Effect of thick barrier in a gapped graphene Josephson junction

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Abstract. We study the Josephson effect in a gapped graphene-based superconductor/barrier/superconductor junction as using the Dirac-Bogoliubov de Gennes (DBdG) equation for theoretical prediction. A massive gap of this regime is induced by fabricating a monolayer graphene on substrate-induced bandgap and superconductivity is acquired by the proximity effect of conventional superconductor (s-wave superconductor) through top gate electrodes. This Josephson junction is investigated in case of thick barrier limit that is pointed out the effect of applying a gate voltage V_G in the barrier. We find that the switching supercurrent can be controlled by the gate voltage V_G and the effect of thick barrier can influence the switching linear curve. When the barrier is adjusted to manner of a potential well which is inside the range of $-mv_F^2 \leq V_G \leq 0$, the supercurrent in thick barrier case is examined to the same behavior as the thin barrier case. The controlling supercurrent through the electrostatic gate is suitable for alternative mechanism into experimental test.

1. Introduction

Graphene is a two-dimensional (2D) single atomic layer that is exfoliated from a stacked layer of graphite linked by van der Waal forces. The existence of graphene has been revealed by Novoselov et al. via micromechanical cleavage on 300-nm SiO_2 [1]. The anomalous properties of graphene are demonstrated to both the electronic [2] and physical [3] properties, e.g., the high mobility of carrier, the half-integer quantum Hall effect, the elastic property, the visual transparency and Klein tunneling. This novel material is expected to the future-nanoelectronic devices but the absence of a bandgap in graphene is the obstacle for developing the devices. Consequently, the epitaxial graphene on substrate-induced bandgap, such as silicon carbide (SiC) [4] and hexagonal boron nitride (h-BN) [5], is deposited for opening the bandgap of monolayer graphene through interacting between graphene and substrate at which the equivalent sublattices in unit cell are broken. The opened gap can be used for tuning off conductance and current that is controlled easily through an electrostatic gate [6] that is suitable for creating transistors and high-speed integrated circuits [7].

In this work, we investigate the Josephson current in a gapped graphene-based superconductor/barrier/superconductor (S/I/S) junction. The fabrication of superconductor on graphene is created by contacting the superconducting electrodes as mostly using aluminum (Al) electrodes [8-10]. The Josephson effect in graphene-based junction is observed in the Dirac fermionic behavior through a gate control [11-12]. The gate-tunable effect can be indicated to the switching current and is led to the application of superconducting electronic devices [10, 12]. We concentrate on the gate-tunable supercurrent under the effect of thick barrier limit. The influence of massive particle behavior is led to the switching current when varying the gate voltage V_G and the Fermi energy E_F.

2. Model and formalism

In our model, we illustrate a gapped graphene Josephson junction in Fig. 1(a), deposited on a substrate-induced bandgap. On the monolayer graphene, these electrodes in the junction can be fabricated by lithography technique [13]. The conventional superconductors under the proximity-induced superconducting state of electrodes are separated by an insulating barrier from $x = 0$ to $x = L$ as the local barrier is controlled via an electrostatic potential V_G.
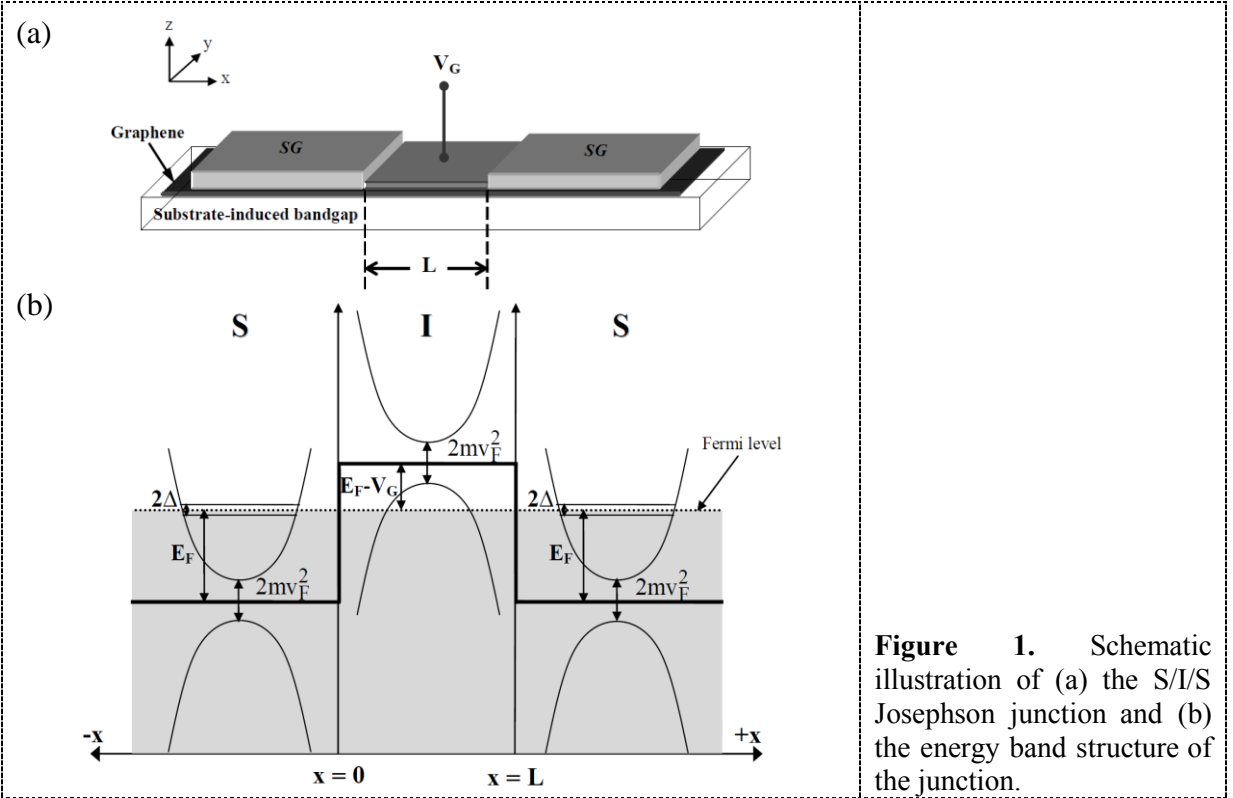
Figure 1. Schematic illustration of (a) the S/I/S Josephson junction and (b) the energy band structure of the junction.

The electron and hole behaviors in gapped graphene are investigated merely at the only K point under the mean-field condition that satisfies with $E_F \gg \Delta_0$, where E_F and Δ_0 are the Fermi energy and the superconducting gap at temperature T = 0 K. We can understand the behavior of these quasiparticles with the massive Dirac-Bogoliubov-de Gennes (DBdG) solutions governed by the DBdG equation [14]

$$\begin{bmatrix} H_0 + U(x,y) & \Delta(x,y) \\ \Delta^*(x,y) & -H_0 - U(x,y) \end{bmatrix} \Psi(x,y) = E\Psi(x,y), \tag{1}$$

where the single-particle Hamiltonain is $H_0 = -i\hbar v_F (\sigma_x \partial_x + \sigma_y \partial_y) + \sigma_z m v_F^2$ as the σ_x, σ_y and σ_z are the Pauli spin matrices, the wave function is formed to $\Psi(x,y) = \psi(x) e^{ik_y y}$, and the pair potential in the superconducting regions is $\Delta(x,y) = \Delta e^{i\phi_L} \Theta(-x) + \Delta e^{i\phi_R} \Theta(x-L)$.

The Boundary conditions are considered at $x = 0$ and $x = L$ interfaces for solving the nonzero solution as assuming E_F, $m v_F^2 \gg E, \Omega = \sqrt{E^2 - \Delta^2}$. Under these boundary conditions, the wave solutions are matched in the conditions. We get the Andreev bound state,

$$E = \Delta(T) \sqrt{1 - \tau(\theta) \sin^2 [\frac{\phi}{2}]}, \qquad (2)$$

where the transmission probability $\tau(\theta) = 4 A_I^2 A_S^2 \cos^2[\theta] \cos^2[\theta_I] / (\alpha_1 + \alpha_2)$,

$$\alpha_1 = \sin^2[k_I L \cos[\theta_I]] \left\{ A_I^2 + A_S^2 - 2 A_I A_S \sin[\theta] \sin[\theta_I] \right\}^2 \text{ and}$$

$$\alpha_2 = 4 A_I^2 A_S^2 \cos^2[k_I L \cos[\theta_I]] \cos^2[\theta] \cos^2[\theta_I].$$
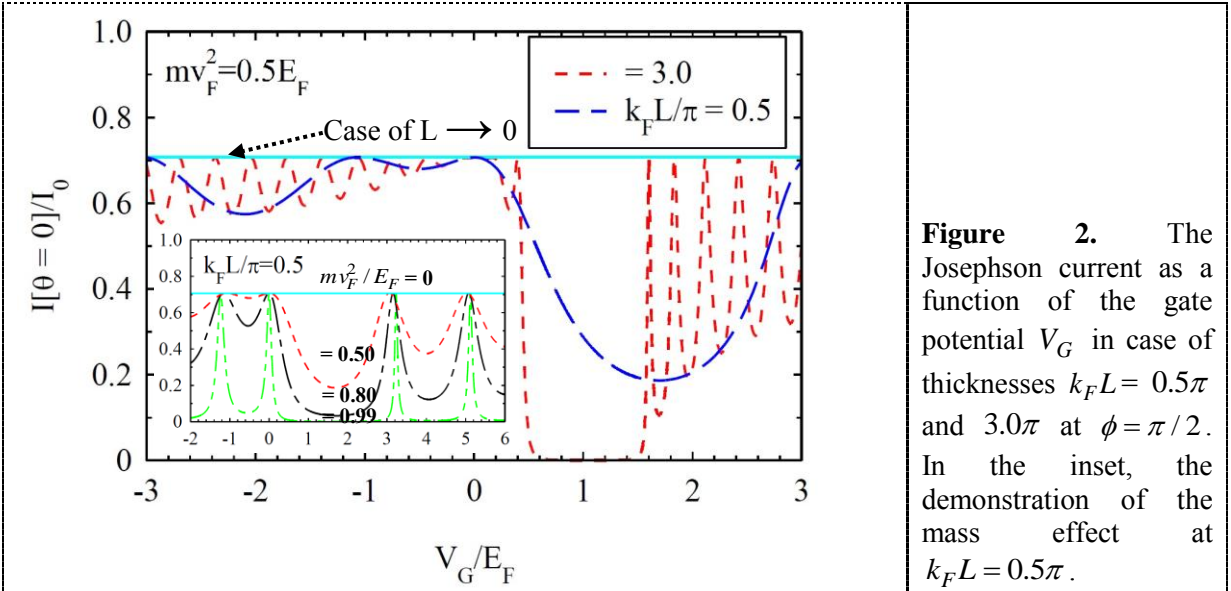
The Josephson current considered only on K valley can be expressed by [14-15]

$$I(\theta, \phi) = -\frac{2e}{\hbar} \left(\frac{\partial E}{\partial \phi} \tanh \left[\frac{E}{2 k_B T} \right] \right) \qquad (3)$$

The total Josephson current is calculated by the summation of all incident angles in the S/I/S junction.

3. Result and discussion

We first investigate the effect of thick barrier in a gapped graphene Josephson junction as setting the temperature $T = 0$ K and the phase difference $\phi = \pi/2$.

Figure 2. The Josephson current as a function of the gate potential V_G in case of thicknesses $k_F L = 0.5\pi$ and 3.0π at $\phi = \pi/2$. In the inset, the demonstration of the mass effect at $k_F L = 0.5\pi$.

In Fig. 2, the dependence of the Josephson current on the gate voltage V_G is studied by varying the different thicknesses at the rest-mass value of massive fermions $m v_F^2 = 0.5 E_F$ and the incident angle $\theta = 0$. We find that the oscillating supercurrent is examined by tuning the electrostatic gate V_G and

this oscillation is dropped with increasing the barrier length from $k_F L = 0.5\pi$ to 3.0π. This is the effect of the propagating wave function through the gap region of barrier sandwich. When the spacing barrier approaches to zero ($L \to 0$), the behavior of supercurrent flow is independent with applying the gate potential. We can clarify this behavior by considering the reduced form of the Andreev energy level in Eq. (2) that one gets

$$E(L \to 0) = \Delta_0 \cos[\phi / 2] . \tag{4}$$

The presence of this bound state in gapped graphene is similar to that in gapless graphene [16]. For the mass effects shown in the inset of Fig. 2, the supercurrent is strong oscillation for increasing the masses $m v_F^2 = 0$, $0.50 E_F$, $0.80 E_F$ and $0.99 E_F$, respectively.
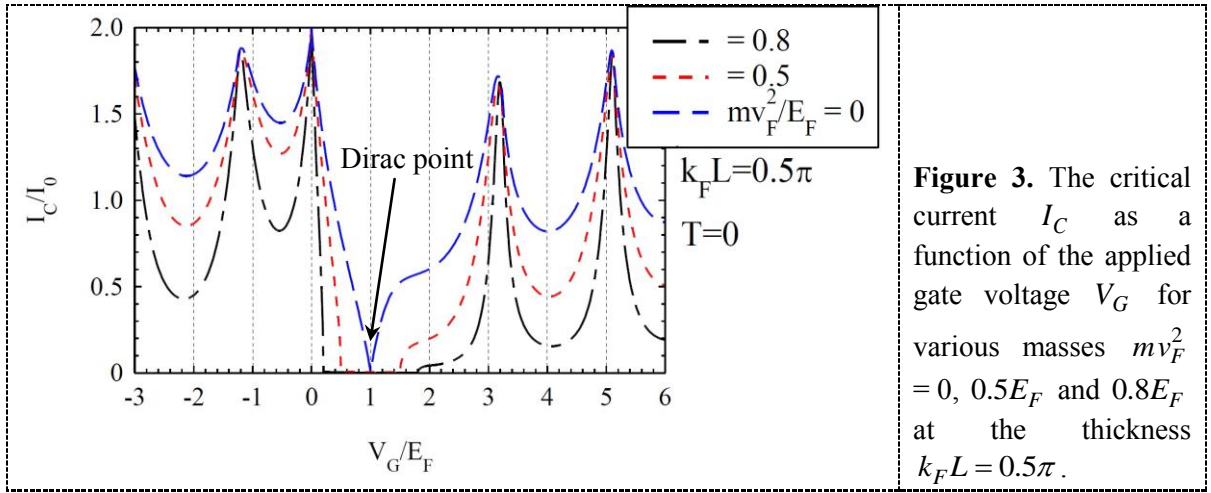
Figure 3. The critical current I_C as a function of the applied gate voltage V_G for various masses $m v_F^2 = 0$, $0.5 E_F$ and $0.8 E_F$ at the thickness $k_F L = 0.5\pi$.

The gate-dependent critical current is investigated for the massless and massive cases, shown in Fig. 3. This consideration is defined at the thickness barrier $k_F L = 0.5\pi$ which is in range of the value suggested for experimental setup [15]. We find that the critical current in the gapped graphene junction can be suppressed in the bandgap of graphene by applying the gate voltage V_G in the range of

$1 - (m v_F^2 / E_F) < \dfrac{V_G}{E_F} < 1 + (m v_F^2 / E_F)$. These carriers in the gap region are evanescent mode that is

the imaginary wave vector, i.e., $k_I = i \sqrt{(m v_F^2 / E_F)^2 - (1 - (V_G / E_F))^2} \big/ \hbar v_F$. For the transfer of the energy level in the conduction band, we can see that in the range of $0 < V_G \leq E_F - m v_F^2$ which is between the undoped voltage ($V_G = 0$) and the bottom conduction band. This is indicated to linear switching of supercurrent. Moreover, the increasing mass gap in graphene affects the linear slope dI_C / dV_G approximated with $-2 \big/ (E_F - m v_F^2)$.

In Fig. 4, the comparison between the gapless and gapped graphene junctions is studied in the thick barrier effect. We see that the slope of switching curve is manipulated by the barrier length, presented by $k_F L = 0.1\pi$, 0.5π, π, and 3π. The switching slope in gapless graphene is still the linear slope for a small thickness ($k_F L = 0.1\pi$). While the thicknesses are increased to $k_F L = 3\pi$, the oscillation of this slope can appear in both gapless and gapped graphene systems.

Finally, we examine the massive Dirac behavior in the graphene Josephson junction by varying the Fermi energy at the different gate voltages (see in Fig. 5). In Fig. 5(a), we tune the barriers with the positive voltages which perform on the potential barrier. In $V_G = 0$ case, we see that the supercurrent can flow through the junction as if the supercurrent flows in a gapless graphene junction [15-16]. This

is the effect of no barrier that the Andreev energy is reduced to be $E(V_G = 0) = \Delta_0 \cos[\phi/2]$. This mass-voltage relation in gapped graphene system is resembled in Ref. [17].
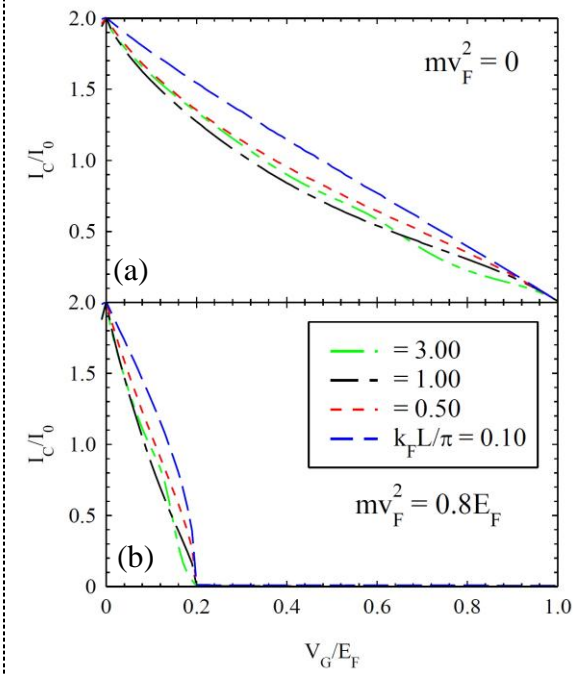
Figure 4. The critical current I_C as a function of the applied gate voltage V_G for various thicknesses $k_F L / \pi$ in case of (a) $m v_F^2 = 0$ and (b) $m v_F^2 = 0.8 E_F$.
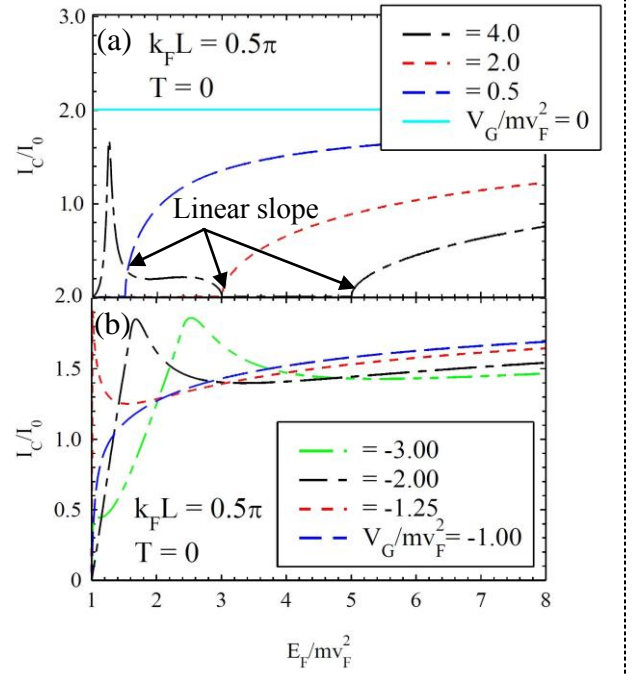
Figure 5. The Fermi energy E_F dependence of the critical current I_C in gapped graphene junction at $k_F L = 0.5\pi$ for various gate potential V_G into (a) potential barriers and (b) potential wells.

When the barriers are increased to $V_G = 0.5 m v_F^2$, $2 m v_F^2$ and $4 m v_F^2$, respectively, the switching limit is decreasing and the gap period is shifting by the gate V_G related to the range of $V_G - m v_F^2 < E_F < V_G + m v_F^2$. Also, the oscillatory behavior of the Josephson current is not appeared for the barrier inside the range $0 \le V_G \le 2 E_F$ but it can oscillate for adjusting $V_G > 2 E_F$. For the potential well, it is revealed in Fig. 5(b). We find that around the bottom band, the carriers located in period of $-m v_F^2 \le V_G \le 0$ behave similarly the case of thin barrier limit [17]. This is the result due to the sufficiently slight value of V_G that the effect of thick barrier $k_F L = 0.5\pi$ cannot affect the trajectory of massive electrons. The well effect can be exhibited for tuning the gate value $V_G < -m v_F^2$.

The linear dependence of supercurrent occurs at $E_F \to m v_F^2$. This is the characteristic behavior can be found near the bottom band in the gapped graphene Josephson junction. With these properties, it is suitable for leading to application of superconducting devices.

4. Summary and conclusion
The Josephson current in a gapped graphene-based S/I/S junction is investigated in case of thick barrier limit as pointing out the thickness of barrier and the electrostatic potential. We find that the oscillatory current in this junction can be observed by varying the barrier thickness. The Dirac fermionic behavior is shown in the same case of thin barrier [17], when the insulating barrier is

modified by the gate potential inside the range of $-mv_F^2 \leq V_G \leq 0$. Moreover, the influence of potential barrier is affected to the slope of the switching Josephson current as observed in the range of $0 < V_G \leq E_F - mv_F^2$. The switching slope can approximate to $-2/(E_F - mv_F^2)$. These interesting behaviors are suitable for alternative mechanism into experimental test.

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