# A novel spectrometer based on the near-field Talbot effect

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**Abstract.**

By making use of advanced technique of the near-field Talbot effect, we design a novel spectrometer. Our method provides a compact and portable spectrometer according to the characteristic of the Talbot effect. Here, we propose the idea with the simulations done with reasonable values. With vibration and temperature adjustments, the high resolution over a range of a few nanometers can be obtained.

## 1. Introduction

Talbot effect, which is an optical near-field effect, has been widely used in many applications especially in measurements and instrumentations. The interference patterns which have the shape similar to the grating itself, so called the self-imaging can be observed at the Talbot distance, $L_T$ as well as the multiples of it. The Talbot effect has been performed with light [1], x-ray [2], plasmonic [3], as well as with quantum objects such as single photons [4], atoms [5], and molecules [6]. The distance measurement was demonstrated by the Talbot effect [7]. Surface profiling can be done using phase shifting Talbot interferometric technique [8]. The holography with the Talbot effect provides a nonscanning optical microscope [9]. The wavefront sensor has also been done with the Talbot effect [10]. Recently, the Talbot effect with the tilt detection plane has been used for miniaturization the spectrometer [11]. All these show that the Talbot effect has potential applications in many optical systems.

Here, we present the spectrometer based on using the optical Talbot effect. The effect is sensitive to the source wavelength. We provide the technique without image detection. We propose the idea with the simulations done with reasonable values. Our spectrometry can be small according to the near-field effect. We show that the optimization of the resolution according to our recent scheme can be as good as in a few nanometers which is in the range of practical applications in physics, chemistry and metrology purposes.

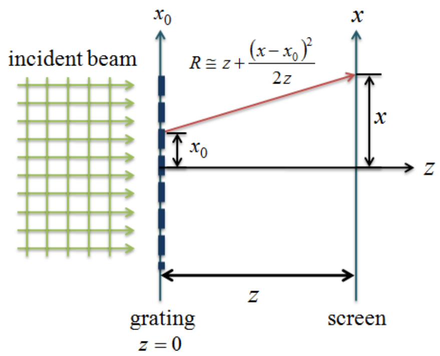


**Figure 1.** A plane wave diffracts through the grating, which has the periodic modulation along the $x_0$-axis, to a screen on the $xz$ plane.

## 2. Theory and method

In this section, we present a short derivation of the near-field Talbot effect and propose for making a spectrometer. Assuming a plane wave falls onto a grating at $z = 0$ as shown in Figure 1.

For the diffraction grating with a period $d$, the transformed wave behind the grating can be represented as

$$\psi_0(x_0, z = 0) = \sum_n A_n \mathrm{exp}\{ink_d x_0\}, \tag{1}$$

where $n = 0, \pm1, \pm2, ...,$ and $k_d = 2\pi/d$. The factor $A_n = \sin(n\pi f)/n\pi$ is the components of the Fourier decomposition of the periodicity for the grating with an opening fraction $f$ [1].

In near-field regime, the wave function at distance $z$ with the transverse axis $x$ can be obtained by applying the Huygens-Fresnel integral. Consider the propagation in $xz$ plane with the leading order approximation $R \simeq z + (x - x_0)^2/2z$, the wave function is given by

$$\psi_\lambda(x, z) = \sqrt{\frac{i}{\lambda z}} \int_{-\infty}^{\infty} dx_0 \mathrm{exp}\{-ik(z + \frac{(x - x_0)^2}{2z})\}\psi_0(x_0, 0), \tag{2}$$

where $k = 2\pi/\lambda$ with $\lambda$ is the wavelength of light. With the field $\psi_0$ in Eq.(1), the field $\psi_\lambda(x, z)$ can be obtained as

$$\psi_\lambda(x, z) = \mathrm{exp}\{-ik(z + \frac{x^2}{2z})\} \sum_n A_n \mathrm{exp}\{\frac{iz}{2k}(nk_d + \frac{kx}{z})^2\}. \tag{3}$$

The intensity distribution behind the grating, which is corresponding to the Talbot effect, $\psi_\lambda^* \psi_\lambda$ can be written in the form

$$I_\lambda(x, z) = \sum_{n,m} A_n A_m \mathrm{exp}\{i\Big((n - m)\frac{2\pi}{d}x + \frac{(n^2 - m^2)\pi z}{L_T}\Big)\}, \tag{4}$$

where $L_T = d^2/\lambda$ is the Talbot distance. The intensity $I_\lambda(x, z)$ gives the near-field interference pattern, so-called the optical carpet. For example, the optical carpets can be simulated with Eq.(4) as shown in Figure 2 (a), and (b).

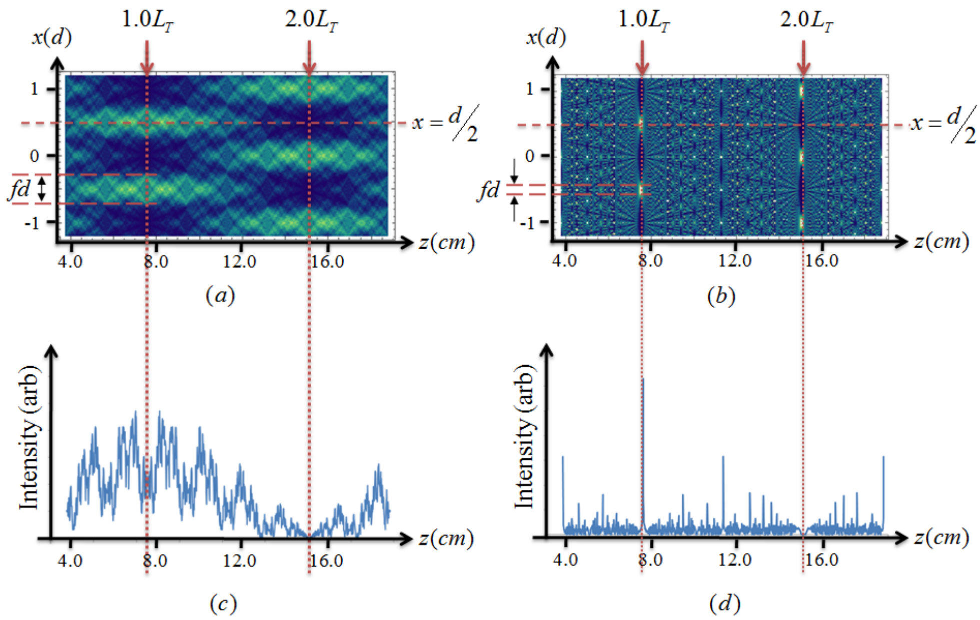


**Figure 2.** Theoretical simulations of light wavelength $\lambda = 532$ nm according to Eq.(4) for the grating with the period $d$=200 $\mu m$, and with the opening fraction (a) $f = 0.5$ and (b) $f = 0.025$. The interference patterns have the period similar to the grating period with the fringe widths equal to $fd = 100$ $\mu$m and 5 $\mu$m, respectively. The graphs (c) and (d) are the intensity as a function of the z distance behind the grating for $f = 0.5$ and 0.025, respectively. The calculations were done with Eq.(4) along $z$-axis at $x = d/2$.

## 3. Numerical results and discussions

Generally, the grating with the opening fraction $f = 0.5$ has been widely used in several optical applications but only small opening fraction can be used for making a spectrometer with high resolution. In comparison to the intensity function along $z$-axis at $x = d/2 = 100$ $\mu m$ between $f = 0.5$ in Figure 2 (c), and $f = 0.025$ in Figure 2 (d), the latter gives the narrow intensity distribution with the full width at half maximum ($\sigma_\lambda$) centered at $z$ around $1.0L_T = 7.52$ cm. In the case of a mixed two-wavelength wave function, one can evaluate the wavelengths from the position of each peak $z_j = L_T$ in the measured intensity distribution by using the relation $\lambda = d^2/z_j$. In other words, $z_j$ can be used to specify only if the distance between the two corresponding Talbot lengths for two wavelengths is fulfilled the condition $\Delta L_T \geq \sigma_\lambda$. According to this condition, we obtain the resolution as $\Delta\lambda \geq \frac{\lambda^2\sigma_\lambda}{d^2}$. Nevertheless, the effect of the wavelength distribution has to be involved. By assuming a Gaussian distribution of the wavelength around $\lambda_j$, the intensity distribution is given by

$$I_{\lambda_j}(x,z) = \sum_{\lambda=0}^{\infty} e^{-\frac{(\lambda-\lambda_j)^2}{\beta^2}} I_\lambda(x,z), \tag{5}$$

where $\beta$ represents the radius of the Gaussian distribution of the wavelength. We clarify our idea with the simulation which is satisfied the above condition with three-wavelength distribution, i.e. $\lambda_j = 536,\ 532$ and $528$ nm. For numerical calculations with $\beta$ about 2 nm, the three wavelengths give the FWHM of the intensity distribution along z-axis in the micrometer scale for the grating period $d = 200\ \mu m$, and the opening fraction $f = 0.025$ as presented in Figure 3 for $\Delta\lambda \approx 4$ nm. The example as shown in Figure 3 has been evaluated for the wavelength distribution $\beta$ about 2 nm. For different wavelength distributions, the intensity distribution $(\sigma_\lambda)$ increases linearly with $\beta$ for $\beta < 5$ nm. And this concerns the resolution of the spectrometer based on this idea.

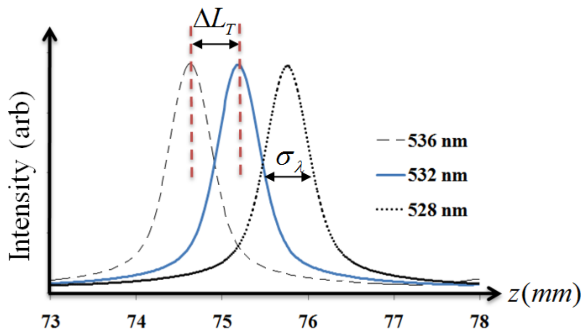


**Figure 3.** Theoretical simulation of the intensity distribution according to Eq.(5) for the wavelength $\lambda_j = 536,\ 532$ and $528$ nm.

## 4. Conclusions

A simple design concept of a spectrometer based on using the optical Talbot effect with the resolution of a few nanometers is proposed. The study carried out in this paper shows that a grating with a small opening fraction $f$, i.e., less than 0.5 is required for a high resolution spectrometer. On the one hand, the resolution increases with decreasing $f$, but on the other hand, the decrease in $f$ leads to the drop in the intensity of the fringe on the screen. $f$ is therefore limited by the source intensity. In addition to $f$, the resolution is influenced by the wavelength distribution of the incident wave but not the the grating period $d$. Despite the influence of $d$ on the Talbot length, the change in $d$ does not effect the resolution. The mechanical stability must be concerned and this can be fulfilled with an isolation such as an optical table.

## References

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