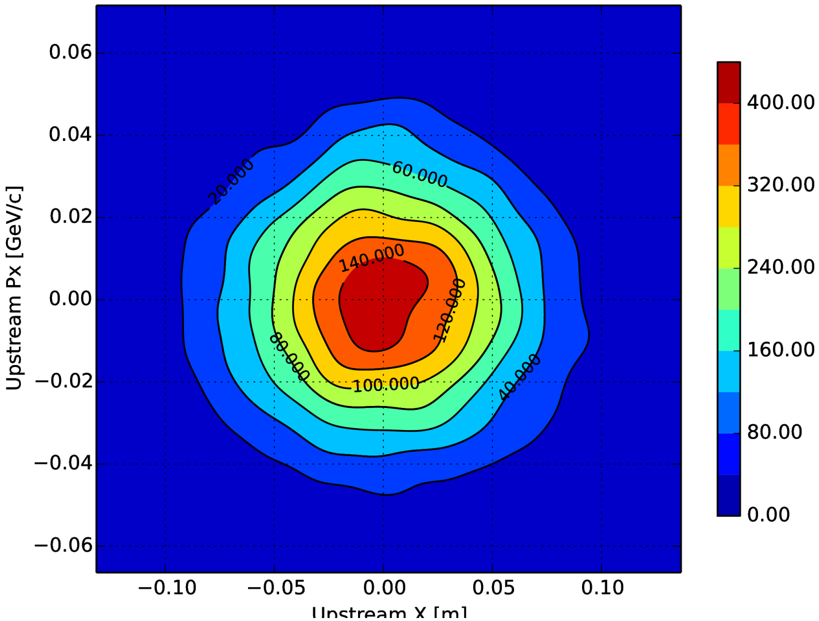


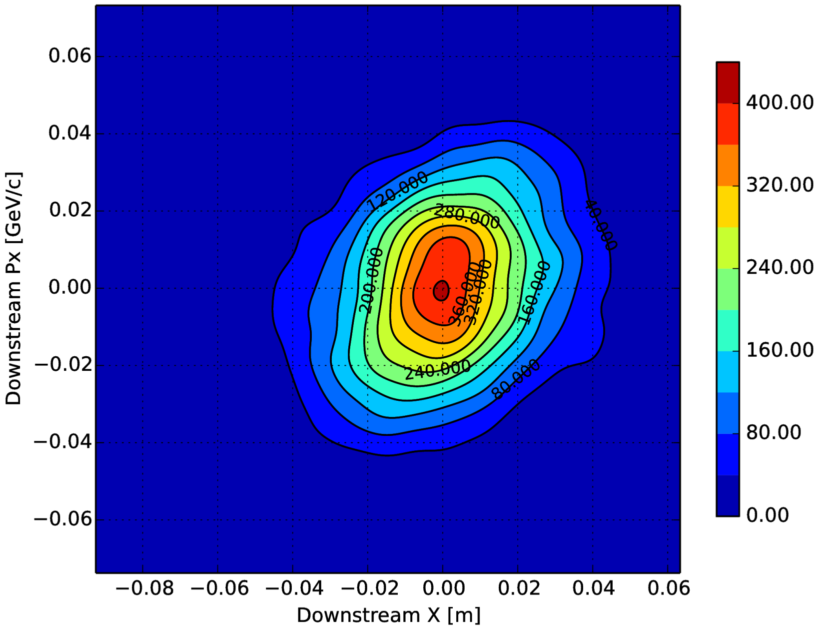
Simulation Parameters	Values
Number of events	10000
Momentum	140 MeV/c
Emittance	4.2 π mm·rad
Beam type	gaussian
Sigma x	30
Sigma y	30
Sigma px	3 / 200
Sigma py	3 / 200
Average p	200 MeV/c
Number of good muons	9571

Preliminary MICE Step IV Plots – (x, px), (y, py)

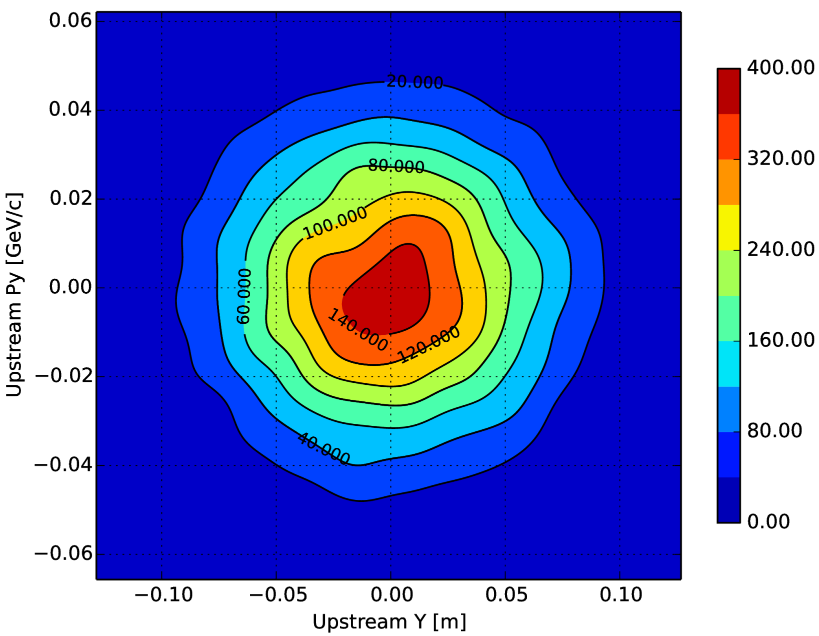
Upstream

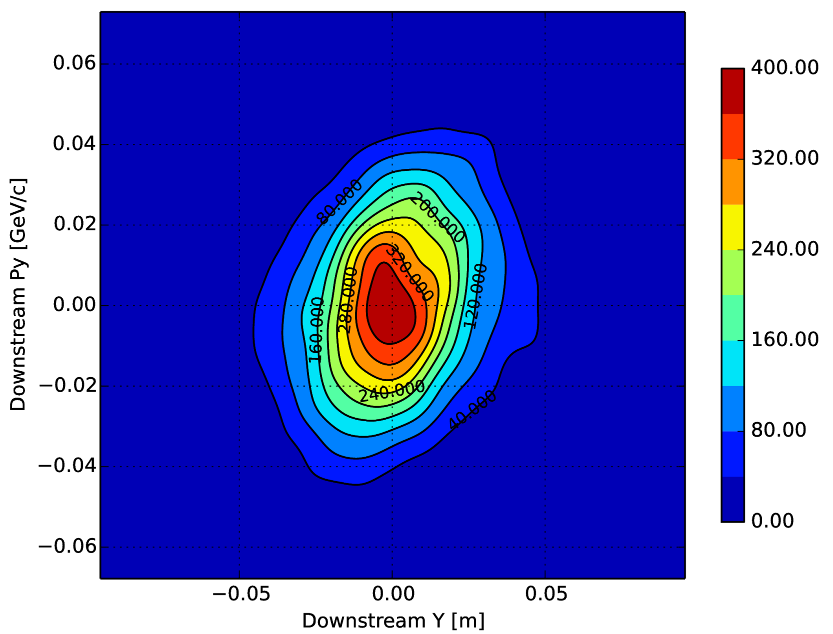
Downstream





(x,x')



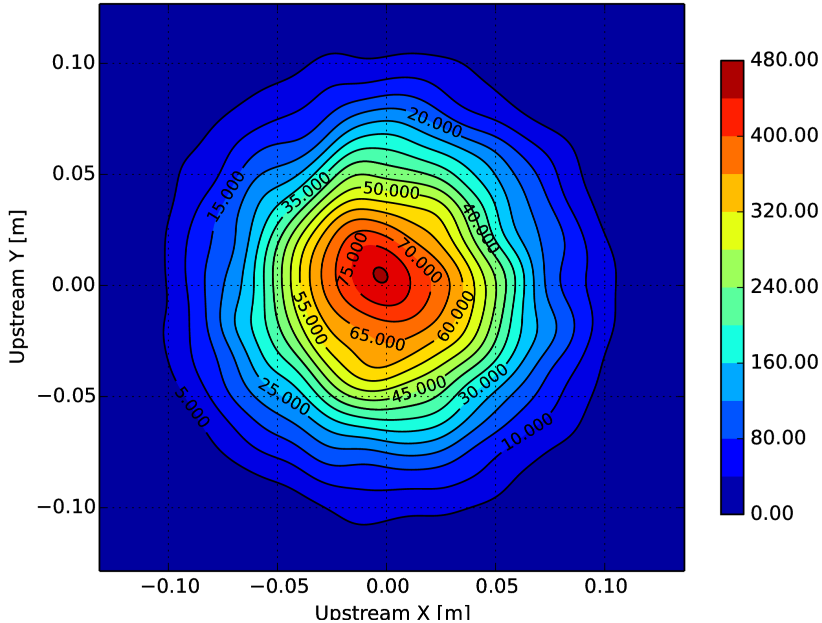


(y,y')

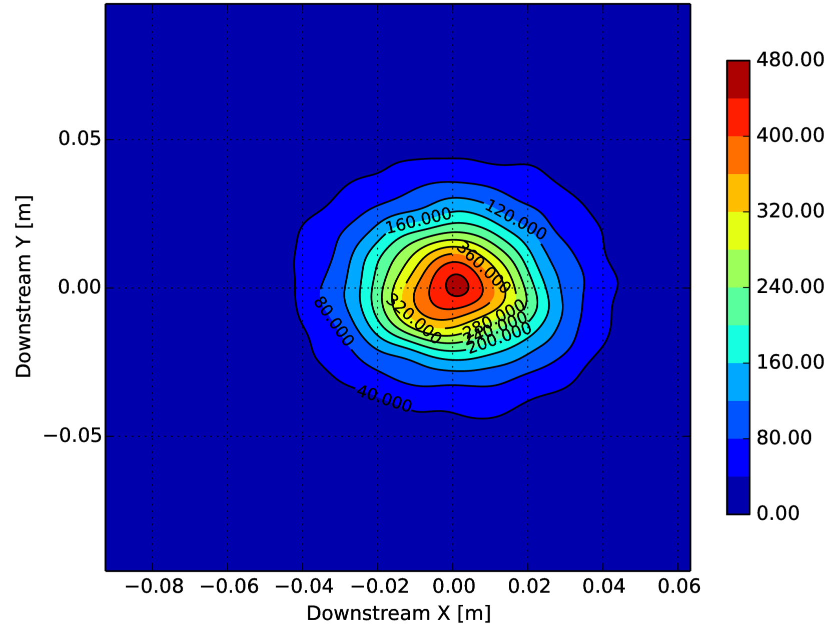
Preliminary MICE Step IV Plots – (y, x), (px, y)

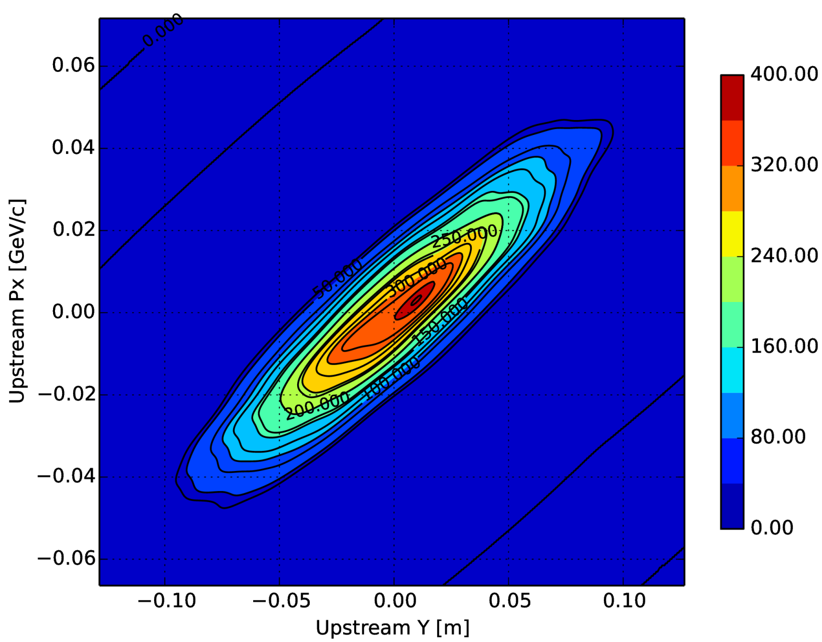
Upstream

Downstream

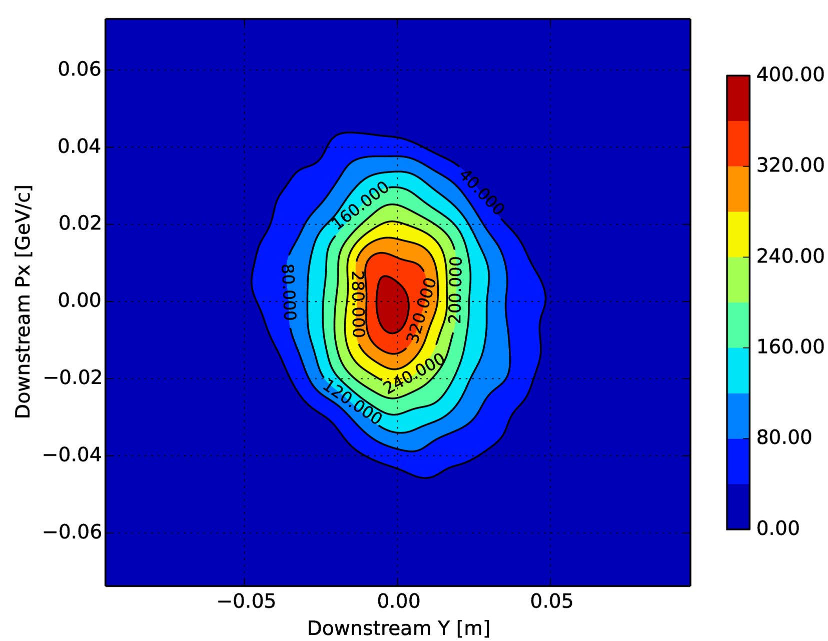


(y,x)





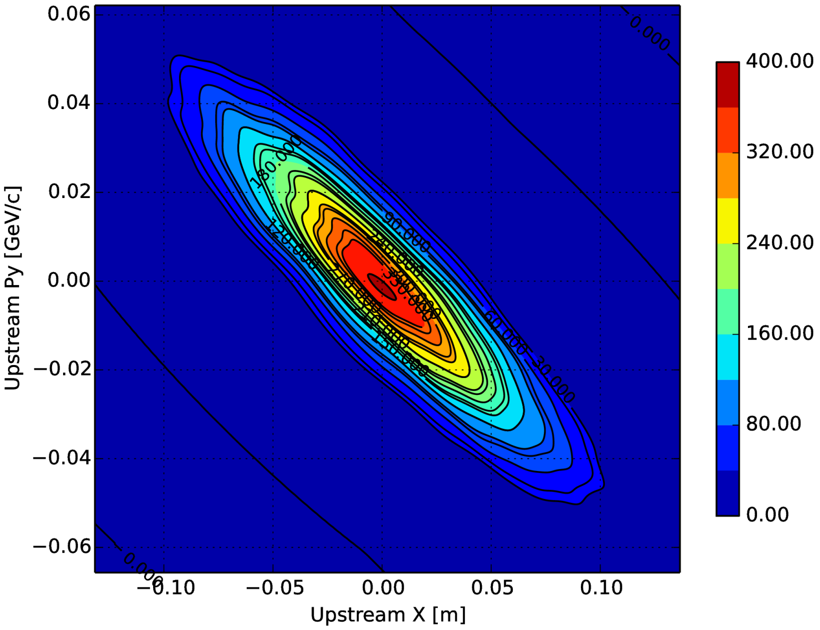
(x',y)



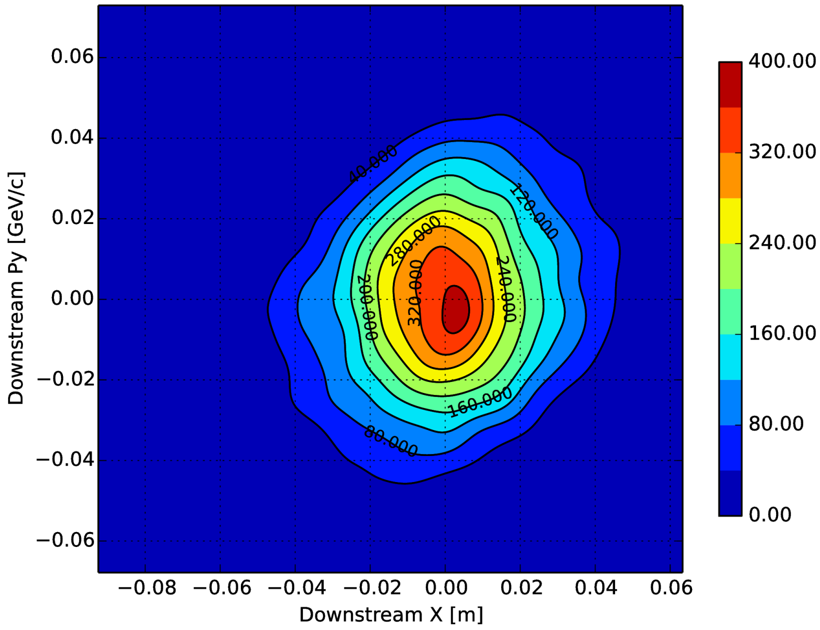
Preliminary MICE Step IV Plots – (x, py), (px, py)

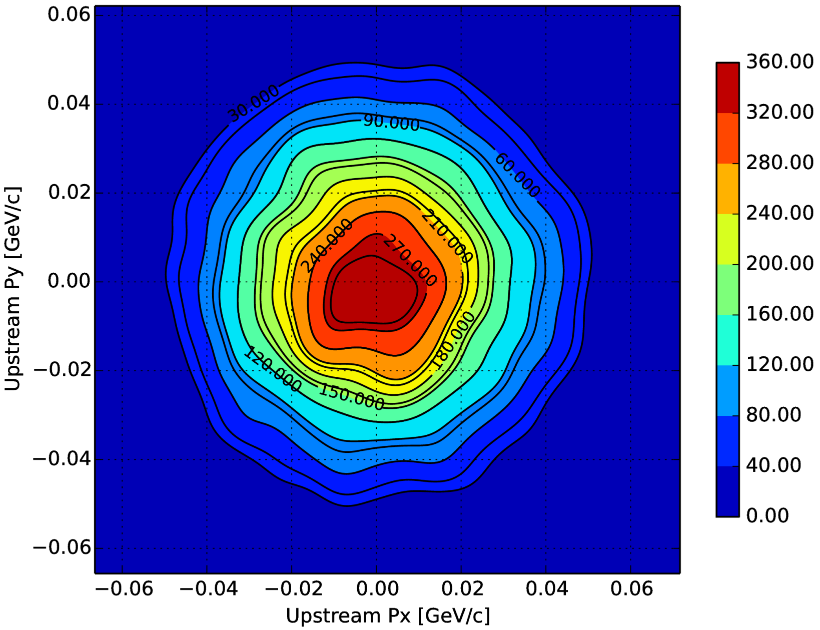
Upstream

Downstream

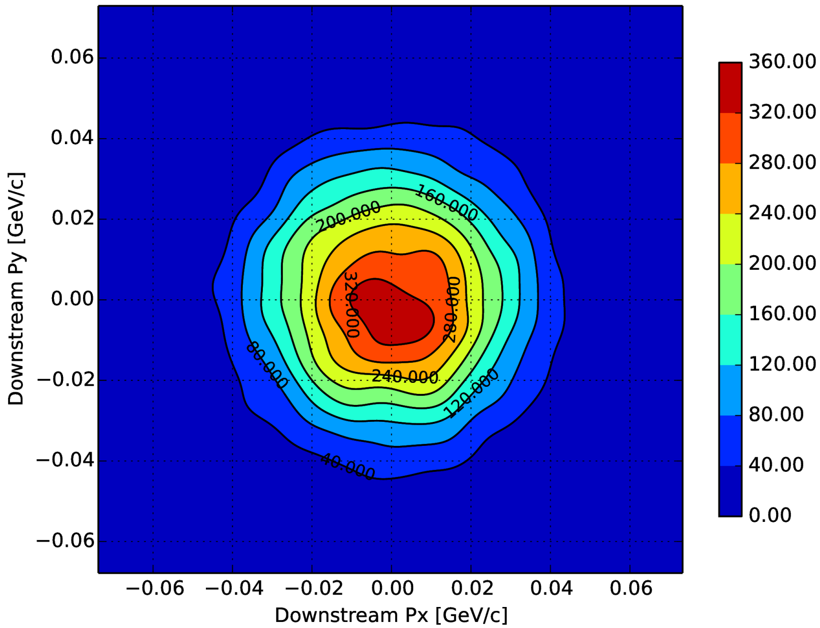


(x,y')





(x',y')



Looking Ahead

- Have started expanding this analysis to 4D. Phase space volume calculation in 4D under investigation.
- Try density estimation using other kernels → especially needed when beam is not fully gaussian.
- Examine the estimated desnsities using different smoothing factors.
- Extend the analysis to MICE Demonstration Step.