Correction of higher order chromaticity – an update

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for the FCC-ee lattice design team

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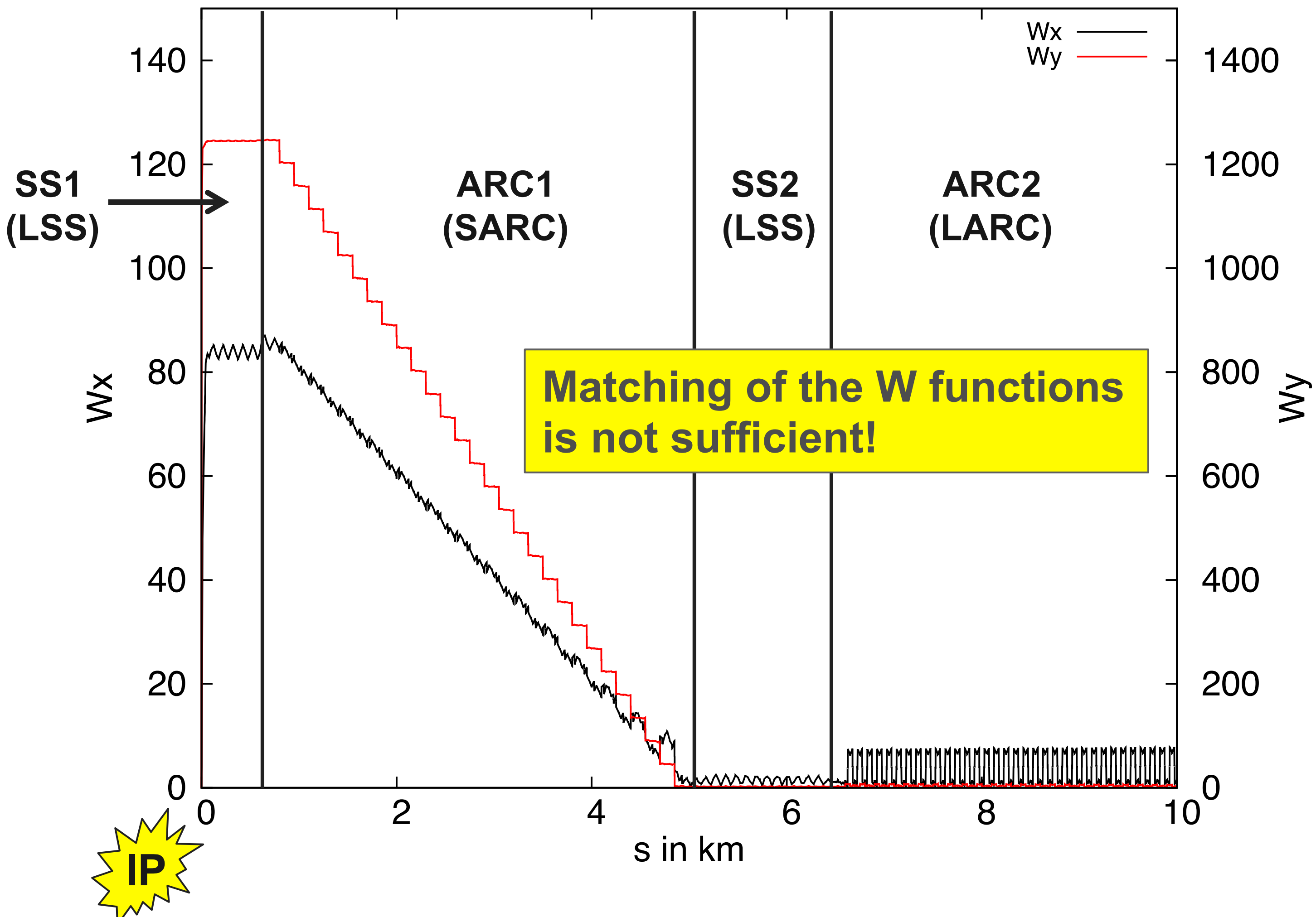
Motivation

Develop analytical approach for
higher order chromaticity correction

I am looking for a way to estimate the effect of additional sextupole pairs on the higher order chromaticities at the beginning/end of the SARCs

W functions: 0-10 km



Derivatives of the β function

The derivatives indicate places for an effective higher order chromaticity correction

$$b_1 = \frac{\partial \beta}{\partial \delta} \qquad b_2 = \frac{1}{\beta}\frac{\partial^2 \beta}{\partial \delta^2}$$

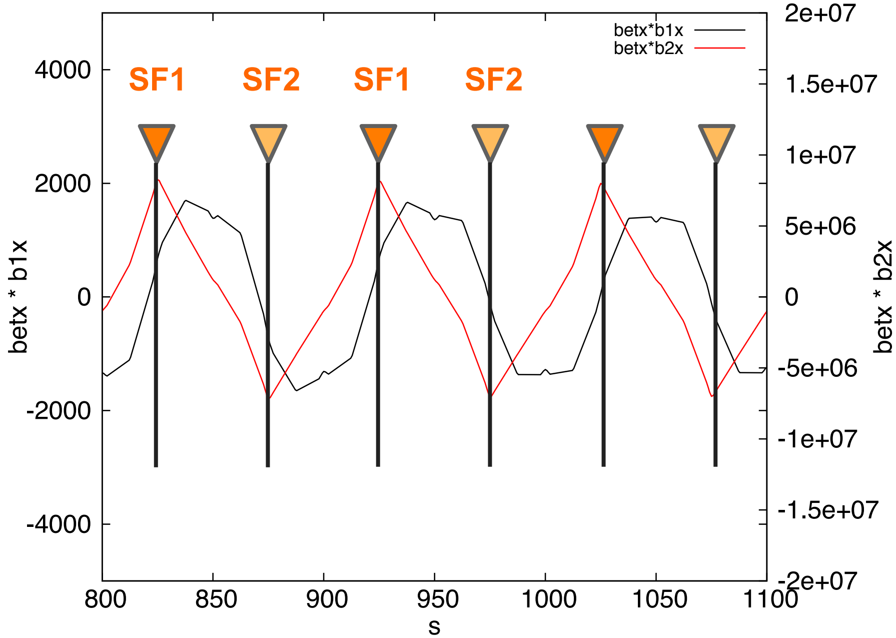
$$\frac{\partial^2 \varphi_y}{\partial \delta^2} = -2\frac{\partial \varphi_y}{\partial \delta} - \int_0^{\Pi} \beta_y K_2 \eta_1 \, ds + \frac{1}{2}\int_0^{\Pi} \beta_y b_{y,1} (K_1 - K_2 \eta_0) \, ds \,,$$

$$\frac{\partial^3 \varphi_y}{\partial \delta^3} = 6\frac{\partial \varphi_y}{\partial \delta} - \int_0^{\Pi} \beta_y (K_1 - K_2 \eta_0)(a_{y,1}^2 + b_{y,1}^2) \, ds + $$

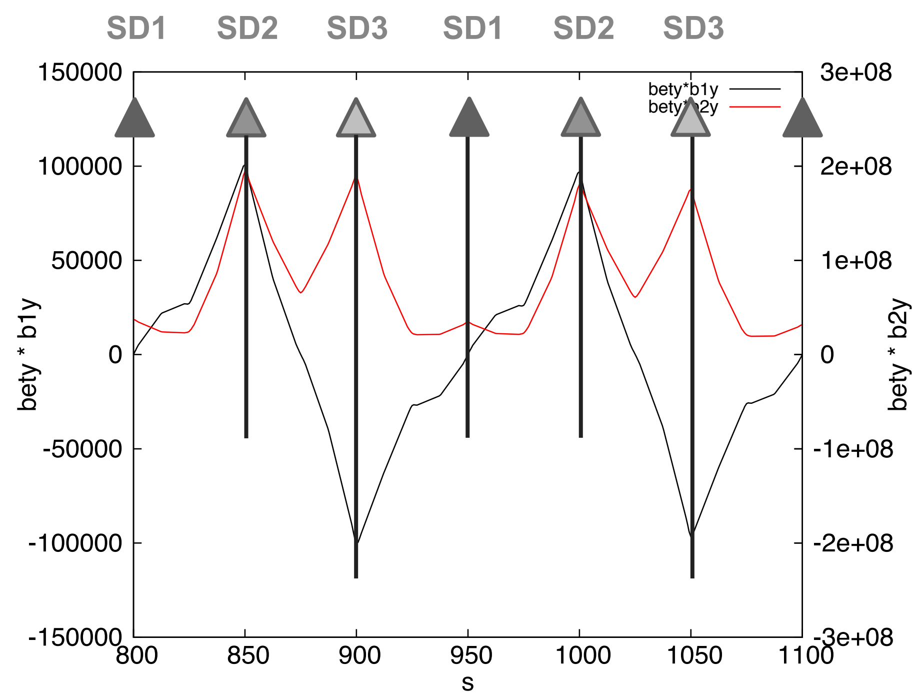
$$+ \; 3\int_0^{\Pi} \beta_y (K_2 \eta_1 - K_2 \eta_2) \, ds + \frac{3}{2}\int_0^{\Pi} \beta_y b_{y,2} (K_1 - K_2 \eta_0) \, ds \,.$$

(A. Bogomyagkov: “Crab waist interaction region for FCC-ee and the arc second attempt”, presentation in the FCC-ee meeting no. 13, 09 Febuary 2015)

$\beta \times b_1$ and $\beta \times b_2$



x plane



y plane

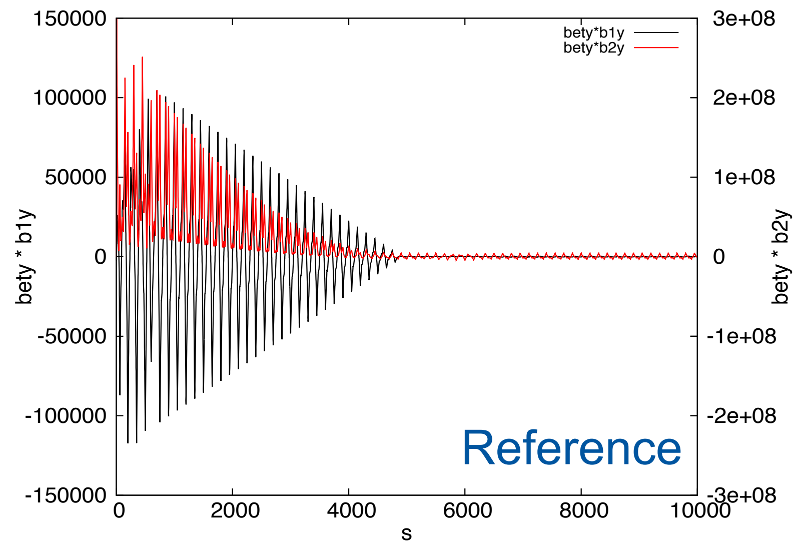
Idea: install free sextupole pairs at the beginning of the SARCs

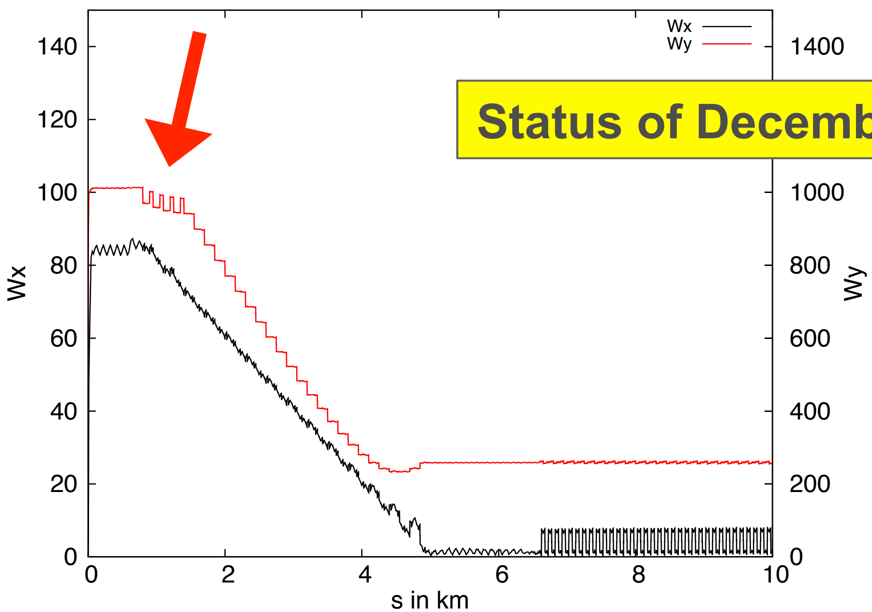
Proposed procedure

1. Matching of W functions in SARCs
2. Calculate strengths for the individual sextupole pairs → Tool needed!
3. Re-matching of the W functions
4. Matching of the linear chromaticity with the LARCs
5. Fine-tuning, Re-iteration

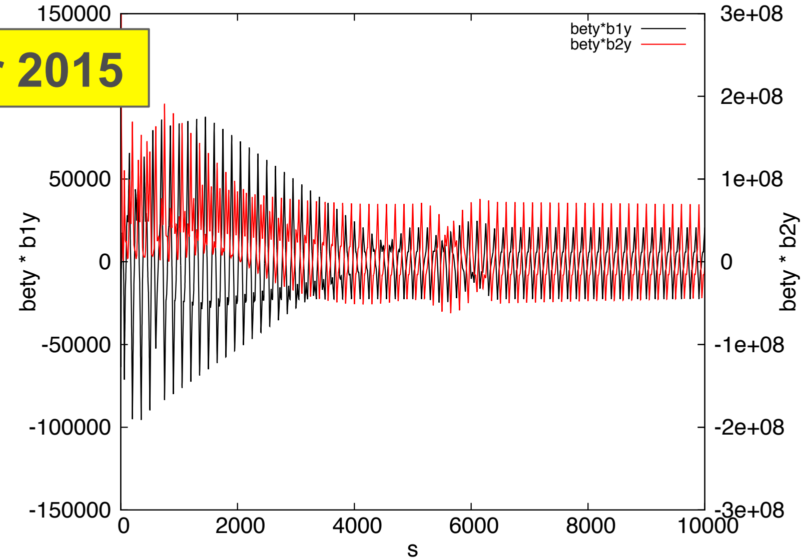
Spoiled optics

- SD3[1,2] ΔK2 = -5
- W function needs to be re-matched

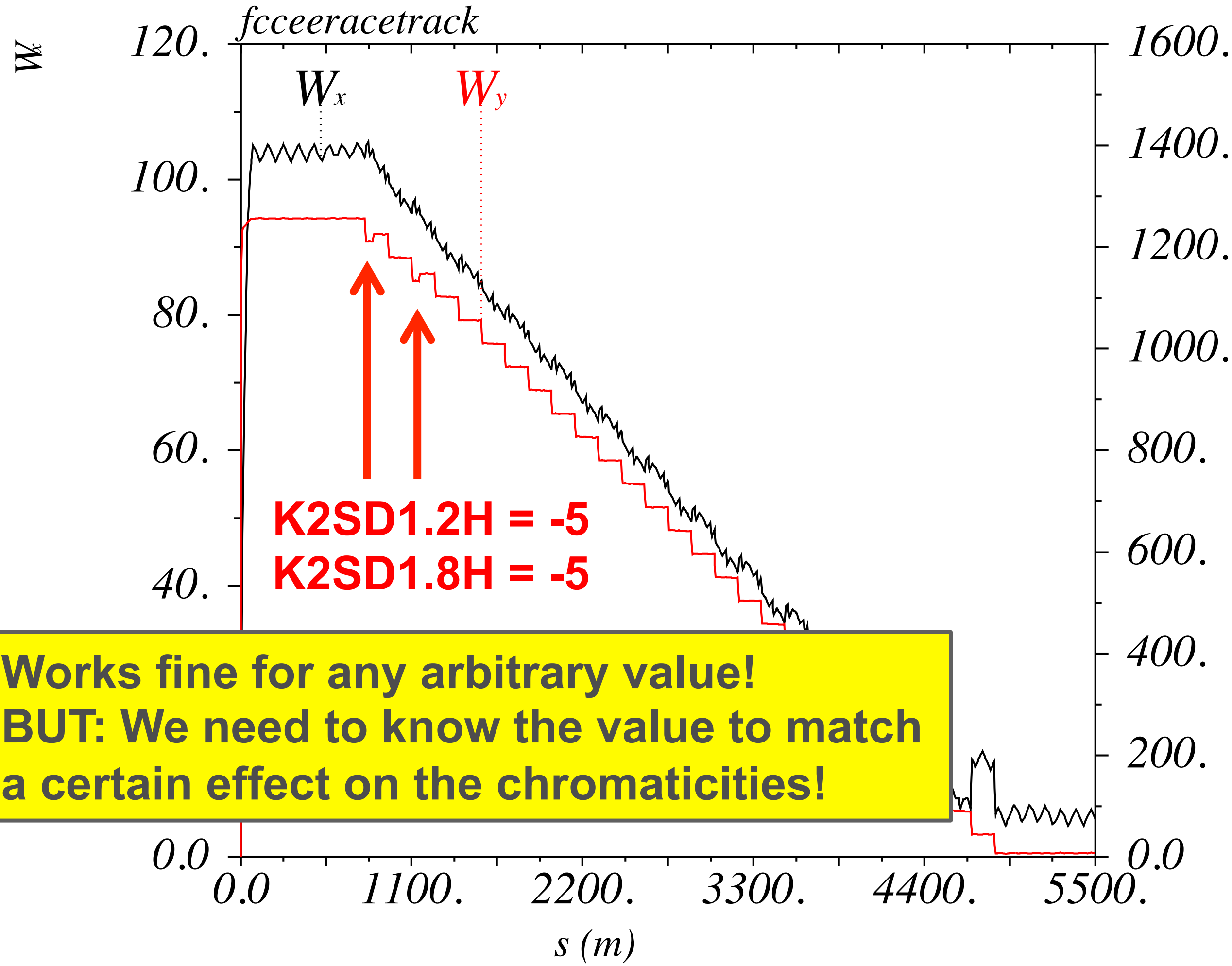




Status of December 2015



Re-matched W functions



Works fine for any arbitrary value!
BUT: We need to know the value to match a certain effect on the chromaticities!

K2SF1.1 = 0.28

K2SF1.2 = 0

K2SD1.1 = -7.66

K2SD1.2 = 0

K2SD1.3 = 0

Past calculations of higher order chromaticities

Macro provided by Anton

- Global method
- Uses difference quotients of the tunes
- Does not use any information about the lattice

→ Not applicable in this case

→ Develop other method using directly Anton's formulae

First and second order

$$\frac{\partial \varphi_y}{\partial \delta} = \frac{1}{2}\int_0^C \beta_y (K_1 - K_2\eta_0)\mathrm{d}s$$
$$= \frac{1}{2}\sum \beta_y (K_1 L_Q - K_2 L_S \eta_0)$$

$$\frac{\partial^2 \varphi_y}{\partial \delta^2} = -2\frac{\partial \varphi_y}{\partial \delta} - \int_0^C \beta_y \left(K_2\eta_1 + K_3\frac{\eta_0^2}{2}\right)\mathrm{d}s + \frac{1}{2}\int_0^C \beta_y b_{1y}(K_1 - K_2\eta_0)\mathrm{d}s$$
$$= -2\frac{\partial \varphi_y}{\partial \delta} - \sum \beta_y \left(K_2 L_S \eta_1 + K_3 L_O \frac{\eta_0^2}{2}\right) + \frac{1}{2}\sum \beta_y b_{1y}(K_1 L_Q - K_2 L_S \eta_0)$$

K_1, K_2, K_3: quadrupole, sextupole, octupole strength
L_Q, L_S, L_O: quadrupole, sextupole, octupole length
η_0: horizontal dispersion, η_1, η_2: chromatic derivatives

Third order

$$\frac{\partial^3 \varphi_y}{\partial \delta^3} = 6\frac{\partial \varphi_y}{\partial \delta} - \int_0^C \beta_y (K_1 - K_2\eta_0)(a_{1y}^2 + b_{1y}^2)\mathrm{d}s$$
$$+ 3\int_0^C \beta_y \left(K_2\eta_1 + K_3\frac{\eta_0^2}{2} - K_2\eta_2 - K_3\eta_0\eta_1 \right) \mathrm{d}s$$
$$+ \frac{3}{2}\int_0^C \beta_y b_{2y}(K_1 - K_2\eta_0)\mathrm{d}s$$

$$= 6\frac{\partial \varphi_y}{\partial \delta} - \sum \beta_y (K_1 L_Q - K_2 L_S \eta_0)(a_{1y}^2 + b_{1y}^2)$$
$$+ 3\sum \beta_y \left(K_2 L_S \eta_1 + K_3 L_O \frac{\eta_0^2}{2} - K_2 L_S \eta_2 - K_3 L_O \eta_0\eta_1 \right)$$
$$+ \frac{3}{2}\sum \beta_y b_{2y}(K_1 - K_2\eta_0)$$

K_1, K_2, K_3: quadrupole, sextupole, octupole strength
L_Q, L_S, L_O: quadrupole, sextupole, octupole length
η_0: horizontal dispersion, η_1, η_2: chromatic derivatives

Estimate the effect of the additional sextupole pairs

Idea for calculation:

- Calculate optical functions needed to
- Calculate the change of chromaticities as a function of the additional ΔK2s
- Use derived functions to match the ΔK2s

Points to check:

- Precision: method uses thin lens approximation
- b_1 & b_2 change behind the modified sextupoles

How precise is the calculation?

1) Compare the results of the global chromaticities obtained by the "sum method" with those using difference quotients

2) Both methods require several TWISS runs with and without energy deviation δ

→ which δ is giving good results?

Characteristics of the lattice

- W functions are minimised in the arcs adjacent to the IR straight section
  → Optimisation of the phase advance
  → 2/3 family scheme of sextupoles
- Tune is matched to .54, .57
- All sextupoles in the LARCs are switched off
  → Linear chromaticity is NOT corrected

Results for different δ

Anton's macro

	$\delta = 10^{-4}$	$\delta = 10^{-5}$	$\delta = 10^{-6}$
Q_x''	-6514.02	-3119.81	-3147.08
$Q_x^{(3)}$	1.22×10^8	-0.75×10^8	-0.74×10^8
Q_y''	2286.69	2275.53	2270.67
$Q_y^{(3)}$	-1.64×10^8	-1.64×10^8	-1.67×10^8

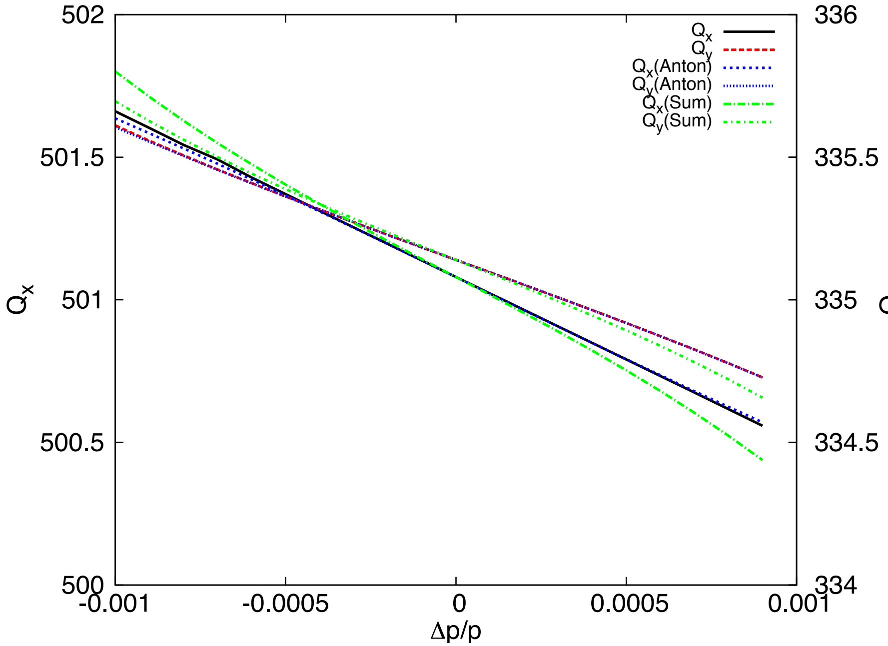
Sum method

MADX: -579.55

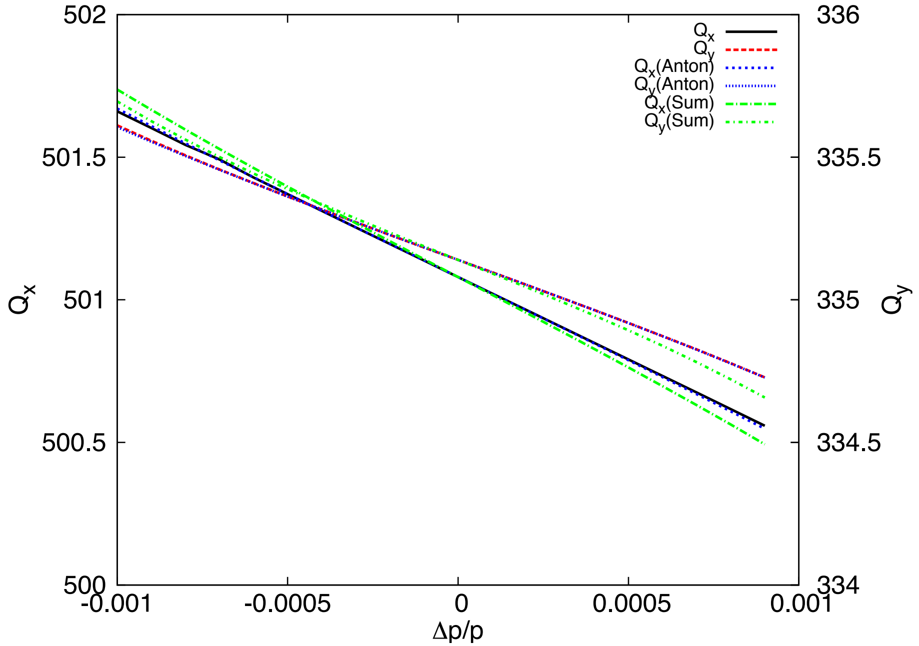
MADX: -436.81

	$\delta = 10^{-4}$	$\delta = 10^{-5}$	$\delta = 10^{-6}$
Q_x'	-626.39	-626.39	-626.39
Q_x''	-12158.61	-690.18	-655.26
$Q_x^{(3)}$	-6.00×10^8	-1.86×10^8	-1.86×10^8
Q_y'	-475.75	-475.75	-475.75
Q_y''	4946.67	4938.94	4938.85
$Q_y^{(3)}$	-4.66×10^8	-4.65×10^8	-4.65×10^8

Comparison with tune scan

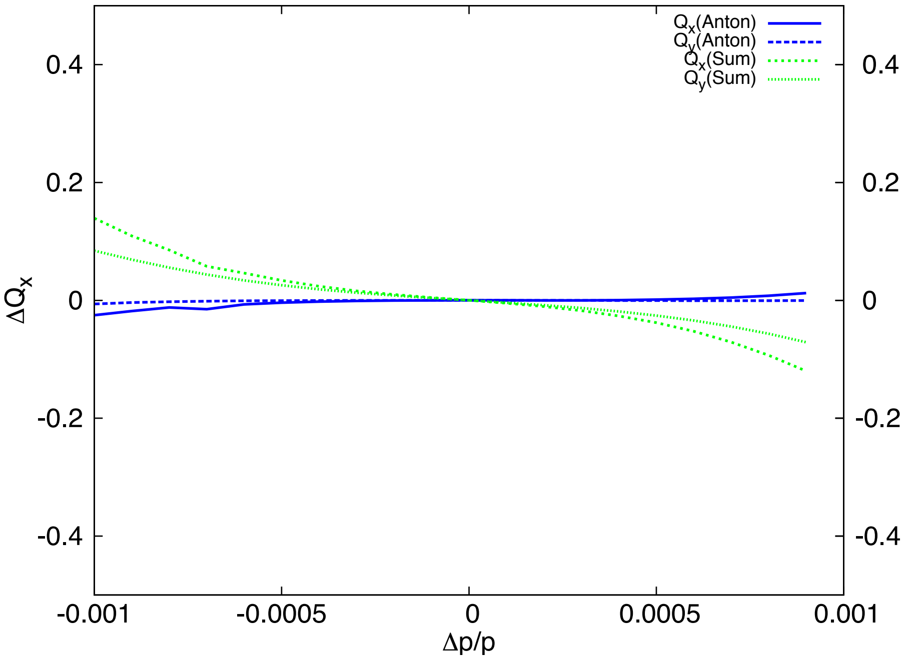


δ = 10^{-4} in TWISS calculations

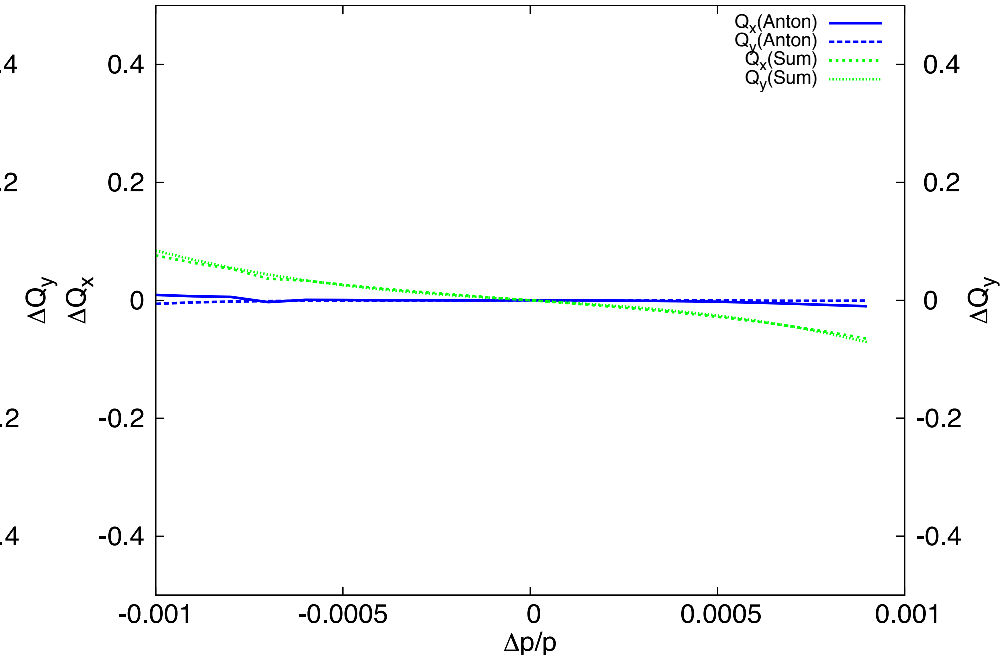


δ = 10^{-6} in TWISS calculations

Difference to simulated tune



δ = 10^{-4} in TWISS calculations



δ = 10^{-6} in TWISS calculations

"Sum method": third order contributions too large!

Which part is dominant?

	$\delta = 10^{-6}$:	$\delta = 10^{-4}$:
$\frac{\partial^2 \varphi_x}{\partial \delta^2} = -2\frac{\partial \varphi_x}{\partial \delta}$	1253	1253
$+\sum \beta_x \left(K_2 L_S \eta_1 + K_3 L_O \frac{\eta_0^2}{2} \right)$	198	198
$-\frac{1}{2}\sum \beta_x b_{1x} (K_1 L_Q - K_2 L_S \eta_0)$	-2106	-13609
$\frac{\partial^3 \varphi_x}{\partial \delta^3} = 6\frac{\partial \varphi_x}{\partial \delta}$	-3758	-3758
$+\sum \beta_x (K_1 L_Q - K_2 L_S \eta_0)(a_{1x}^2 + b_{1x}^2)$	53,103,088	220,739,988
$-3\sum \beta_x \left(K_2 L_S \eta_1 + K_3 L_O \frac{\eta_0^2}{2} - K_2 L_S \right.$	-5308	-5229
$-\frac{3}{2}\sum \beta_x b_{2x} (K_1 L_Q - K_2 L_S \eta_0)$	-239,250,697	-821,228,377

Summary

- For an analytic integration of the free sextupole pairs a tool is required which estimates their effect on the chromaticities

- Such a tool is being developed using Anton's equations for the chromaticities with following assumptions:

  thin lens approximation,

  optical functions are linear inside the sextupoles,

  sextupole field is discrete and has the length L_S

Summary II

- The method is being benchmarked calculating the global chromaticities
- The choice of δ in the calculations has a huge effect on the resulting chromaticities
- It has to be studied, why the effect is observed only in the x-plane (dispersion?)
- $\delta = 10^{-6}$ seems to give better results (“sum method”) than $\delta = 10^{-4}$

Summary III

- After comparison of the calculated tunes with those obtained by a tune scan the third order seems to be too strong
- The chromaticities calculated with the difference quotients of the tune are more precise than the ones obtained by the “sum method”
- The dominant terms driving the second and third order are those containing b_1, b_2 and a_1

Conclusion

- I am developing a tool to calculate the effect of the additional sextupole pairs
- The analytic study helps me to understand, how the non-linear dynamics work.

Next steps

- Improve method to calculate effect of additional sextupole pairs
  → Effect on chromatic derivatives b_1 and b_2 behind the sextupoles
  → Understand effect of δ in the x-plane
- Find tolerable maximum for each order
- Optimise additional sextupole pairs with the goal to replace the local CCS in x-plane

Thank you for your attention!