

Generation of spiral waves pinned to obstacles in a simulated excitable system.

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Abstract. Pinning phenomena emerge in many dynamical systems. They are found to stabilize extreme conditions such as superconductivity and super fluidity. The dynamics of pinned spiral waves, whose tips trace the boundary of obstacles, also play an important role in the human health. In heart, such pinned waves cause longer tachycardia. In this article, we present two methods for generating pinned spiral waves in a simulated excitable system. In method A, an obstacle is set in the system prior to an ignition of a spiral wave. This method may be suitable only for the case of large obstacles since it often fails when used for small obstacles. In method B, a spiral wave is generated before an obstacle is placed at the spiral tip. With this method, a pinned spiral wave is always obtained, regardless the obstacle size. We demonstrate that after a transient interval the dynamics of the pinned spiral waves generated by the methods A and B are identical. The initiation of pinned spiral waves in both two- and three-dimensional systems is illustrated.

1. Introduction

Spiral waves are dominant dynamical structures in excitable media [1]. They stay longer when their organizing center so-called spiral tips [2] are pinned to obstacles in the media [3-5]. Such pinned spiral waves are found to maintain the superconductor state [3] and the super fluidity [4]. For human health, spiral waves of electrical excitation are related to some cardiac arrhythmia like tachycardia and fibrillation which can lead to sudden cardiac death [6]. The tachycardia can last longer when the spiral waves are pinned to anatomical heterogeneities (e.g., veins or scars) [5]. In this article, we illustrate two different methods for generating pinned spiral waves in both 2D and 3D simulated excitable systems.

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2. Methods

We use the two-variable Oregonator model [7] which is a well-known model for excitable media. It describes two variables u and v , whose dynamics are governed by a set of reaction-diffusion equations

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon} \left(u - u^2 - fv \frac{u - q}{u + q} \right) + D_u \nabla^2 u, \quad (1)$$

$$\frac{\partial v}{\partial t} = u - v + D_v \nabla^2 v$$

The parameter values are set as in ref. [8]: $q = 0.002$, $f = 1.4$, diffusion coefficients $D_u = 1.0$ and $D_v = 0.6$, the ionic mobilities $M_u = -1.0$, $M_v = 2.0$ and $\varepsilon^{-1} = 100$. The spiral tip rotates around a circle of 0.9 system unit (s.u.) in diameter in the absence of obstacles. The Laplace operator in Eq. (1) is approximated using a 9-point and 27-point discretization for 2D and 3D systems, respectively [9]. The simulations are performed on a uniform grid space of $\Delta x = 0.1$ s.u. with a time step $\Delta t = 3.0 \times 10^{-3}$ time unit (t.u.) and no-flux boundary conditions are used at the boundaries of both the system and the obstacle.

Pinned spiral waves are initiated by two distinguish methods. In method A, the obstacle is set into the system before the generation of a spiral wave while in method B a spiral wave is generated before the obstacle is placed into the system.

3. Results

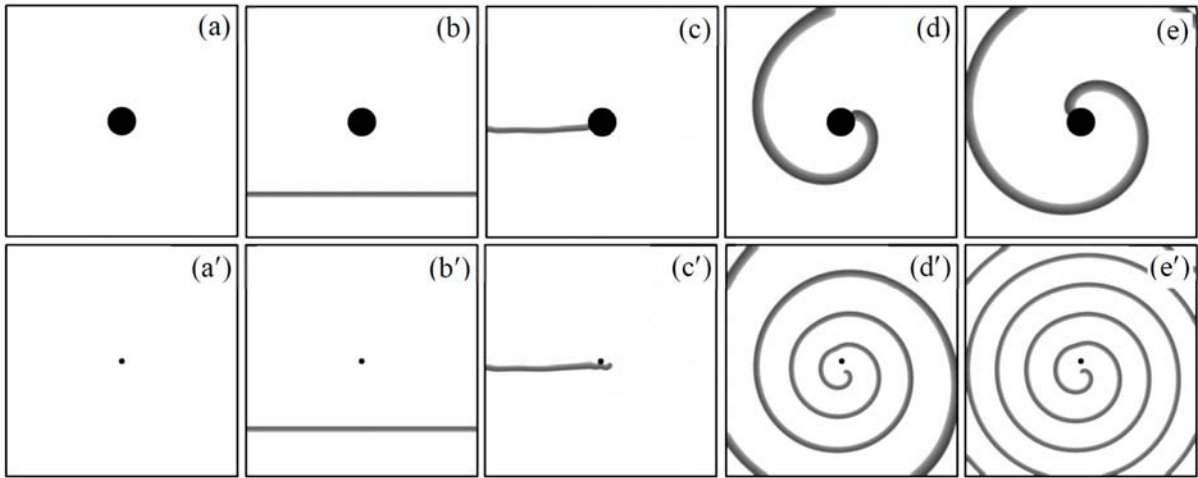


Figure 1. Sequential images of the initiation of pinned spiral waves by method A. (a)-(e) A spiral wave is successfully pinned to a large obstacle (5.0 s.u. in diameter) but (a')-(e') it fails to be pinned to a small obstacle (1.0 s.u. in diameter).

Figure 1 illustrates the generation of pinned spiral waves by using method A for a large and a small obstacle (top and bottom rows, respectively). At the beginning, the obstacles are set at the middle of the systems with a size of $80.0 \text{ s.u.} \times 80.0 \text{ s.u.}$ [figures 1(a) and 1(a')]. Then, a planar wave is triggered by setting a five-grid-point slab at the left edge of the medium to an excited state (e.g., $u = 1.0$ and $v = 0$ for $0.0 \leq x \leq 0.5$) [see in figures 1(b) and 1(b')]. When the wave front reaches the obstacle (around the middle of the medium), a half of the medium is reset to an excitable state (e.g., $u = 0$ and $v = 0$ at the time step = 275) leading to a planar wave with two ends attached to the edge of the obstacle and the system [figures 1(c) and 1(c')]. Subsequently, the spiral tip is remained attached to the large obstacle (5.0 s.u. in diameter), forming a pinned spiral wave [figures 1(d) and 1(e)]. In the other hand,

the spiral tip rotates near the small obstacle (1.0 s.u. in diameter) without pinning [figures 1(d') and 1(e')].

Initiation of pinned spiral waves by using method B is shown in figure 2. In the absence of obstacle, free spiral waves, as in figures 2(a) and 2(a'), are created at the middle of the medium by triggering a planar wave and subsequently cutting half of it as described above. Then an obstacle is placed very close to the spiral tip resulting in a pinned spiral wave around large [figures 2(b) - 2(e)] and small [figures 2(b') - 2(e')] obstacles. In the course of time, their wavelength, period and wave velocity transiently increase to stable values. As reported earlier [10], the wavelength, period and wave velocity of the pinned spiral waves increase with the obstacle size [e.g. compare the wavelength in figures 2(e) and 2(e')]. Note that after a transient interval the dynamics of the pinned spiral waves initiated by method A and B are identical for a given obstacle size [e.g. compare the wavelength in figures 1(e) and 2(e)].

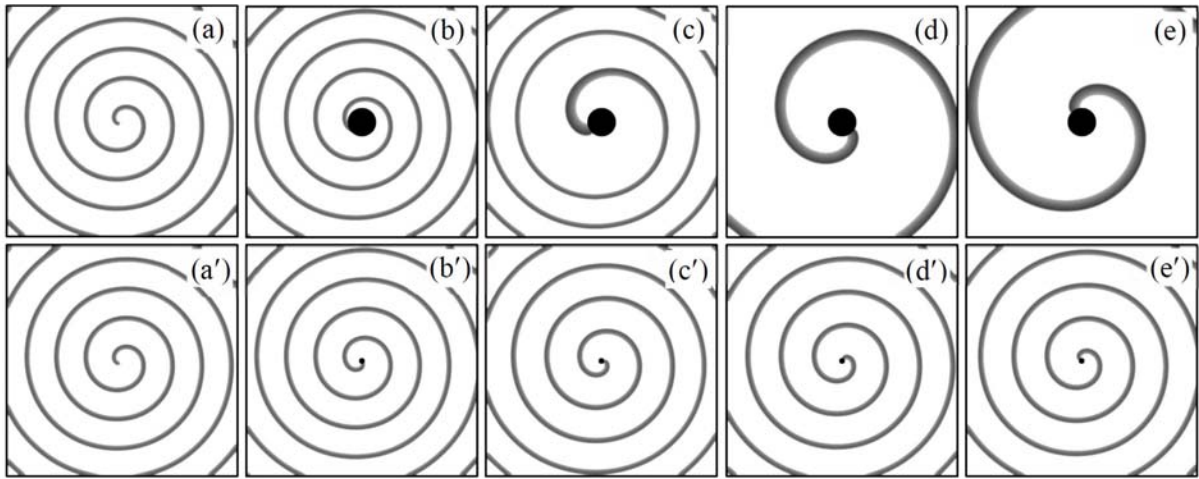


Figure 2. Sequential images of the initiation of pinned spiral waves by method B. (a)-(e) A spiral wave is successfully pinned to both a large obstacle (5.0 s.u. in diameter) and (a')-(e') a small obstacle (1.0 s.u. in diameter).

For 3D systems, the simulations are performed using $10.0 \text{ s.u.} \times 10.0 \text{ s.u.} \times 10.0 \text{ s.u.}$ in dimensions. In these cases, both the spiral waves and the obstacles used in the 2D cases are extended in the third dimension so that the obstacles become cylinders. We tested methods A and B for cylindrical obstacles with small and large diameters (as used in figures 1 and 2). As found in the 2D cases, method A is suitable only for pinning a spiral wave to a cylindrical obstacle with a large diameter while method B can be used to create a pinned spiral wave in both cases of small and large cylindrical obstacles.

Finally, we investigate a more complicated 3D situation so-called a partial pinning spiral wave, i.e., one part is pinned while the other part can freely move. Figure 3 shows the initiation of such partial pinned spiral waves by using method A and B. A cylinder with 1.6 s.u. in diameter acts as the obstacle. The length of the cylinder is set to a half of the system height. In method A, the obstacle is placed into the system and subsequently a spiral wave is created [figure 3(a)] using the same manner as shown in figure 1. As time goes on, the top part rotates freely while the bottom part pinned to the obstacle with a longer wave period, resulting in a cone-shaped wave structure [figure 3(b)]. In method B, a 3D spiral wave is initiated in the absence of the obstacle. Then the obstacle is inserted into the system [figure 3(c)]. Like in method A, the obstacle induces a change in the wave period of the pinned part and a cone-shaped wave structure [figure 3(d)].

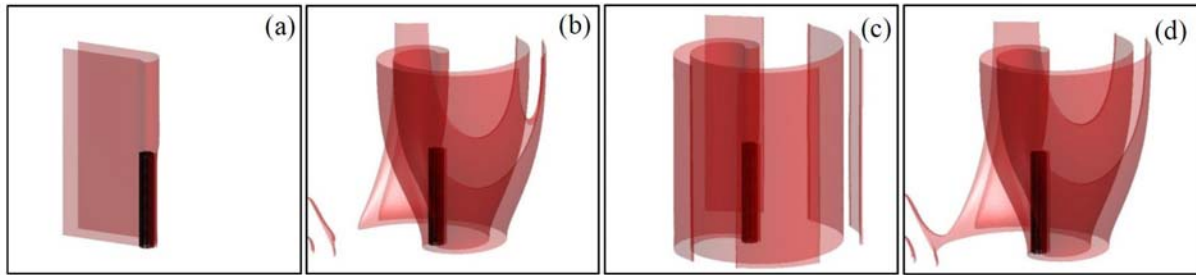


Figure 3. The initiation of partial pinned spiral waves in 3D systems by using (a-b) method A and (c-d) method B. The system size is $10.0 \text{ s.u.} \times 10.0 \text{ s.u.} \times 10.0 \text{ s.u.}$ and the cylindrical obstacle is 1.6 s.u. in diameter and 5.0 s.u. in length.

4. Discussion and Conclusion

We have presented two methods of initiation of pinned spiral waves in 2D and 3D simulated excitable systems. The two methods have different procedures of the ignition of spiral wave and the settle of obstacle, i.e., method A: setting an obstacle before creating a spiral wave and method B: adding an obstacle after creating a spiral wave. When the initiation is successful, the dynamics of pinned spiral waves generated by both methods are very similar. However, the initiation of pinned spiral waves by method A often fails especially for small obstacles. Therefore, we suggest using method B to initiate a pinned spiral waves.

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6. References

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