

Unpinning of spiral waves from rectangular obstacles by stimulated wave trains

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Abstract. Pinned spiral waves are exhibited in many excitable media. In cardiology, lengthened tachycardia correspond to propagating action potential in forms of spiral waves pinned to anatomical obstacles including veins and scars. Thus, elimination such waves is important particularly in medical treatments. We present study of unpinning of a spiral wave by a wave train initiated by periodic stimuli at a given location. The spiral wave is forced to leave the rectangular obstacle when the period of the wave train is shorter than a threshold T_{unpin} . For small obstacles, T_{unpin} decreases when the obstacle size is increased. Furthermore, T_{unpin} depends on the obstacle orientation with respect to the wave train propagation. For large obstacles, T_{unpin} is independent to the obstacle size. It implies that the orientation of the obstacle plays an important role in the unpinning of the spiral wave, especially for small rectangular obstacles.

1. Introduction

Spiral waves have been observed in various excitable media [1-5]. In the heart tissues, the spiral waves are the origin of ventricular tachycardia and fibrillation which potentially leading to sudden cardiac death [6]. They will survive much longer if they are pinned to anatomical inhomogeneities or obstacles, e.g., dead cells or scars [1]. Cardioversion shocks for eliminating spiral waves from the heart are applications of high electrical voltage. The method results in damage of the cardiac tissues.

To avoid this problem, many studies have been performed to eliminate the spiral waves with low voltage, e.g., applications of wave trains. It is found that high-frequency wave trains induce drift of free spiral waves [7] and they can unpin spiral waves from circular obstacles [8,9]. Such drifting spiral waves are annihilated when the spiral tip hit the boundary.

In this article, we present an elimination of spiral waves pinned to rectangular obstacles with different orientations with respect to the direction of the applied wave trains in simulations by using the Oregonator model [10].

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2. Methods

In our simulations, the Oregonator model is used to describe the dynamics of the activator u and the inhibitor v in excitable media as shown in Eq. 1

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{1}{\varepsilon} \left(u - u^2 - fv \frac{u - q}{u + q} \right) + D_u \nabla^2 u, \\ \frac{\partial v}{\partial t} &= u - v + D_v \nabla^2 v.\end{aligned}\tag{1}$$

As in a study by Jahnke and Winfree [11], the parameters were chosen as $\varepsilon = 0.01$, $q = 0.002$, $f = 1.4$, and the diffusion coefficients as $D_u = 1.0$ and $D_v = 0.6$. For this parameter set, the system supported spiral waves with a circular spiral core of 0.9 space units (s.u.) in diameter, wavelength = 10.5 s.u., period = 1.55 time units (t.u.), and velocity = 6.76 s.u. t.u.⁻¹.

The variables u and v in Eq. 1 were calculated using an explicit Euler method with a 9-point approximation of the two-dimensional Laplacian operator on a discrete system of a dimensionless size = 40×40 s.u. with a uniform grid space of $\Delta x = \Delta y = 0.1$ s.u. and a time step $\Delta t = 3.0 \times 10^{-3}$ t.u., as required for numerical stability ($\Delta t \leq (3/8)(\Delta x)^2$ [12]). A single unexcitable rectangle was defined as the obstacle in each simulation. The boundaries of both the medium and the obstacle had no-flux conditions.

We tested totally twenty rectangles consist of two series: The first series (fixed height) 0.5×2.0, 1.0×2.0, 1.5×2.0, 2.0×2.0, 2.5×2.0, 3.0×2.0, 3.5×2.0, 4.0×2.0, 4.5×2.0 and 5.0×2.0 s.u.² and the second series (fixed width) 2.0×0.5, 2.0×1.0, 2.0×1.5, 2.0×2.0, 2.0×2.5, 2.0×3.0, 2.0×3.5, 2.0×4.0, 2.0×4.5 and 2.0×5.0 s.u.².

To create a spiral wave, a planar wave was triggered by setting a 5-grid-point strip at a boundary to an excited state ($u = 1.0$ and $v = 0.0$) and subsequently reset a half of it to an excitable state ($u = 0.0$ and $v = 0.0$) when it reaches the obstacle at the middle of the medium. A periodic wave train is simulated by setting a semicircular area (4.0 s.u. in diameter) to an excited state at the left edge of the medium.

3. Results

Figure 1 shows three examples of spiral waves eliminated by wave trains. The top row [figures 1(a)-1(d)] illustrates the phenomenon in the absence of obstacles. At the beginning, the tip of the free spiral wave rotates around a circular core (diameter 0.9 s.u.) at the middle of the medium [figure 1(a)]. When the fronts of the stimulated wave train with a period = 1.51 t.u. (slightly shorter than the period T_{sp} of the free spiral wave 1.55 t.u.) reach the tip of free spiral wave [figure 1(b)], the position of the spiral tip drifts to the right of the medium [see the red line in figure 1(c)] and then the spiral wave is terminated at the boundary [figure 1(d)].

In the middle row [figure 1(e)-1(h)], the spiral wave is initially pinned to a rectangular obstacle with 2.0 s.u. × 0.5 s.u. in dimensions [figure 1(e)]. This pinned spiral wave has a period T_{sp} of 1.59 t.u. which longer than that of the free spiral wave, as reported earlier [13]. To unpin the spiral wave i.e. its tip is detached from the obstacle [figure 1(g)], the period of the wave train must be lower than a critical value called T_{unpin} of 1.48 t.u. which is shorter than that in the case of the free spiral wave (the top row). Eventually, the spiral wave is forced to be annihilated at the boundary. Note that this pinned spiral wave is harder to be eliminated in comparison to the free spiral wave since the wave train is required to have a shorter period.

In the bottom row [figure 1(i)-1(l)], the spiral wave is pinned to a rectangular obstacle with 0.5 s.u. × 2.0 s.u. in dimensions [figure 1(e)]. This obstacle has the same size but different orientation to that in the middle row so that these two pinned spiral waves has the same wave period. Surprisingly, the unpinning of spiral wave occurs at a period of the wave train $T_{unpin} = 1.40$ t.u., i.e., shorter than that in the middle row.

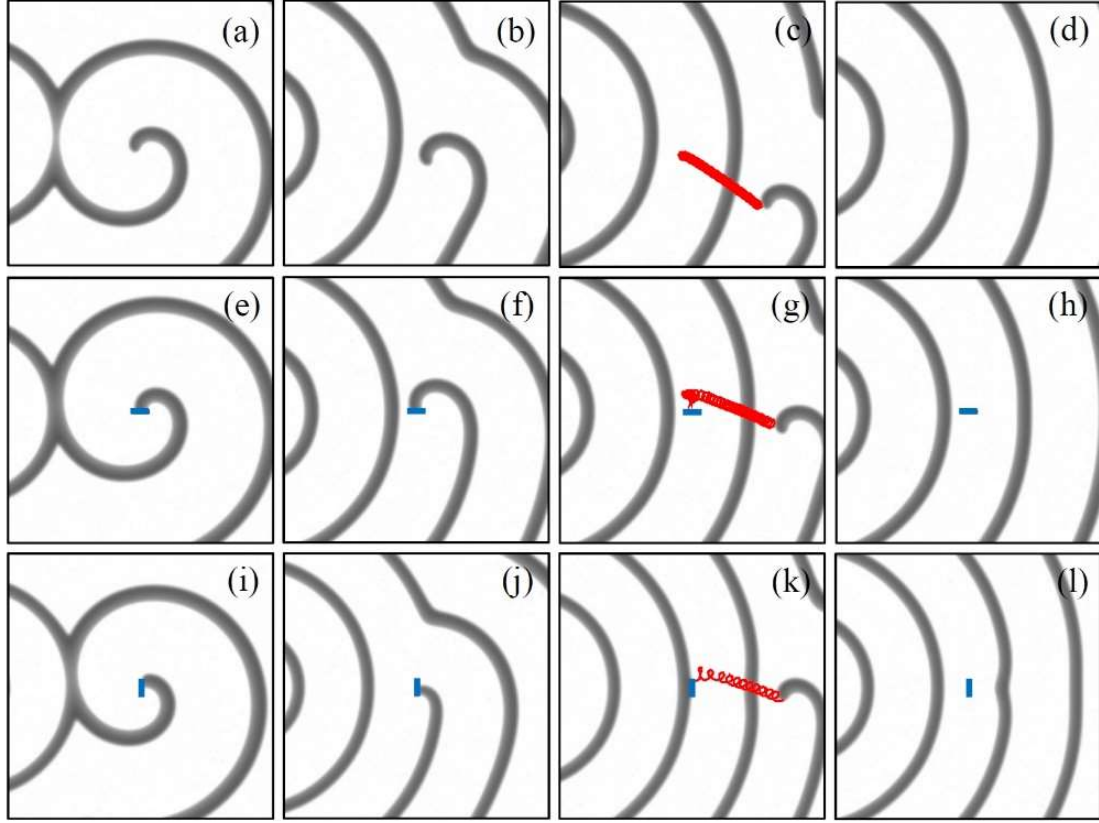


Figure 1. Elimination of spiral waves by a wave train application: (a)-(d) A free spiral wave is forced to drift and be annihilated by a wave train with a period of 1.51 t.u., Pinned spiral wave are forced to unpin from rectangles with dimensions (e)-(h) 2.0 s.u. \times 0.5 s.u. and (i)-(l) 0.5 s.u. \times 2.0 s.u. and subsequently to be annihilated by wave trains with different periods of 1.48 t.u. and 1.40 t.u. The red paths indicate the trajectories of the spiral tips.

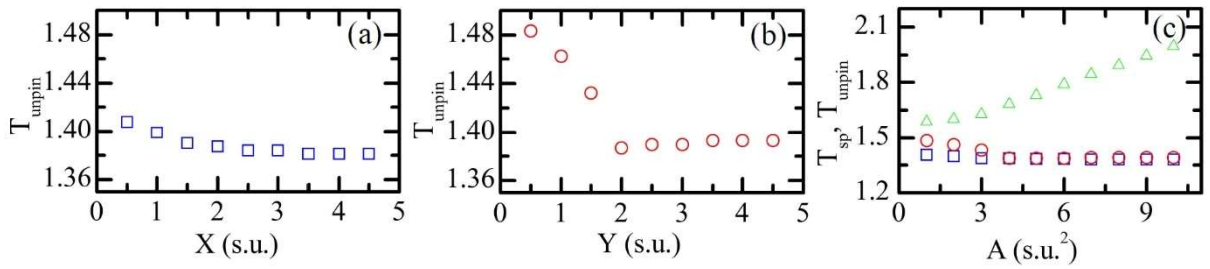


Figure 2. The critical period T_{unpin} of the wave train for unpinning of spiral wave vs the obstacle (a) width x , (b) height y , and (c) area A with squares and circles indicating T_{unpin} taken from (a) and (b), respectively. Triangles in (c) represent T_{sp} the period of the spiral waves pinned to different obstacles.

The detailed results of the unpinning of spiral waves by wave trains in our simulations are presented in figure 2. For the first series of obstacles with a fixed height of 2.0 s.u., the critical value of wave train for unpinning the spiral wave T_{unpin} decreased slightly while the obstacle width X is increased [figure 2(a)]. The simulations with the second series of obstacles with a fixed width of 2.0 s.u. give different results [figure 2(b)]. As the obstacle height Y is increased, T_{unpin} decreased very much for $Y < 2.0$ s.u. but T_{unpin} is approximately constant for $Y > 2.0$ s.u.

To compare results from all obstacles, T_{unpin} is plotted against the obstacle area A [circles and squares in figure 2(c)]. Clearly, in these unpinning phenomena the obstacle height is more crucial in

comparison to the obstacle width especially for small rectangle obstacles. This implies that for such small rectangles, the success of spiral elimination depends on also the orientation of the obstacles with respect to the wave train propagation.

Finally, the plot of the period T_{sp} of pinned spiral waves [triangles in figure 2(c)] shows that it increases with the obstacle size as described earlier in [13]. Termination of these pinned spiral waves is distinguish from the case of free spiral waves which can be illuminated by a wave train with a period slightly shorter than T_{sp} of the free spiral waves. When the obstacle size is increased, the difference between T_{sp} and T_{unpin} also increases [compare triangles to circles and squares in figure 2(c)].

4. Discussion and Conclusion

We have presented an investigation on the termination of spiral waves pinned to unexcitable rectangular obstacles with different sizes and orientations by an application of wave trains. The results show that the elimination of the pinned spiral wave requires the period of the wave train T_{unpin} shorter than in the case of the free spiral wave. For a given small rectangular obstacle, its orientation with respect to the wave train propagation also plays an important role, i.e., the longer rectangle facing to the wave train, the shorter T_{unpin} . For the case of large obstacles, T_{unpin} is almost independent to the size of the rectangles.

Recently, Tanaka et al. [8,9] presented simulations of spiral unpinning from circular obstacles by wave trains. The pinned spiral waves have a period, wavelength, and velocity that increase with the obstacle diameter. The period T_{unpin} of the wave train for unpinning decreases when the obstacle diameter is increased. Based on these results, they proposed a theory to describe the relation of the velocity of the pinned spiral wave and the required period T_{unpin} . The results of spiral unpinning from rectangular obstacles, presented in our study, are more complicated so that we suggest further investigations on this topic to obtain deeper understanding which leads to a generalization of the unpinning theory.

5. Acknowledgments

We thank the Faculty of Science, the Research and Development Institute (KURDI), the Center for Advanced Studies of Industrial Technology, and the Graduate School, Kasetsart University; and the Office of the Higher Education Commission and King Mongkut's University of Technology North Bangkok (contract no. KMUTNB- NRU-58-03) for financial support.

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