Classical Nuclear Simulations of the $\pi^+(^3He, ppn)\pi^+$ Reactions with Quantum Corrections

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Abstract

We investigated quantum corrections to classical nuclear simulations of the $\pi^+(^3He, ppn)\pi^+$ reactions.These simulations are often used to describe nuclear reactions which lead to many final states.The ratio of the quantum multiple scattering to the classical cross section for the same process is used as a correction to the classical model calculation.The single,double,triple and all scatterings for the scattering protons of different angles are presented.

Introduction

To solve Schrödinger equation for many-body scattering problems is difficult, for this reason the quantum probabilities are often replaced with classical ones. The idea was first suggested by Serber[1] and implemented in the very early days of computers by Goldberger[2]. Metropolis *et al.*[3] used pion beams as incident projectiles. RQMD [4],[5] and the Liège [6],[7],[8] code used proton and antiproton beams for treating intermediate energy heavy ion reactions. The code developed by the group at Valencia [9] is capable of treating proton and pion projectiles. Almost all of these models rely on the treatment of the scattering from the point of view of classical probabilities. In this work we investigated the consequences of the quantum effects on the reactions.

Pion Interaction with Nucleons

The pion-nucleon scattering reactions can be either elastic scattering or charge exchange. There are ten possible reactions:

$$\pi^+ p \to \pi^+ p \qquad (1)$$
$$\pi^- p \to \pi^- p \qquad (2)$$
$$\pi^- p \to \pi^0 n \qquad (3)$$
$$\pi^+ n \to \pi^+ n \qquad (4)$$
$$\pi^+ n \to \pi^0 p \qquad (5)$$
$$\pi^- n \to \pi^- n \qquad (6)$$
$$\pi^0 p \to \pi^0 p \qquad (7)$$
$$\pi^0 n \to \pi^0 n \qquad (8)$$
$$\pi^0 n \to \pi^- p \qquad (9)$$
$$\pi^0 p \to \pi^+ n\ . \qquad (10)$$

In our model we allowed pion interacts with one nucleon once, thus there can only be single, double or triple scattering.

Single Scattering

The pion single scattering amplitude operator can be written as

$$F_S(\mathbf{k}, \mathbf{k}', \mathbf{r}) = e^{-i\mathbf{k}'\cdot\mathbf{r}} f(\mathbf{k}, \mathbf{k}') e^{i\mathbf{k}\cdot\mathbf{r}} \quad .$$

Which can be interpreted as an incident plane wave with wave number $\mathbf{k}$ which enters and interacts with the particle at $\mathbf{r}$, with amplitude $f(\mathbf{k}, \mathbf{k}')$ then leaves as a plane wave $e^{-i\mathbf{k}'\cdot\mathbf{r}}$.

Double Scattering

The pion double scattering amplitude operator can be written as [10]

$$F_D(\mathbf{k}, \mathbf{k}', \mathbf{r}_1, \mathbf{r}_2) =$$

$$\frac{1}{2\pi^2}\int_0^\infty d\mathbf{q} \frac{e^{-i\mathbf{k}'\cdot\mathbf{r}_2} f_2(\mathbf{q}, \mathbf{k}') e^{i\mathbf{q}\cdot(\mathbf{r}_2-\mathbf{r}_1)} f_1(\mathbf{k}, \mathbf{q}) e^{i\mathbf{k}\cdot\mathbf{r}_1}}{q^2 - k^2} \quad .$$

Which can be read from right to left as, the incident plane wave with wave number $\mathbf{k}$ enters and interacts with the first particle at $\mathbf{r}_1$, with amplitude $f_1(\mathbf{k}, \mathbf{q})$, then propagates from $\mathbf{r}_1$ to $\mathbf{r}_2$ as expressed by the propagator $\frac{e^{i\mathbf{q}\cdot(\mathbf{r}_2-\mathbf{r}_1)}}{q^2-k^2}$, interacts with the second particle at $\mathbf{r}_2$ with amplitude $f_2(\mathbf{q}, \mathbf{k}')$ then leaves as a plane wave $e^{-i\mathbf{k}'\cdot\mathbf{r}_2}$.

Triple Scattering

The pion triple scattering amplitude operator can be written as

$$F_{triple}(\mathbf{k}, \mathbf{k}', \mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) =$$

$$\frac{1}{(2\pi^2)^2}\int_0^\infty d\mathbf{q}d\mathbf{q}' e^{-i\mathbf{k}'\cdot\mathbf{r}_3} f_3(\mathbf{q}', \mathbf{k}') \frac{e^{i\mathbf{q}'\cdot(\mathbf{r}_3-\mathbf{r}_2)}}{q'^2 - k_{23}^2} \times$$

$$f_2(\mathbf{q}, \mathbf{q}') \frac{e^{i\mathbf{q}\cdot(\mathbf{r}_2-\mathbf{r}_1)}}{q^2 - k_{12}^2} f_1(\mathbf{k}, \mathbf{q}) e^{i\mathbf{k}\cdot\mathbf{r}_1} \quad .$$

The three πN amplitudes are

$$f_1(\mathbf{k},\mathbf{q}) = v(k)v(q)[\lambda_0(1) + \lambda_1(1)\mathbf{k}\cdot\mathbf{q}]$$
$$f_2(\mathbf{q},\mathbf{q}') = v(q)v(q')[\lambda_0(2) + \lambda_1(2)\mathbf{q}\cdot\mathbf{q}']$$
$$f_3(\mathbf{q}',\mathbf{k}') = v(q')v(k')[\lambda_0(3) + \lambda_1(3)\mathbf{q}'\cdot\mathbf{k}'] \quad .$$

The index 1, 2, 3 on f and λ denotes that the scattering will occur at different energies and can be of different types (elastic or charge exchange). To evaluate this integral we make use of the angular-momentum expansion of the scattering amplitude,

$$\lambda_0(i) + \lambda_1(i)\mathbf{q}\cdot\mathbf{q}' = 4\pi\sum_{l=0}^{1}\sum_{m}\frac{\lambda_l(i)q^l q'^l}{(2l+1)}Y_l^m(\hat{\mathbf{q}})Y_l^{m^*}(\hat{\mathbf{q}}') \quad .$$

Bauer's series expansion for the plane wave,

$$e^{i\mathbf{q}\cdot\mathbf{R}} = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l} i^l Y_l^m(\hat{\mathbf{q}})Y_l^{m^*}(\hat{\mathbf{R}})j_l(qR)$$

and technique of contour integration. The triple scattering amplitude finally can be written as

$$F_{triple} = \frac{(4\pi)^3}{(2\pi^2)^2}e^{-i\mathbf{k}'\cdot\mathbf{r}_3}e^{i\mathbf{k}\cdot\mathbf{r}_1}\times$$
$$\sum_{l=0}^{1}\sum_{m}\frac{\lambda_l(2)}{(2l+1)}\times$$
$$\sum_{l'=0}^{1}\sum_{m'}\frac{\lambda_{l'}(1)k^{l'}}{(2l'+1)}Y_{l'}^{m'^*}(\hat{\mathbf{k}})\sum_{l''=0}^{\infty}\sum_{m''=-l''}^{l''} i^{l''}$$
$$\times Y_{l''}^{m''}(\hat{\mathbf{R}}_{12})\sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2l''+1)}}C_{ll'l''}^{000}C_{ll'l''}^{mm'm''}$$
$$\times\frac{1}{4}(\alpha^2+k_{12}^2)^2\{Q_{ll'l''}^{+}(-k_{12},\alpha,R_{12}) - Q_{ll'l''}^{-}(-k_{12},\alpha,R_{12})\}\times$$

a similar factor from the second and third scatterings. $Q_{ll'l''}^{+}(-k,\alpha,R)$ has been defined as $Q_{ll'l''}^{+}(-k,\alpha,R) =$

$$\frac{2\pi i}{(\alpha^2+k^2)^2}\left[\frac{(-k)^{l+l'+2}h_{l''}^{+}(-kR)}{(-2k)}\right.$$
$$-\frac{1}{4}(i\alpha)^{l+l'}\left\{(\alpha^2+k^2)R\frac{dh_{l''}^{+}(qR)}{d(qR)}|_{q=i\alpha}\right.$$
$$\left.\left.+\frac{h_{l''}^{+}(i\alpha R)}{(i\alpha)}\left[(\alpha^2+k^2)(l+l'+1) - 2\alpha^2\right]\right\}\right]$$

$C_{ll'l''}^{mm'm''}$ is Clebsch-Gordon coefficients, $Y_{l''}^{m''}$ is spherical harmonics and $h_{l''}^{+}$ is spherical Hankel function.

The Quantum Correction

We take the ratio of the quantum cross section to the classical cross section as a correction to the weight, w, in the code. The new weight W is then given by

$$W = w\cdot\frac{\sigma_{qm-n}}{\sigma_{cl-n}}$$

where σ_{qm-n} is the quantum cross section for n scatterings(n=1,2,3)

$$\sigma_{qm-3} = |<\sigma_1',\sigma_2',\sigma_3'|\mathbf{F}(\mathbf{k}_1,\mathbf{k}_3)|\sigma_1,\sigma_2,\sigma_3>|^2$$
$$\equiv \left|\mathbf{F}_{\sigma_1,\sigma_2,\sigma_3}^{\sigma_1',\sigma_2',\sigma_3'}(\mathbf{k}_1,\mathbf{k}_3)\right|^2$$

Simililary for n=1 and 2.
σ_{cl-n} is the classical cross section for n scatterings.
For single scattering the weight does not change since $\sigma_{qm-1} = \sigma_{cl-1}$.
For double scattering

$$\sigma_{cl-2} = \frac{AB}{|\mathbf{r}_2-\mathbf{r}_1|^2}$$

where

$$A = |\lambda_0(1) + k_1k_{12}\lambda_1(1)x|^2 + |k_1k_{12}\lambda_2(1)|^2(1-x^2)$$

$$B = |\lambda_0(2) + k_{12}k_2\lambda_1(2)x'|^2 + |k_{12}k_2\lambda_2(2)|^2(1-x'^2)$$

and

$$x = \hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_{12}\ ;\ x' = \hat{\mathbf{k}}_{12}\cdot\hat{\mathbf{k}}_2 \quad .$$

For triple scattering

$$\sigma_{cl-3} = \frac{ACD}{|\mathbf{r}_2-\mathbf{r}_1|^2|\mathbf{r}_3-\mathbf{r}_2|^2}$$

where

$$C = |\lambda_0(2) + k_{12}k_{23}\lambda_1(2)y'|^2 + |k_{12}k_{23}\lambda_2(2)|^2(1-y'^2)$$

$$D = |\lambda_0(3) + k_{23}k_3\lambda_1(3)y''|^2 + |k_{23}k_3\lambda_2(3)|^2(1-y''^2)$$

and

$$y' = \hat{\mathbf{k}}_{12}\cdot\hat{\mathbf{k}}_{23}\ ;\ y'' = \hat{\mathbf{k}}_{23}\cdot\hat{\mathbf{k}}_3 \quad .$$

The quantum corrections are shown in Figure1 and Figure2.

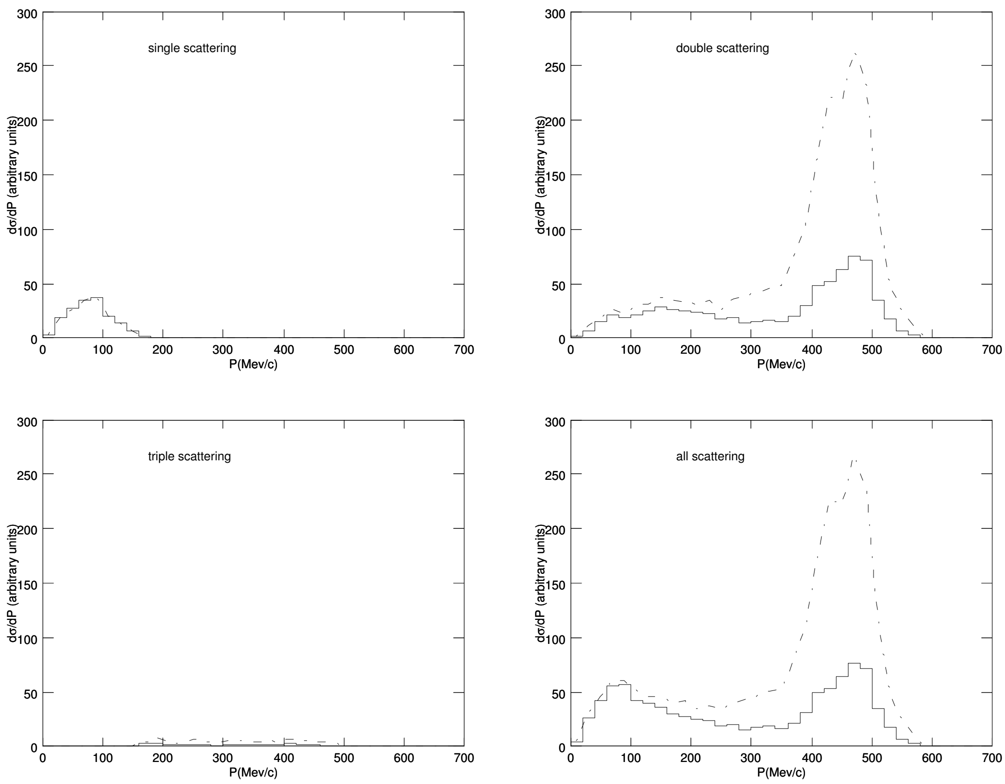

Figure 1: Comparison of quantum vs. classical $\pi^+(^3He, ppn)\pi^+$ reactions for single, double, triple, and all scatterings. The solid curve shows the result of the classical calculation and the dashed curve include quantum corrections. The spectra shown are for protons at 0^o- 30^o coincident with the pion at 0^o- 20^o.

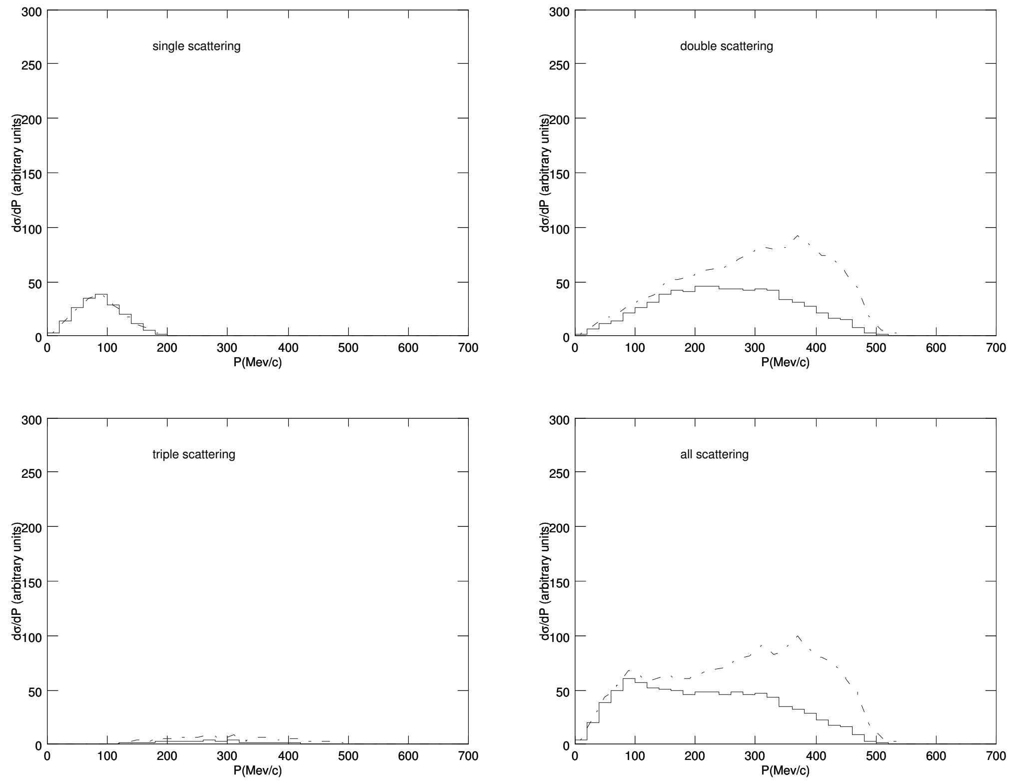

Figure 2: Comparison of quantum vs. classical $\pi^+(^3He, ppn)\pi^+$ reactions for single, double, triple, and all scatterings. The solid curve shows the result of the classical calculation and the dashed curve include quantum corrections. The spectra shown are for protons at 30^o- 60^o coincident with the pion at 0^o- 20^o.

Conclusion

The quantum corrections to classical nuclear simulation for $\pi^+(^3He, ppn)\pi^+$ reactions are visible in certain regions. In our case it is the region where pion is observed between 0-20 degrees and the forward protons are observed between 0-30 degrees. The double scattering contributes most.

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