The Background Evolution in General Disformal Gravity

Saksith Jaksri
Naresuan University
saksith@tuta.io

Introduction

The disformal transformation for gravity has been introduced by Bekenstein in 1992 [1]. It is the most general mapping between the metric involving one scalar field and preserve diffeomorphisms of spacetime. This transformation can be written as

$$\bar{g}_{\mu\nu} = C(\phi,Y)g_{\mu\nu} + D(\phi,Y)\partial_\mu\phi\partial_\nu\phi \,, \qquad (1)$$

where $Y \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ a kinetic term of the scalar field ϕ. C, D are arbitrary functions of the scalar field and its kinetic terms.

These called the conformal and disformal factor, respectively. In order to study the influence of the disformal factor on the evolution of background universe, we will set $C(\phi, Y) = 1$ in this paper. We call this transformation the general disformal because the disformal factor depends also on the kinetic terms of the scalar field beside a single scalar field.

The disformal gravity action

The disformal gravity action is induced from disformal transformation of the metric in the Einstein-Hilbert action ,

$$S_{disf}[g_{\mu\nu}] = S_{EH}[\bar{g}_{\mu\nu}] = \frac{1}{2\kappa}\int d^4x\sqrt{-\bar{g}}R(\bar{g}_{\mu\nu}) \,, \qquad (2)$$

where we have set $c = 1$ and $\kappa = 8\pi G$. From disformal metric transformation

$$\boxed{g_{\mu\nu} \to g_{\mu\nu} + D(\phi,Y)\phi_\mu\phi_\nu,} \qquad (3)$$

where $(\phi_\mu \equiv \nabla_\mu\phi = \partial_\mu\phi)$,

Conclusion

In this work we have studied the gravity theory generated by general disformal transformation which can be shown that it fits into the class of GLPV theories. By analyzing the background evolution equations we have found that this theory does not provide the self-accelerating solution as generally expected from GLPV theories.

References

[1] Bekenstein J D 1993 *Phys. Rev.* **D48** 3641 (*Preprint* `gr-qc/9211017`)

[2] Gleyzes J, Langlois D, Piazza F and Vernizzi F 2015 *JCAP* **1502** 018 (*Preprint* `arXiv:1408.1952`)

[3] De Felice A, Koyama K and Tsujikawa S 2015 *JCAP* **1505** 058 (*Preprint* `arXiv:1503.06539`)

[4] Gannouji R and Sami M 2010 *Phys. Rev. D* **82** 024011 (*Preprint* `arXiv:1004.2808`)

[5] De Felice A and Tsujikawa S 2010 *Phys. Rev. Lett.* **105** 111301 (*Preprint* `arXiv:1007.2700`)

[6] Kobayashi T, Watanabe Y and Yamauchi D 2015 *Phys. Rev.* **D 91** 064013 (*Preprint* `arXiv:1411.4130`)

Acknowledgements

I would like to thanks to Thailand Center of Excellence in Physics (THEP) and Neresuan University. thanks so much to Dr. Khamphee Karwan for advising me.

The disformal gravity action 2

After a somewhat tedious but straightforward calculation, we obtain the disformal action
This action can be recast into the form of covariant GLPV action [2] (setting $2\kappa = 1$): $S = \int d^4x\sqrt{-g}\sum_{i=2}^{5}\mathcal{L}_i$, where

$$\begin{aligned}
\mathcal{L}_3 &= (C_3 + 2Y\,C_{3Y})\Box\phi \,, & (4)\\
\mathcal{L}_4 &= (B_4 + C_5)\,R - 2(B_4 + C_5)_Y(\Box\phi^2 - \phi_{\mu\nu}^2) + F_4\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\beta\gamma\delta}\phi_\nu\phi^\beta\phi_\rho^\gamma\phi_\sigma^\delta \,, & (5)\\
\mathcal{L}_5 &= \tilde{G}_5\,G_{\mu\nu}\phi^{\mu\nu} + \frac{1}{3}\tilde{G}_{5Y}\left(\Box\phi^3 - 3\Box\phi\,\phi_{\mu\nu}^2 + 2\,\phi_{\mu\nu}^3\right) + F_5\epsilon^{\mu\nu\rho\sigma}\epsilon_{\alpha\beta\gamma\delta}\phi^\alpha\phi_\mu\phi_\nu^\beta\phi_\rho^\gamma\phi_\sigma^\delta \,, & (6)
\end{aligned}$$

where $\epsilon^{\mu\nu\rho\sigma}$ is a Levi-Civita pseudotensor. In our case, it can be shown that $B_4 = 1/\gamma$, $C_3 = -\frac{1}{2}\int\gamma D_\phi dY$, $A_2 = -\frac{1}{2}D_\phi Y^2 - YC_{3\phi}$, $A_4 = \gamma DY - \frac{1}{\gamma}$, $C_5 = \tilde{G}_5 = F_5 = 0$, $F_4 \equiv Y^{-2}(B_4 + A_4 - 2Y B_{4Y}) = -\gamma D_Y$.

The dynamics of background spacetime

For simplicity we will study the disformal gravity obtained from the disformal transformation of FLRW metric $ds^2 = -N^2(t)dt^2 + a^2(t)\delta_{ij}dx^i dx^j$. After calculate the Euler-Lagrange equations we respectively obtain

$$\begin{aligned}
0 &= (A_2 - 2YA_{2,\mathrm{Y}}) - \rho_m + 3H^2\gamma\frac{1 - Y^2D_{,Y}}{1 + DY} \,, & (7)\\
0 &= H\gamma^3\dot{\phi}(D_{,\phi}Y - 2(D + YD_{,Y})\ddot{\phi}) + \gamma(2\frac{\ddot{a}}{a} + H^2) + A_2 + p_m \,, & (8)
\end{aligned}$$

From these equations we can roughly analysis, we expect that for the disformal gravity considered here, the accelerated expansion of the universe cannot be driven by kinetic terms of the scalar field. To confirm this analysis we solve the equations of motion for the background universe numerically. By varying the action with respect to ϕ we obtain the equation of motion for ϕ

$$\begin{aligned}
0 &= \ddot{\phi}[A_{2,\mathrm{Y}} + 2YA_{2,\mathrm{YY}} + \frac{3}{2}H^2\gamma^5[D(1 - Y^2D_{,Y} + 2Y^3D_{,YY}) - 2YD^2]]\\
&\quad + Y(5D_{,Y} - 3Y^2D_{,Y}^2 + 2YD_{,YY}) + 3H\dot{\phi}\left(A_{2,\mathrm{Y}} - \gamma^3Y(D + YD_{,Y})(\frac{1}{2}H^2 + \frac{\ddot{a}}{a})\right)\\
&\quad + \frac{1}{2}\left(A_{2,\phi} - 2YA_{2,\mathrm{Y}\phi} + \frac{3}{2}H^2\gamma^3[3Y^2D_{,\phi}\frac{D + YD_{,Y}}{1 + DY} - 2Y^2D_{,\phi Y} - YD_{,\phi}]\right). \qquad (9)
\end{aligned}$$

For concreteness, we choose the disformal coupling and A_2 as

$$D \equiv M^{-4\lambda_2-4}\mathrm{e}^{-\lambda_1\phi}(-Y)^{\lambda_2} \,,\; A_2 \equiv \frac{1}{2}M_k^{4-4\lambda_3}(-Y)^{\lambda_3} - M_v^4\mathrm{e}^{-\lambda_4\phi} \,. \qquad (10)$$

Here, M, M_k and M_v are the constant parameter with dimension of mass, while λ_1, λ_2, λ_3 and λ_4 are the dimensionless constant parameters.

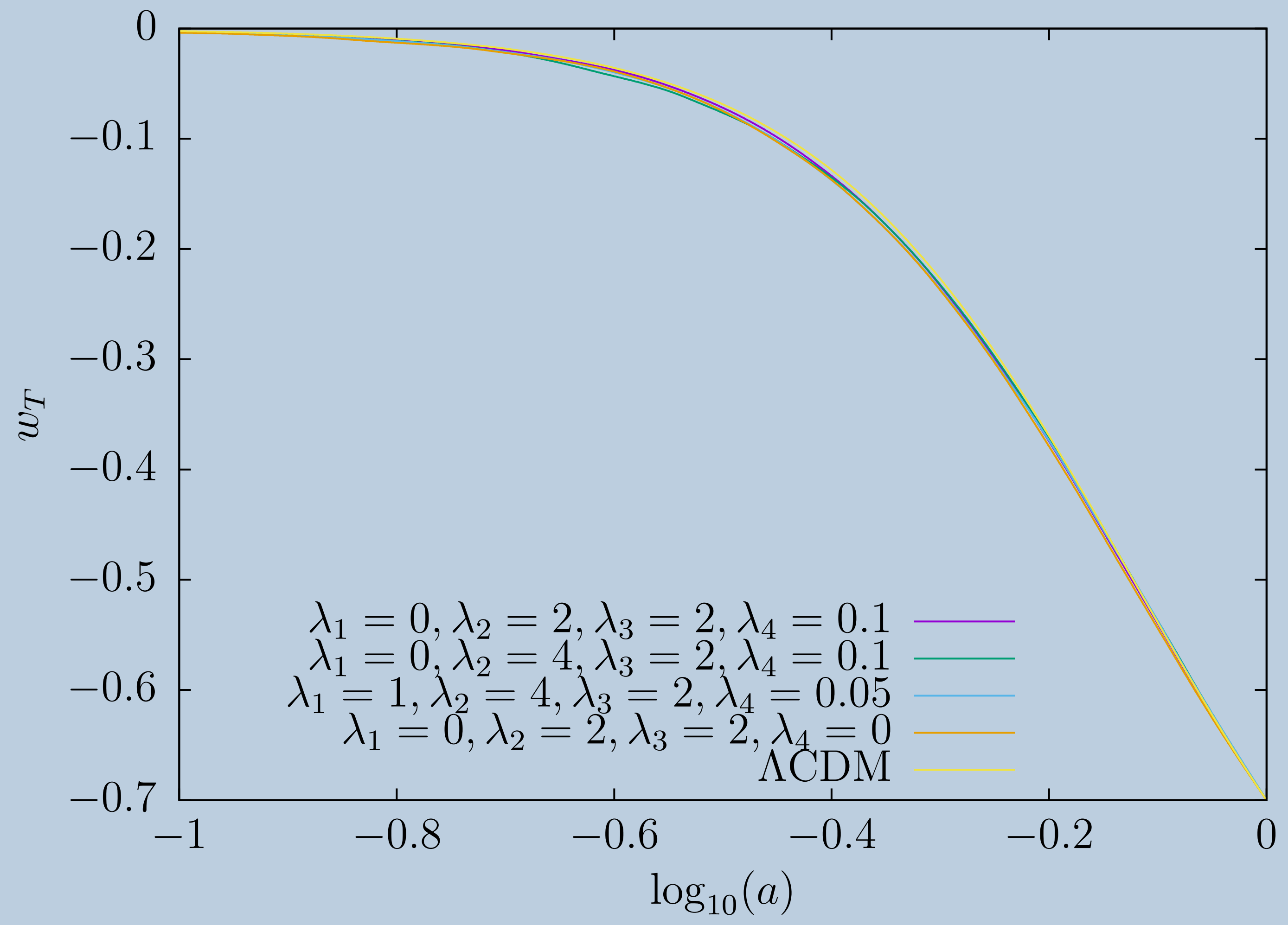

Figure 1: The equation of state parameter w_T as a function of $\log_{10} a$ for various values of λ_1, λ_2, λ_3 and λ_4.

We setting $M^2 = M_k^2 = M_v^2 = H_0$ where H_0 is the present value of the Hubble parameter, we have found that the acceleration of the universe at late time can occur only if scalar field ϕ slowly evolves, i.e. $\dot{\phi} \ll H$. Hence, the accelerated expansion of the universe is driven by the potential terms rather than the kinetic terms of the scalar field. As a result the evolution of $w_T \equiv -2\dot{H}/(3H^2) - 1$ always mimics the evolution of w_T for ΛCDM model as shown in a figure
From this analysis, we conclude that for the disformal gravity considered here, the accelerated expansion of the universe cannot be driven by kinetic terms of the scalar field.