

Investigation of L - H and ITB Transitions in Fusion Plasma Based on Bifurcation Concept

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Abstract. The criteria for the L - H transition and ITB transition in fusion plasmas is studied based on a bifurcation concept. Three transport equations including thermal, particle and toroidal momentum density are solved simultaneously, resulting in the prediction of plasma pressure, plasma density and toroidal velocity profiles at steady state. The thermal and particle transport include both neoclassical and anomalous effects with the velocity shear dependent suppression effect. The results show that the flux (thermal/particle/momentum) versus gradient (pressure/density/velocity) space of each field independently exhibits s -curve bifurcation nature in which a forward L - H and ITB transitions requires higher flux than that of the respective backward transitions, hence hysteresis behaviors. In addition, it is found that there exist certain regimes where the transitions are possible. In particular, the ratios of anomalous over neoclassical transport must exceed certain thresholds.

1. Introduction

Technology have been growing rapidly, resulting in a demand for more energy. It is expected that the main energy sources, mostly fossil fuels, will not be enough to supply human demand in the future. Fusion energy could be the answer. In fusion research, it is generally known that formation of an edge transport barrier (ETB) can significantly improve plasma performance, including central pressure, density and confinement time [1]. The barrier formation causes the plasma to abruptly transit from low (L -mode) to high confinement modes (H -mode), exhibiting high gradients in the pedestal region near plasma edge. The improvement is so important that the international ITER project had designed its experiments to operate in H -mode [2]. Additionally, the plasma performance can be even further improved with formation of an internal transport barrier (ITB) [3]. Simultaneous formations of both transport barriers can lead to steady state operation of fusion plasma. It is well accepted that the formation of transport barriers is a result of turbulent transport reduction or suppression. The physical mechanism for turbulence quenching is still in research study. One of the possible ideas involves shear of radial electric field or plasma velocity [4]. Bifurcation concept explains that the plasma transition, as least for L - H transition, is an intrinsic property of the plasma [5]. Specifically, the turbulent suppression is caused by the velocity shear resulting in sudden increase of pressure or density gradients. Malkov *et al.* solved the thermal and particle transport equations simultaneously to show that there exists bifurcation regime in heat/thermal flux versus pressure/particle gradient regime for ETB formation [5]. Later, Jhang *et al.* showed that this concept can be applied to ITB formation as well by solving thermal and toroidal momentum density transport equations [6]. Recently, it was found that magnetic shear also plays role in both ETB and ITB formations [4]. This research aims to study the criteria for the L - H and

ITB transitions (low to high confinement transition) in tokamak fusion plasmas based on bifurcation concept. The model in this work adapts the concept proposed by Malkov *et al.* and Jhang *et al.*, which is to solve all three transport equations during stationary state, simultaneously and self-consistently. The transitions criteria are studied by using the three transport equations including thermal, particle and toroidal momentum density, consisting the effects of neoclassical and anomalous transports, which are solved simultaneously. These criteria are important to analyze because they demonstrate the possible regimes where the transitions can occur. For example, it was experimentally found that *L-H* transition takes place as soon as the heating power surpasses a certain threshold, regardless of the heating scheme [1].

2. Three-Field Bifurcation Model and Transition Points

In this research, the simplified 1D transport equations including thermal, particle and toroidal momentum density during steady state, are shown respectively as:

$$\left(\chi_0 + \frac{\chi_1}{1+\alpha\gamma_E^2}\right)P = \frac{1}{r} \int_0^r r^* H(r^*) dr^* = Q(r), \quad (1)$$

$$\left(D_0 + \frac{D_1}{1+\alpha\gamma_E^2}\right)N = \frac{1}{r} \int_0^r r^* S(r^*) dr^* = \Gamma(r), \quad (2)$$

$$\left(v_0 + \frac{v_1+v_2}{1+\alpha\gamma_E^2}\right)V - \frac{G}{1+\alpha\gamma_E^2}NP = \frac{1}{r} \int_0^r r^* U_\phi(r^*) dr^* = F(r), \quad (3)$$

where $P \equiv -\frac{\partial p}{\partial r}$, $N \equiv -\frac{\partial n}{\partial r}$, $V \equiv -\frac{\partial v_\phi}{\partial r}$, $v_2 \equiv \frac{\epsilon}{q} \xi v_1$ and $G \equiv -\frac{\xi v_1}{eBn^2}$. The variable p , n , v_ϕ , $H(r)$, $S(r)$ and $U_\phi(r)$ represent the pressure, density, toroidal flow velocity, thermal, particle and toroidal momentum density sources, respectively. The plasma transports are calculated from the neoclassical transport coefficients (χ_0 , D_0 and v_0) and the turbulent transport coefficients (χ_1 , D_1 and v_1). In this work, the turbulence is assumed to be suppressed by the velocity shear, which can be calculated from the force balance equation. The constant α , representing the strength of the suppression, is related to the turbulence correlation time. The γ_E represents the velocity shear, which can be computed as:

$$\gamma_E = \frac{\partial v_E}{\partial r} = -\frac{1}{eBn^2} \frac{\partial n}{\partial r} \frac{\partial p}{\partial r} + \frac{\epsilon}{q} \frac{\partial v_0}{\partial r}, \quad (4)$$

where q is safety factor, B is magnetic field, ϵ is inverse aspect ratio and e is electron charge. The terms on the right hand side of equations (1) - (3), $Q(r)$, $\Gamma(r)$ and $F(r)$ are heat flux, particle flux and momentum toroidal flux, respectively. The full transport equations can be seen in Refs [5] and [6].

Equations (1), (2) and (3) can be decoupled to demonstrate bifurcation (*s*-curve) of each field. As generally found in experiments, the transition could occur both simultaneously or individually among all fields. Algebraically, for example, a subtraction of equation (1) times ND_I by equation (2) times $P\chi_I$ can be used as a decoupling substitution. All three decoupling substitutions are as follows: $P = \frac{Q(r)D_1N}{\Gamma(r)\chi_1 + N(D_1\chi_0 - D_0\chi_1)}$, $N = \frac{\Gamma(r)\chi_1P}{Q(r)D_1 + P(D_0\chi_1 - D_1\chi_0)}$ and $V = P \left(\frac{-\chi_1F(r) + G(\chi_0P - Q(r))N}{(\chi_0(v_1+v_2) - \chi_1v_0)P - (v_1+v_2)Q(r)} \right)$, where they can be substituted back into equations (1) to (3). Consequently, the flux versus gradient of each field can be numerically solved. Examples are shown in figures 1 and 2 below. In this research, the constant $\xi = -1$ is a dimensionless parameter involving the shift of the intensity fluctuation profile due to $\mathbf{E} \times \mathbf{B}$ shear, $\sqrt{T_i/T_e}$ and the ratio of density to magnetic shear scaling length, at which ξ depends on turbulence modes [3]. Additionally, $\epsilon = 1/4$ is used based on standard and high field tokamak reactor designs [7].

Table 1. Transport coefficients of standard simulation

	χ_0	χ_1	D_0	D_1	v_0	v_1
<i>L-H</i> transition	2.20	12.00	0.35	2.30	1.00	12.00
ITB transition	2.00	12.00	0.28	2.50	1.65	20.00

3. Transition Criteria

In this section, we analyze the possible regimes where *L-H* and ITB transitions can occur. The parameters and constants of the model are arbitrarily chosen but the selection is based on physical observation in which the anomalous transport coefficients are one (ion) to two (electron) order of magnitude higher than those of neoclassical coefficients. The details of the numerical values used can

be seen in Ref. [4]. The study in this section is as follows: based on the transport coefficients of standard simulation shown in table 1, each parameter is separately varied to locate the regime where bifurcation nature of the plasma exist, signifying as *s*-curve or non-monotonic behavior in flux versus gradient space. Example of these are shown in figures 1 and 2. The difference between ITB and *L-H* transitions is the numerical value of q . Experimentally, the q profile is found to be non-monotonic in ITB plasma with ITB location near the rational q surface. Whereas, ETB is found where q has high value.

3.1 *L-H* transition

The bifurcation curves for each field are shown in figure 1. The horizontal lines illustrate where the *L-H* transitions take place. These results show that the transition can be triggered separately, that is once the threshold flux (thermal/particle/toroidal momentum density) is reached the respective gradient (pressure/density/toroidal velocity) abruptly increases. However, it is possible that more than one transition can occur at the same time. This is illustrated in detailed analysis as shown in table 2. The table shows that there exist minimum and maximum values of thermal, particle and momentum density transport coefficients ranges where *L-H* transition of the respective field is possible. This table can be read as follows: for example, for $15 < \chi_0 < 20$, *L-H* transition is possible solely in density profile, while for $4 < \chi_0 < 15$, the transition is possible both in density and pressure profiles. Additionally, there exist the minimum values of anomalous transport and maximum values of neoclassical transport. Previous works on bifurcation model concluded that the transition is possible only if the ratio of anomalous over neoclassical transport coefficients must be greater than a certain value [5]. Overall, the transition window is widest in particle and thermal fields and narrow in momentum field.

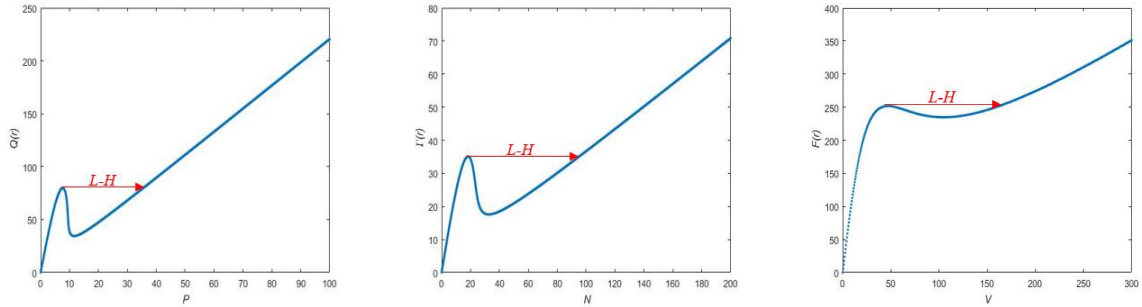


Figure 1. Illustrates *L-H* transition in thermal (left), particle (middle) and toroidal momentum density (right) fields.

Table 2. Show the criteria of ETB transition

Coefficient	Thermal		Particle		Momentum	
	MIN	MAX	MIN	MAX	MIN	MAX
χ_0	0.5	15.0	1.2	20.0	1.5	4.0
χ_1	3.0	60.0	2.0	18.0	6.0	>60.0
D_0	0.3	4.0	0.2	1.5	0.2	>4.0
D_1	0.5	2.8	1.0	6.0	1.8	4.5
v_0	0.0	>1.2	0.0	>1.2	0.6	1.2
v_1	<10.0	>25.0	<10.0	>25.0	10.0	25.0

3.2 ITB transition

The bifurcation curves representing ITB transition for each field are shown in figure 2. They demonstrate that there also exist transitions in ITB formation with respect to flux versus gradient space. The analysis results are similar to those discussed in section 3.1, as shown here in table 3. But it appears that the transition window for momentum field is as wide as the others. This confirms the experiment evidences that ITB formation is more related to toroidal momentum drive.

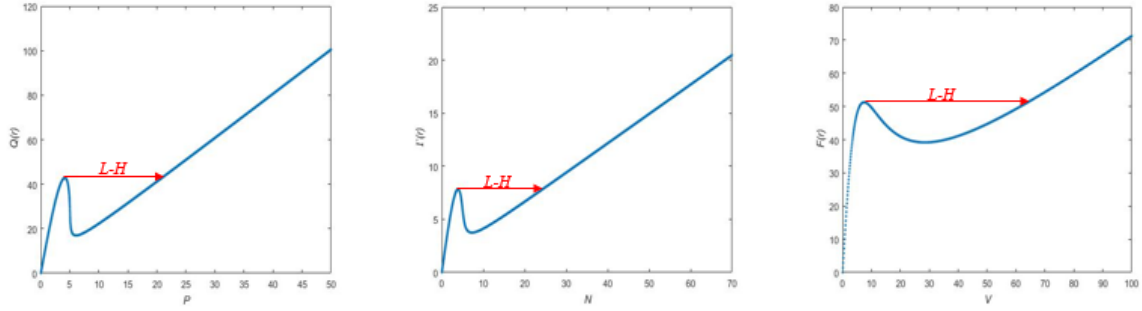


Figure 2. Illustrate ITB transition in thermal (left), particle (middle) and toroidal momentum density (right) fields.

Table 3. Show the criteria of ITB transition

Coefficient	Thermal		Particle		Momentum	
	MIN	MAX	MIN	MAX	MIN	MAX
χ_0	0.60	7.00	0.90	20.00	0.50	5.00
χ_1	4.00	40.00	3.00	30.00	7.50	>40.00
D_0	0.24	3.00	0.08	1.50	0.10	>3.00
D_1	0.30	2.90	1.00	10.00	< 0.30	>10.00
ν_0	0.40	2.00	0.10	1.50	0.45	1.00
ν_1	0.05	>31.00	0.05	>31.00	9.00	31.00

4. Conclusion

In this work, three transport equations are solved simultaneously and self-consistently. The transport effects include both neoclassical and anomalous transports, where the latter is suppressed by the flow shear. The results show that the bifurcation s -curve for L - H and ITB transitions are found either individually or simultaneously among all fields. The transitions of each field are possible only when the respective flux surpasses a threshold. The criteria of L - H and ITB transition are investigated, demonstrating that there exists certain ranges of transport coefficients where the transitions are possible. In particular, the L - H transition in momentum field is more challenging to achieve than that for ITB transition.

References

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