Effect of External Economic-Field Cycle and Market Temperature on Stock-Price Hysteresis: Monte Carlo Simulation on the Ising Spin Model.

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Abstract. In this work, the stock-price versus economic-field hysteresis was investigated. The Ising spin Hamiltonian was utilized as the level of 'disagreement' in describing investors' behaviour. The Ising spin directions were referred to an investor's intention to perform his action on trading his stock. The periodic economic variation was also considered via the external economic-field in the Ising model. The stochastic Monte Carlo simulation was performed on Ising spins, where the steady-state excess demand and supply as well as the stock-price were extracted via the magnetization. From the results, the economic-field parameters and market temperature were found to have significant effect on the dynamic magnetization and stock-price behaviour. Specifically, the hysteresis changes from asymmetric to symmetric loops with increasing market temperature and economic-field strength. However, the hysteresis changes from symmetric to asymmetric loops with increasing the economic-field frequency, when either temperature or economic-field strength is large enough, and returns to symmetric shape at very high frequencies. This suggests competitive effects among field and temperature factors on the hysteresis characteristic, implying multi-dimensional complicated non-trivial relationship among inputs-outputs. As is seen, the results reported (over extensive range) can be used as basis/guideline for further analysis/quantifying how economic-field and market-temperature affect the stock-price distribution on the course of economic cycle.

1. Introduction

Stock market is where investors trade their ownership (stock) giving rise to fluctuation of the wealth of the referred asset [1]. In general, the stock-price changes according to demand and supply. The stock-price increases with excess demand and decreases with excess supply. In addition, the investors' decision in buying/selling stock is usually inspired by financial influences or news. For instance, if there are some bad news on an asset, it could influence investors to release their stock on that asset giving to more supply level, but if there are some good news, it could encourage investors to seek for more ownership leading to more demand level. One form of the good and bad news (external influence) comes in periodic pattern caused by economic cycle. The economic cycle is economy fluctuation between economic growth and recession periods [2]. The understanding of economic cycle then help realizing economic crisis. One aspect that can be used to investigate the effect of economic cycle in stock market is the stochastic econophysics, where stock-price and market situation relationship can be predicted [3]. Though number of previous works was performed on the topic but external influences considered were usually but unrealistically set as constant values (e.g. see [4]), or demand-supply temporal fluctuation was not considered [5]. Therefore, this work aims to investigate the effect of the time dependent economic cycle, treated as external economic-field, on buying/selling decision of the investors with Monte Carlo simulation on Ising spin Hamiltonian.

2. Background theories and methodologies

In this work, the considered system contains magnetic Ising spins, where each spin can have only 2 possible discrete states, e.g. +1 and -1. These states can be referred to different intentions of an investor in stock market, i.e. "+1" for an intention to buy and "-1" for an intention to sell. Therefore, with this Ising characteristic, the state of investors' decision on buying and selling stocks can be ascribed by the Hamiltonian [6]

$$H = -J\sum_{<ij>} s_i s_j - h\sum_i s_i \,. \tag{1}$$

In physics, at a fixed temperature, the Hamiltonian should be minimized at its equilibrium/steady state. However, in economics, this Hamiltonian could be assigned as the level of 'disagreement' [5], where it needs to be minimized unless the market may fail to form due to the failure to meet between demand and supply. Also, in equation (1), the first term $-J\sum_{\langle ij \rangle} s_i s_j$ describes interaction among investors (spins), where the interaction strength J = 1 was set as Hamiltonian unit, and $\langle ij \rangle$ covers only first neighboring pairs. The second term $-h\sum_i s_i$ is due to external economic-field influences on the spin, where $h = h_0 \sin(2\pi f t)$ was periodically assumed to match typical economic cycle. The parameter h_0 is the economic-field strength (corresponding to financial situation, market trends, etc.), f is frequency, and t is time. The magnetization $m = (1/N)\sum_i s_i$ then denotes excess demand ($m > 0$) or supply ($m < 0$), on the average. Then, the current price $p(t)$ can be extracted from $p(t) = p_{fund}\, e^{km(t)}$, where k is a constant and P_{fund} is the fundamental (reasonable) price. Full derivation was given elsewhere [5,7]. Note that for $m(t) = 0$, the market price $p(t)$ stay at the fundamental price p_{fund}. However, for $m(t) > 0$, the excess demand increases $p(t)$ over the p_{fund}, reflecting the bull market, but for $m(t) < 0$, $p(t)$ decreases below the p_{fund}, reflecting the bear period in the market.

In the simulation, the system was prepared on two-dimensional square lattice, where the investors (Ising spins) were located on the lattice points, of sizes $N = L\times L$ with periodic boundary condition. In this work, the varying parameters are the market temperature T (from 1.00 to 3.00), the system size L (from 40 to 100), the economic-field strength h_0 (from 1.00 to 3.00), and the economic-field frequency f (from 0.01 to 1.00). For each simulation, at a fixed set of considered parameters, each investors/spins were randomly assigned their intentions to perform trading action, i.e. +1 (for to buy) and -1 (for to sell). Then, they interacted with their first nearest neighbors, according to equation (1), and got updated via the heat bath algorithm with probability $P = \dfrac{\exp(-\Delta E / 2k_B T)}{\exp(\Delta E / 2k_B T) + \exp(-\Delta E / 2k_B T)}$, where ΔE is the energy difference due to the update and k_B is Boltzmann constant. The unit time was defined from one full simulation update of all sites of the lattice, i.e. 1 Monte Carlo step per site (mcs). Also, in each period, at the interval of period/100, the magnetization was recorded as a function of the field h to extract information about how decision of the investor varies with external influence. Then, the period average magnetization $Q = \left(\int_0^{\text{period}} m(t)\,dt\right)\Big/\text{period}$ was calculated to judge number of field periods to discard for obtaining steady state, i.e. when $dQ/dt \to 0$ [8]. From investigations, 10000 field periods discarded were enough to extract steady state m-h loops for making an average one. After that, the average m-h loop was used to calculate the average p-h loop. The whole Monte Carlo procedure was repeated for each considered set of $\{T, L, h_0, f\}$ parameters.

3. Results and discussions

From the simulations, how the results vary with field parameters (h_0,f) and temperature T was found, which agrees well with previous mean-field works in both physics and economic point of views [5,9-10]. For instance, in figure 1a, with increasing T, the stock-price hysteresis loop changes and gains more symmetric behavior. Specifically, at low T (i.e. $T \rightarrow 0$), the system arrives in its extreme states either overloading demand ($m \rightarrow +1$) or supply ($m \rightarrow -1$) states, causing the price to locate close to their optimum values, i.e. either $P_{min} = P_{fund}\times\exp(-1) \approx 0.368$ or $P_{min} = P_{fund}\times\exp(1) \approx 2.718$. Note that T is the average wealth per agent [11]. Therefore, investors have more money to spend with increasing T and hence both excess m and price p drop in magnitude due to more transactions are allowed. Consequently, the excess demand/supply becomes more susceptible to the economic-field direction. This results in the hysteresis loops changing from asymmetric to symmetric with slimmer shaped, i.e. less phase-lag between m (or p) and h signals.

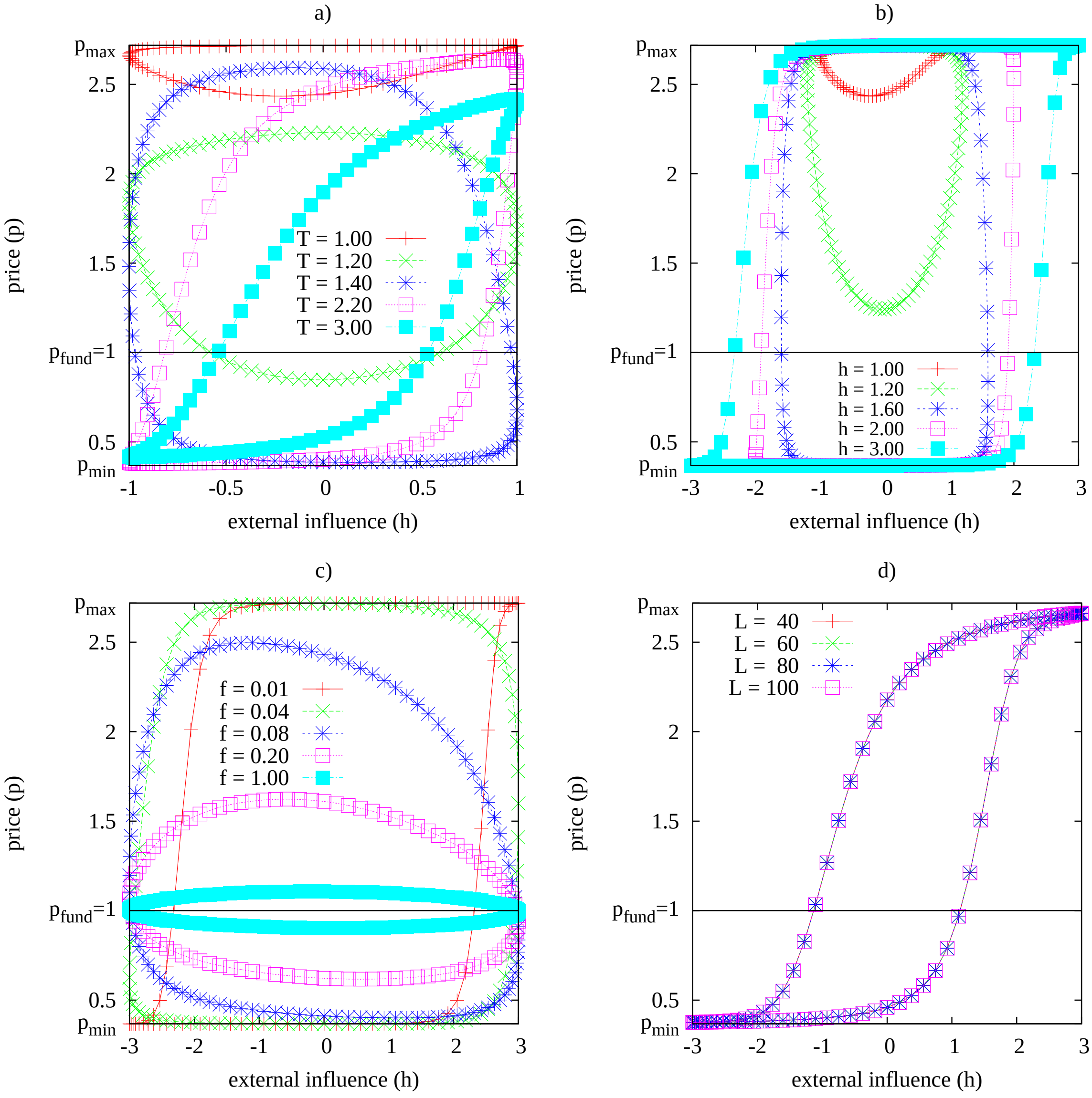

Figure 1. The price versus economic-field (*p-h*) hysteresis loops with a) varying temperature *T* (*L* = 100, *h* = 1, *f* = 0.01), b) varying influence *h* (*L* = 100, *f* = 0.01, *T* = 1), c) varying frequency *f* (*L* = 100, *h* = 3, *T* = 1), and d) varying system size *L* (*h* = 3, *T* = 3, *f* = 0.01).

Apart from T effects, the role of economic-field can be found in figures 1b and 1c. For instance, with increasing h_0, the hysteresis loops again change from an asymmetric loop to broad symmetric loop. This is expected as the greater the field h_0, the higher economic influence to enhance m in both positive and negative directions. This then grants more options for the price to curve between two optimum values. Also note that the greater the h_0 the broader of the loops. This is as when investors have a stronger credence in their own foreseen economic trends, it becomes more difficult to change minds in a totally opposite way. However, the increase of f seems to present more intriguing effects. Particularly, at low f, the loop is quite well saturated as investors have more time (long period) to respond to the economic situation and tune their intention to comply with the economic-field direction (though with some phase-lag between signals). However, with increasing f, the field sweeps more frequently and investors have less time to change their intention in following the economic situation, causing the loop to become less symmetric and can cycle only in sub-region of the price-space. Next, when f is very high, the price can only slight changes around the P_{fund}, the initial starting price, as the economic-field changing is too fast for the investor to follow and to perform mid- to long-term investment. Either better stay away from the market or do nothing is the results of this very high f cases. Finally, for the effect of system size L, it was found that the size does not have significant effect on the normalized excess demand/supply and price. This is as the discrete set of considered parameters is not in the vicinity of the critical region (typically a narrow region on the phase diagram) where finite size effect becomes important [12], complying with that the finite size effect is beyond scopes of this work.

4. Conclusions

In this work, the impact of economic cycle (via external economic-field) on the stock-price variation was investigated via the excess demand and supply of a stock. The economic-field was prototypically prescribed as external financial influence on investors' decision and took a sinusoidal from. The excess demand and supply, extracted via Ising magnetization, was then used to predict stock-price as a function of time, market temperature and economic-field influences. The hysteresis loop of the field and the corresponding parameters were drawn and discussed. It was found that the economic-field characteristic and market temperature have competitive significant effect on the price changes resulting in different characteristic of the stock-price variation in the hysteresis loops, implying multi-dimensional complicated relationship among inputs-outputs. This suggests data engineering may be needed for conclusive formulating these non-trivial relationships. However, results reported in this work can serve as an elementary base of modelling stock-price when considering the external influence as dynamically changeable and periodic parameters, and suggest how the stock-price reacts to that dynamic behavior of the external influence.

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