Higuchi’s Method applied to detection of changes in timbre of digital sound synthesis of string instruments with the functional transformation method

Manorth Kanjanapen[1], Cherdsak Kunsombat[1] and Surasak Chiangga[1,a]

[1] Department of Physics, Kasetsart University, Bangkok 10900, Thailand
[a]Corresponding’s author: fscissc@ku.ac.th

Abstract. The functional transformation method (FTM) is a powerful tool for detailed investigation of digital sound synthesis by the physical modeling method, the resulting sound or measured vibrational characteristics at discretized points on real instruments directly solves the underlying physical effect of partial differential equation (PDE). In this paper, we present the Higuchi’s method to examine the difference between the timbre of tone and estimate fractal dimension of musical signals which contains information about their geometrical structure that synthesizes by FTM. With the Higuchi’s method we obtain the whole process is not complicated, fast processing, with the ease of analysis without expertise in the physics or virtuoso musicians and the easiest way for the common people can judge that sounds similarly presented.

1. Introduction

Since sound is a wave, it has many properties in the physical sense. A complex sound waves can be decomposed into a family of simple sine waves, each of which is characterized by its frequency, amplitude, and phase. These are called the partials or the overtones of the sound.

The sound produced by different the physical behavior that make up musical instruments is usually complex. Two or more sounds may have the same loudness and the same pitch, but that they may differ in number and relative intensities of overtones. It also agrees well to the analytic listening of humans as they possible to recognize each sound. This characteristic of sound is called quality, tone color, or timbre of a tone.

Nevertheless, it will be hard to distinguish especially the auditory sense of string instruments which two or more sounds have the same pitch, the same playing techniques, the same musical instruments but playing by different positions. For example, open B note on the second string and musical note B on the fourth fret of the third string of the guitar.

To study the development of the different timbre, many computational methods have been used for digital sound synthesis to produce musical sounds with digital devices. One of these approaches is physical modeling synthesis methods, are often adapted from the areas of numerical mathematics and digital signal processing. Examples are finite difference methods, mass-spring networks, modal decomposition, digital waveguides, wave digital filters, and source-filter models [1].

A method which models the physical effects to the connection between the original physical parameters and the low order transfer functions is the so-called functional transformation method. The derivation of the multidimensional transfer function model is based on suitable functional transformations for the time and space variable and does not involve any approximations. The strength of this method is not only the coefficients of the synthesis algorithm depend explicitly on the parameters of the physical model, but also the variability of the sound synthesis through exploring by parameter variations.

Complex systems as the digital sound synthesis generate time series showing the combination of fractal and periodic components commonly appear in the experimentation when carrying out the measure of a physical quantity. There are several ways to display results with regard to sound synthesis. For example, the harmonic spectrum, spectrogram, the pole of the transfer function, etc. Each of the methods, results is a different pattern and depending on the difficulty and complexity of the process.

The fractal dimension is a statistical measure indicating the complexity of an object or a measure that is self-similar over some region of space or time interval. It has been effectively applied in several domains to distinguish such objects and quantities [2], [3], [4]. In the analysis of time series or non-stationary nature of sound synthesis signals, the Higuchi's method presents the advantage of being a quite fast and reliable method for the determination of the correlation of the series [5], [6], [7].

In this paper, we present an explanation for the observed timbre of tone behavior in the graph corresponding to the Higuchi's method and its subsequent application of digital sound synthesis waveform. We apply the Higuchi's method of the detection of periodic components of vibrating string with the functional transformation method by analyzing the sound quality that arises from the same musical note but is a difference between physical parameters.

2. Higuchi's method

The algorithm presented by Higuchi [8] measures the FD of discrete time series directly from the time domain. It is established on a measure of the length $L(k)$ of the signal or curve that represents the considered time series while using a segment of k samples. The assumption is that a fractal signal scales according to the following:

$$L(k) \infty k^{-D} \tag{1}$$

In order to obtain the fractal dimension D, considered a finite set of regular interval:

$$X(1), X(2), X(3), ..., X(N) \tag{2}$$

From the time series, the new series X_k^m are obtained

$$X_k^m; X(m), X(m+k), X(m+2k), ..., X\left(m+\left[\frac{N-m}{k}\right]k\right), \quad (m=1,2,3,...,k) \tag{3}$$

Where m and k are integers, indicating initial time and interval time respectively. Assuming that the series has only N = 100 and k = 3, three new time series are obtained

$$X_3^1 : X(1), X(4), X(7), ..., X(97)$$
$$X_3^2 : X(2), X(5), X(8), ..., X(98)$$
$$X_3^3 : X(3), X(6), X(9), ..., X(99) \tag{4}$$

The length of the curve associated to each time series X_k^m is defined as:

$$L_m(k) = \frac{1}{k}\left(\sum_{i=1}^{\frac{N-m}{k}} \left(X\left(m+ik\right) - X\left(m+\left(i-1\right)k\right)\right)\right)\left(\frac{N-1}{\left[\frac{N-m}{k}\right]k}\right) \tag{5}$$

where

$$\frac{N-1}{\left[\frac{N-m}{k}\right]k}$$

represents the normalization factor for the length of the subset.

A sum of all the lengths $L_m(k)$ for each *k* is determine by:

$$L(k) = \sum_{m=1}^{k} L_m(k) \tag{6}$$

Finally, the slope of the curve is fractal with dimension *D*, estimated using the best fit by linear least squares. Higuchi's algorithm can be applied even over time series that are not stationary. We have used *k* = 5 to compute Higuchi's fractal dimension.

2. The functional transformation method

A vibrating string is used here as an example of a physical system to emphasize the connection between the parameters of the physical model and the resonating frequencies of the digital reproduction. For a more extensive introduction to the functional transformation method see [9].

We consider a string of length *l* with is fixed at both ends. The deflection of the string is denoted by $y(x,t)$ depends on the space and time coordinates are *x* and *t*. The string is excited by a certain force per unit length $f_e(x,t)$. It is formulated as a PDE for the deflection of the vibrating string by

$$m\frac{\partial^2 y(x,t)}{\partial t^2} + EI\frac{\partial^4 y(x,t)}{\partial x^4} - T_s\frac{\partial^2 y(x,t)}{\partial x^2} + d_1\frac{\partial y(x,t)}{\partial t} + d_3\frac{\partial^3 y(x,t)}{\partial t \partial x^2} = f_e(x,t) \tag{7}$$

This PDE depends on certain physical parameters of a nylon guitar B string are shown in Table 1. Because the string is fixed at both ends, the deflection and the curvature are zero at *x* = 0, *l*. The initial conditions of the deflection at t = 0 are supposed to be zero for being easy to understand.

Table 1. Physical parameters of a nylon guitar B string.

m	mass per unit length	$0.5914.10^{-3} kg/m$
E	Young's modulus	$5.4GPa$
l	length	$0.65m$
I	Moment of inertia	$0.171\ mm^4$
d_1	Frequency independent damping	$8.10^{-5} kg/(ms)$
d_3	Frequency dependent damping	$-1.4.10^{-5} kg\ m/s$
T_S	Tension of the string	$60.97N$

The outline of the FTM is given in Fig. 1. To accomplish a model that can be effected for sound synthesis in the computer, the continuous initial-boundary-value problem has to be discretized. This procedure is well known from the simulation of one-dimensional systems.

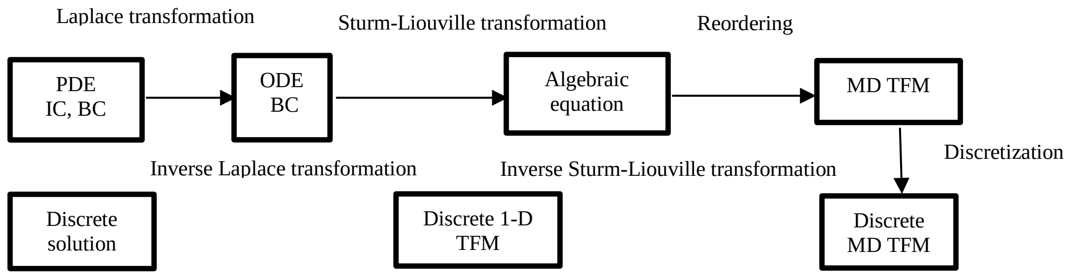

Figure 1. Procedure of the FTM

4. Higuchi's analysis of time series of string instruments with the functional transformation method

We used Higuchi's method of time series, according to sound synthesis waveform by FTM from two directions.

First, we calculate the fractal dimension of different harmonics from linear physical based PDE by reduces to the wave equation $(m\partial^2 y / \partial t^2 - T_S \partial^2 y / \partial x^2 = 0)$. The string has zero stiffness (E=0) and zero damping no decay variables d_1 and d_3 that includes the losses in the air, the string material and the coupling to the resonance body [10].

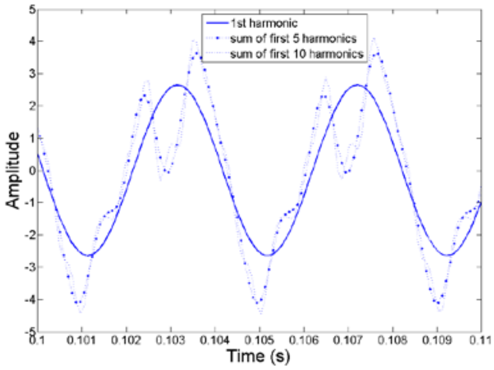

Figure 2. Zero damping and zero stiffness waveform of three different cases

As an example of applying the Higuchi's method we present in Fig. 2 waveforms of discrete time corresponding poles lies on the imaginary axis (have a real part equal to zero) so the dynamic system has marginally stability properties which proportional physical effects of ignoring damping and stiffness. Thus the value of fractal dimension D equal to 1.0114, 1.0740 and 1.1645 for sound synthesis of the fundamental frequency, sum of the first 5 and 10 harmonics. Although the fractal dimension seems to very similar but the results of listening are clearly distinguishable.

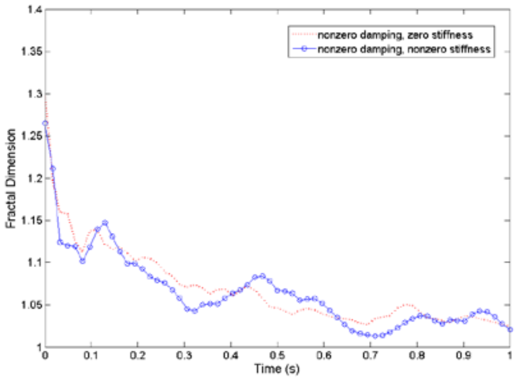

Figure 3. Evaluation of the fractal dimension of string vibrations with the FTM

Secondly, we synthesize the FTM with the parameters from Table 1 by classified these output signals into 2 cases, zero and nonzero stiffness, study of the fractal dimension of the two timbres to be very similar. The solution was calculated with M = 50 harmonics.

Nonzero damping, zero stiffness:

Which include the damping variables d_1 and d_3 but ignore the stiffness (E =0). In the beginning, there is a sufficiently rich signal, but then the higher harmonics decay faster than the lower harmonics. The effect of damping changes the timbre to get closer the real-sounding instruments, compatible with the fractal dimension is consistent with high value at first and gradually decreased in later time based on the characteristics of timbre.

Nonzero damping, nonzero stiffness:

In this case all parameters from Table 1 are used. The effect of string's stiffness will exhibit inharmonicity corresponding the fractal dimension is shown in Fig. 3, which create a more lively sound than a purely harmonic spectrum and produced the quality of the audio as close to the real instrument than all above cases.

5. Conclusion

At first sight, It may seem that the fractal dimension of musical instrument tones requires a lot of efforts to understand the elements of that complexity and not beneficial from physical reality. However, we have shown that it is possible to link the digital generation of sound samples of vibrating structures in terms of mathematical physics and the analysis tool of their characteristics and properties with the fractal dimension. With a further analysis, we can see the Higuchi's method provides useful information by varying the sum of the harmonic number or the overtone of a musical note's sound.

We have found very interesting result in the fractal dimension that possibly indicate the effect of inharmonicity on pitch for guitar-like tones, which produces slow beats between the different harmonics, corresponding sense of the listener can analyze the difference between tone-colors of instruments underlying physical factors.

The fractal dimension of digital audio synthesis is something fresh on today's sound design equipment, it allows for a new method for synthesizing sound based on timbre and is about to become an entire component of fractal multimedia applications.

References

[1] Välimäki V, Pakarinen J, Erkut C, and Karjalainen M 2006 Discrete-time modelling of musical instruments *Rep. Prog. Phys*. **69** pp 1-78

[2] Nakamura Y, Yamamoto Y, and Muraoka I 1993 Autonomic control of heart rate during physical exercise and fractal dimension of heart rate variability *Journal of Applied Physiology*. **74** pp 875-881.

[3] Peters E E 1994 Fractal Market Analysis *Applying Chaos Theory to Investment and Economics* (Wiley)

[4] Turunen J, Lipping T, Tanttu J T 2005 Speech analysis using Higuchi fractal dimension *Models and Analysis of Vocal Emissions for Biomedical Applications, 4th International workshop* pp 171-174

[5] Chaigne A, Askenfelt A 1994 Numerical simulations of piano strings. I. A physical model for a struck string using finite difference methods *J. Acoust. Soc. Am.* **95** pp 1112-1118

[6] Gigerelle M, Iost A 2000 Fractal dimension and classification of music *Chaos, Solitons and* Fractals **11** pp 2179-2192

[7] Esteller R, Vachtsevanos G, Echauz J, Litt B 2001 A comparison of waveform fractal dimension algorithms *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications* **48** pp 177-183

[8] Higuchi T 1988 Approach to an irregular time series on the basis of the fractal theory *Physica D Nonlinear Phenomena*. **31** pp 277-283

[9] Rabenstein R and Trautmann L 2003 Digital sound synthesis of string instruments with the functional transformation method *Signal Processing*. **83** pp 1673-1688