The disformal gravity action

The disformal gravity action is induced from disformal transformation of the metric in the Einstein-Hilbert action, 

\[ S_{\text{disf}}[g_{\mu\nu}] = S_{\text{EH}}[\bar{g}_{\mu\nu}] = \frac{1}{2\kappa} \int d^4x \sqrt{-\bar{g}} R(\bar{g}_{\mu\nu}), \]

where we have set \( c = 1 \) and \( \kappa = 8\pi G \). From disformal metric transformation

\[ g_{\mu\nu} \to \bar{g}_{\mu\nu} + D(\phi, Y)\partial_\mu \phi \partial_\nu \phi, \]

we find that the disformal factor depends also on the kinetic terms of the scalar field besides a single scalar field.

The dynamics of background spacetime

For simplicity we will study the disformal gravity obtained from the disformal transformation of FLRW metric \( ds^2 = -N^2(t)dt^2 + a^2(t)dx_i dx^i \). After calculate the Euler-Lagrange equations we respectively obtain

\[
\begin{align*}
0 &= (A_2 - 2Y A_{2,Y}) - \rho_m + 3H^2\gamma \frac{1 - Y^2D_Y}{1 + D_Y}, \\
0 &= H^2\delta_y (D_\rho Y - 2(D + DY D_Y)\delta_y + \gamma (D + H^2) + A_2 + p_m, \\
\end{align*}
\]

From these equations we could roughly analyze, we expect that for the disformal gravity considered here, the accelerated expansion of the universe cannot be driven by kinetic terms of the scalar field. To confirm this analysis we solve the equations of motion for the background universe numerically. By varying the action with respect to \( \phi \) we obtain the equation of motion for \( \phi \)

\[
0 = \delta_y (A_{2,Y} + 2Y A_{2,Y} + 0) \frac{3}{2}H^2\gamma_3[D(1 - Y^2D_Y + 2Y^2D_Y Y - Y^2D_Y')]
\]

\[
+ Y(5D_Y - 3Y^2D_Y + 2Y D_Y Y) + 3H\delta_y (A_{2,Y} - \gamma A_Y (D + D_Y) (A_5^2 + 2\frac{H^2}{a}))
\]

\[
+ \frac{1}{2} (A_{2,Y} - 2Y A_{2,Y} + 0) \frac{3}{2}H^2\gamma_3 [3Y^2D_Y D + YD_Y - 2DY D_Y - Y D_y].
\]

For concreteness, we choose the disformal coupling and \( A_2 \) as

\[
D \equiv M^{-4\lambda_4 - \lambda_5 - \lambda_1 \phi} (-Y)^{\lambda_3}, \quad A_2 \equiv \frac{1}{2} M_5^{4-4\lambda_4} (-Y)^{\lambda_3} - M_5^{4-4\lambda_4} e^{-\lambda_1 \phi}.
\]

Here, \( M, M_5 \) and \( M_6 \) are the constant parameter with dimension of mass, while \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \) are the dimensionless constant parameters.

Figure 1: The equation of state parameter \( w_T \) as a function of \( \log_{10}a \) for various values of \( \lambda_1, \lambda_2, \lambda_3 \) and \( \lambda_4 \).

We setting \( M_5^2 = M_5^2 = H_0 \) where \( H_0 \) is the present value of the Hubble parameter, we have found that the acceleration of the universe at late time can occur only if scalar field \( \phi \) slowly evolves, i.e. \( \phi \ll H \). Hence, the accelerated expansion of the universe is driven by the potential terms rather than the kinetic terms of the scalar field. As a result the evolution of \( w_T \approx -2H/(3H^2) \) – 1 always mimics the evolution of \( w_T \) for \( \Lambda \)CDM model as shown in a figure.

From this analysis, we conclude that for the disformal gravity considered here, the accelerated expansion of the universe cannot be driven by kinetic terms of the scalar field.