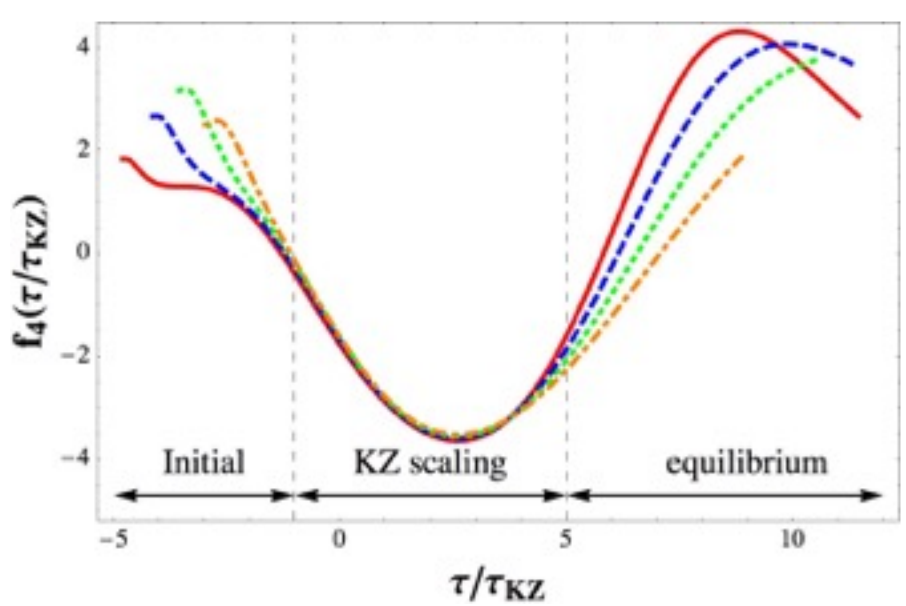
Kibble-Zurek dynamics and non-equilibrium scaling for critical cumulants

Yi Yin

BROOKHAVEN NATIONAL LABORATORY Nuclear Theory Group



Topical Workshop on Beam Energy Scan,
Indiana University, May. 9-11, 2016

Motivations

Critical Fluctuations and Static Universality

- Fluctuations of critical mode σ : scale with correlation length near a critical point.

- Higher cumulants: stronger dependence on $\xi_{\rm eq}$, universal pattern in sign (Stephanov, 2009, 2011) from static universality (the same as 3d Ising model).

$$\kappa_2^{eq} \equiv \langle \delta\sigma^2 \rangle \sim \xi_{eq}^2$$

$$\kappa_3^{eq} \equiv \langle \delta\sigma^3 \rangle \sim \xi_{eq}^{4.5} \qquad \kappa_4^{eq} \equiv \langle \delta\sigma^4 \rangle - 3\left(\kappa_2^{\rm eq}\right)^2 \sim \xi_{eq}^7$$

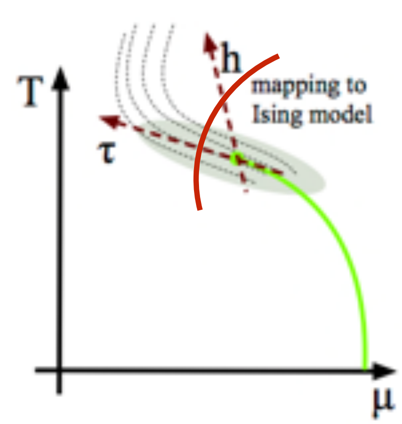
Static universality is not enough

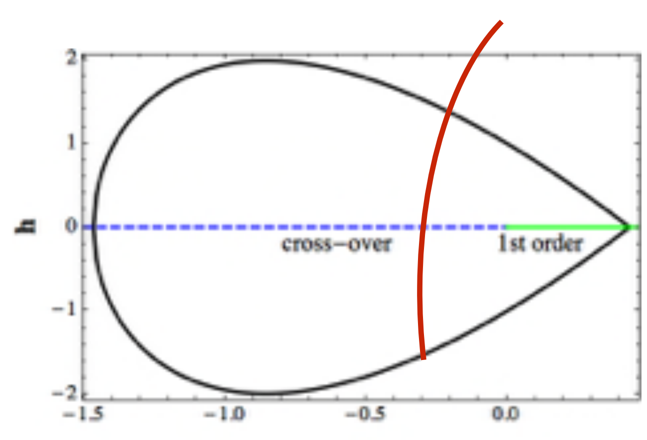
- Non-equilibrium effects are unavoidable: $\tau_\sigma \sim \xi_{eq}^z, \qquad z \approx 3$

- We need to determine cumulants $\kappa_n(\tau)$ along a trajectory (parametrized by the proper time τ) passing the QCD critical regime.

- We derived a set of novel evolution equations of cumulants (S. Mukherjee, R. Venugopalan and YY, 1506.00645, PRC; 1512.08022, QM proceedings).

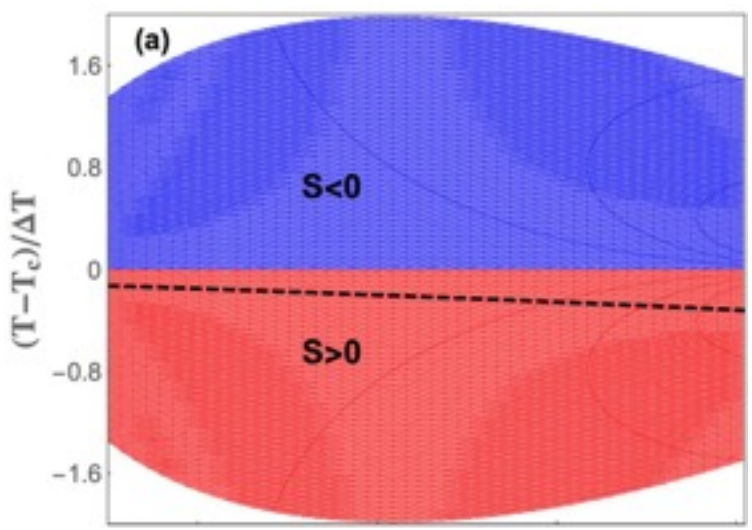
(useful trick: mapping between QCD variable and Ising variables)



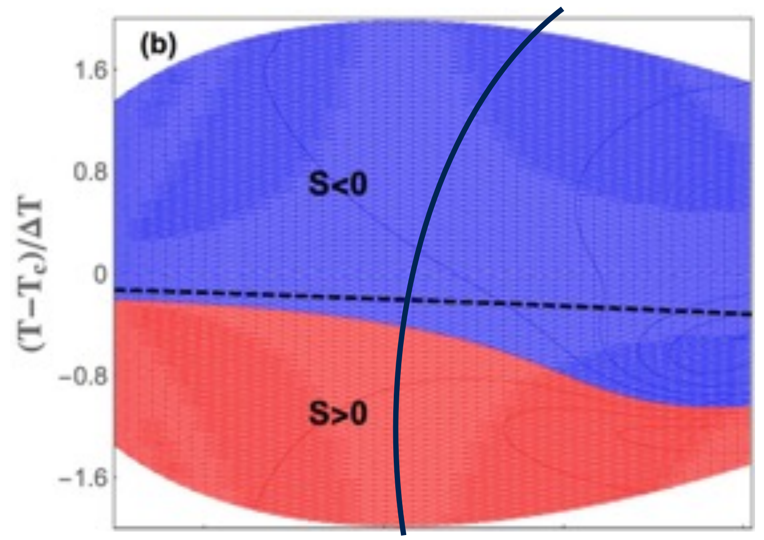


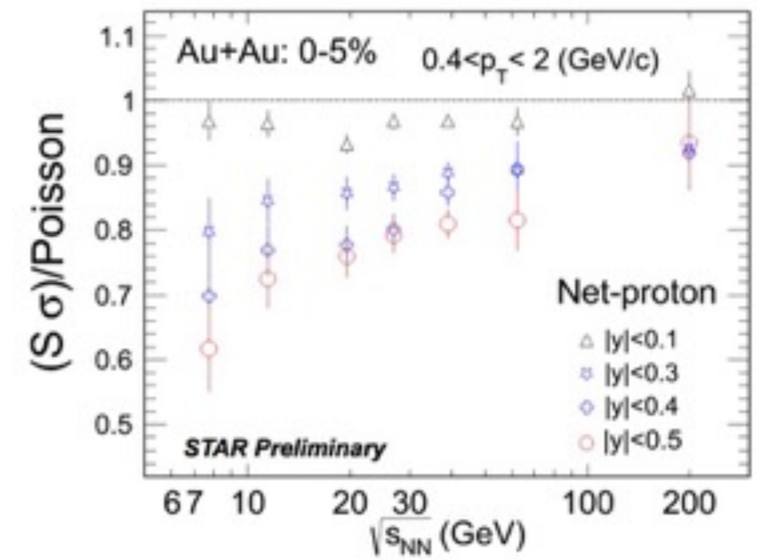
Example: non-equilibrium Skewness

Equilibrium

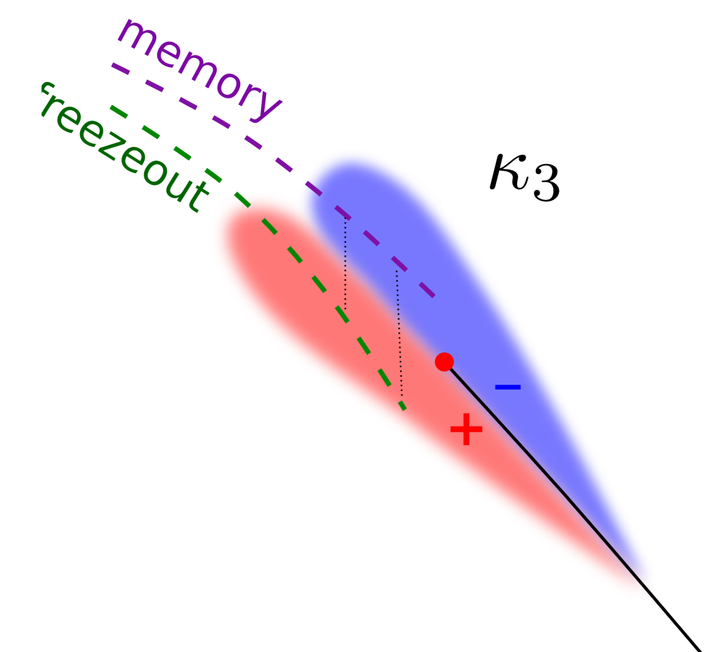


non-equilibrium





Decreasing beam energy

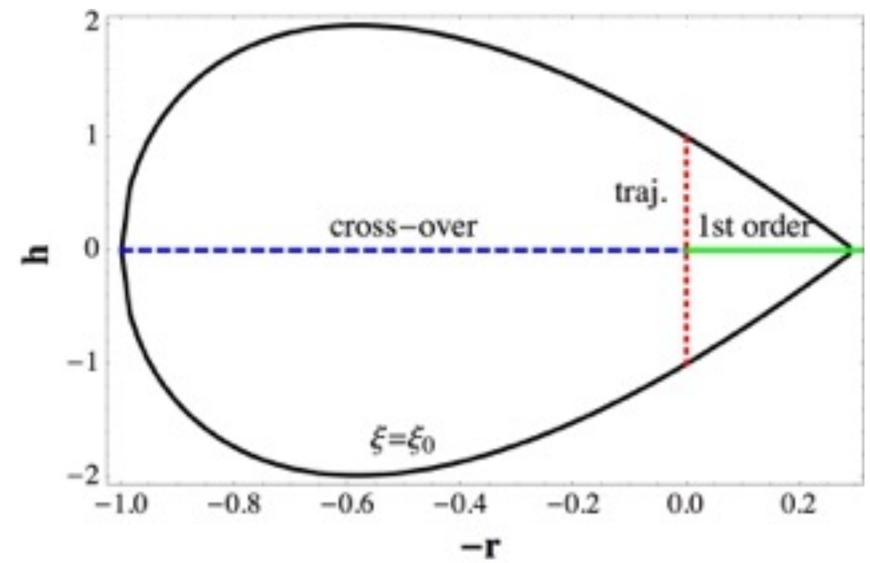


- "Sign puzzle" of skewness: "remembrance of things past".
- Non-equilibrium critical cumulants could be qualitatively different from equilibrium expectations.

- When do non-equilibriums effects become important?
- Which non-universal inputs (collectively denoted by Γ) dominate the dynamics?
- Are there any universal features of non-equilibrium cumulants which suggest the presence of a critical point?
- Answers to those questions are connected by: *Kibble-Zurek dynamics*.

Kibble-Zurek dynamics

An illustrative example



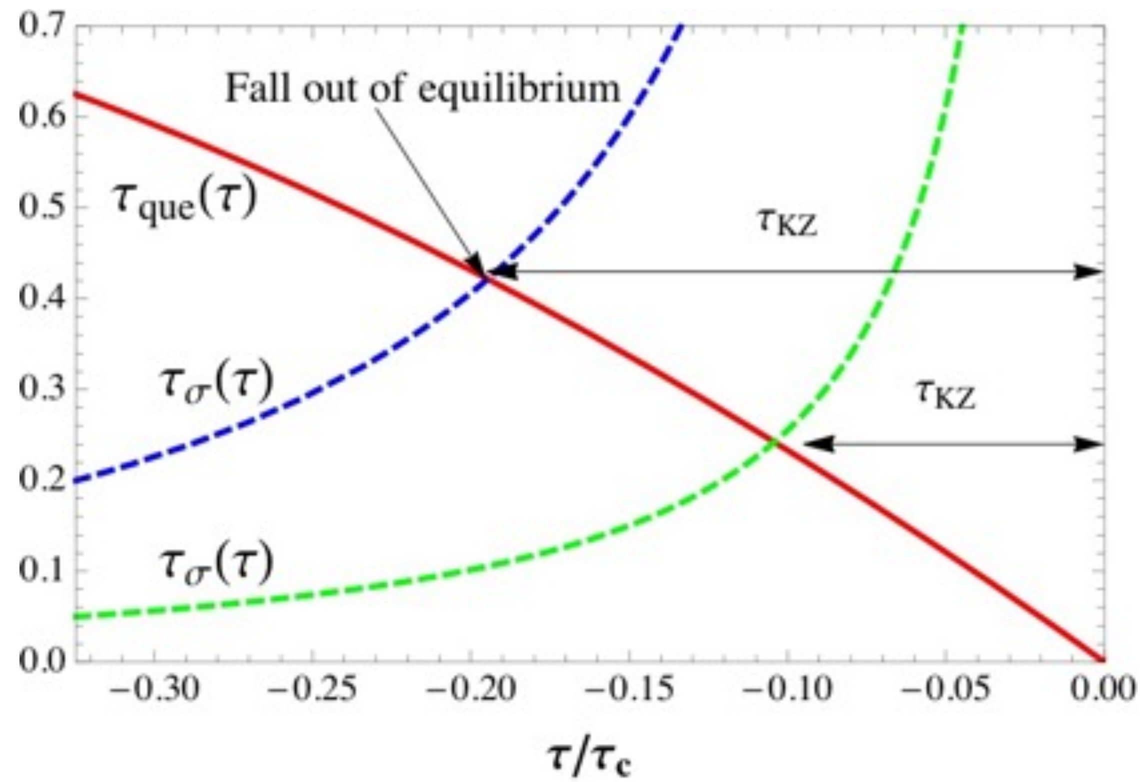
- Consider a trajectory passing the critical point first discussed in Berdnikov-Rajagopal-1999).

- Parametrizing trajectory as:

$$h(\tau) \sim [\,T(\tau) - T_c\,] \qquad T(\tau) = T_c \left(\frac{\tau}{\tau_c}\right)^{-n_V c_s^2}$$

  - The evolution of relaxation time is now known:

$$\tau_\sigma(\tau) = \tau_\sigma^0 \left[\frac{\xi_{\rm eq}(\tau)}{\xi_0}\right]^3 \qquad \xi_{\rm eq}(\tau) \sim |h(\tau)|^{-\frac{\nu}{\beta\delta}}$$



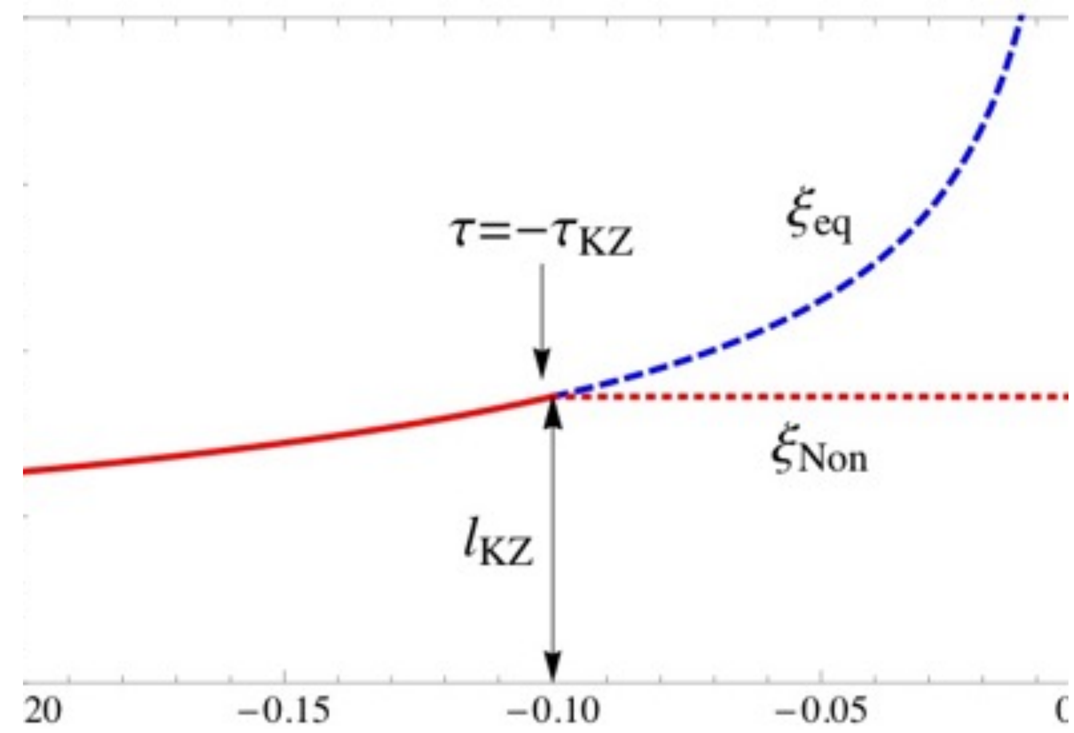
Q: when do non-equilibrium effects become important?

A: relaxation time becomes longer than the "quench" time.

$$\tau_\sigma(\tau) > \tau^{\xi}_{\text{que}} \equiv \left|\frac{\xi_{\text{eq}}}{\partial_\tau \xi_{\text{eq}}}\right|$$

- Kibble-Zurek time (Kibble, domain growth in early universe, 1976, Zurek, Superfluid, 1993): an emergent time scale for non-equilibrium dynamics.

$$\tau_\sigma(\tau_{\text{KZ}}) = \tau_{\text{que}}(\tau_{\text{KZ}})$$



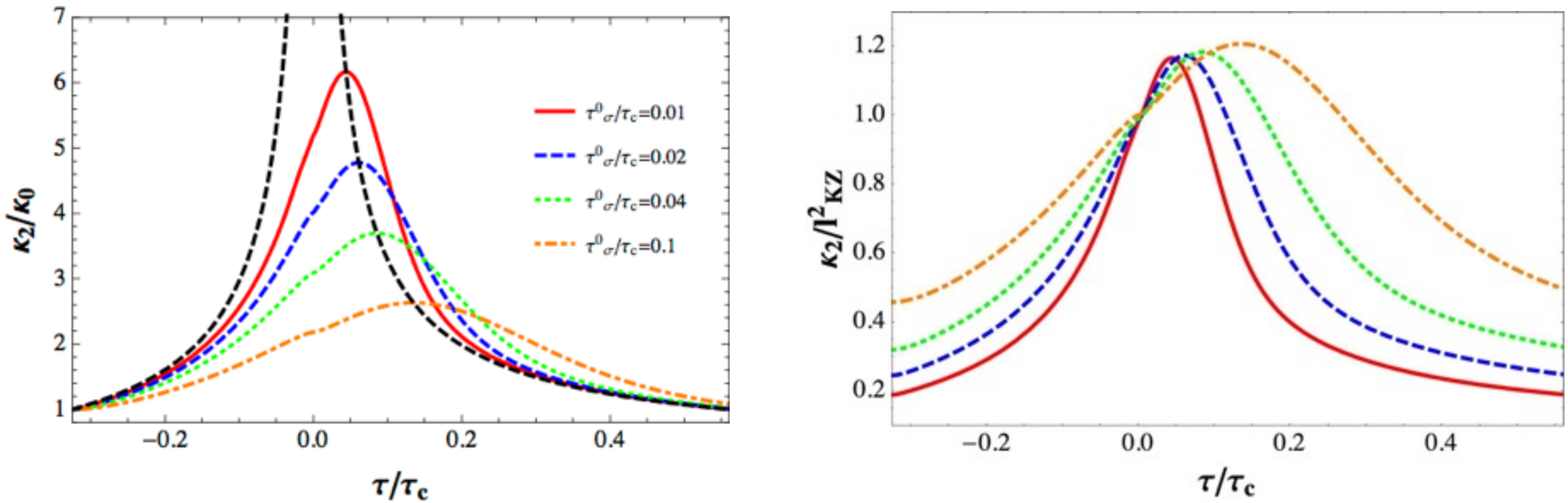
- A simple approximation: the evolution is frozen .

$$l_{KZ} = \xi_{eq}(\tau_{KZ})$$

- Kibble-Zurek dynamics: l_{KZ}, τ_{KZ} determine the length and time scale of the non-equilibrium evolution.

- For example: $\kappa_2 \sim l_{KZ}^2$ and $\kappa_3 \sim l_{KZ}^{9/2}$ $\kappa_4 \sim l_{KZ}^7$

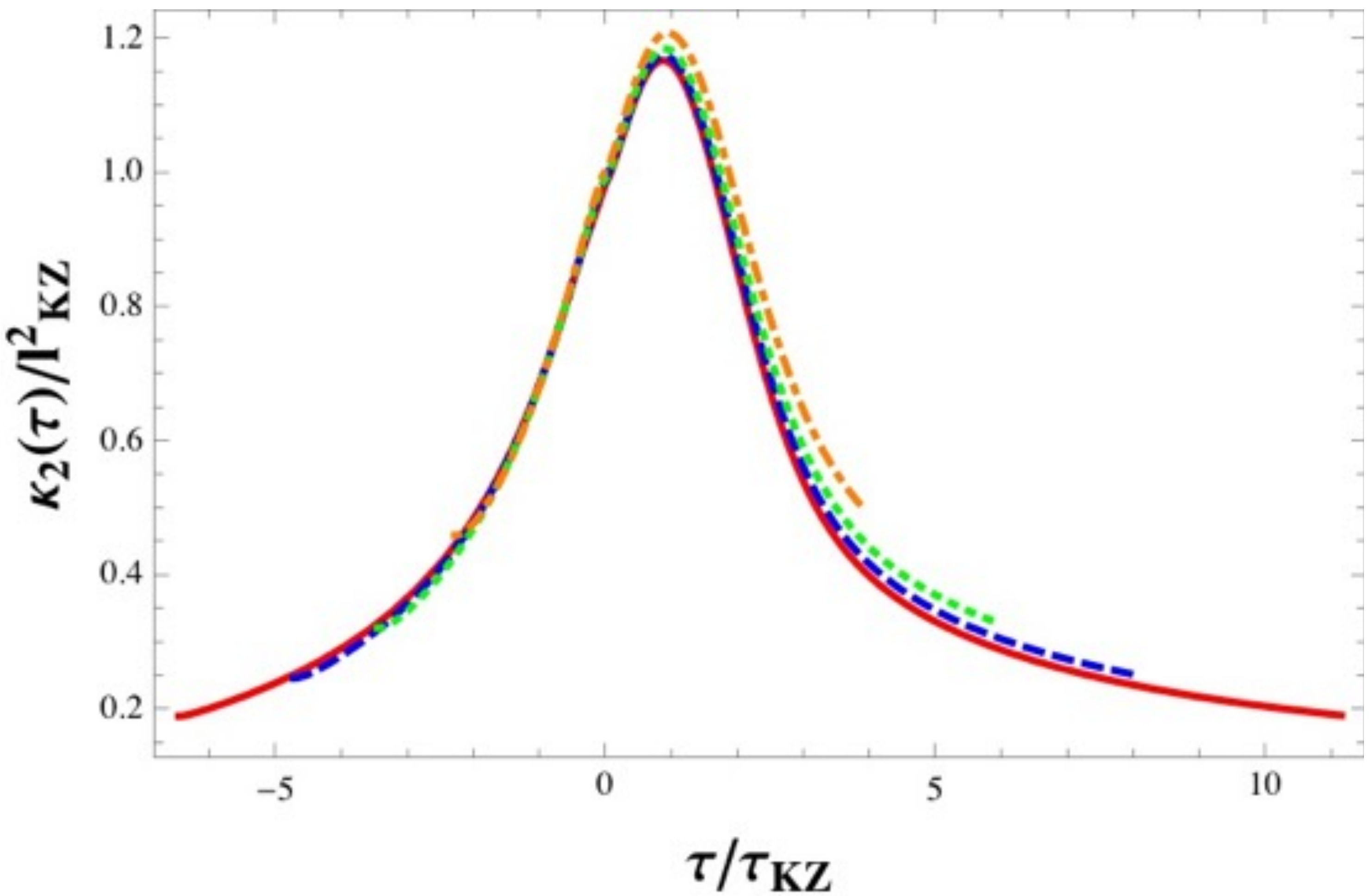
Scaling with length is not enough



- Let us rescale Gaussian cumulants determined from Berdnikov-Rajagopal model by l_{KZ}^2 .

- The peak value now looks universal, but time-dependence does not.

- A step forward: let us rescale time by τ_{KZ} !



- We illustrated the existence of a scaling function:

$$\kappa_2(\tau;\Gamma) \sim l^2_{\mathrm{KZ}}(\Gamma)\,\underbrace{f_2\left(\tau/\tau_{\mathrm{KZ}}(\Gamma)\right)}_{\text{Universal}} \quad (\Gamma\text{: non-universal inputs})$$

- NB: the study of non-equilibrium dynamical scaling is a new frontier in critical dynamics.

The Kibble-Zurek Problem: Universality and the Scaling Limit

Anushya Chandran
Department of Physics, Princeton University, Princeton, NJ 08544

Amir Erez*
Department of Physics, Ben Gurion University of the Negev, Beer-Sheva 84105, Israel

Steven S. Gubser and S. L. Sondhi
Department of Physics, Princeton University, Princeton, NJ 08544

PUPT-2405

(Dated: September 20, 2012)

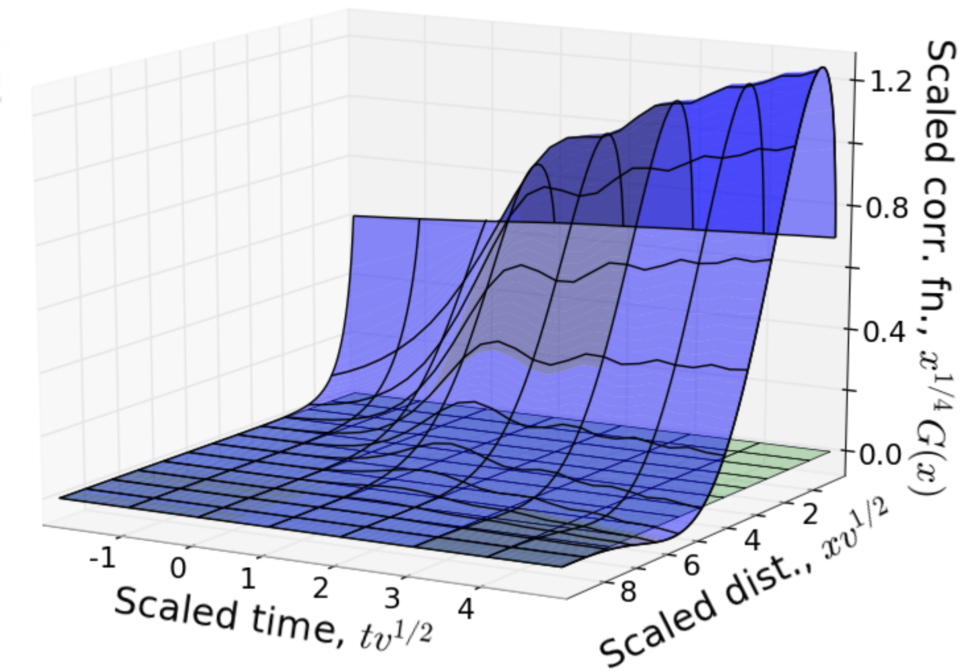
PRL **109,** 015701 (2012) PHYSICAL REVIEW LETTERS week ending 6 JULY 2012

Nonequilibrium Dynamic Critical Scaling of the Quantum Ising Chain

Michael Kolodrubetz,[1] Bryan K. Clark,[1,2] and David A. Huse[1,2]
[1]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*
[2]*Princeton Center for Theoretical Science, Princeton University, Princeton, New Jersey 08544, USA*

(Received 2 February 2012; published 2 July 2012)

We solve for the time-dependent finite-size scaling functions of the one-dimensional transverse-field Ising chain during a linear-in-time ramp of the field through the quantum critical point. We then simulate Mott-insulating bosons in a tilted potential, an experimentally studied system in the same equilibrium universality class, and demonstrate that universality holds for the dynamics as well. We find qualitatively



PRL **116,** 080601 (2016) PHYSICAL REVIEW LETTERS week ending 26 FEBRUARY 2016

Universality in the Dynamics of Second-Order Phase Transitions

G. Nikoghosyan,[1,2] R. Nigmatullin,[3] and M. B. Plenio[1]
[1]*Institut für Theoretische Physik, Albert-Einstein Allee 11, Universität Ulm, 89069 Ulm, Germany*
[2]*Institute of Physical Research, 378410 Ashtarak-2, Armenia*
[3]*Department of Materials, University of Oxford, Oxford OX1 3PH, United Kingdom*

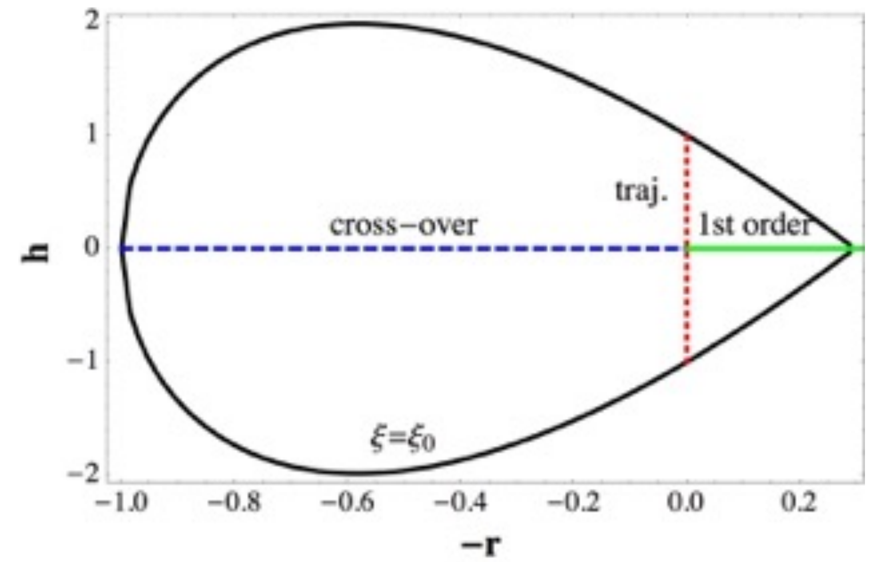
(Received 18 November 2013; revised manuscript received 10 February 2015; published 26 February 2016)

- What does Kibble-Zurek scaling imply for search for QCD critical point via Beam energy scan program?
- Missing gaps in literature:
  - Formulating and testing scaling hypothesis for non-Gaussian cumulants.
  - Generalizing non-equilibrium scaling for trajectories away from the critical point.
- The remainder of this talk: report recent results (S. Mukherjee, R. Venugopalan and YY, in preparation).

Non-equilibrium scaling hypothesis for critical cumulants

$$M(\tau;\Gamma) \sim l_{\rm KZ}^{-1/2}(\Gamma)\, f_1(\tilde{t})$$
$$\kappa_2(\tau;\Gamma) \sim l_{\rm KZ}^{2}(\Gamma)\, f_2(\tilde{t})$$
$$\kappa_3(\tau;\Gamma) \sim l_{\rm KZ}^{9/2}(\Gamma)\, f_3(\tilde{t})$$
$$\kappa_4(\tau;\Gamma) \sim l_{\rm KZ}^{7}(\Gamma)\, f_4(\tilde{t})$$

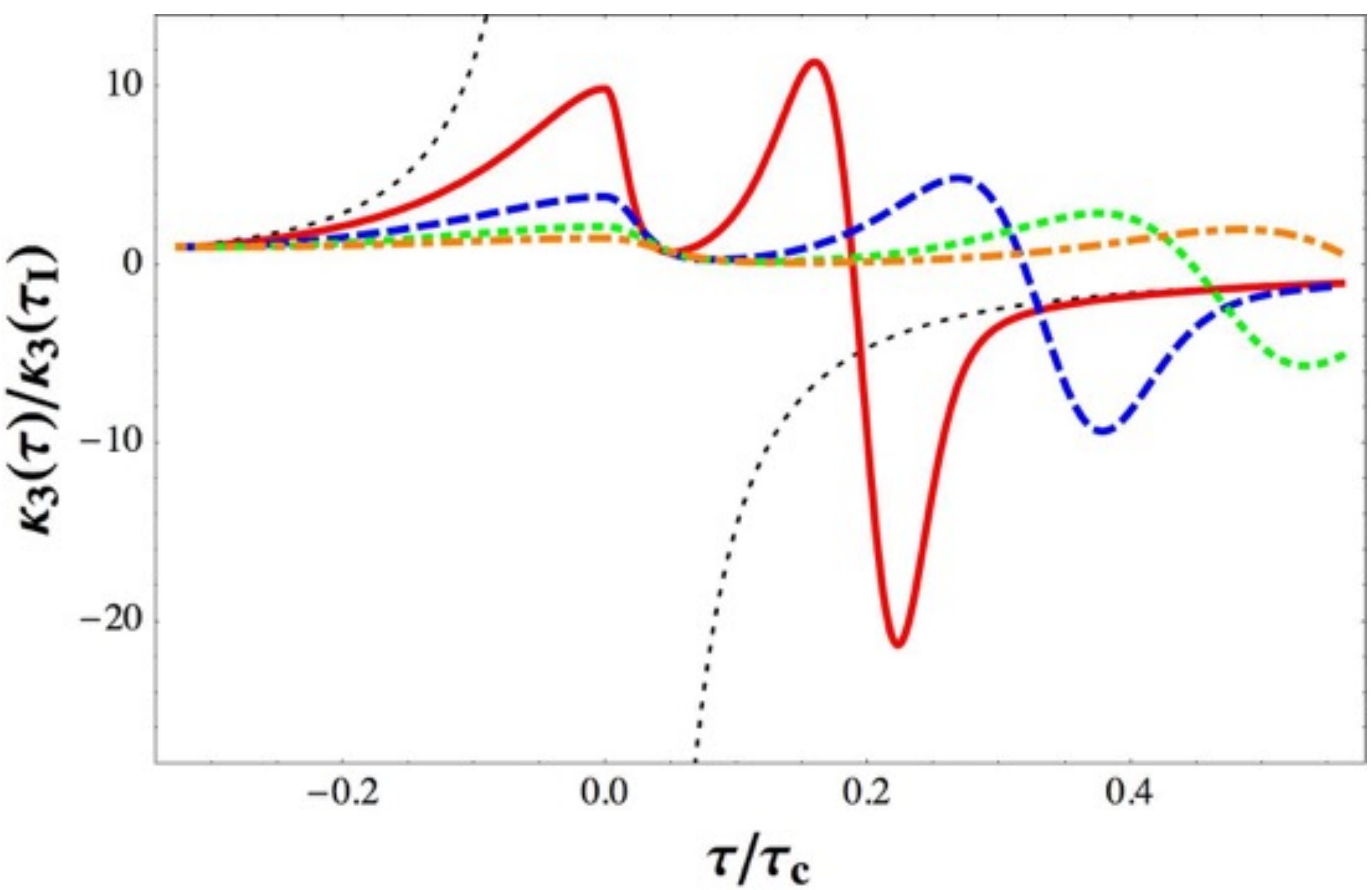
$$\tilde{t} = \frac{\tau}{\tau_{\rm KZ}(\Gamma)}$$



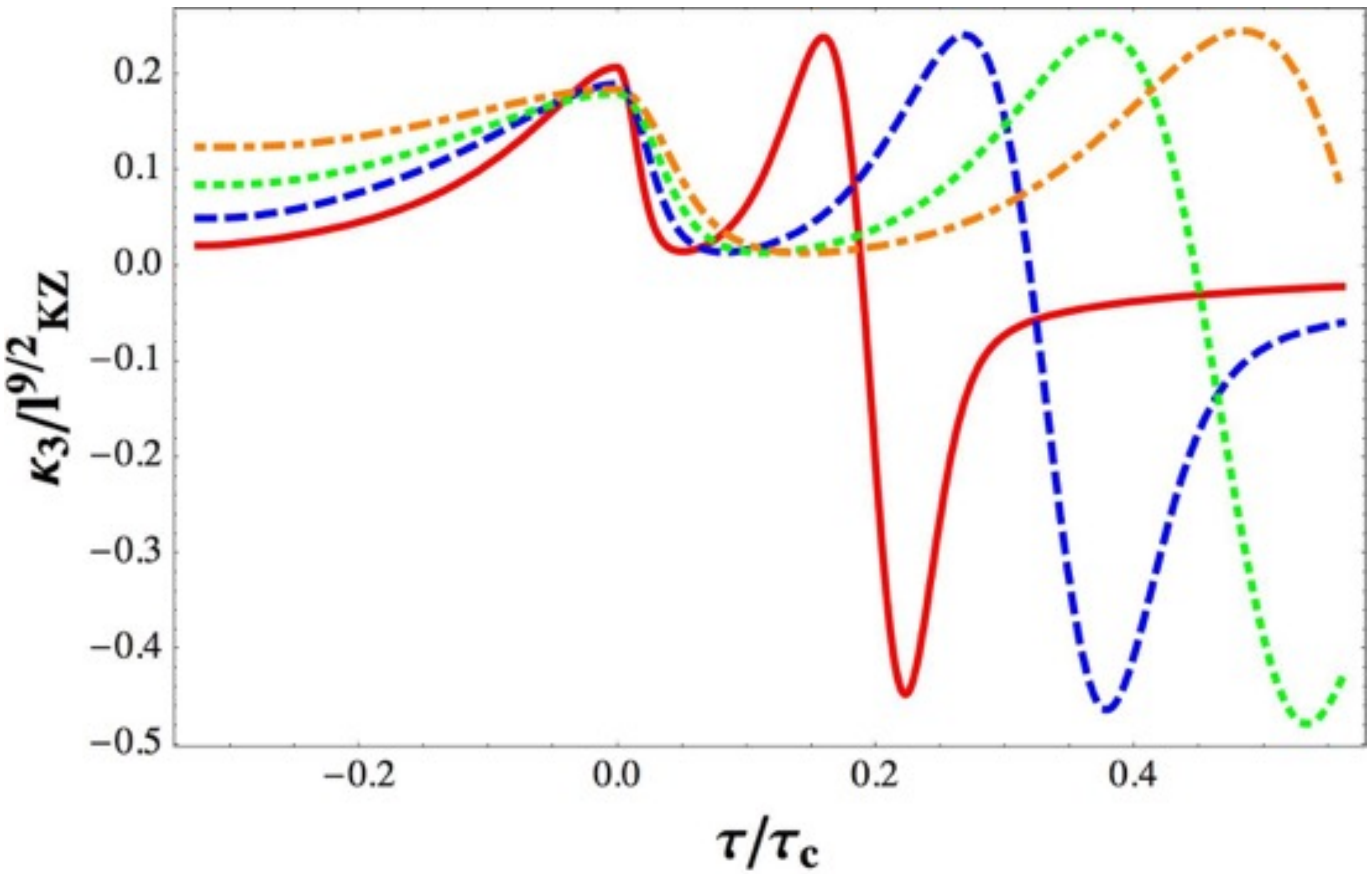
(Γ: non-universal inputs)

- Scaling behavior can be shown exists analytically (under mild technical assumption) for trajectories passing the critical point.

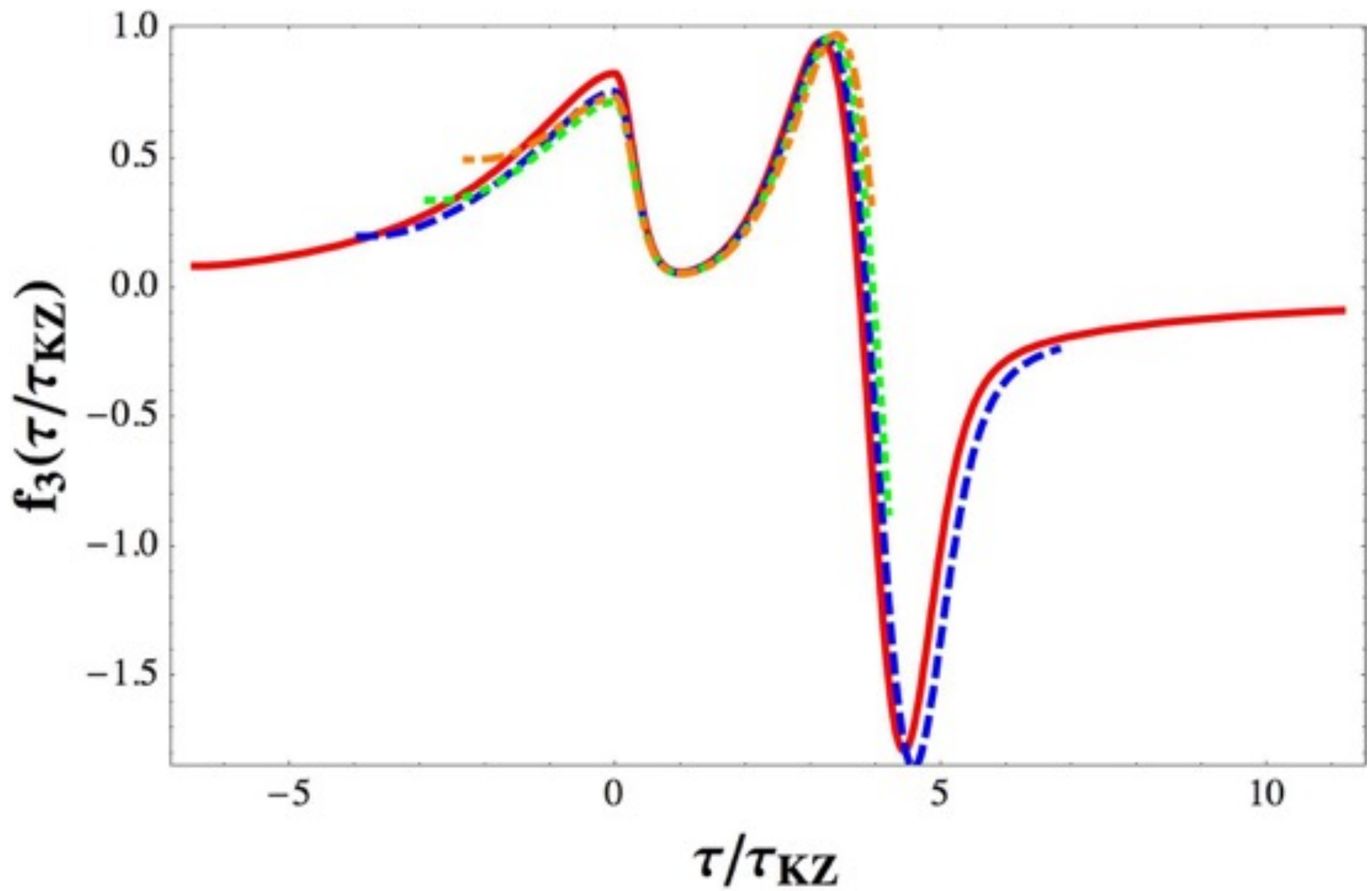
- We also test it numerically for third and fourth cumulants.



10
0
−10
−20
−0.2
0.0
0.2
0.4
$\kappa_3(\tau)/\kappa_3(\tau_I)$
τ/τ_c



Rescaled by l_{KZ}



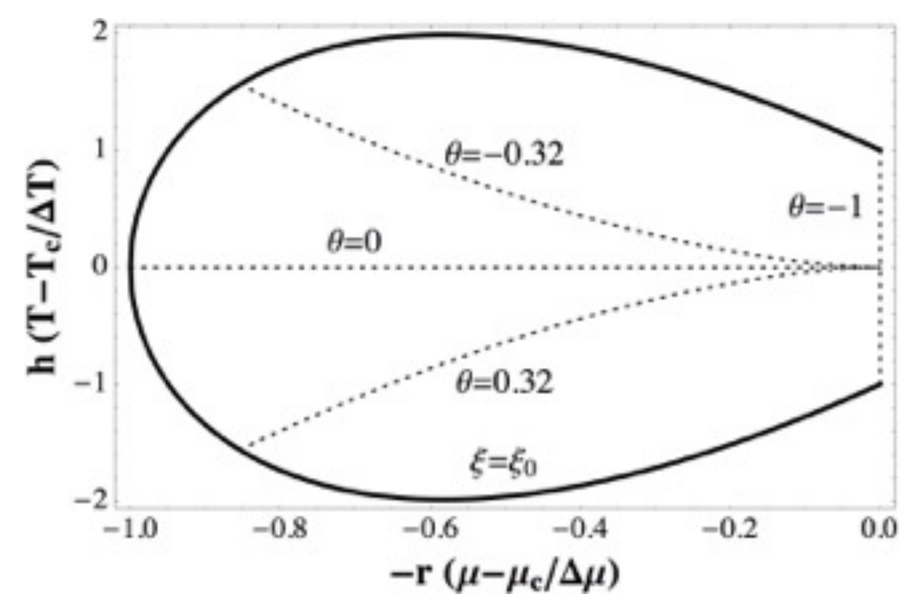
1.0
0.5
0.0
−0.5
−1.0
−1.5
−5
0
5
10
$f_3(\tau/\tau_{KZ})$
τ/τ_{KZ}

Trajectories away from the critical point

- A point in critical regime can be mapped to r, h or $\xi_{\rm eq}(r,h), \theta(r,h)$. The latter will be convenience.

$$M_{\rm eq}(r,h) \sim \xi_{\rm eq}^{-1/2}(r,h)\,\tilde{M}(\theta)$$

- θ is the scaling variable, which changes sign when passing cross-over line. It controls sign of magnetization as well as non-Gaussian cumulants.

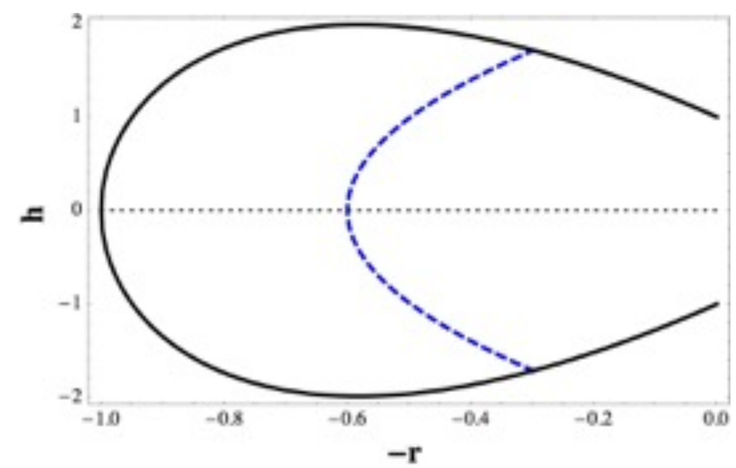


- Two different quench times accordingly:

$$\tau^{\xi}_{\rm que}(\tau) \equiv \left|\frac{\xi_{\rm eq}(\tau)}{\partial_\tau \xi_{\rm eq}(\tau)}\right|, \qquad \tau^{\theta}_{\rm que}(\tau) \equiv \left|\frac{\theta(\tau)}{\partial_\tau \theta(\tau)}\right|$$

A new realization of KZ dynamics

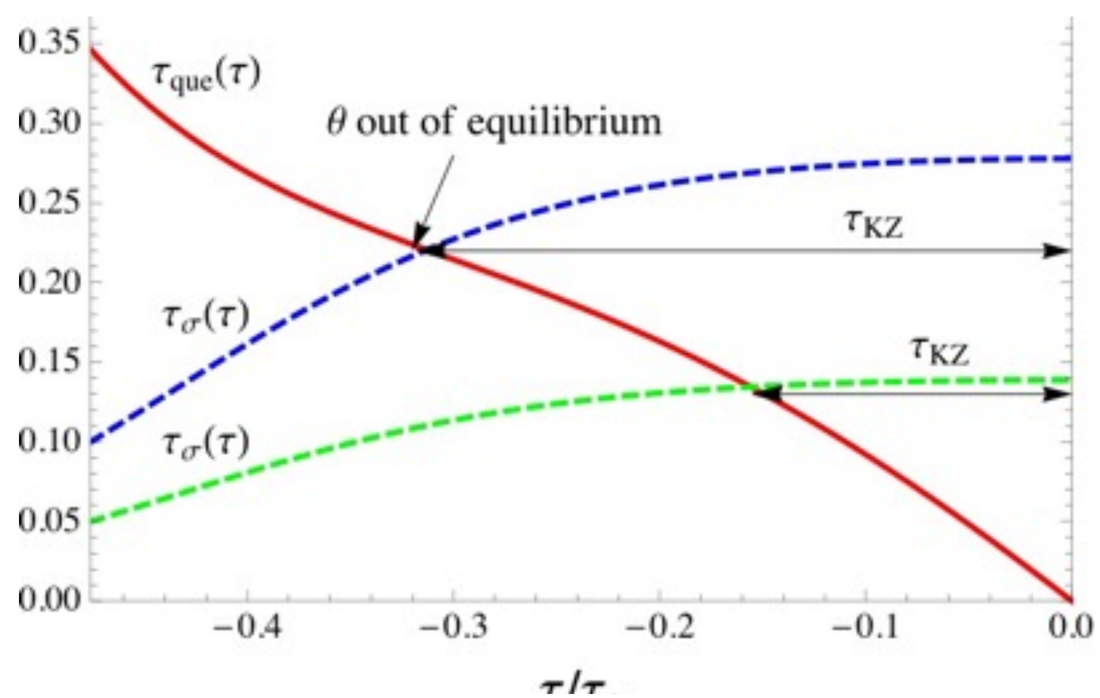
- For a generic trajectory near the cross-over line: (we choose $\tau = 0$ when passing the cross-over line .)



$$\theta(\tau) \propto \tau$$

Consequently, $\tau^{\theta}_{\rm que}(\tau) \sim |\tau| \ll \tau^{\xi}_{\rm que}$

- Relaxation time remains finite, but θ changes too fast!



- A new non-equilibrium scaling variable: ("memory of spin orientation"). $\theta_{\rm KZ} = \theta(-\tau_{\rm KZ})$

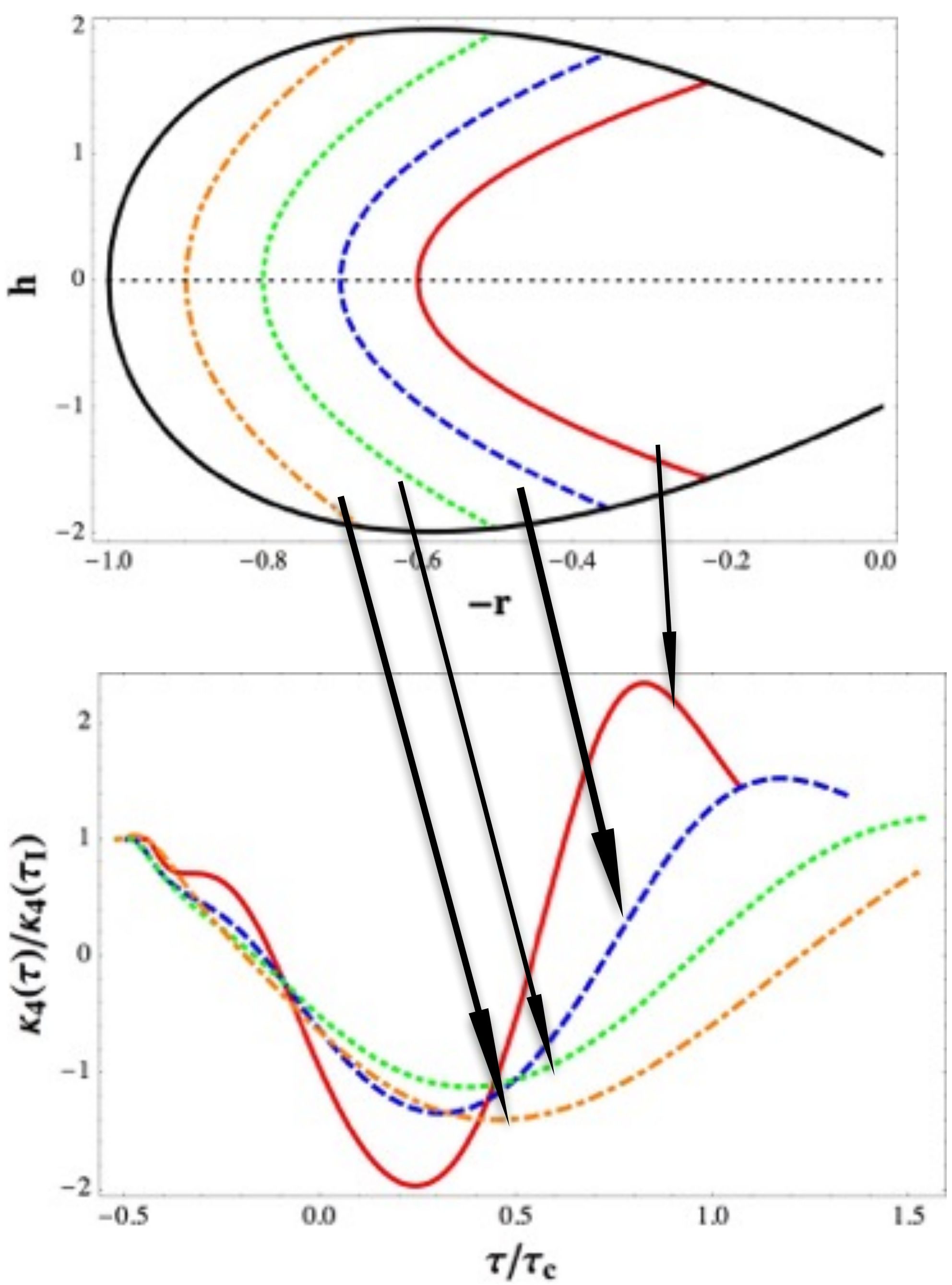
- Generalized scaling hypothesis (S. Mukherjee, R. Venugopalan and YY, in preparation, including *analytic* insights):

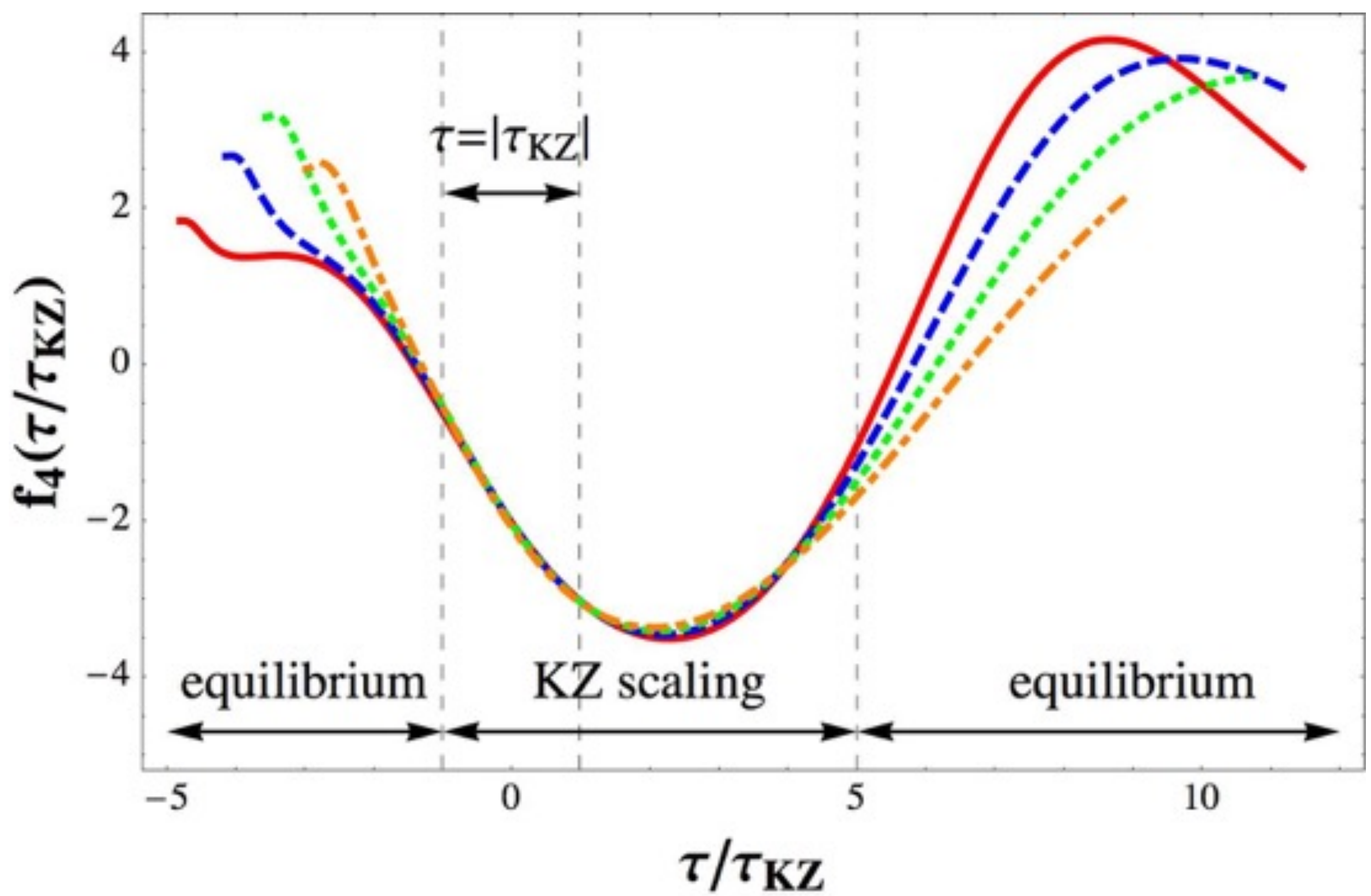
$$\kappa_n(\tau;\Gamma) \sim l_{\rm KZ}^{\#}(\Gamma)\, f_n(\tilde{t}, \theta_{\rm KZ})$$

- Testing scaling hypothesis: different trajectories, same θ_{KZ} .

- Equilibrium cumulants, $\tau_{\mathrm{KZ}}, l_{\mathrm{KZ}}$ are different for those trajectories.

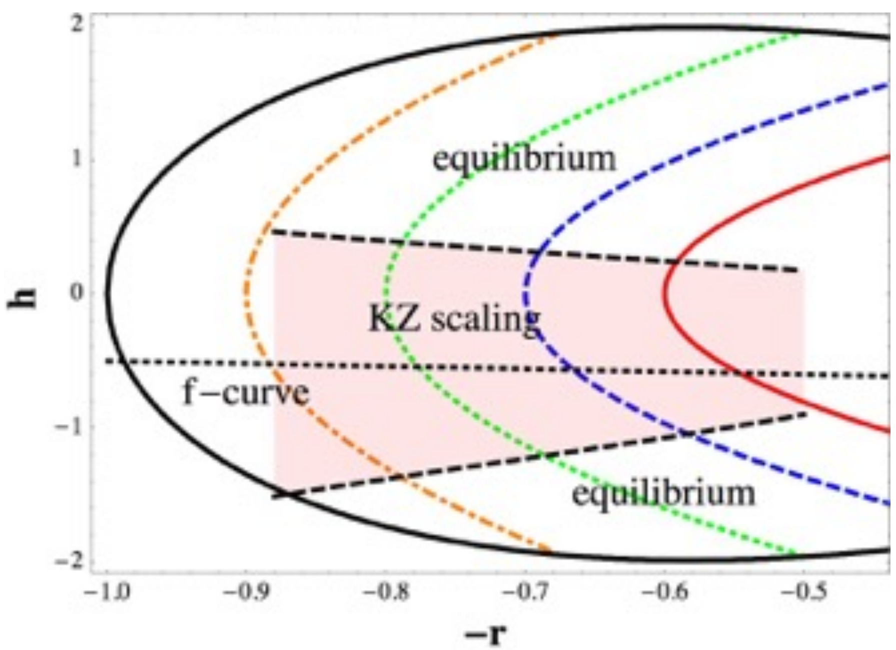
- Expectation from the scaling hypothesis: scaling functions are independent of trajectories.





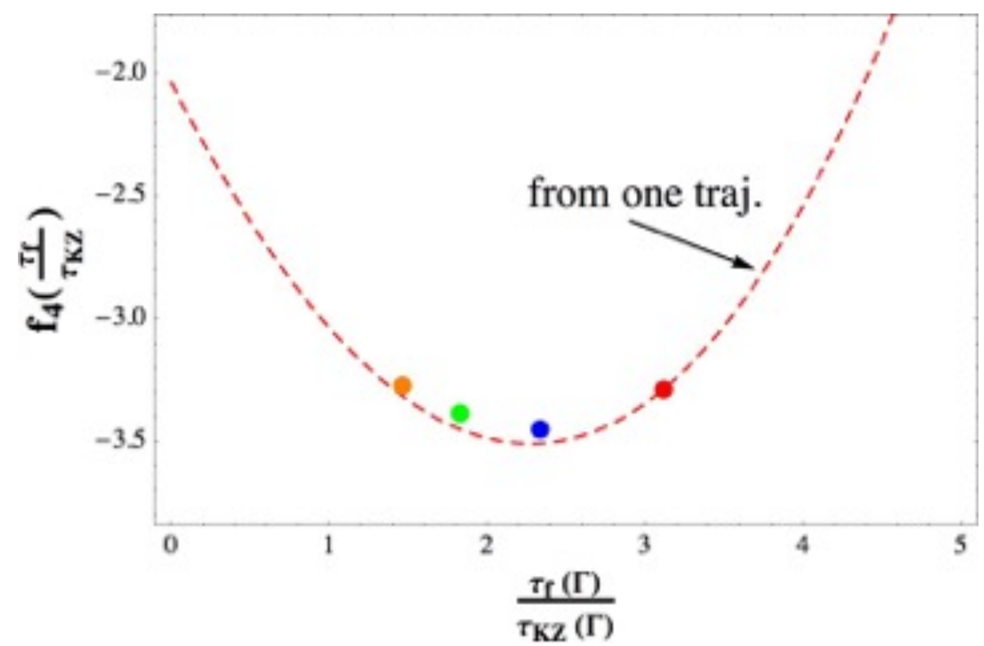
- The system stays much longer in KZ scaling regime than naively expectation (an attractor?).

This extended scaling regime below cross-over curve might potentially be probed by freeze-out curve.



Step 1: finding scaling function from one representative trajectory.

Step 2: determining $\tau_{\rm KZ}(\Gamma), l_{\rm KZ}(\Gamma)$ for other trajectories.



Step 3: check if rescaled cumulants at (rescaled) freeze-out time matches to scaling functions.

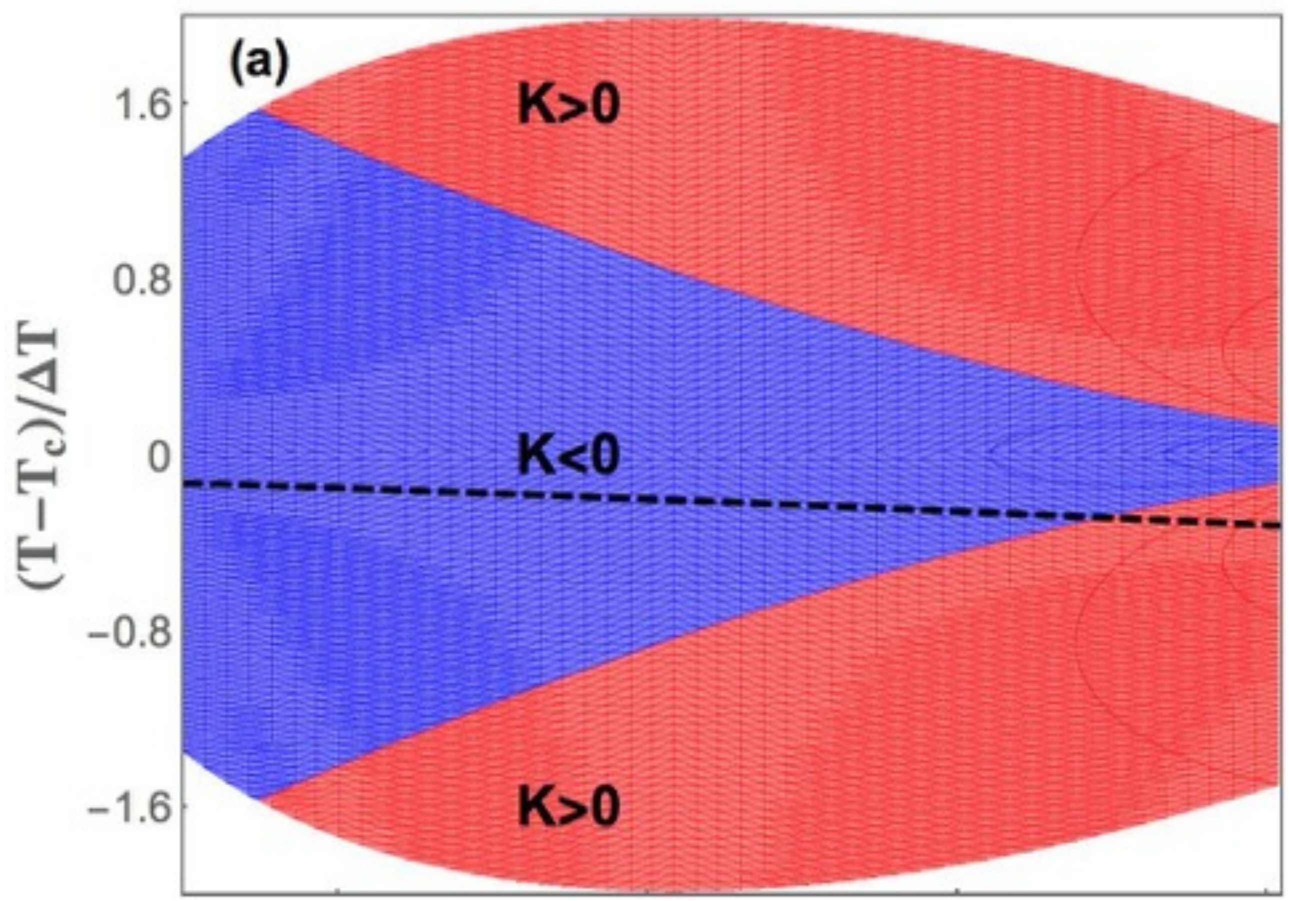
Hydro. modeling is essential for determining $\tau_{\rm KZ}(\Gamma), l_{\rm KZ}(\Gamma)$

A vision: if scaling behavior based on Kibble-Zurek dynamics has been observed in data, this would be a convincing evidence for the existence of critical point.

Summary

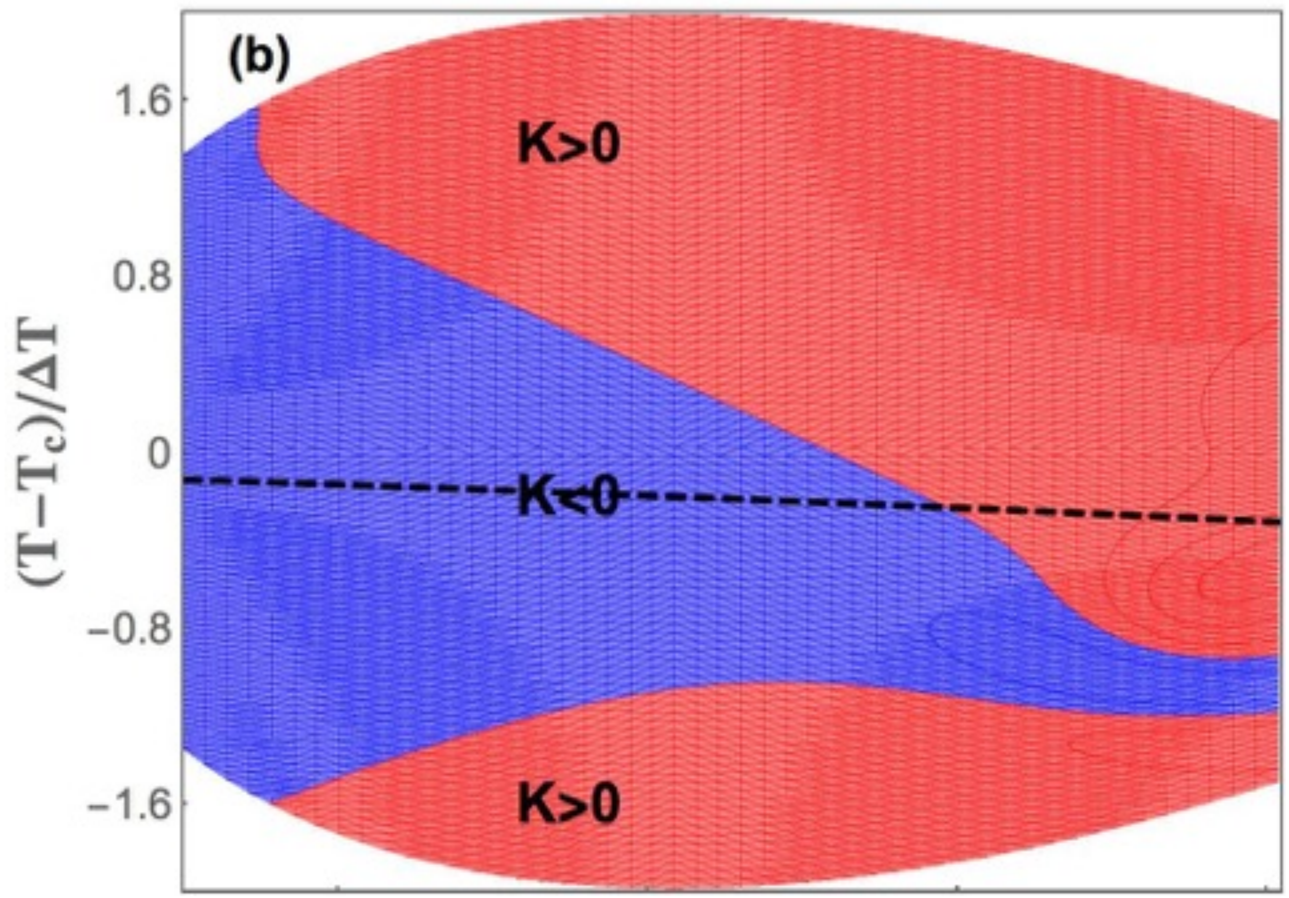
(or how to produce a better cartoon)

Static universality: zeroth order approximation



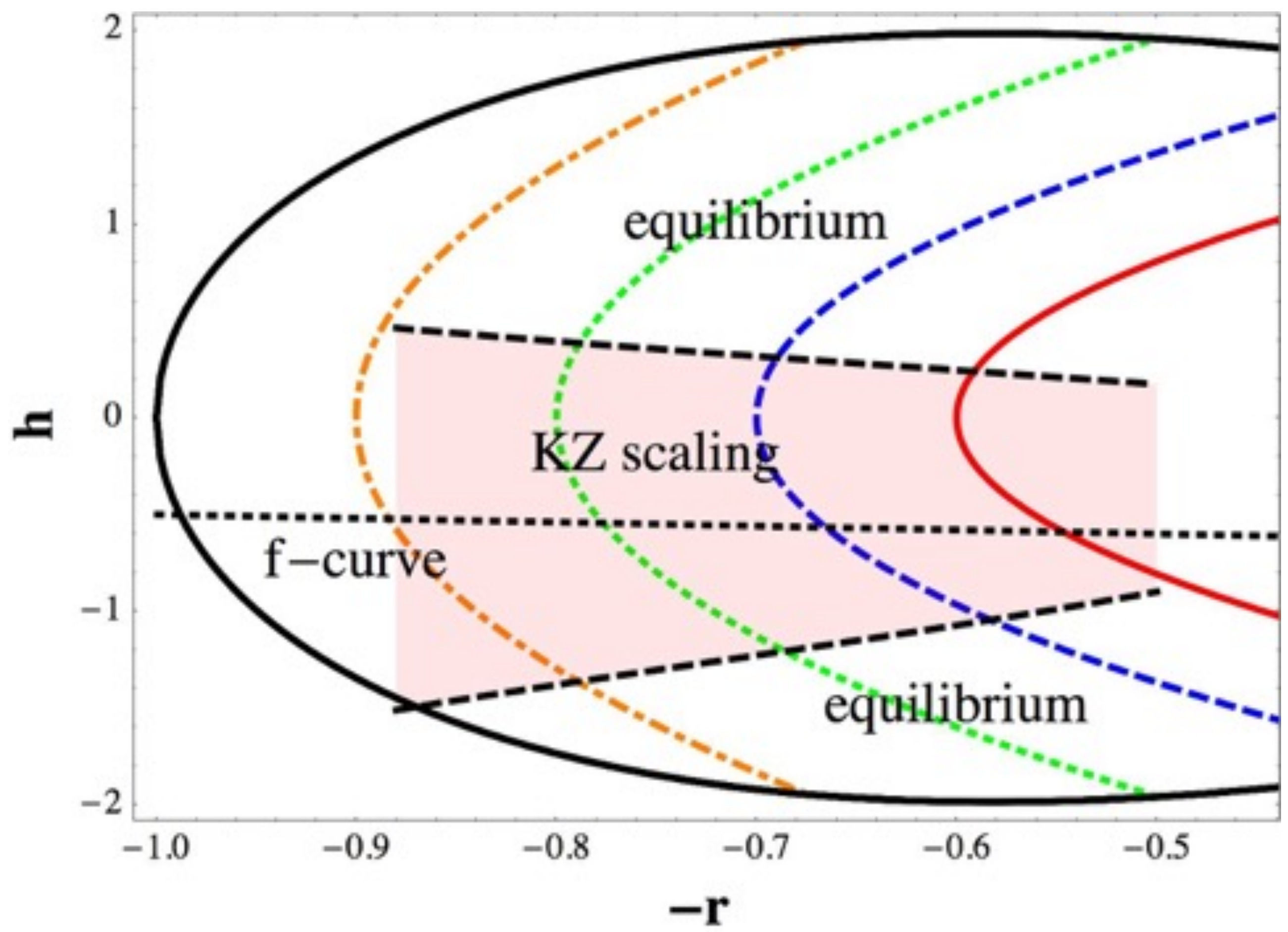
(equilibrium kurtosis, translated from Stephanov, PRL 2011)

Non-equilibrium effects: a step forward!



(non-equilibrium Kurtosis, from S. Mukherjee, R. Venugopalan and YY, 1506.00645, PRC, 2015)

Emergent scale ($\tau_{\rm KZ}, l_{\rm KZ}\ \theta_{\rm KZ}$), emergent new physics.

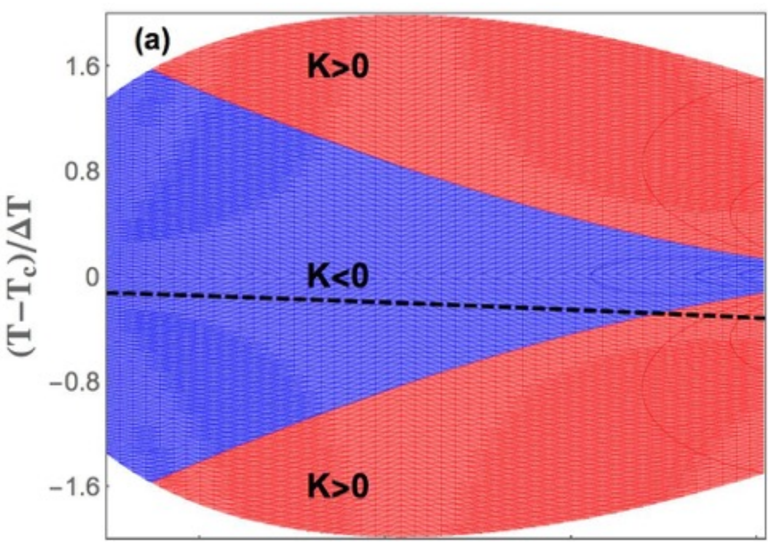


(This talk, S. Mukherjee, R. Venugopalan and YY, in preparation)

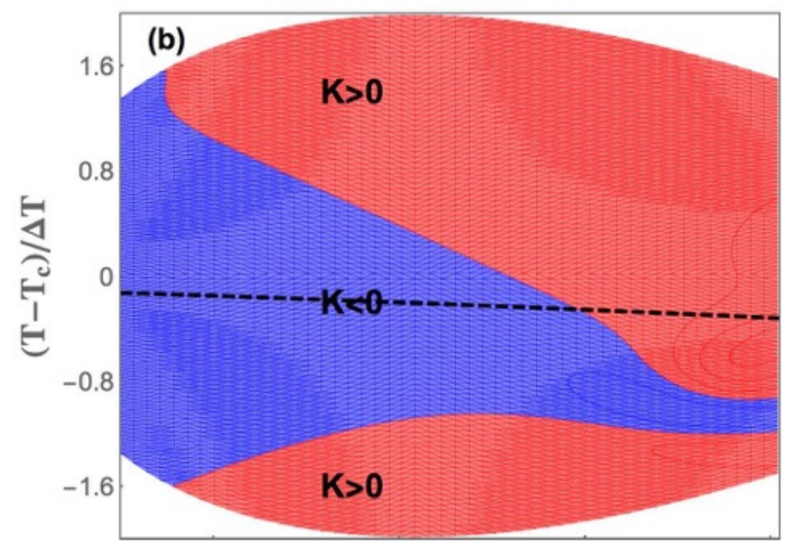
Back-up slides

- We report progress on non-equilibrium evolution of critical dynamics.
- We discuss characterize time and length scale and illustrate the possible existence of non-equilibrium scaling behavior.
- Non-equilibrium scaling behavior might signal the presence of a critical point.

Kurtosis in (cross-over side of) critical regime

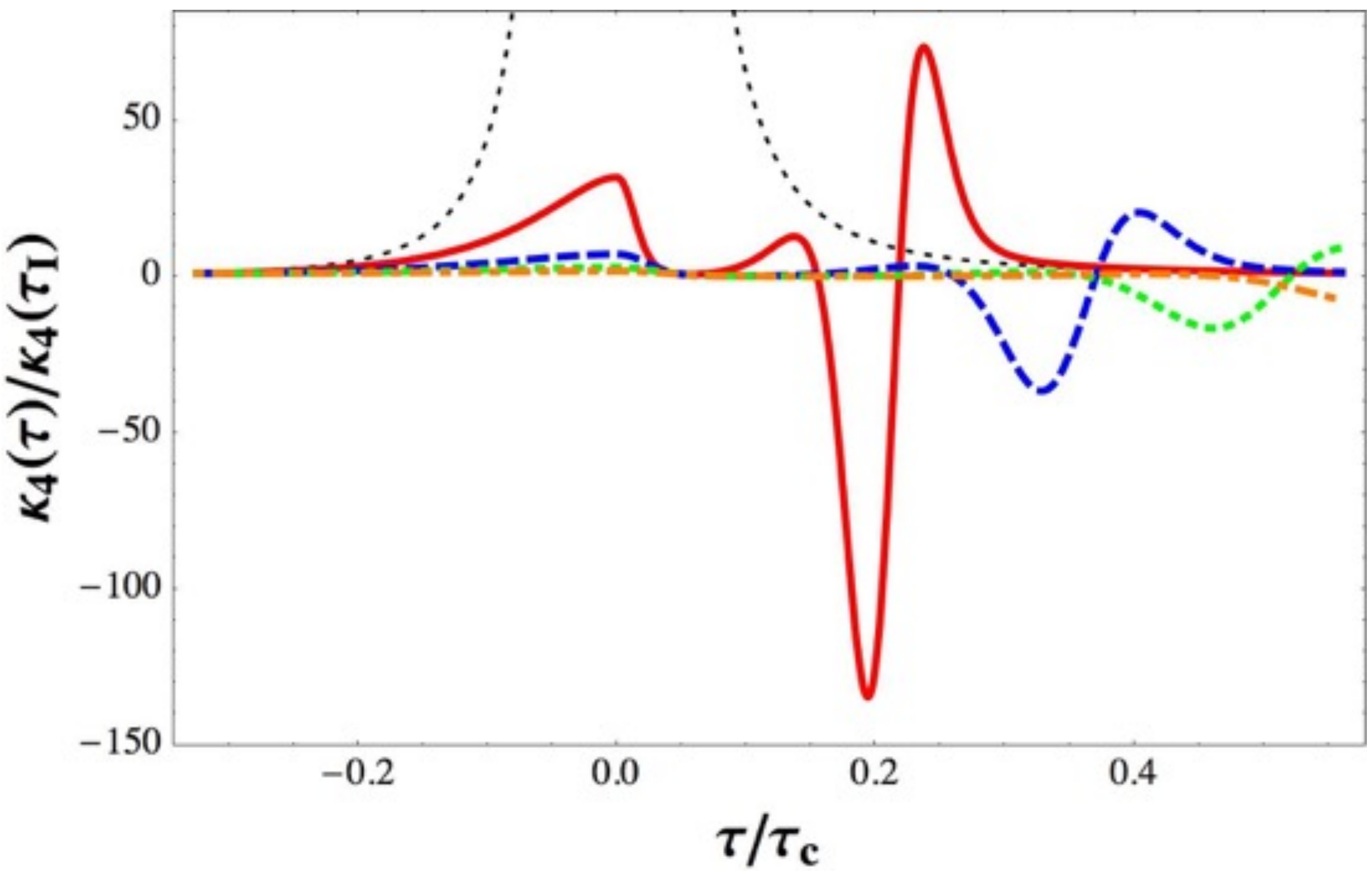


Equilibrium

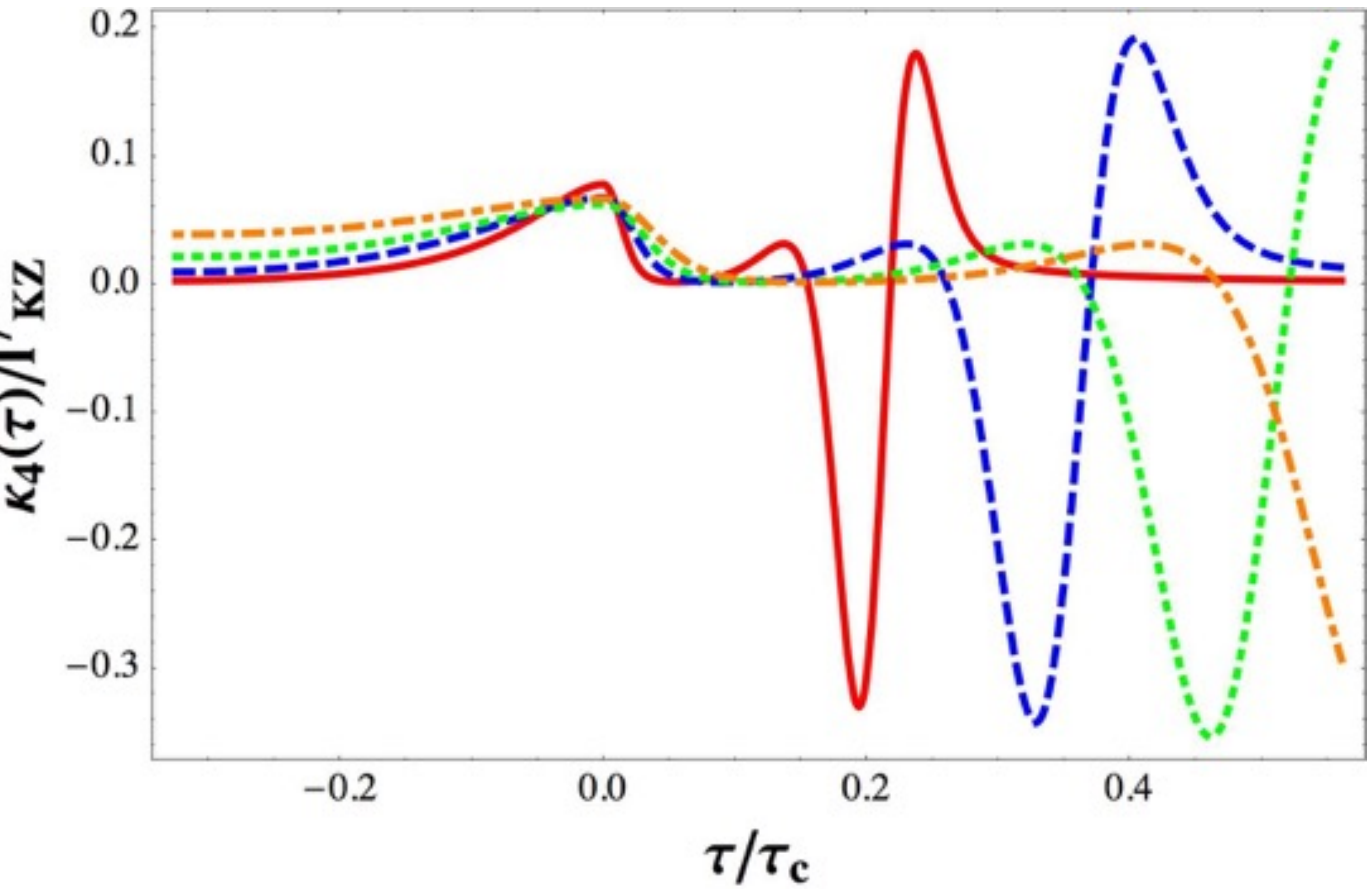


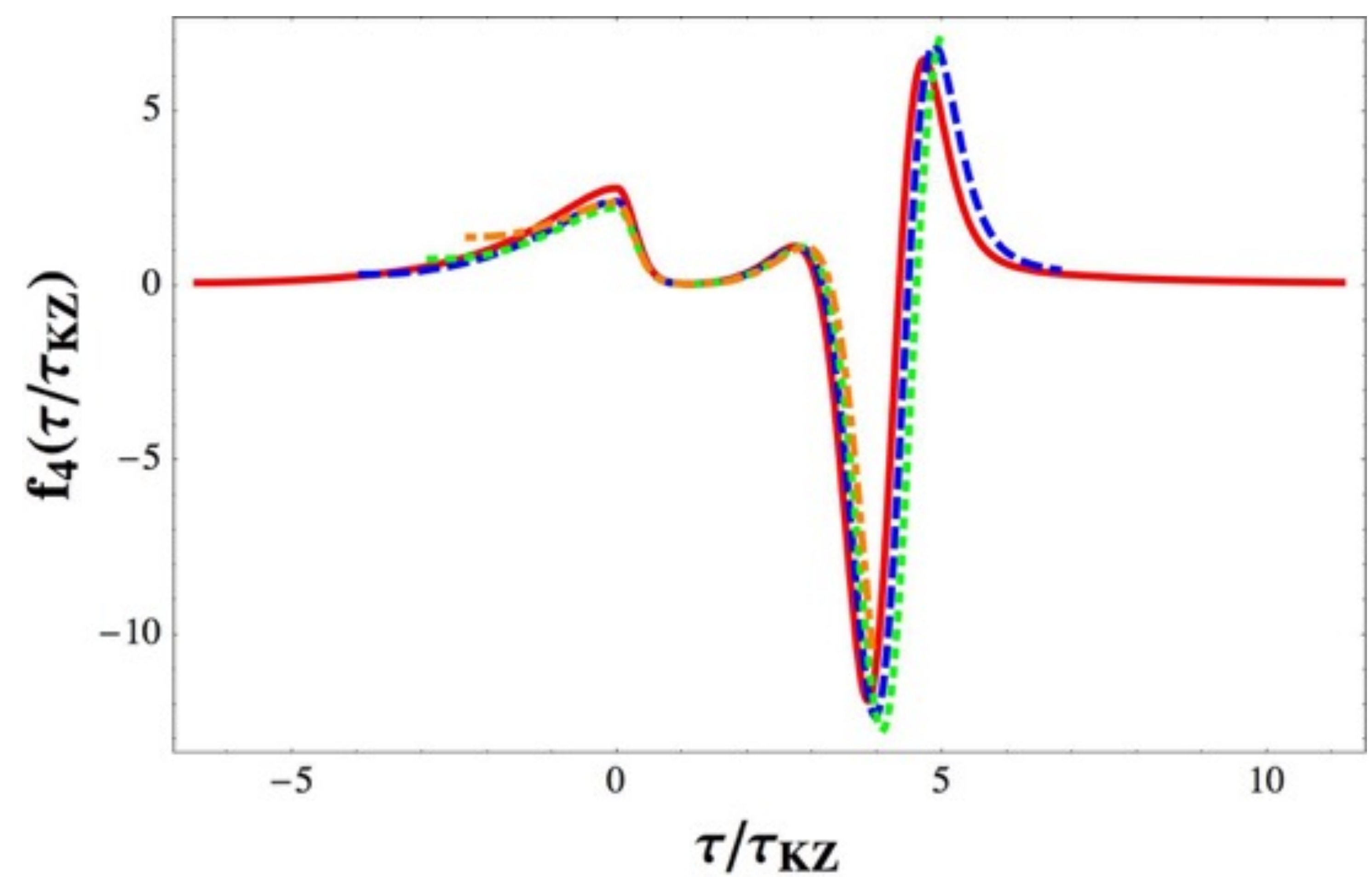
non-equilibrium

- The boundary deforms.



Kurtosis


0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.2
0.0
0.2
0.4
$\kappa_4(\tau)/l^7_{KZ}$
τ/τ_c



5
0
−5
−10
−5
0
5
10
$f_4(\tau/\tau_{KZ})$
τ/τ_{KZ}