Theoretical overview on bulk properties and search for critical point at BES

Topical Workshop on Beam Energy Scan - BEST 2016

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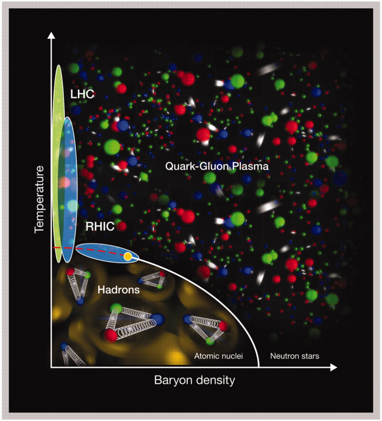
May 9, 2016

Duke University

Introduction

Ideas about the QCD phase diagram

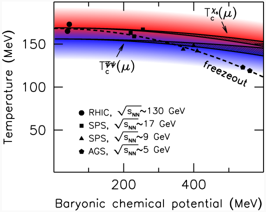
- Properties of strongly interacting many-body systems.
- Phases of hot and dense nuclear matter.
- Phase transition from the quark-gluon plasma (QGP) to a hadron gas.
- Is there a critical point in the phase diagram of QCD?



B. Jacak and B. Müller Science 337 (2012)

Lattice QCD calculations

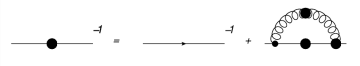
- Crossover at $\mu_B = 0$ and $T = [145, 165]$ MeV
  WB JHEP1009 (2010), HotQCD PoS LATTICE2010 (2010)
- Fermionic sign problem at $\mu_B \neq 0 \Rightarrow$ usual importance sampling fails.
- Methods to extend to finite μ_B, e.g. **Taylor expansion**, etc.
  $\Rightarrow$ no critical point for small $\mu_B/T < 1$.



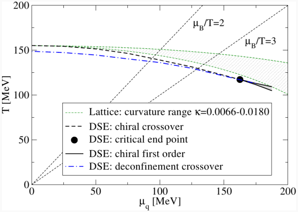
G. Endrodi et al. JHEP1104 (2011)

Functional RG/Dyson-Schwinger equations

- Solve RG-flow/DSE equations for the entire phase diagram.



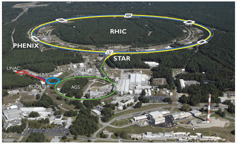
- Sophisticated truncation schemes and approaches to the vertex strength.
- $\Rightarrow$ a critical point for large $\mu_B/T > 3$.



C. Fischer, J. Luecker, C. Welzbacher, 1405.4762

QCD phase diagram: the experimental perspective

- Highest energies at LHC, CERN: PbPb at $\sqrt{s_{\mathrm{NN}}} = 2.76$, 5 TeV
  $\Rightarrow$ Energy deposition at the highest beam energies $\rightarrow$ temperature.
- Beam energy scan at RHIC, BNL: AuAu at $\sqrt{s_{\mathrm{NN}}} = 200 - 7.7$ GeV
  $\Rightarrow$ Baryon stopping at lower beam energies $\rightarrow$ baryochemical potential.
- Measure particle species at chemical freeze-out (instance where inelastic collisions become rare) $\rightarrow$ success of statistical hadronization models
- Measure particle spectra at kinetic freeze-out (instance where elastic collisions become rare) $\rightarrow$ success of fluid dynamics



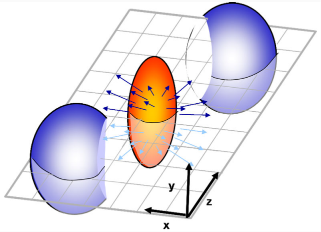
Fluid dynamical modeling of the BES

Fluid dynamical description of heavy-ion collisions

- The discovery of RHIC: The QGP is an almost ideal strongly coupled fluid.
- Early fluid dynamical calculations reproduce spectra and elliptic flow.

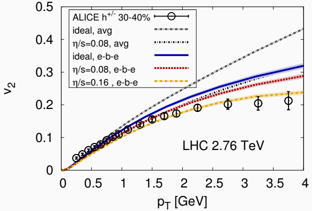
  P. Kolb, U. Heinz, QGP (2003)
- Numerous improvements during the last decade:

  $(3+1\mathrm{d})$, viscosity, initial conditions, initial state fluctuations, hybrid models



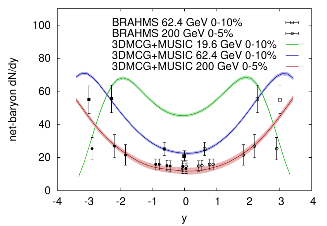
Spatial eccentricity $\Rightarrow$ momentum anisotropy
via fluid dynamical pressure

elliptic flow at LHC

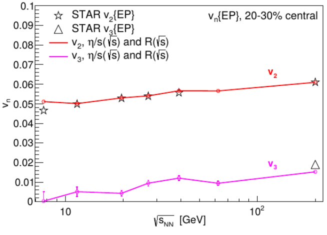


MUSIC by B. Schenke, S. Jeon, C. Gale PLB702 (2011)

Fluid dynamics at finite μ_B



QM2015 talk by B. Schenke

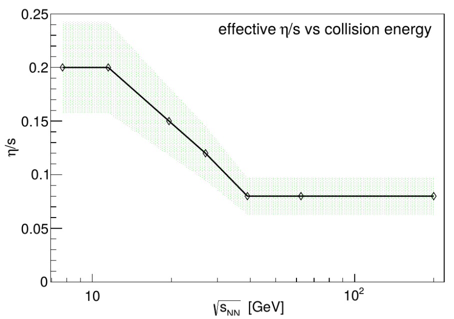


QM2015 talk by Y. Karpenko

- inclusion of net-baryon diffusion into fluid dynamical simulations:
  (baryon diffusion, baryon-shear and baryon-bulk coupling, δf corrections)
- initial state and initial baryon stopping
  $\Rightarrow$ explore net-baryon rapidity correlations and fluctuations
- is there a fluid dynamical phase at high-baryon densities?

Transport coefficients at finite μ_B: estimate

- hybrid model: 3 + 1d viscous fluid dynamics, including baryon and electric charge current + UrQMD
- chiral equation of state with crossover transition for all μ_B

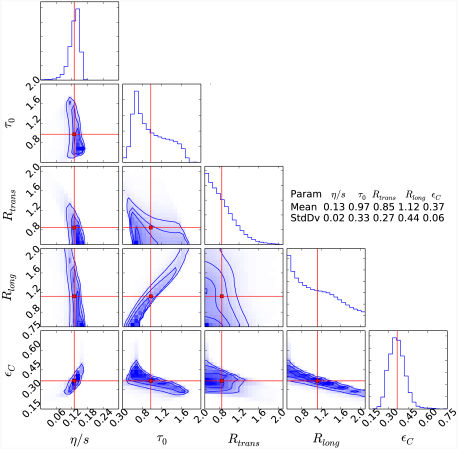


I. Karpenko, P. Huovinen, H. Petersen and M. Bleicher, Phys. Rev. C 91 (2015)

- too many parameters to tune in hybrid models to obtain reliable values → model-to-data comparison via Bayesian statistical analysis

Transport coefficients at finite μ_B: determination

- same hybrid model (fluid dynamics + UrQMD, chiral eos)
- model emulation via Gaussian processes + sampling with Markov Chain Monte Carlo



$\sqrt{s_{\rm NN}} = 62.4$ GeV

QM2015 poster by J. Auvinen, I. Karpenko, J. Bernhard, S. Bass

Transport coefficients at finite μ_B: determination

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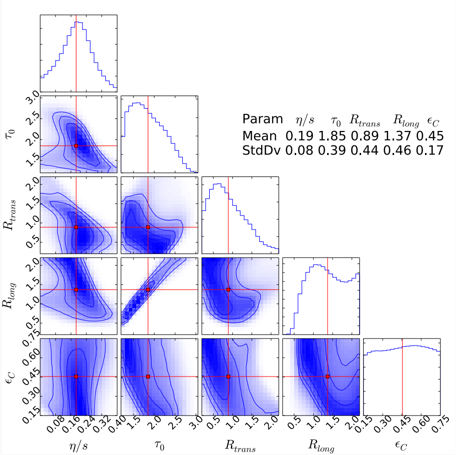


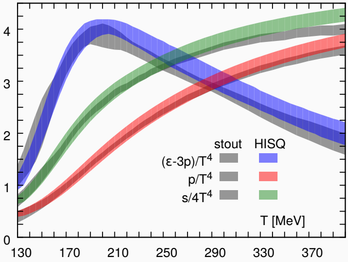
$\sqrt{s_{\mathrm{NN}}} = 19.6$ GeV

p_T spectra are important for constraining parameters!

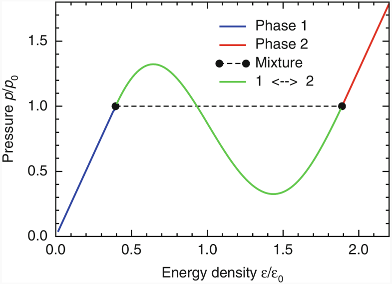
QM2015 poster by J. Auvinen, I. Karpenko, J. Bernhard, S. Bass

Equation of state and phase transitions

HotQCD Coll. PRD90 (2014); WB Coll. PLB730 (2014)



CBM physics book, Lecture Notes in Physics 814 (2011)

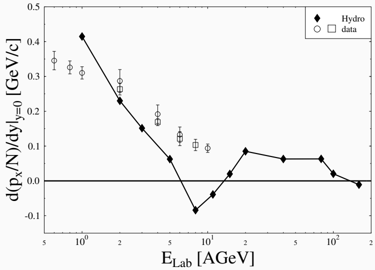


- Thermodynamic quantities change characteristically at the phase transition.
- Speed of sound $c_s^2 = (\partial p/\partial e)_S \rightarrow$ minimum at the phase transition/crossover.
- Compressibility $\kappa_S = -1/V(\partial V/\partial p)_S \rightarrow$ maximum at the phase transition/crossover.

"softest point"
anomaly in the pressure

Phase transitions in fluid dynamics

- Describing a phase transition fluid dynamically is simple!
- Need to know the equation of state and transport coefficients!



H. Stöcker, NPA780 (2005)

- A pronounced minimum in the slope of the directed flow v_1 is observed in a first-order phase transition.

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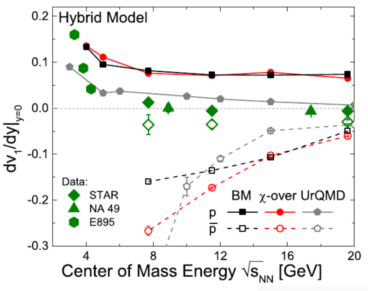


J. Steinheimer, J. Auvinen, H. Petersen, M. Bleicher, H. Stöcker, PRC89 (2014)

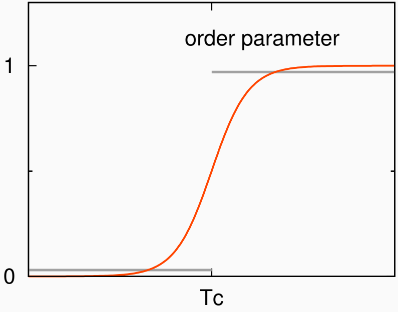
- A pronounced minimum in the slope of the directed flow v_1 is not observed in a first-order phase transition?
- In dynamical simulations: no clear sensitivity on a phase transition in the equation of state yet...

Fluctuations matter at the phase transition!

Fluctuations & phase transitions

Phase transitions: order parameter & derivatives

- An order parameter changes characteristically at the phase transition - discontinuously or continuously.



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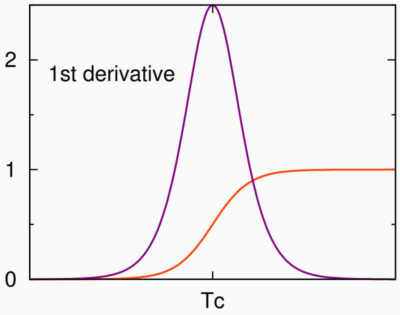

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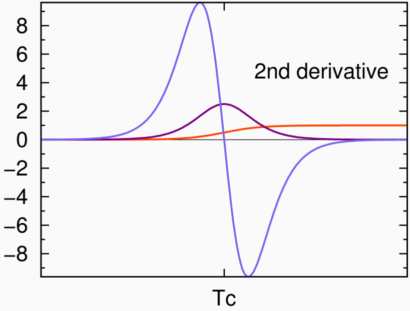

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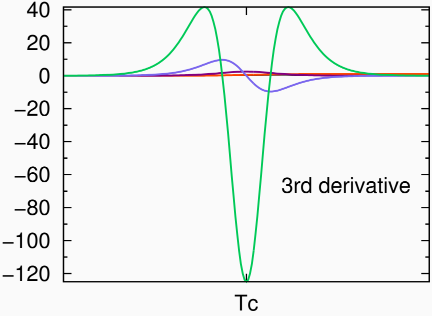

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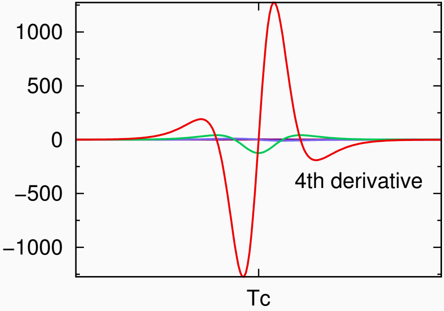


- Derivatives reveal more details!
- Derivatives of thermodynamic quantities are related to fluctuations!

What are fluctuation observables?

- Susceptibilities $\chi_n = \frac{\partial^n (P/T^4)}{\partial(\mu/T)^n}\Big|_T$ relate to fluctuations in multiplicity

$$\chi_1 = \frac{1}{VT^3}\langle N\rangle\,, \quad \chi_2 = \frac{1}{VT^3}\langle(\Delta N)^2\rangle\,, \quad \chi_3 = \frac{1}{VT^3}\langle(\Delta N)^3\rangle\,,$$

$$\chi_4 = \frac{1}{VT^3}\langle(\Delta N)^4\rangle_c \equiv \frac{1}{VT^3}\left(\langle(\Delta N)^4\rangle - 3\langle(\Delta N)^2\rangle^2\right)\,.$$

- To zeroth-order in volume fluctuations:

$$\frac{\chi_2}{\chi_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \kappa\sigma^2$$

variance — Skewness — Kurtosis

- M, σ^2, S and κ are obtained from measured event-by-event multiplicity distributions.

STAR Coll. PRL112 (2014), PRL113 (2014); PHENIX Coll. arxiv:1506.07834

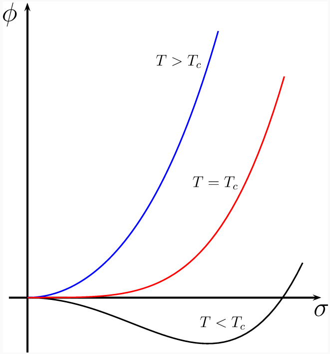
Fluctuations at the critical point

- Universal behavior of the long-wavelength modes.
- Correlation length diverges $\xi \to \infty$.
- Fluctuations of the critical mode σ diverge.
- Higher moments more sensitive to ξ:

$$\langle \Delta\sigma^2 \rangle \propto \xi^2 , \quad \langle \Delta\sigma^3 \rangle \propto \xi^{9/2}$$

$$\langle \Delta\sigma^4 \rangle_c \propto \xi^7 .$$

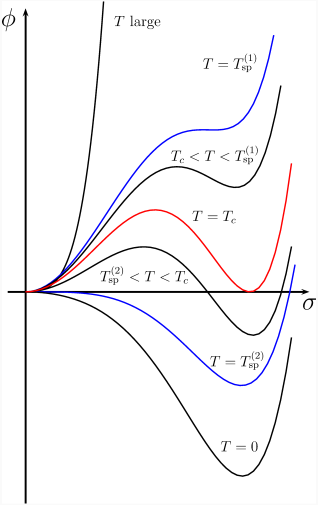
- For QCD: parameters from 3d Ising universality class.
- Relaxation time $\tau_{\rm rel} \propto \xi^z$ diverges $\Rightarrow$ critical slowing down!



$\Rightarrow$ **LARGE fluctuations in equilibrated systems!**

P. Hohenberg, B. Halperin, RMP49 (1977); T. Hatsuda, T. Kunihiro, PRL55 (1985); M. Stephanov, K. Rajagopal, E. Shuryak, PRL81 (1998), PRD60 (1999); S. Jeon, V. Koch, PRL83 (1999); B. Berdnikov and K. Rajagopal, PRD61 (2000); Y. Hatta, T. Ikeda, PRD67 (2003); M. Stephanov, PRL102 (2009); PRC82 (2010); M. Stephanov, PRL107 (2011)

Fluctuations at the first-order phase transition



- Coexistence of two stable thermodynamic phases at $T = T_c$.
- Metastable states above and below $T_c \Rightarrow$ supercooling and -heating.
- Nucleation & spinodal decomposition.

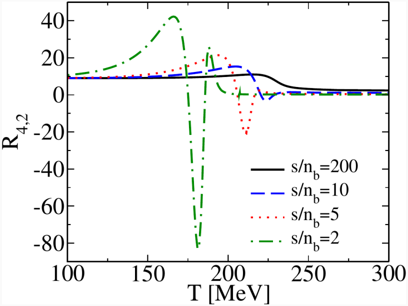
$\Rightarrow$ Domain formation and large inhomogeneities.

$\Rightarrow$ **LARGE fluctuations in nonequilibrium systems!**

L Csernai, I Mishustin, PRL74 (1995); J. Randrup, PRC79 (2009), PRC82 (2010)

Critical fluctuations in QCD effective models

- QCD effective models (e.g. (P)QM, (P)NJL) include some aspects of the QCD phase transition
- excellent opportunity to study critical fluctuations in conserved-charge densities at finite μ_B.



V. Skokov, B. Friman, K. Redlich, PRC83 (2011)

- Strong T-μ_B-dependence of $R_{4,2} = \chi_4/\chi_2$ toward critical point in mean-field (MF) approach.

- Clear signals for the critical point in effective models!

C. Ratti, M. Thaler, W. Weise, PRD73 (2006); B.-J. Schaefer, J. Pawlowski, J. Wambach, PRD76 (2007); C. Sasaki, B. Friman, K. Redlich, PRD75 (2007); C. Ratti, S. Roessner, W. Weise, PLB649 (2007); K. Fukushima, PRD77 (2008); E. Nakano, B.-J. Schaefer, B. Stokic, B. Friman, K. Redlich, PLB682 (2010); T. Herbst, J. Pawlowski, B.-J. Schaefer, PLB696 (2011); K. Morita, V. Skokov, B. Friman, K. Redlich, EPJC74 (2014)

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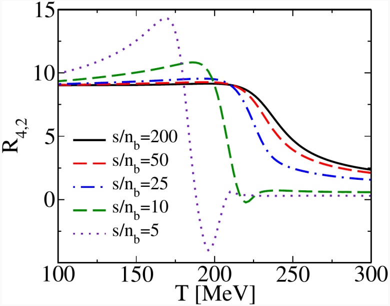


V. Skokov, B. Friman, K. Redlich, PRC83 (2011)

- Reduced but still significant T-μ_B-dependence of $R_{4,2} = \chi_4/\chi_2$ toward critical point after including mesonic fluctuations (via FRG).

- Clear signals for the critical point in effective models!

C. Ratti, M. Thaler, W. Weise, PRD73 (2006); B.-J. Schaefer, J. Pawlowski, J. Wambach, PRD76 (2007); C. Sasaki, B. Friman, K. Redlich, PRD75 (2007); C. Ratti, S. Roessner, W. Weise, PLB649 (2007); K. Fukushima, PRD77 (2008); E. Nakano, B.-J. Schaefer, B. Stokic, B. Friman, K. Redlich, PLB682 (2010); T. Herbst, J. Pawlowski, B.-J. Schaefer, PLB696 (2011); K. Morita, V. Skokov, B. Friman, K. Redlich, EPJC74 (2014)

From chiral to net-proton fluctuations

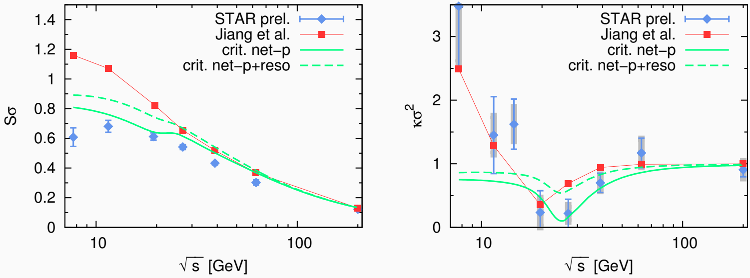
couple order parameter to measurable particles: $g_p \bar{p} \sigma p$

M. Stephanov, K. Rajagopal, E. Shuryak, PRL81 (1998), PRD60 (1999); C. Athanasiou, K. Rajagopal, M. Stephanov, PRD82 (2010)

- finite expectation value of σ in the chirally broken phase contributes to the mass of the proton
- fluctuations $\Delta\sigma$ lead to fluctuations in the proton mass $m_p \to m_p + g\Delta\sigma$,
- modification of fluctuations (statistical + critical) in the distribution functions:

$$\delta f = \delta f^0 + g \frac{\partial f^0}{\partial m_p} \Delta\sigma \, .$$

Critical net-proton fluctuations - phenomenology



MN, QM2015 proceedings, 1601.07437

- equilibrium 3d Ising model assumptions for $\Delta\sigma$
- fluctuations in net-protons at chemical freeze-out
- critical fluctuations are reduced but survive when resonance decays are included

  M. Bluhm, MN, S. Bass, T. Schaefer work in progress
- particle emission during Cooper-Frye freeze-out over a hypersurface from fluid dynamical evolution

  L. Jiang, P. Li and H. Song, arXiv:1512.06164

Still no dynamical fluctuations...

Non-critical effects on fluctuation observables

- Limited acceptance & detector efficiency. A. Bzdak, V. Koch, PRC86 (2012); PRC91 (2015)
- Isospin randomization. M. Kitazawa, M. Asakawa, PRC85, PRC86 (2012)
- Volume fluctuations V. Skokov, B. Friman, K. Redlich, PRC88 (2013)
  ($\rightarrow$ strongly intensive measures).
  E. Sangaline, arxiv:1505.00261; M. Gorenstein, M. Gazdzicki, PRC84 (2011)
- Global net-baryon number conservation.
  MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012); A. Bzdak, V. Koch, V. Skokov, PRC87 (2013)

$\Rightarrow$ These effects are or can be included in microscopic transport models, e.g. UrQMD, (P)HSD, or hybrid models = valuable baseline studies!

- Initial fluctuations due to baryon stopping.

$\Rightarrow$ Need to be well understood!

Dynamical modeling

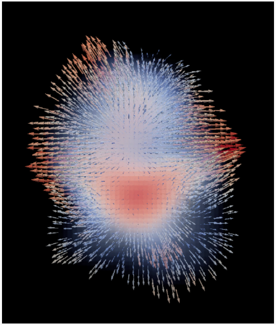
Why is dynamical modeling important?

In a grand-canonical ensemble, the system is

- in thermal equilibrium (= long-lived),
- in equilibrium with a particle heat bath,
- spatially infinite
- and static.

Systems created in heavy-ion collisions are

- short-lived,
- spatially small,
- inhomogeneous,
- and highly dynamical!



plot by H. Petersen, madai.us

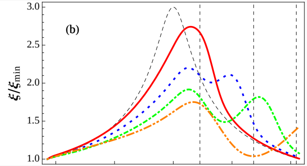
Toward dynamics: memory effects

real-time evolution of non-Gaussian cumulants in the scaling regime, where

$$L_{\mathrm{micro}} \ll \xi \ll L_{\mathrm{sys}}$$

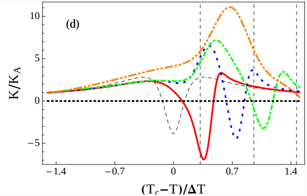
- memory effects are important
- magnitude and sign can be different in non-equilibrium compared to equilibrium expectations
- different trajectories, chemical freeze-out conditions and τ_{rel} can give similar results
- needs dynamical space-time evolution

cf: B. Berdnikov, K. Rajagopal, PRD 61 (2000)



black dashed: equilibrium

colors: different τ_{rel}



S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015)

Nonequilibrium chiral fluid dynamics (NχFD)

IDEA: combine the dynamical propagation of fluctuations at the phase transition with fluid dynamical expansion!

- Relaxational equation for the sigma field: damping and noise from the interaction with the quarks

$$\partial_\mu \partial^\mu \sigma + \frac{\delta U}{\delta \sigma} + g \rho_s + \eta \partial_t \sigma = \xi$$

- Phenomenological dynamics for the Polyakov-loop

$$\eta_\ell \partial_t \ell T^2 + \frac{\partial V_{\rm eff}}{\partial \ell} = \xi_\ell$$

- Fluid dynamical expansion of the quark fluid = heat bath, including energy-momentum exchange

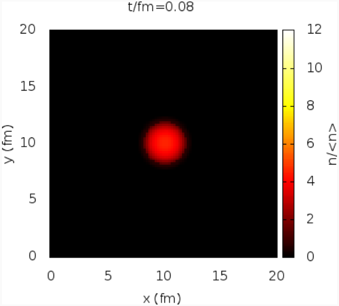
$$\partial_\mu T^{\mu\nu}_{\rm q} = S^\nu = -\partial_\mu T^{\mu\nu}_\sigma \,, \quad \partial_\mu N^\mu_{\rm q} = 0$$

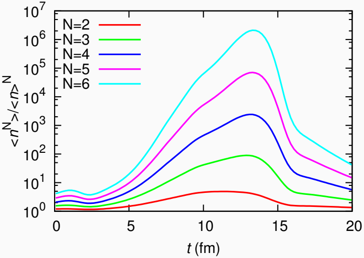
$\Rightarrow$ includes a stochastic source term!

MN, S. Leupold, I. Mishustin, C. Herold, M. Bleicher, PRC **84** (2011); PLB **711** (2012); JPG **40** (2013)
C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013); NPA**925** (2014), C. Herold, MN, Y. Yan, C. Kobdaj JPG **41** (2014)

Domain formation & decay at the QH phase transition

- use a chiral effective model with correct low-temperature degrees of freedom in NχFD!





- droplets of quark density decay in the hadronic phase due to non-vanishing large pressure (cf. also **J. Steinheimer, J. Randrup, V. Koch PRC89 (2014)**)
- future: combine initial and dynamical fluctuations, include particlization and late hadronic interactions

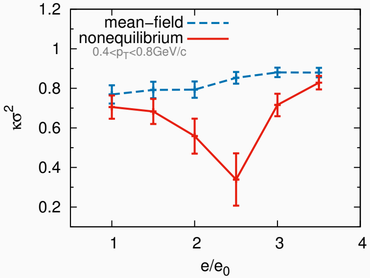
MN and C. Herold, 1602.07223

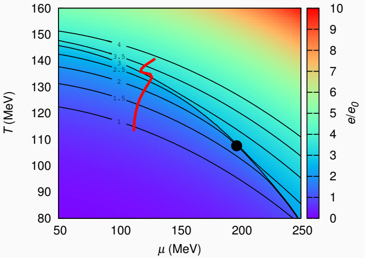
Net-Proton fluctuations

- from densities to particle via Cooper-Frye particlization:

$$E\frac{\mathrm{d}N_i}{\mathrm{d}^3p} = \int \mathrm{d}\sigma^\mu p_\mu (f_i^{\mathrm{eq}}(p) + \delta f)$$

- here: couple the densities of the order parameter field with the fluid dynamical densities





C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2

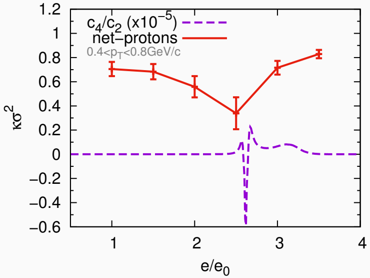
- future: include δf in the particlization and perform calculations for the BES!

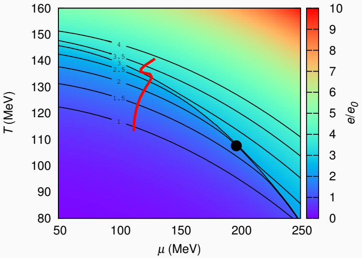
Net-Proton fluctuations

- from densities to particle via Cooper-Frye particlization:

$$E\frac{\mathrm{d}N_i}{\mathrm{d}^3p} = \int \mathrm{d}\sigma^\mu p_\mu (f_i^{\mathrm{eq}}(p) + \delta f)$$

- here: couple the densities of the order parameter field with the fluid dynamical densities





C. Herold, MN, Y. Yan and C. Kobdaj, PRC93 (2016) no.2

- future: include δf in the particlization and perform calculations for the BES!

Fluid dynamical fluctuations

Conventional fluid dynamics propagates thermal averages of the energy density, pressure, velocities, charge densities, etc.

However, ...

- ... already in equilibrium there are thermal fluctuations
- ... the fast processes, which lead to local equilibration also lead to noise!

Conventional ideal fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}}$$
$$N^{\mu} = N^{\mu}_{\mathrm{eq}}$$

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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Conventional viscous fluid dynamics:

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Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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Fluctuating viscous fluid dynamics:

$$T^{\mu\nu} = T^{\mu\nu}_{\mathrm{eq}} + \Delta T^{\mu\nu}_{\mathrm{visc}} + \Xi^{\mu\nu}$$
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Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

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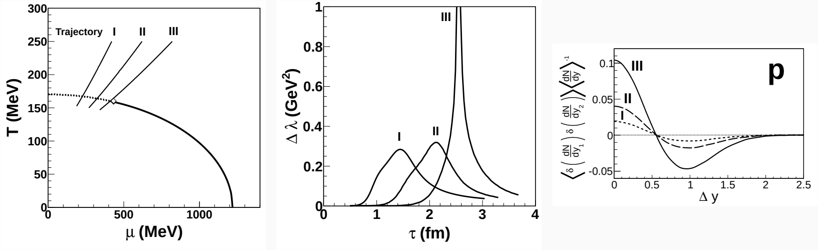
- $\langle T^{\mu\nu} T^{\mu\nu} \rangle$ give viscosities (Kubo-formula), consistently with dissipation-fluctuation theorem fluctuations need to be included as well!
- This is important at the critical point, because the true critical mode is a mixture of chiral mode and net-baryon density!

Y. Minami, T. Kunihiro, PTP122 (2010); P. Kovtun, G. Moore, P. Romatschke, PRD84 (2011); J. Kapusta, B. Müller, M. Stephanov PRC85 (2012); C. Chafin, T. Schäfer, PRA87 (2013); J. Kapusta, C. Young, PRC90 (2014); P. Romatschke, R. Young, PRA87 (2013); P. Kovtun, G. Moore, P. Romatschke, JHEP1407 (2014); C. Young, J. Kapusta, C. Gale, S. Jeon, B. Schenke, PRC91 (2015)

Fluid dynamical fluctuations

Bjorken expansion example with a critical point:

J. Kapusta, J. Torres-Rincon PRC86 (2012)

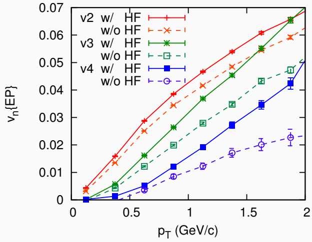


- near the CP the thermal conductivity is enhanced ⇒ enhancement of the rapidity correlator of protons
- how to implement in a $3+1$d relativistic causal fluid dynamical evolution?

Fluid dynamical fluctuations

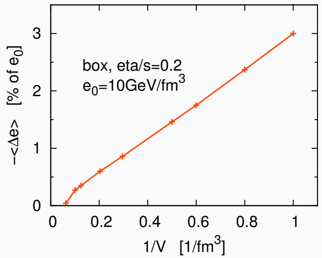
$$\partial_\mu T^{\mu\nu} = \partial_\mu \left(T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu} \right) = 0$$

- Enhancement of flow due to additional fluctuations?



QM2015 talk by K. Murase, T. Hirano; arxiv:1304.3243

- Important check: equilibrium expectations for fluctuations and nonlinear effects.

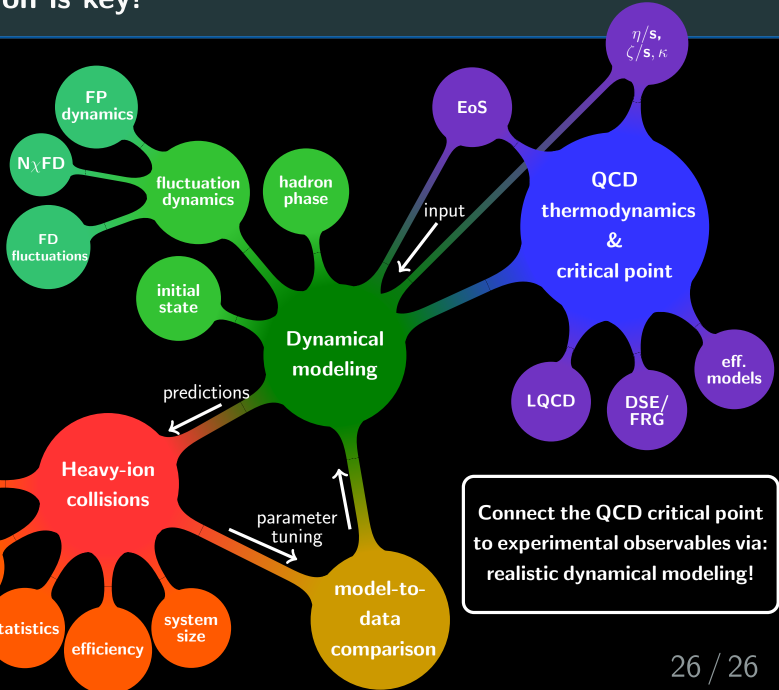


MN, M. Bluhm, Y. Karpenko, T. Schäfer, S. Bass, work in progress

- Implementing fluid dynamical fluctuations is important,

but requires a sustained and systematic effort!

Collaboration is key!



RHIC BES I&II

NICA

FAIR CBM

JPARC

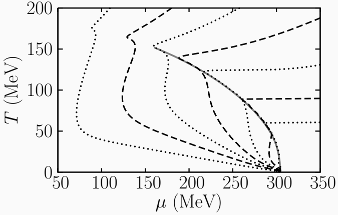
NA61/SHINE

Connect the QCD critical point to experimental observables via: realistic dynamical modeling!

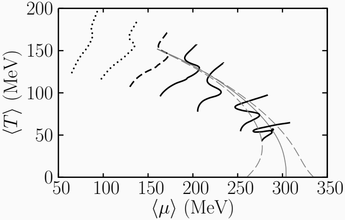
backup

Trajectories and isentropes at finite μ_B

Isentropes in the PQM model



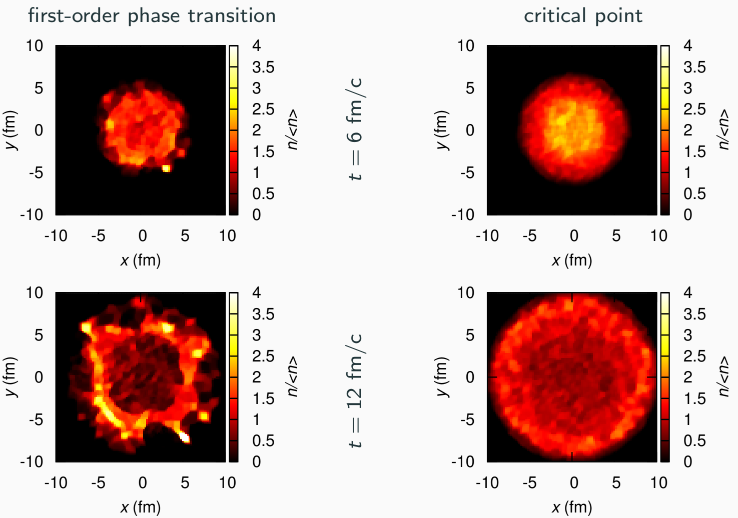
Fluid dynamical trajectories



- Fluid dynamical trajectories similar to the isentropes in the crossover region.
- No significant features in the trajectories left of the critical point.
- Right of the critical point: trajectories differ from isentropes and the system spends significant time in the spinodal region! $\Rightarrow$ possibility of spinodal decomposition!

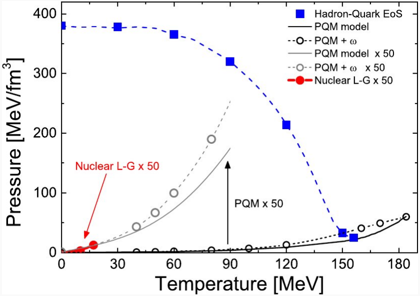
C. Herold, MN, I. Mishustin, M. Bleicher NPA**925** (2014)

Domain formation in net-baryon density - PQM



C. Herold, MN, I. Mishustin, M. Bleicher NPA925 (2014)

EoS: PQM versus QH



J. Steinheimer, J. Randrup, V. Koch PRC89 (2014)

- several eos lead to similar pressures at $\mu_B \approx 0$, but differ at large μ_B
- with coexistence between dense quark matter and compressed nuclear matter (HQ-EoS) : $\partial p_c/\partial T < 0$
- from effective models, like PNJL, PQM etc.: $\partial p_c/\partial T > 0$

V. Dexheimer, S. Schramm, PRC81 (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC88 (2013)

Critical point: nonequilibrium correlation length

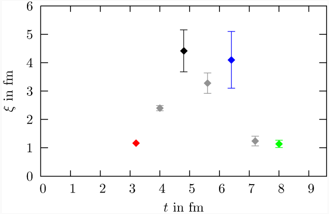
Phenomenological equation: $\frac{\mathrm{d}}{\mathrm{d}t} m_\sigma(t) = -\Gamma[m_\sigma(t)] \left(m_\sigma(t) - \frac{1}{\xi_{\mathrm{eq}}(t)} \right)$

with input from the dynamical universality class $\Rightarrow \xi \sim 1.5 - 2.5$ fm

B. Berdnikov and K. Rajagopal, PRD 61 (2000)

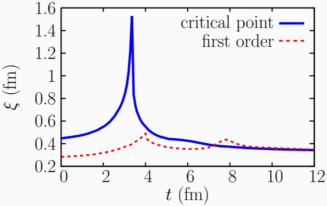
$$G(r) = \int \mathrm{d}^3x \mathrm{d}^3y \langle \sigma(x) - \sigma_0 \rangle \langle \sigma(y) - \sigma_0 \rangle$$
$$\sim \exp(-r/\xi)$$

Assume σ_0 is the volume averaged field.



From the curvature of V_{eff}:

$$\langle \xi^2 \rangle = \langle 1/m_\sigma^2 \rangle = \left\langle \left(\frac{\mathrm{d}^2 V_{\mathrm{eff}}}{\mathrm{d}\sigma^2} \right)^{-1} \right\rangle$$



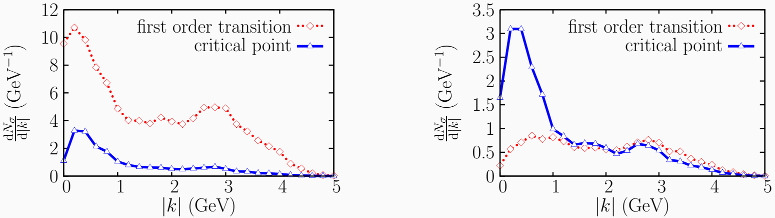
C. Herold, MN, I. Mishustin, M. Bleicher PRC87 (2013)

Definition of ξ in inhomogeneous systems involves averaging!

$\Rightarrow$ Similar magnitude of $\xi \sim 1.5 - 3$ fm!

Dynamics versus equilibration

- Static box with temperature quench to $T < T_c$.
- Fluctuations of the order parameter:



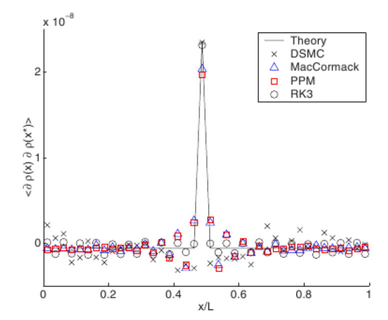
- Strong enhancement of the intensities for a first-order phase transition during the evolution.
- Strong enhancement of the intensities for a critical point scenario after equilibration.

C. Herold, MN, I. Mishustin, M. Bleicher PRC **87** (2013)

Fluid dynamical fluctuations

- In a numerical treatment → discretization: $\langle \xi^2 \rangle \propto \frac{1}{\Delta V}$
- ⇒ large fluctuations from cell to cell ⇒ coarse-graining, smearing, etc. compare to expectations from equilibrium and MC kinetic theory!

- Example: non-relativistic Navier-Stokes + fluctuations
- 1d, dilute gas, periodic boundary conditions



J. Bell, A. Garcia, S. Williams, PRE**76** (2007)

- Different algorithms treat fluctuations differently, third-order methods seem to work best.

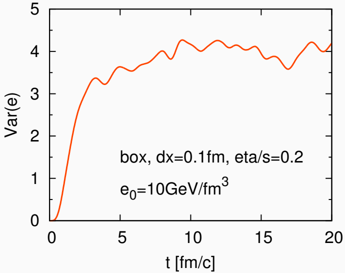
Fluid dynamical fluctuations

- Static box with periodic boundary conditions in relativistic $3 + 1$d fluid dynamics

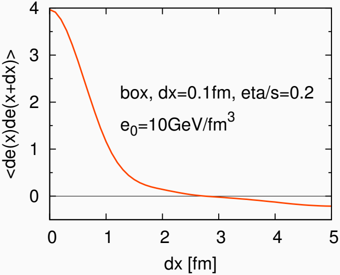
  based on $3 + 1$d viscous fluid dynamical code by Y. Karpenko.

- Noise correlated over 1 fm^3

time evolution of the variance $\langle \delta e^2 \rangle$:



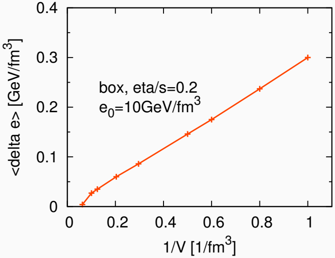
$\langle \delta e(x) \delta e(x + dx) \rangle$ correlation function:

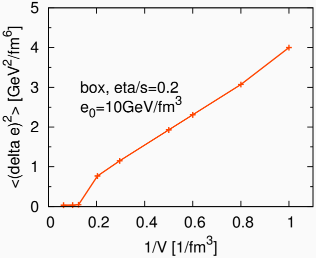


- Average energy(-momentum) conserved within 5%.
- Variance of the energy density fluctuations are approximately $30 - 40\%$ of what is expected in a grandcanonical ensemble.

work in progress

Fluid dynamical fluctuations

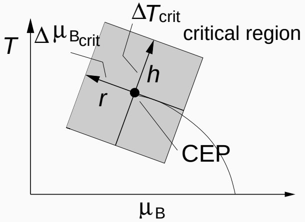




- Reduction of the pressure due to the nonlinearities in the fluctuations.
- Variance of energy density fluctuations approximately $30-40\%$ of what is expected in a grandcanonical ensemble.
- NEXT: include net-baryon number density, diffusion and fluctuations.

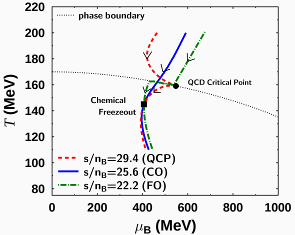
work in progress

Equation of state: critical point



- construct an eos with CP from the universality class of the 3d Ising model
- map the temperature and the external magnetic field (r, h) onto the (T, μ)-plane $\Rightarrow$ critical part of the entropy density S_c
- match with nonsingular entropy density from QGP and the hadron phase:

$$s = 1/2(1 - \tanh S_\mathrm{c})s_\mathrm{H} + 1/2(1 + \tanh S_\mathrm{c})s_\mathrm{QGP}$$



- focussing of trajectories ... or not? Strongly depends on mapping & matching!

C. Nonaka, M. Asakawa PRC71 (2005); M. Asakawa, S. Bass, B. Mueller, C. Nonaka PRL101 (2008)

Equation of state - effective models

- hadronic SU(3) non-linear sigma model including quark degrees of freedom

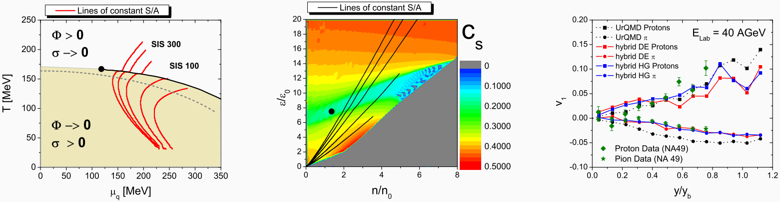
$$\mathcal{L} = \sum_i \bar{\psi}_i (i\gamma^\mu \partial_\mu - \gamma^0 (g_{i\omega}\omega + g_{i\phi}\phi) - M_i)\psi_i + \mathcal{L}_{\mathrm{M,kin}} - U(\sigma,\zeta,\omega) - \mathcal{U}(\ell)$$

  and effective masses generated by

$$M_q = g_{q\sigma}\sigma + g_{q\zeta}\zeta + M_{0q} + g_{q\ell}(1-\ell)$$
$$M_B = g_{B\sigma}\sigma + g_{B\zeta}\zeta + M_{0B} + g_{qB}\ell^2$$

  V. Dexheimer, S. Schramm, PRC81 (2010); M. Hempel, V. Dexheimer, S. Schramm, I. Iosilevskiy PRC88 (2013)

- the scalar Polyakov field ℓ suppresses the baryons at high temperatures/density and the quarks at low temperatures/densities
- realistic structure of the phase diagram and phenomenologically correct results for saturated nuclear matter



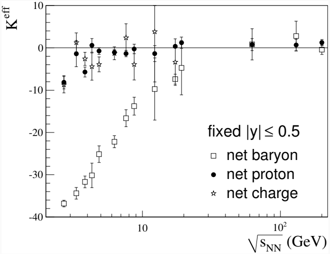
J. Steinheimer, V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm, H. Stoecker, PRC81 (2010)

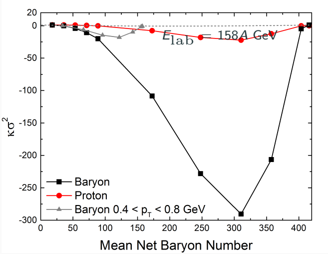
Non-critical fluctuations: charge conservation

- Global net-baryon number conservation.

  MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012); A. Bzdak, V. Koch, V. Skokov, PRC87 (2013)

- In a microscopic transport model the microcanonical nature of individual scatterings is preserved.
- Strongly negative kurtosis of net-baryon number due to global conservation and volume fluctuations.
- Net-proton fluctuations follow this trend slightly.





by J. Steinheimer

MN, T. Schuster, M. Mitrovski, R. Stock, M. Bleicher, EPJC72 (2012)

What's the critical mode?

- At $\mu_B \neq 0$ σ mixes with the net-baryon density n (and e and $\vec{m}$)
- In a Ginzburg-Landau formalism:

$$V(\sigma, n) = \int \mathrm{d}^3x (\sum_m (a_m \sigma^m + b_m n^m) + \sum_{m,l} c_{m,l} \sigma^m n^l) - h\sigma - jn$$

- $V(\sigma, n)$ has a flat direction in $(a\sigma, bn)$ direction
- Equations of motion (including symmetries in $V(\sigma, n)$):

$$\partial_t^2 \sigma = -\Gamma \delta V / \delta \sigma + \ldots$$
$$\partial_t n = \gamma \vec{\nabla}^2 \delta V / \delta n + \ldots$$

- two time scales (with $D \to 0$ at the critical point)

$$\omega_1 \propto -i\Gamma a$$
$$\omega_2 \propto -i\gamma D \vec{q}^2$$

- The diffusive mode becomes the critical mode in the long-time dynamics. These fluctuations need to be included at the critical point!

H. Fuji, M. Ohtani PRD**70** (2004); M. Stephanov, D. Son PRD**70** (2004)

Fluid dynamical fluctuations

- Linearized fluid dynamical equations: small fluctuations $\bar{e} + \delta e$, $\bar{p} + \delta p$ and δv^i with: $\delta T^{00} = \delta e$ and $\delta T^{ij} = m^i = (\bar{e} + \bar{p}) v^i = \bar{w} v^i$

$$\begin{aligned} \partial_t \mathbf{m}_\perp + \eta / \bar{w} \mathbf{k}^2 \mathbf{m}_\perp &= 0 \\ \partial_t \delta e + i \mathbf{k} \cdot \mathbf{m}_{||} &= 0 \\ \partial_t \mathbf{m}_{||} + i v_s^2 \mathbf{k} \delta e + \gamma_v \mathbf{k}^2 \mathbf{m}_{||} &= 0 \end{aligned}$$

- retarded Green's function for δe and $\mathbf{m}_{||}$:

$$G^{\mathrm{ret}}_{ab}(\omega, \mathbf{k}) = \frac{\bar{w}}{\omega^2 - v_s^2 \mathbf{k}^2 + i \omega \gamma_s \mathbf{k}^2} \begin{pmatrix} \mathbf{k}^2 & \omega |\mathbf{k}| \\ \omega |\mathbf{k}| & v_s^2 \mathbf{k}^2 - i \omega \gamma_s \mathbf{k}^2 \end{pmatrix}$$

- including the transverse momentum density:

$$G^{\mathrm{ret}}_{m_i, m_j}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \frac{\eta \mathbf{k}^2}{i \omega - \gamma_\eta \mathbf{k}^2} + \frac{k_i k_j}{\mathbf{k}^2} \frac{\bar{w} (v_s^2 \mathbf{k}^2 - i \omega \gamma_s \mathbf{k}^2)}{\omega^2 - v_s^2 \mathbf{k}^2 + i \omega \gamma_s \mathbf{k}^2}$$

- Kubo-formulas for viscosities:

$$\begin{aligned} \eta &= - \frac{\omega}{2 \mathbf{k}^2} \left(\delta_{ij} - \frac{k_i k_j}{\mathbf{k}^2} \right) \Im G^{\mathrm{ret}}_{m_i m_j}(\omega, \mathbf{k} \to 0) \\ \zeta + \frac{4}{3} \eta &= - \frac{\omega^3}{\mathbf{k}^4} \Im G^{\mathrm{ret}}_{ee}(\omega, \mathbf{k} \to 0) \end{aligned}$$

Fluid dynamical fluctuations

$$\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu 0}(x)\Xi^{\mu 0}(x')\rangle^S = -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu 0}(x)T^{\mu 0}(x')\rangle^S$$

$$= \int \frac{\mathrm{d}\omega}{2\pi}\int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')}e^{-i\omega(t-t')}\times$$

$$\times\left(\omega^2 \underbrace{G^S_{ee}(\omega,\mathbf{k})}_{\text{FDT}} - 2\omega|\mathbf{k}|\underbrace{G^S_{em_{||}}(\omega,\mathbf{k})}_{\text{FDT}} + \mathbf{k}^2 \underbrace{G^S_{m_{||}m_{||}}(\omega,\mathbf{k})}_{\text{FDT}}\right)$$

$$G^S_{ab}(\omega,\mathbf{k}) = -\frac{2T}{\omega}\Im G^{\mathrm{ret}}_{ab}(\omega,\mathbf{k})$$

$$= 0$$

$$\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle \Xi^{\mu i}(x)\Xi^{\mu j}(x')\rangle^S = -\frac{\partial}{\partial_x^\mu}\frac{\partial}{\partial_{x'}^\mu}\langle T^{\mu i}(x)T^{\mu j}(x')\rangle^S$$

$$= 2T\left[\left(\zeta + \frac{4}{3}\eta\right)\partial_i\partial_j + \eta(\delta_{ij}\nabla^2 - \partial_i\partial_j)\right]\delta^4(x-x')$$

Then boost to arbitrary frame:

Fluid dynamical fluctuations

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Delta T^{\mu\nu}_{\rm visc} + \Xi^{\mu\nu}$$
$$N^{\mu} = N^{\mu}_{\rm eq} + \Delta N^{\mu}_{\rm visc} + \mathrm{I}^{\mu}$$

with

$$\langle \Xi^{\mu\nu}(x)\Xi^{\alpha\beta}(x')\rangle = 2T[\eta(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) + (\zeta - 2/3\eta)\Delta^{\mu\nu}\Delta^{\alpha\beta}]\delta^4(x - x')$$

- In second-order fluid dynamics there are relaxation equations for $\Xi^{\mu\nu}$:

$$u^\gamma \partial_\gamma \Xi^{\langle\mu\nu\rangle} = -\frac{\Xi^{\mu\nu} - \xi^{\mu\nu}_{\rm gauss}}{\tau_\pi}$$

- In white noise approximation and ignoring bulk viscosity ($\zeta = 0$):

$$\langle \xi^{\mu\nu}_{\rm gauss}(x)\xi^{\alpha\beta}_{\rm gauss}(x')\rangle = 4T\eta\Delta^{\mu\nu\alpha\beta}\delta^{(4)}(x - x')$$