Effect of a modified sinusoidal forcing on spiral wave in a simulated reaction-diffusion system.

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Abstract. Spiral waves are often found in excitable media. In the hearts, they are abnormal forms of action potential propagation. Under an external forcing, the spiral waves drift and are subsequently terminated at the boundary. Spiral waves can be studied in simulations using a discrete reaction-diffusion system; thereby the time step must not exceed a numerical stability limit (t_s). In this article, we present the dynamics of spiral waves in a simulated system under an external forcing as a modified sinusoidal function of time. The spiral waves are forced to drift along a straight line with a velocity and an angle depending on the time step. An optimization study provides the optimal time step of $0.2t_s$, where further reductions of the time step do not alter the drifting of the spiral waves.

1. Introduction

Spiral waves are often found in excitable media, e.g., cell aggregation in slime mold colonies [1], during CO-oxidation on a platinum surface [2], electrical wave propagation in cardiac tissues [3], and the Belousov-Zabotinsky (BZ) reaction [4]. Action potential propagation in forms of spiral waves in heart tissues related to some cardiac arrhythmias and can lead to sudden cardiac death. Under an external forcing, the spiral waves drift and are subsequently terminated at the boundary of the medium. Drifting of spiral waves can be induced by an applied electric field and the drifting velocity and angle of the spiral waves increase with the field strength [5]. Under a sufficient strong field, the spiral waves are forced to hit the boundary and eventually annihilated [6].

Dynamics of spiral waves and the effect of forcing in excitable media are often studied in a discrete reaction-diffusion system [7,8]. To avoid errors from calculations, the time step used in the simulations must not exceed a numerical stability limit (t_s) [9]. In this article, we present the effect of the time step on the dynamics of spiral waves in a simulated system.

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2. Methods

In our numerical simulations, we use the two-variable Oregonator model [7] to describe the dynamics of the activator u and inhibitor v in excitable media. The advection terms for both *u* and *v* account for electric field for sinusoidal function E applied in x-direction.

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon}\left(u - u^2 - fv\frac{u-q}{u+q}\right) + D_u\nabla^2 u - M_u E\frac{\partial u}{\partial x}$$

$$\frac{\partial v}{\partial t} = u - v + D_v\nabla^2 v - M_v E\frac{\partial v}{\partial x}$$

(1)

As in References [8], the parameters are chosen as $\varepsilon = 0.01$, $q = 0.002$, $f = 1.4$, the diffusion coefficients $D_u = 1.0$ and $D_v = 0.6$, and the ionic mobilities $M_u = -1.0$ and $M_v = 2.0$. In the absence of an electric field, the tip of a free spiral wave rotates around a circular core (diameter = 0.9 s.u.).

The simulations are performed using the explicit Euler method with 9-point approximation of the two-dimensional Laplacian operator and a centered-space approximation of the gradient term. We use the uniform grid space $\Delta x = \Delta y = 0.1$ system unit (s.u.). The time step Δt is varied between 0.01 - 0.9 t_s, where the numerical stability limit $t_s = (3/8)(\Delta x)^2$ [9]. The dimensionless size of the system is 40 x 40 s.u. (400 x 400 grid points). To create a spiral wave, a planer wave is triggered by setting 5-grid-point strip at an edge of the medium to an excited state. The wave front is allowed to propagate into the middle of the medium, before half of the medium is reset to an excitable state. Then we apply the forcing *E* as a sinusoidal function of time t

$$E = \frac{E_0}{2}\sin\left(\frac{2\pi t}{T}\right) + \frac{E_0}{2}$$

(2)

where the amplitude E_0 and the period (*T*) are kept constant at 0.4, 0.3 t.u. We observe the dynamics of the spiral wave both with and without the applied forcing and search for the optimal time step i.e., the largest time step that further reduction of the time step does not change the spiral wave dynamics.

3. Results

In the absence of the applied forcing, the free spiral wave, as shown in figure 1a, rotates around a small circular circle. When the time step Δt is decreased, the wavelength is approximately constant while the wave period T_w are increased [figure 1 (b)] but the wave velocity v_w decreased as shown in figures 1(b) and 1(c). The results imply that the optimal time step is $\Delta t = 0.4\ t_s$ since the smaller time steps give very similar results.

The motion of a spiral wave under an applied sinusoidal forcing with the time step $\Delta t = 0.8\ t_s$ is shown in figure 2. The forcing causes the spiral wave whose tip is at the middle of the system [figure 2(a)], to drift along a straight path to the left with an angle with respect to the forcing direction pointing to the right [figures 2(a)-2(d)]. It is subsequently terminated at the boundary of the medium [figure 2(e)].

Figure 3 illustrates the detailed analysis of all simulations with an applied sinusoidal forcing. A graph of modified sinusoidal forcing (see equation 2) plotted against time is shown in figure 3(a). Even though the spiral wave drift linearly under the applied forcing, the drifting velocity and angle depend on the time step as shown in figures 3(b) and 3(c). When the time step Δt *is* decreased until $\Delta t = 0.2\ t_s$, both the drifting velocity and angle increase. However, reductions to smaller time steps do not change the drifting velocity and angle. Thus, the optimal time step of $\Delta t = 0.2\ t_s$ for the simulations with sinusoidal forcing.
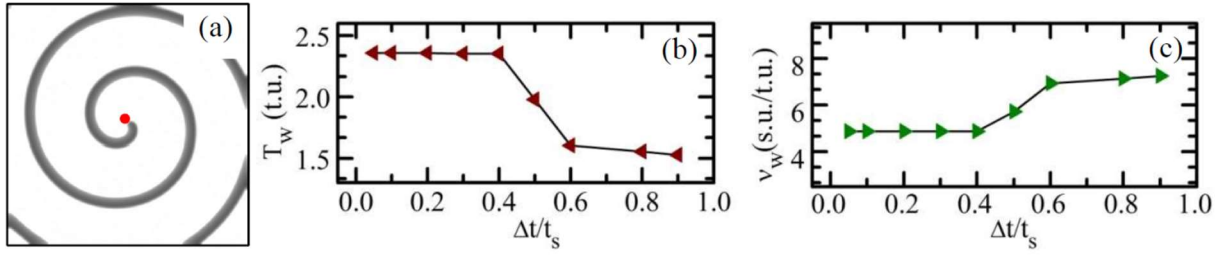
Figure 1. Dynamics of a spiral wave in the absence of applied forcing. (a) The free spiral wave rotates around a small circle (the filled circle) with (b) wave period T_w and (c) wave velocity v_w as a function of the time step Δt.
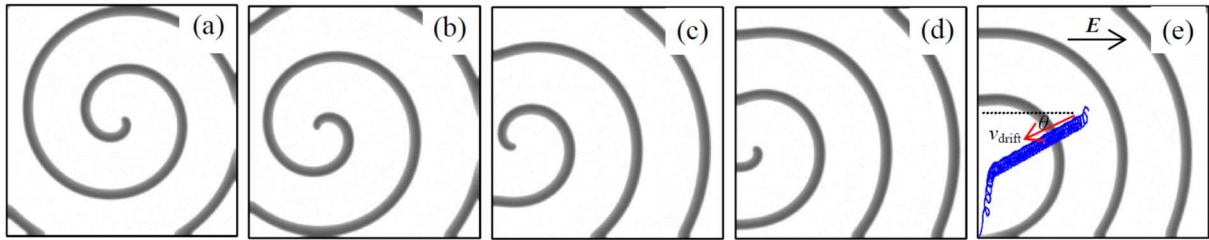

Figure 2. Sequential images of a spiral wave under a sinusoidal forcing. The spiral tip (a) initially located at the middle of the system (b-d) is gradually drifts to the left with an angle (e) until hits the boundary where the spiral wave is eliminated. The blue line indicates the drifting trajectory of the spiral tip. When the tip is located near the boundary, it drifts approximately with an angle θ_{drift} to the direction of forcing.
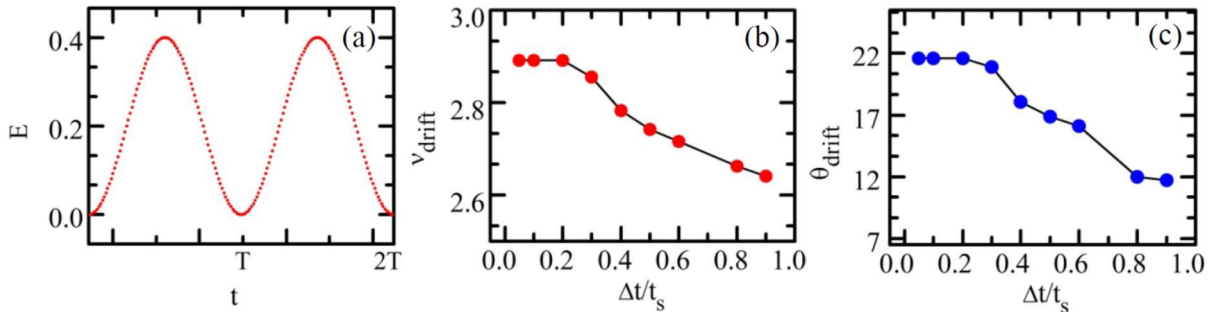

Figure 3. Effect of time step on the spiral wave under a sinusoidal forcing. (a) The sinusoidal forcing E vs time t. The spiral tip drifts with (b) velocity v_{drift} and (c) angle θ_{drift} depending of the time step Δt.

4. Discussion and Conclusion

We have presented an investigation on the effect of the time step on the dynamics of spiral waves at different time step in a simulated system. Both the motion of free spiral waves (in the absence of forcing) and the drifting of spiral waves under an applied sinusoidal forcing depend on the time step. The optimal time step in the case of time varying forcing $(0.2t_s)$ which is smaller than that for the simulations of free spiral waves $(0.4t_s)$. This comes from the fact that the applied time-dependent forcing is discretized and becomes a step-wise function which depends on the time step. Therefore, simulations of spiral waves under an applied forcing as a function of time should be performed with a fine time step to obtain promising results for comparison and confirmation of experimental results.

It has been shown that the lifetime of tachycardia is extended by pinning of spiral waves to anatomical obstacles like veins and scars [3] and the elimination of such pinned spiral waves become a tough task especially for large obstacles [10]. Even though we show that the sinusoidal forcing induce a linear drift of spiral waves very similar to the case of constant forcing, it is interesting to investigate

that whether an application of time varying forcing as in this article can improve the success of unpinning of spiral waves.

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6. References

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