Fluid dynamics for the anisotropically expanding quark-gluon plasma

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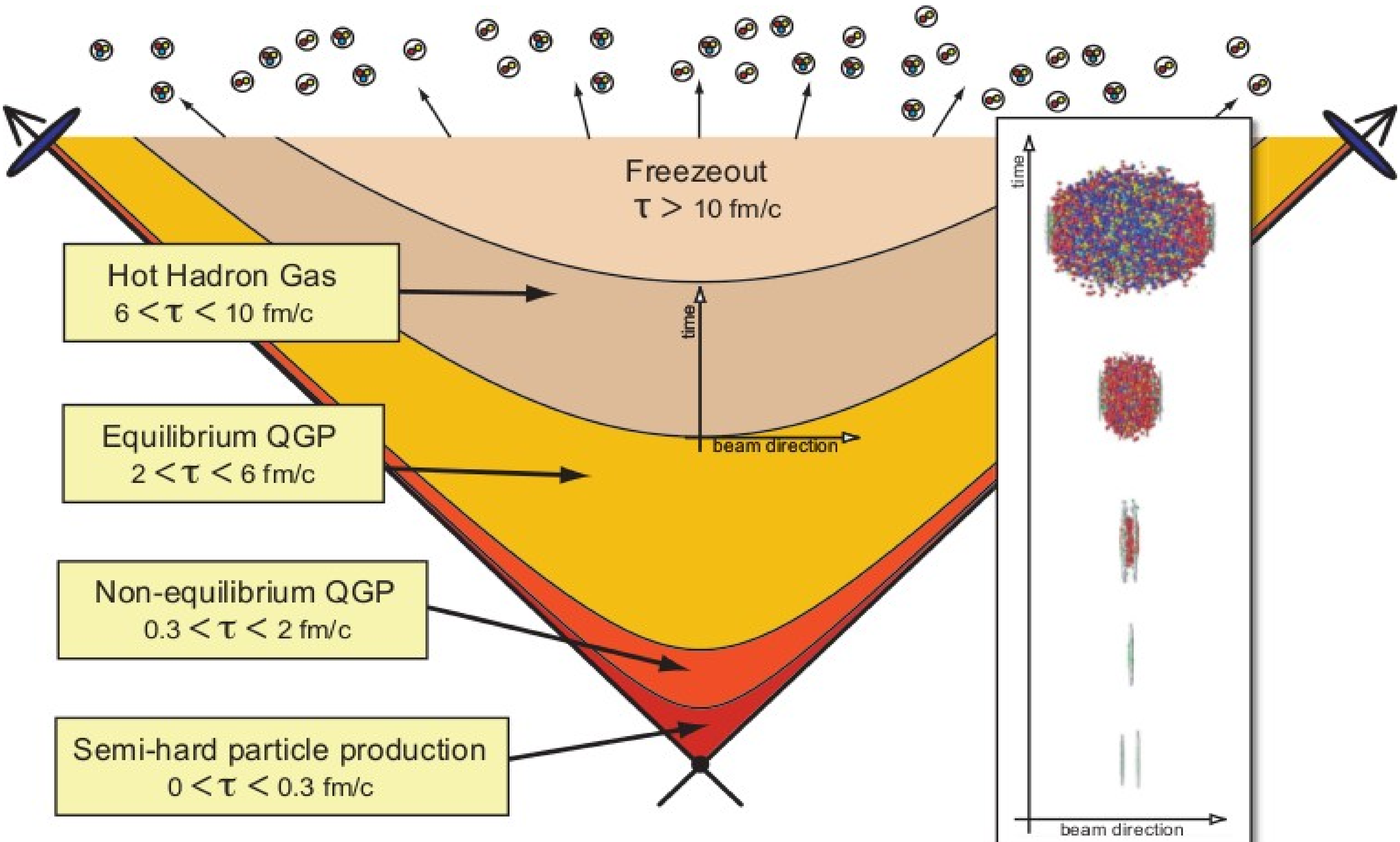
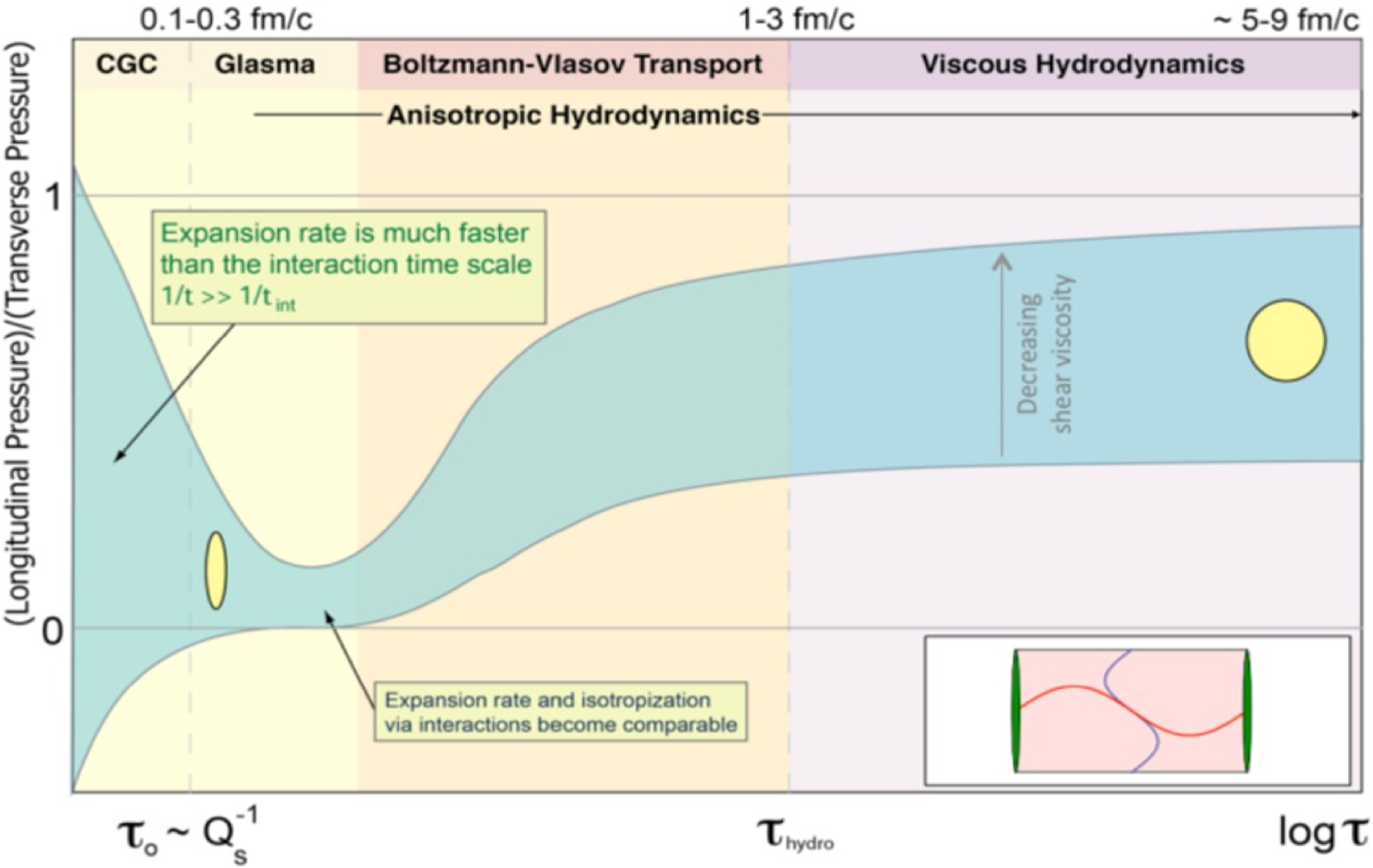
BEST 2016 Workshop, May 9-11 Indiana University Bloomington

Rough estimate of QGP timescales at LHC



Plot taken from Strickland, arxiv:1410.5786

Momentum-space anisotropy



Plot taken from Strickland, arxiv:1410.5786

Estimates of early-time momentum anisotropies

Consider Navier-Stokes solution in Bjorken $\pi_{\rm NS} = -\frac{4\eta}{3\tau}$

$$\left(\frac{P_{\rm L}}{\mathcal{P}_\perp}\right)_{\rm NS} = \frac{3\tau T - 16\eta/\mathcal{S}}{3\tau T + 8\eta/\mathcal{S}}$$

RHIC-like initial conditions:

$$T_0 = 400\,{\rm MeV\,at}\,\tau_0 = 0.5\,{\rm fm/c} \implies (\mathcal{P}_{\rm L}/\mathcal{P}_\perp)_{\rm NS} \simeq 0.5$$

LHC-like initial conditions:

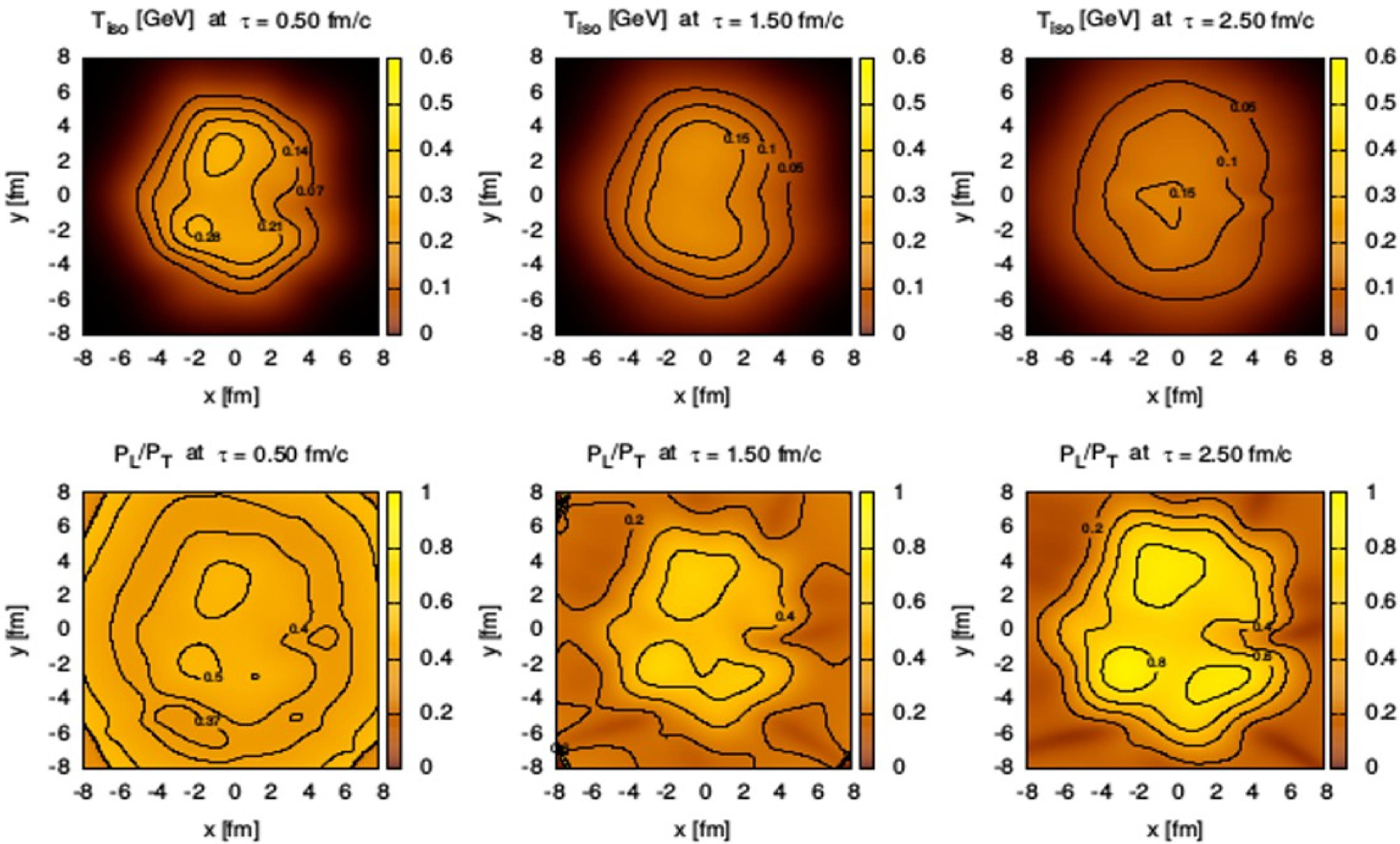
$$T_0 = 600\,{\rm MeV\,at}\,\tau_0 = 0.25\,{\rm fm/c} \implies (\mathcal{P}_{\rm L}/\mathcal{P}_\perp)_{\rm NS} \simeq 0.35$$

Both use $4\pi\eta/\mathcal{S} = 1$

Viscous hydro predicts sizable momentum-space anisotropies

Late-time momentum anisotropies

2+1d, fluctuating ICs. Martinez&Ryblewski&Strickland, arxiv:1204.1473



Large anisotropies even at the center due to fluctuating ICs.

$b = 7\,\mathrm{fm}$ $\quad T_0 = 0.6\,\mathrm{GeV}\,, \tau_0 = 0.25\,,\mathrm{fm/c}$ $\quad 4\pi\eta/\mathcal{S} = 1$

Lets look at hydrodynamics from the Boltzmann equation.

Hydrodynamic expansion in from kinetic theory

- linearize around a local equilibrium distribution function

$$y_0 \equiv \frac{u \cdot p}{T} - \frac{\mu}{T}\,, \qquad y - y_0 \equiv \delta y \ll 1$$

$$f(y) = \underbrace{f_{\rm eq}(y_0)}_{\equiv f_0} + \underbrace{f_{\rm eq}(1 - a f_{\rm eq})\delta y}_{\equiv \delta f} + \mathcal{O}(\delta y^2)$$

Expand in terms of a complete functional space a la Grad

Particle momentum-space is approximated at leading-order by a sphere

$$u \cdot p \underset{LRF}{\equiv} \sqrt{m^2 + |\mathbf{p}|^2}$$

Validity of the distribution function for non-equilibrium systems

$$f = f_{\rm eq}\left(1 + \frac{p^\alpha p^\beta \pi_{\alpha\beta}}{2(\mathcal{E}+\mathcal{P})T^2}\right)$$

anisotropic in momentum-space

For Navier-Stokes

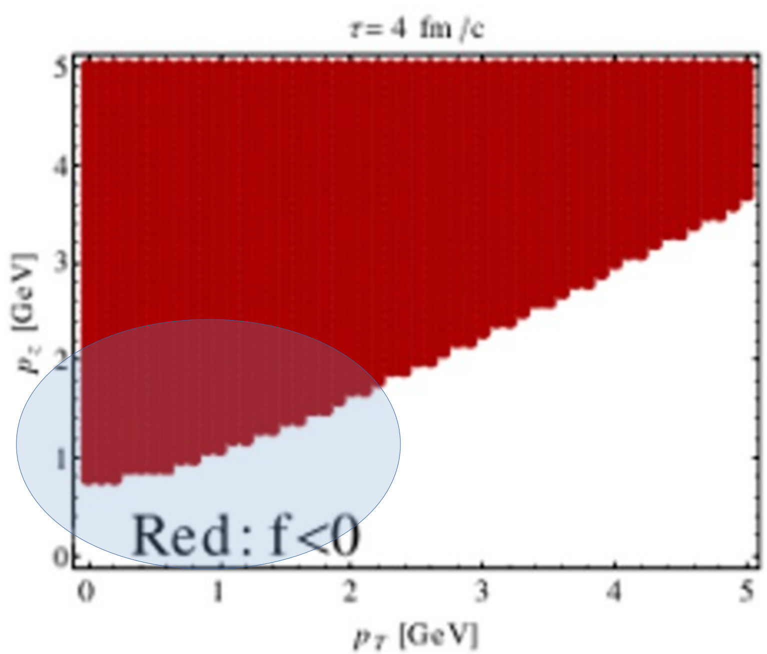
$$f^{0+1d}_{NS} = f_{\rm eq}\left[1 + \frac{\eta}{\mathcal{S}}\left(\frac{p_x^2 + p_y^2 - 2p_z^2}{3\tau T^3}\right)\right]$$

Take

$$\tau_0 = 0.2\,{\rm fm/c}\,,\, T_0 = 0.3\,{\rm GeV}\,,\, 4\pi\eta/\mathcal{S} = 1$$

Plot distribution function at time 4 fm/c for IS hydro

f<0 for regions of phase space where hydro is valid



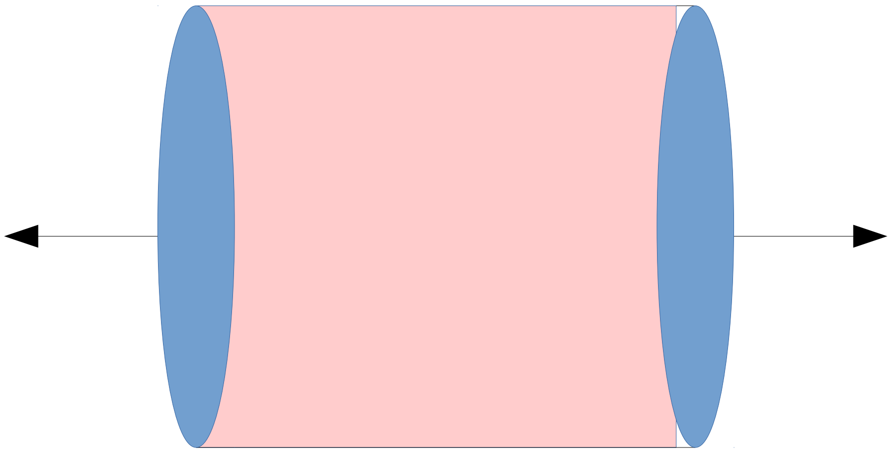
Hydrodynamic expansion revisited: a reorganized approach

- Generalized solution

$$f(x,p) = f_0(x,p) \sum_{\ell,\alpha} a_\alpha(x) P_\alpha^{(\ell)}(p)$$

- f_0 is LO approximation (arbitrary weight factor)
- In order to obtain the most rapid convergence, choose f_0 such that it is as close as possible to the exact solution f
- The choice of f_0 is guided by general insights into the properties of f for the problem at hand

Early time QGP



Looks like a tiny one-dimensionally expanding universe

Longitudinal expansion scalar behaves as $1/\tau$

Takes some time to generate significant transverse expansion

Is this still a reasonable approximation outside of the early-time limit?

Anisotropic expansion

- In HIC, rapid longitudinal expansion suggests to use an f_0 distorted along the p_z (beam)-direction with azimuthal momentum-space symmetry
- Expansion around a “local anisotropic equilibrium” momentum-space distribution function with spheroidal symmetry

$$f(x,p) = f_{\rm iso}\left(\frac{\sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\Lambda(x)}\right) + \delta\tilde{f} \equiv f_{\rm aniso} + \delta\tilde{f}$$

$$\Xi^{\mu\nu} \equiv u^\mu u^\nu + \xi z^\mu z^\nu$$

Temperature-like scale

Martinez&Strickland, arxiv:1007.0889

Terminology:

$$\text{aHydro} : f \equiv f_{\rm aniso}$$

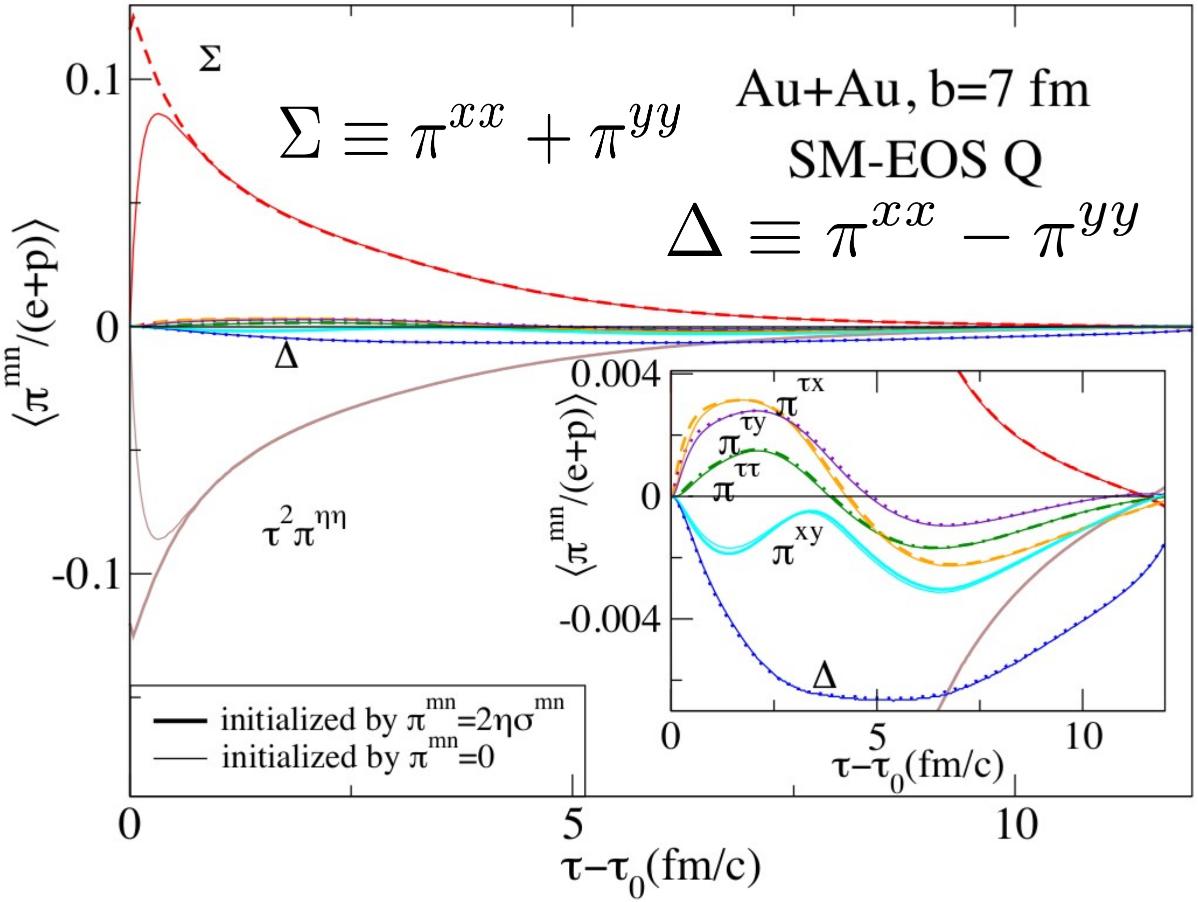
$$\text{vaHydro} : f \equiv f_{\rm aniso} + \delta\tilde{f}$$

In the local rest frame take Romatschke-Strickland form

$$f^{\rm LRF}_{\rm aniso} = f_{\rm iso}\left(\frac{\sqrt{m^2 + p_\perp^2 + (1+\xi(x))p_z^2}}{\Lambda(x)}\right)$$

Realistic 2+1d HIC simulation

Plot taken from Song, arxiv:0908.3656



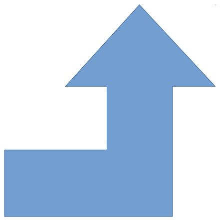
Σ and $\tau^2\pi^{\eta\eta}$

Are the largest components, related by the traceless condition

$$\Sigma \equiv -\tau^2\pi^{\eta\eta}$$

$$\tau_0 = 0.6\,\mathrm{fm/c}$$

No need to over complicate things.



Good approximation is to assume spheroidal form in momentum-space at LO. Evolution of these non-hydrodynamic DOF's are treated non-perturbatively

All off-diagonal components are small. Treat them as a perturbation.

Hydrodynamic tensor decomposition for anisotropic systems

Expansion around a spheroidal momentum-space distribution function

$$f(x,p) \equiv f_{\rm aniso} + \delta\tilde{f}$$

Gives rise to dissipative currents which (mostly) account for viscous effects other than those of spheroidal form

$$\Longrightarrow \mathcal{P}_\perp \equiv \mathcal{P}_x = \mathcal{P}_y \neq \mathcal{P}_L$$

Particle current and energy-momentum tensor are:

$$J^\mu = \mathcal{N} u^\mu + \tilde{V}^\mu$$

$$T^{\mu\nu} = \mathcal{E} u^\mu u^\nu - (\mathcal{P}_\perp + \tilde{\Pi})\Delta^{\mu\nu} + (\mathcal{P}_{\rm L} - \mathcal{P}_\perp) z^\mu z^\nu + \tilde{\pi}^{\mu\nu}$$

- Large portion of dissipative currents caused by spheroidal deformation of particle momentum-space are treated non-perturbatively

Conservation laws

Dynamical Landau matching conditions

$$\mathcal{E}(\Lambda, \xi, \tilde{\mu}) \equiv \mathcal{E}_{\rm eq} \;, \qquad \mathcal{N}(\Lambda, \xi, \tilde{\mu}) \equiv \mathcal{N}_{\rm eq}$$

As with viscous hydro, the particle density, energy density, and fluid velocity can be obtained from solving the conservation laws

$$\partial_\mu J^\mu = 0 \;, \qquad \partial_\mu T^{\mu\nu} = 0$$

$$\xi \,, \tilde{\Pi} \,, \tilde{V}^\mu \,, \tilde{\pi}^{\mu\nu}$$

Need additional equations of motion to solve for the anisotropy parameter and dissipative currents

Strategy

- Follow method from Denicol&Koide&Rischke, arxiv:1004.5013

$$\dot{\tilde{\Pi}} \equiv -\frac{m^2}{3}\int dP\delta\dot{\tilde{f}}\,, \dot{\tilde{V}}^{\langle\mu\rangle} \equiv \int dPp^{\langle\mu\rangle}\delta\dot{\tilde{f}}\,, \dot{\tilde{\pi}}^{\langle\mu\nu\rangle} \equiv \int dPp^{\langle\mu}p^{\nu\rangle}\delta\dot{\tilde{f}}$$

- Use Boltzmann equation for

$$\delta\dot{\tilde{f}} \equiv -\dot{f}_{\rm aniso} - (u{\cdot}p)^{-1}[p{\cdot}\nabla(f_{\rm aniso} + \delta\tilde{f}) - C[f]]$$

- Gives perturbative transport equations

How do we obtain an equation of motion for ξ ?

Evolution equation for ξ

No kinetic definition for ξ

- For now, what's the easiest thing to do?
- Assume relaxation time approx. $C \equiv -\frac{u \cdot p}{\tau_{\rm eq}}(f - f_{\rm eq})$
- High-energy limit, we ignore chemical potential;

$\tilde{V}^\mu \equiv 0$

$\partial_\mu J^\mu$ no longer couples to dissipative currents. Use it.

Caveat: no way to conserve particle number

$$\partial_\mu J^\mu = 0 \to \partial_\mu J^\mu = \mathcal{C}$$

Non-vanishing source term.

$$\mathcal{C} \equiv \int dP\, C[f]$$

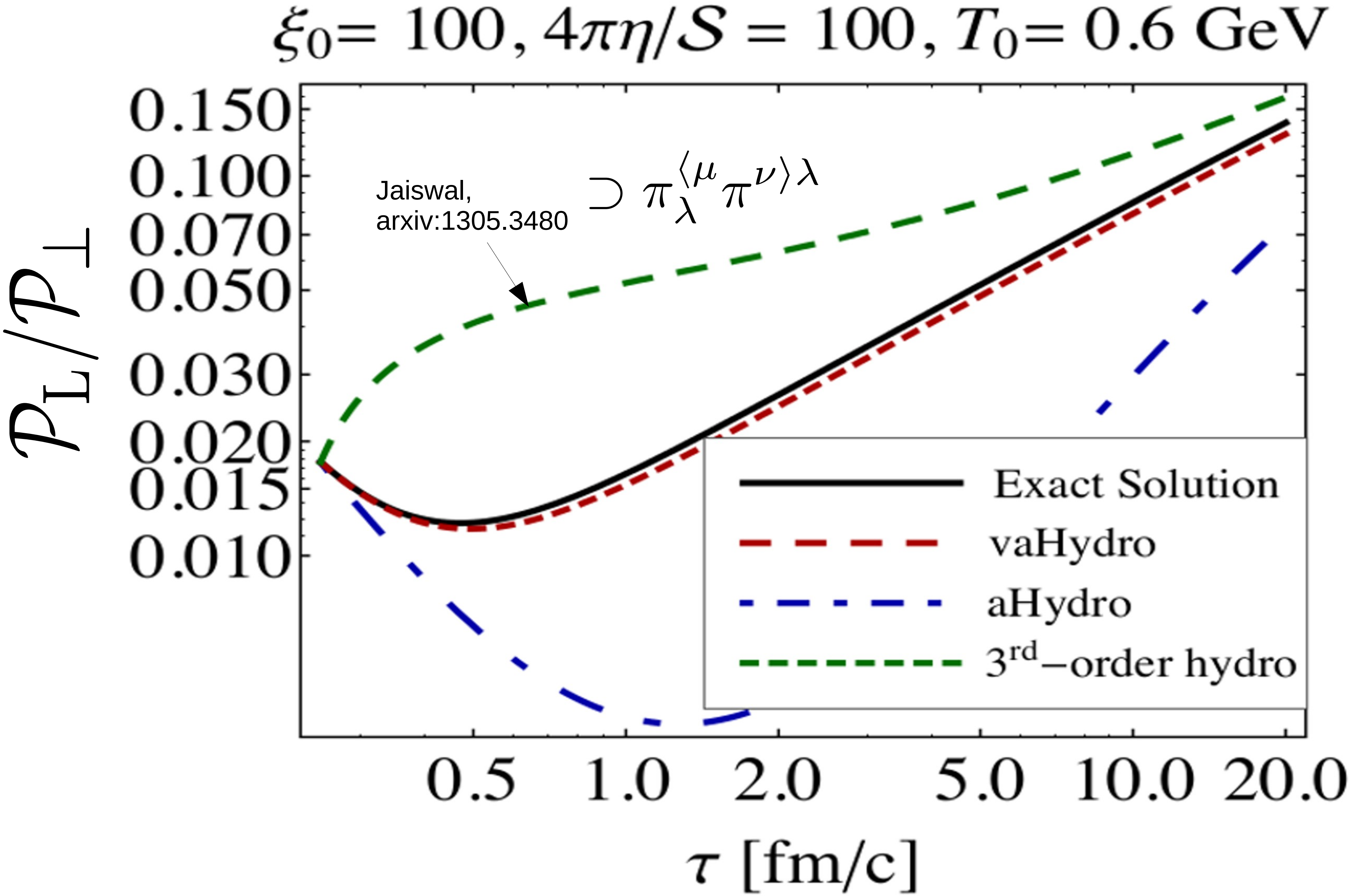
$$\frac{\dot{\xi}}{1+\xi} + 6\frac{\dot{\Lambda}}{\Lambda} - 2\theta = 2\frac{1}{\tau_{\rm eq}}\left(1 - \sqrt{1+\xi}\mathcal{R}^{3/4}(\xi)\right)$$

Is there a way to test vaHydro?

Focus here

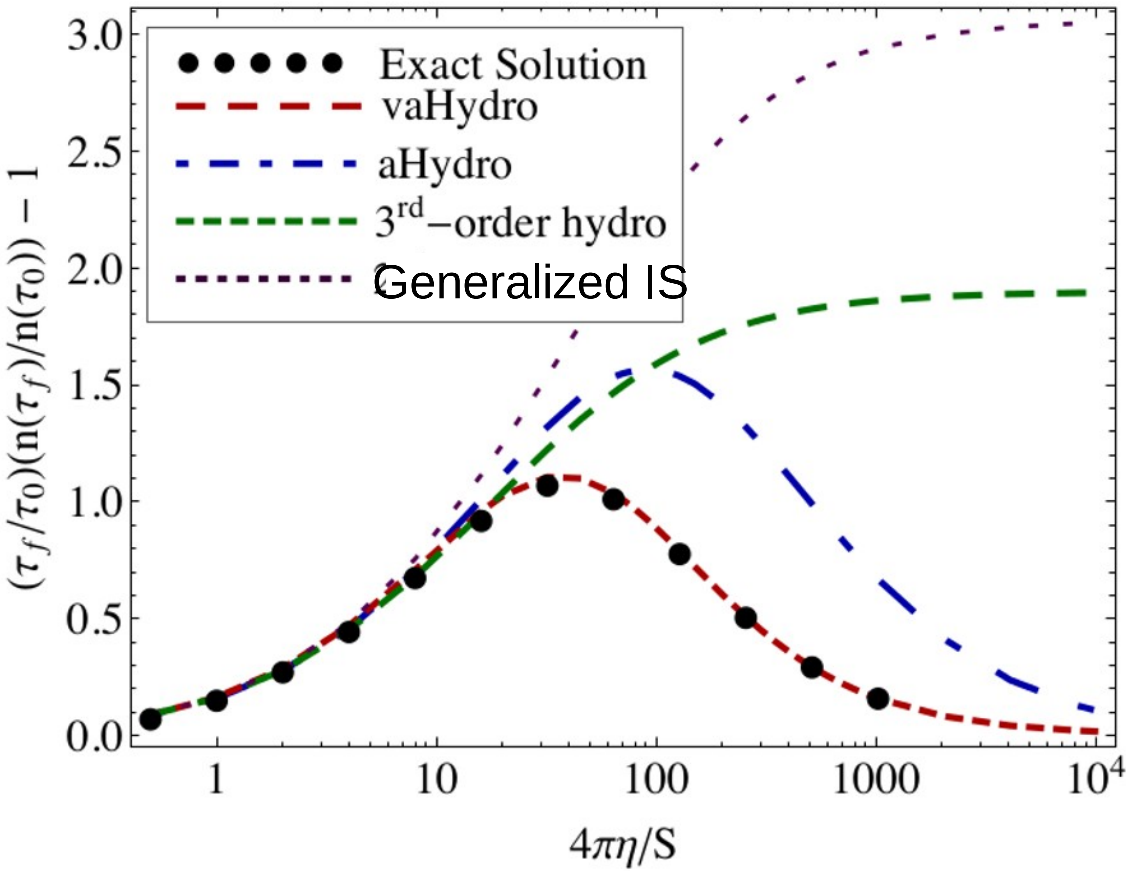
- Exact solutions to BE in simplified situations
- For Bjorken flow in relaxation time approximation (RTA)
  - Baym, Phys. Lett B128, 18 (1984)
  - Zero particle masses: Florkowski&Ryblewski&Strickland, arxiv:1305.7234
  - Finite particle masses: Florkowski&Maksymiuk&Ryblewski&Strickland, arxiv:1402.7348
- For Gubser flow in RTA:
  - Denicol&Heinz&Martinez&Noronha&Strickland, arxiv:1408.5646
  - Denicol&Heinz&Martinez&Noronha&Strickland, arxiv:1408.7048
  - Nopoush&Ryblewski&Strickland, arxiv:1410.6790

First consider gas of massless particles.



$\xi_0 = 100, 4\pi\eta/\mathcal{S} = 100, T_0 = 0.6$ GeV
$\mathcal{P}_L/\mathcal{P}_\perp$
0.150
0.100
0.070
0.050
0.030
0.020
0.015
0.010
0.5
1.0
2.0
5.0
10.0
20.0
τ [fm/c]
Jaiswal,
arxiv:1305.3480
$\supset \pi_\lambda^{\langle\mu}\pi^{\nu\rangle\lambda}$
Exact Solution
vaHydro
aHydro
3rd−order hydro

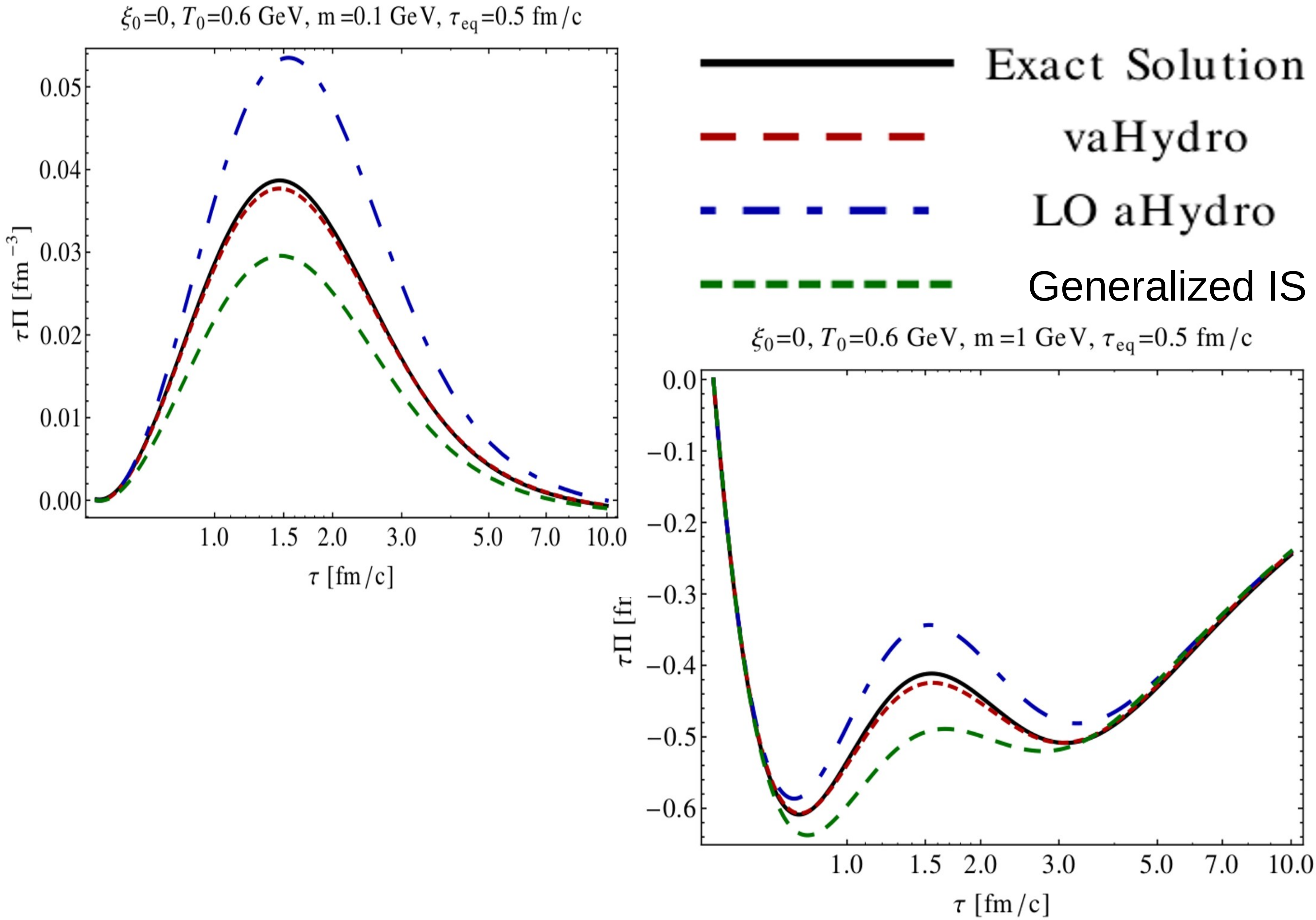
Entropy (particle) production



Generalized Isreal-Stewart hydrodynamics, Denicol&Koide&Rischke, arxiv:1004.5013

DB&Heinz&Strickland, arxiv:1311.6720

Now consider nonconformal systems.


ξ_0=0, T_0=0.6 GeV, m =0.1 GeV, τ_{eq}=0.5 fm/c
0.05
0.04
0.03
0.02
0.01
0.00
$\tau\Pi$ [fm^{-3}]
1.0
1.5
2.0
3.0
5.0
7.0
10.0
τ [fm/c]
Exact Solution
vaHydro
LO aHydro
Generalized IS
ξ_0=0, T_0=0.6 GeV, m =1 GeV, τ_{eq}=0.5 fm/c
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
$\tau\Pi$ [fr
1.0
1.5
2.0
3.0
5.0
7.0
10.0
τ [fm/c]

Conclusions and outlook

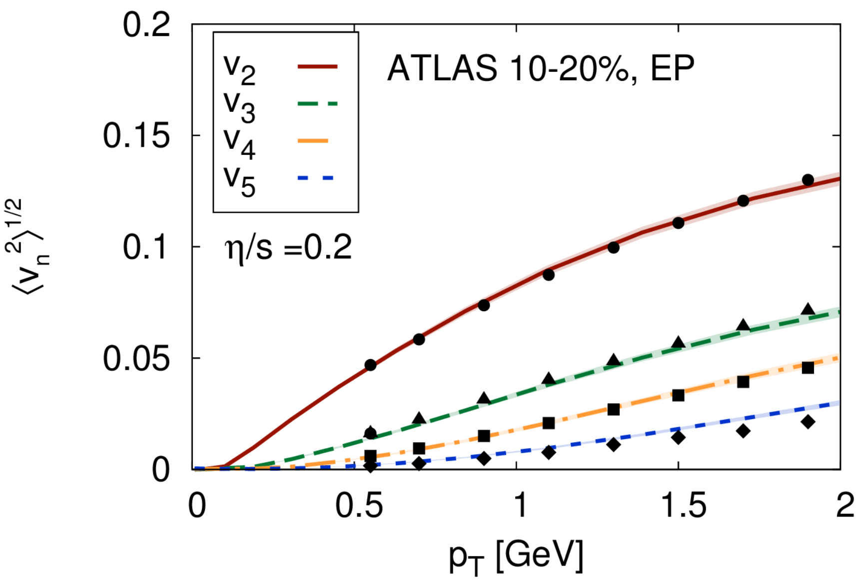
- Viscous anisotropic hydrodynamics is a more efficient way to solve relativistic fluid dynamics for HIC
- It Improves the validity of fluid dynamic approach to heavy ion physics
  - For early time dynamics
  - Large values of shear viscosity and/or low temperatures
  - Near transverse edge of overlap region
- Fully tested 3+1d relativistic hydrodynamic code [DB&Heinz&Strickland, in preparation]
  - KT algorithm
  - Fluctuating ICs, shear and bulk, QCD EoS
  - On GPUs using CUDA C, ~88x speedup from serial C code
- Ready to extend 3+1d code to vaHydro (include baryon chemical potential)
- Port Cooper-Frye freeze-out prescription to GPUs ← Major bottleneck
  - Do easiest case first (vhydro), then anisotropic case

Backup

Success of viscous hydro

Experimental data of anisotropic flow coefficients v_n are very well described by viscous hydro with a small

$$\frac{\eta}{\mathcal{S}} = \frac{2}{4\pi} + 50\%$$



Gale&Jeon&Schenke&Tribedy&Venugopalan, arxiv:1209.6330

Need

- Pre-equilibrium evolution: IP-Glasma model
- EoS: lattice+hadron resonance gas
- Hydrodynamic evolution of the fields
- Freeze-out/hadronization prescription

Large anisotropies from IP-Glasma (gradients of fluid velocity are large). Matching full pre-equilibrium energy-momentum tensor to hydro energy-momentum tensor can be problematic

Lets revisit this issue

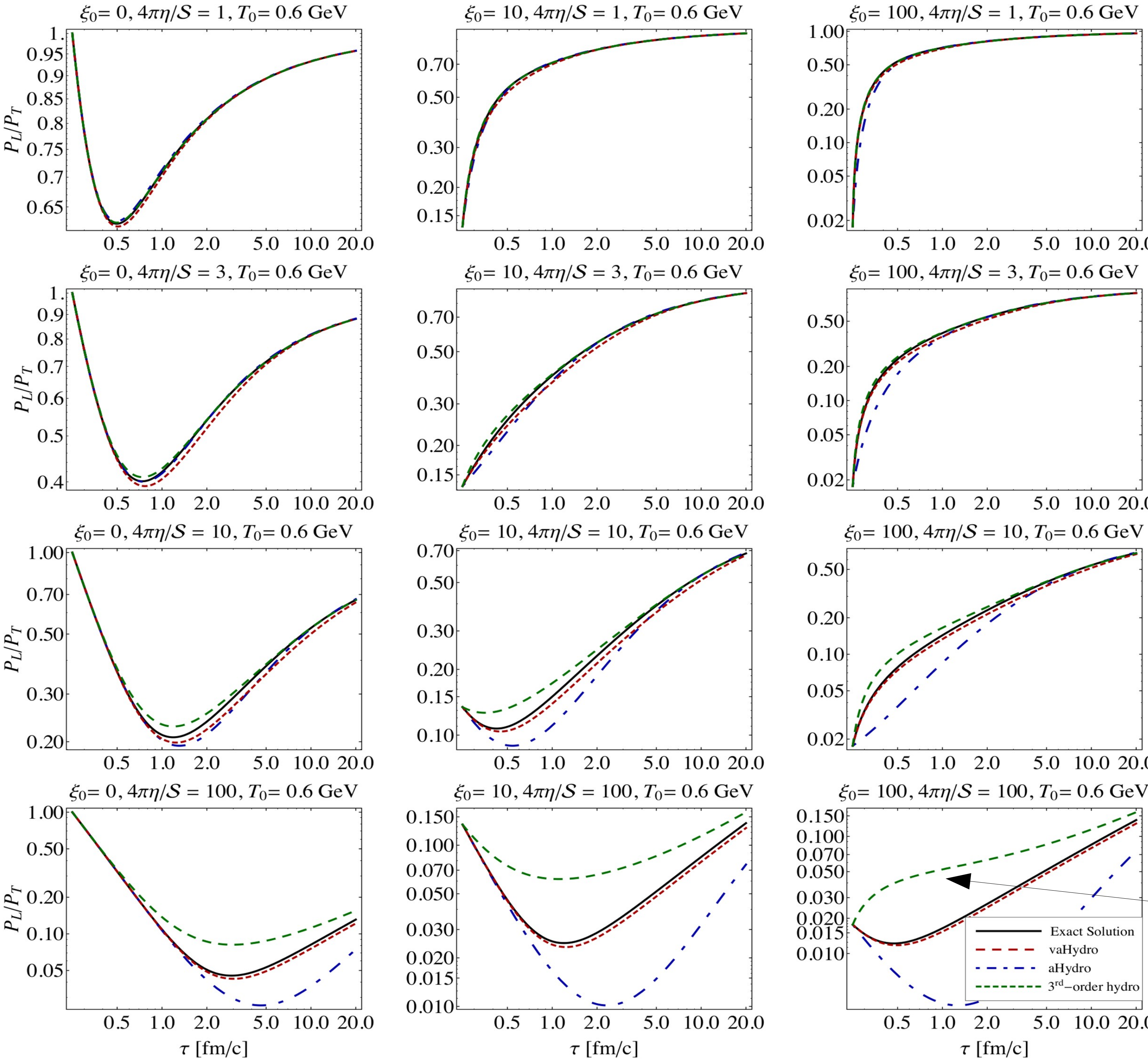
vaHydro dissipative currents

$$-\frac{3}{m^2}\dot{\tilde{\Pi}} = \mathcal{C}_{-1} + \mathcal{W} + \beta_{\Pi\perp}\theta + \beta_{\Pi L} z^\mu z^\nu \sigma_{\mu\nu} - \tilde{\Pi}\theta - \lambda^{\mu\nu}_{\Pi V}\nabla_\mu \tilde{V}_\nu - \tau^{\mu}_{\Pi V}\tilde{V}_\mu$$

$$- \delta^{\mu\nu}_{\Pi\Pi}\tilde{\Pi}\nabla_\mu u_\nu - \tilde{\pi}_{\alpha\beta}\delta^{\mu\nu\alpha\beta}_{\Pi\pi}\nabla_\mu u_\nu \ .$$

$$\begin{aligned}\dot{\tilde{V}}^{\langle\mu\rangle} &= \mathcal{C}^{\langle\mu\rangle}_{-1} + \mathcal{Z}^\mu - \tilde{V}^\lambda\nabla_\lambda u^\mu - \tilde{V}^\mu\theta - \ell^{\mu\nu}_{V\Pi}\nabla_\nu\tilde{\Pi} - \tau^{\mu}_{V\Pi}\tilde{\Pi} - \delta^{\mu\nu\alpha\beta}_{VV}\tilde{V}_\nu\nabla_\alpha u_\beta \\ &+\ell^{\mu\mu\alpha\beta}_{V\pi}\nabla_\nu\tilde{\pi}_{\alpha\beta} + \tau^{\mu\alpha\beta}_{V\pi}\tilde{\pi}_{\alpha\beta} \ , \\ \dot{\tilde{\pi}}^{\langle\mu\nu\rangle} &= \mathcal{C}^{\langle\mu\nu\rangle}_{-1} + \mathcal{K}^{\mu\nu} + \mathcal{L}^{\mu\nu} + \mathcal{H}^{\mu\nu\lambda}\left(\dot{z}_\lambda + u^\alpha\nabla_\lambda z_\alpha\right) + \mathcal{Q}^{\mu\nu\lambda\alpha}\nabla_\lambda u_\alpha \\ &-\frac{5}{3}\tilde{\pi}^{\mu\nu}\theta - 2\tilde{\pi}^{\langle\mu}_\lambda\sigma^{\nu\rangle\lambda} + 2\tilde{\pi}^{\langle\mu}_\lambda\omega^{\nu\rangle\lambda} + 2\tilde{\Pi}\sigma^{\mu\nu} \\ &-\ell^{\mu\nu\alpha\beta}_{\pi V}\nabla_\alpha\tilde{V}_\beta - \tau^{\mu\nu\lambda}_{\pi V}\tilde{V}_\lambda - \tilde{\Pi}\delta^{\mu\nu\alpha\beta}_{\pi\Pi}\nabla_\alpha u_\beta - \delta^{\mu\nu\alpha\beta\sigma\lambda}_{\pi\pi}\tilde{\pi}_{\sigma\lambda}\nabla_\alpha u_\beta .\end{aligned}$$

See DB&Heinz&Strickland, arxiv:1311.6720 for transport coefficients



Pressure ratio

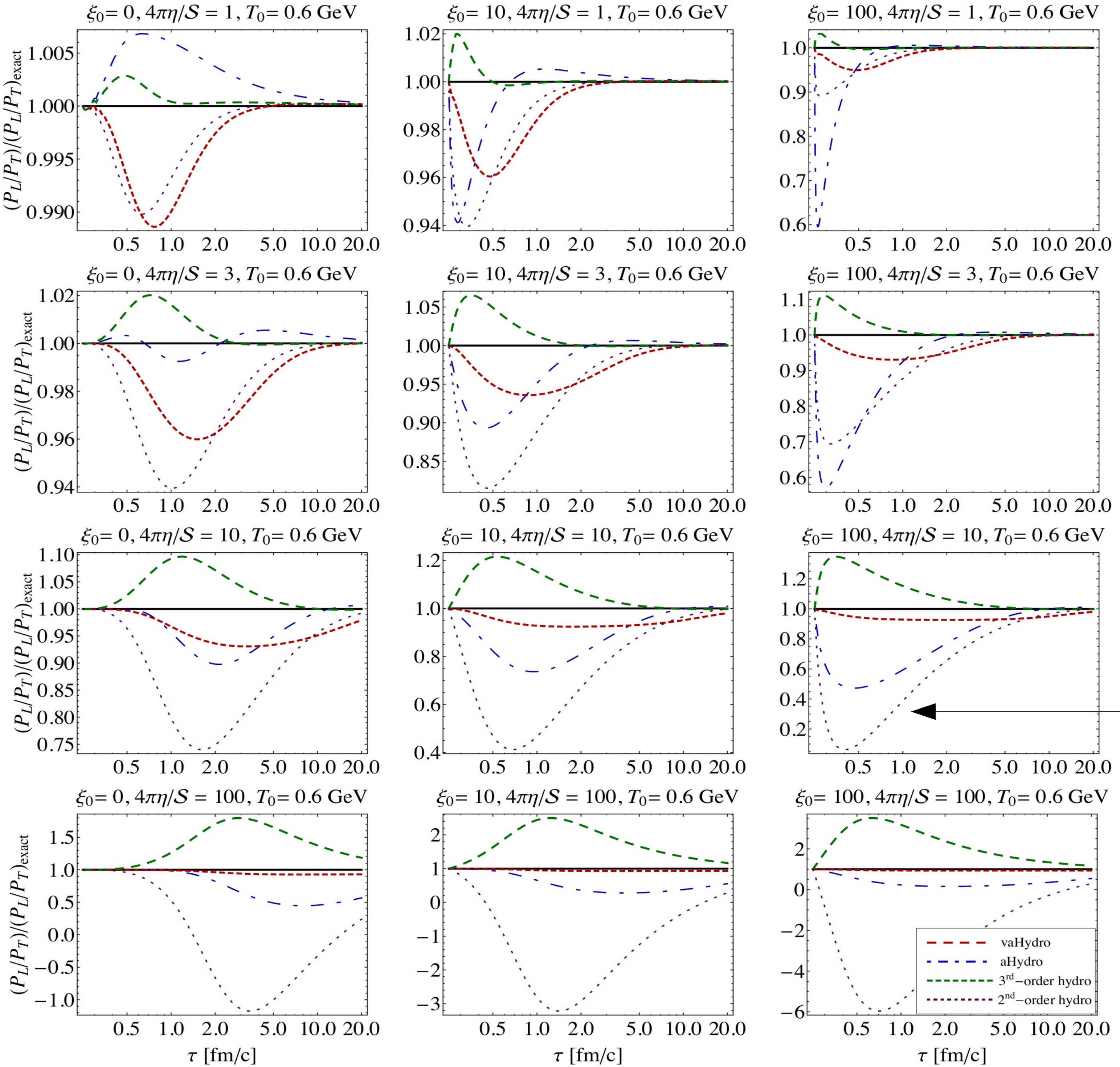
Initial time is 0.25 fm/c

Black: Exact solution to BE
Red: vaHydro
Blue: ahydro
Green: 3rd order viscous hydro

$\supset \pi_{\lambda}^{\langle\mu}\pi^{\nu\rangle\lambda}$

Jaiswal, arxiv:1305.3480

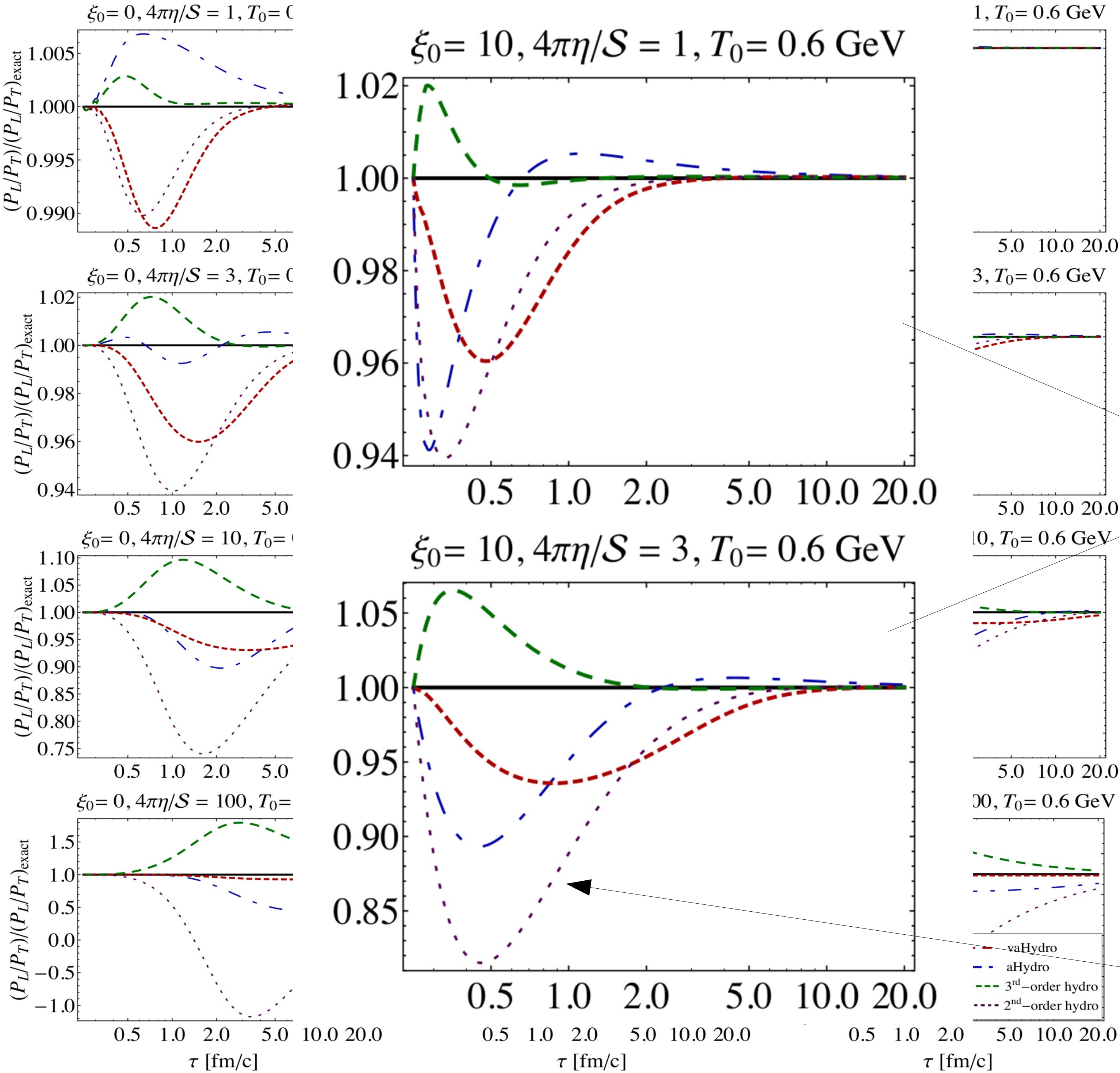
DB&Heinz&Strickland, arxiv:1311.6720



Relative error of pressure ratio

“Generalized Isreal-Stewart”
Denicol&Koide&Rischke, arxiv:1004.5013

DB&Heinz&Strickland, arxiv:1311.6720



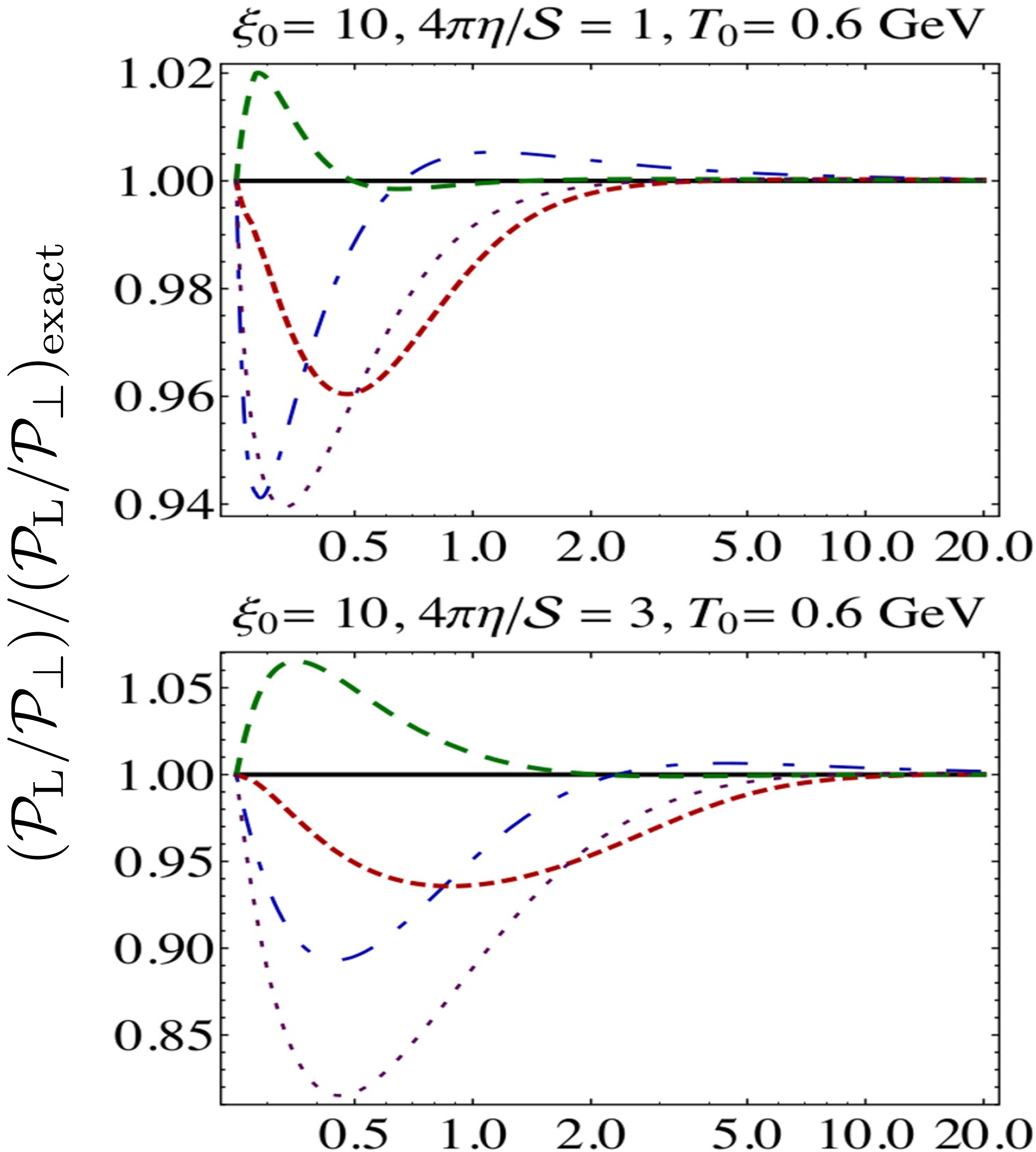
Relative error of pressure ratio

Order of anisotropy parameter and range of shear viscosity to entropy density ratios expected for LHC-like initial conditions

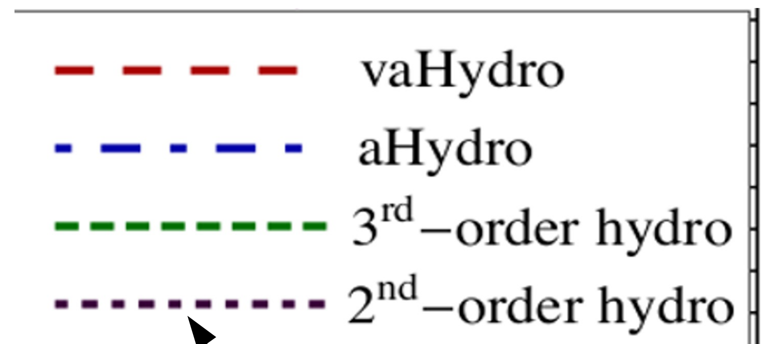
Generalized Isreal-Stewart hydrodynamics, Denicol&Koide&Rischke, arxiv:1004.5013

DB&Heinz&Strickland, arxiv:1311.6720

Relative error of pressure ratio



Order of anisotropy parameter and range of shear viscosity to entropy density ratios expected for LHC-like initial conditions



Generalized Isreal-Stewart hydrodynamics, Denicol&Koide&Rischke, arxiv:1004.5013

Finite particle masses

- We can also implement in this framework bulk viscous effects which arise in nonconformal systems.
  - Add additional scalar degree of freedom at LO

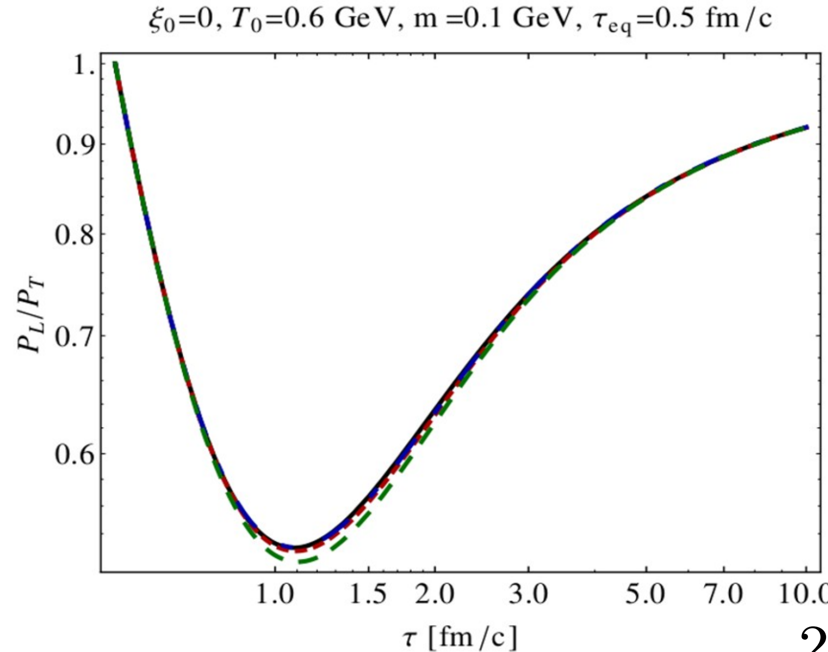
$$\Xi^{\mu\nu} \equiv u^\mu u^\nu - \boxed{\Phi \Delta^{\mu\nu}} + \xi z^\mu z^\nu$$

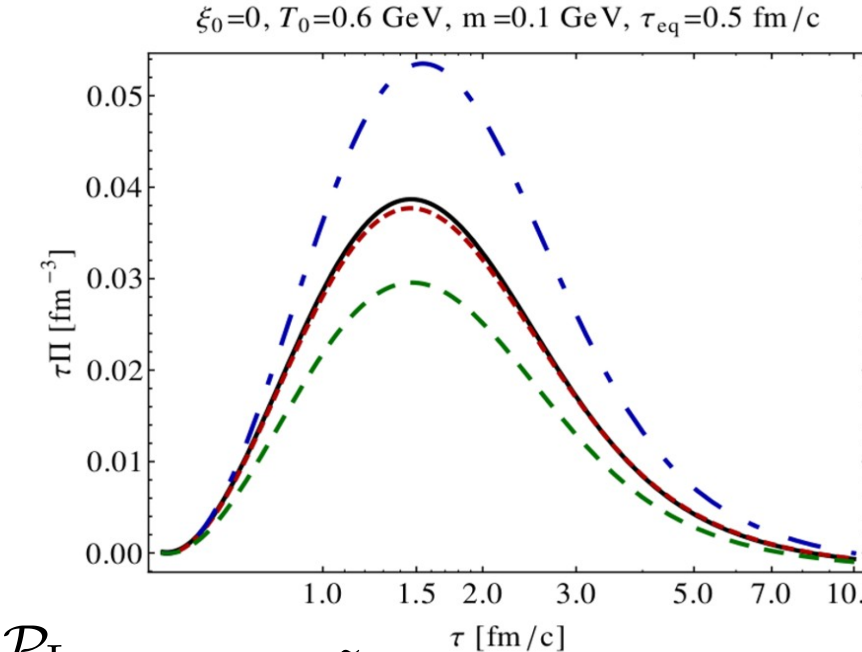
Accounts for largest dissipative effects from bulk viscous pressure

Leads to

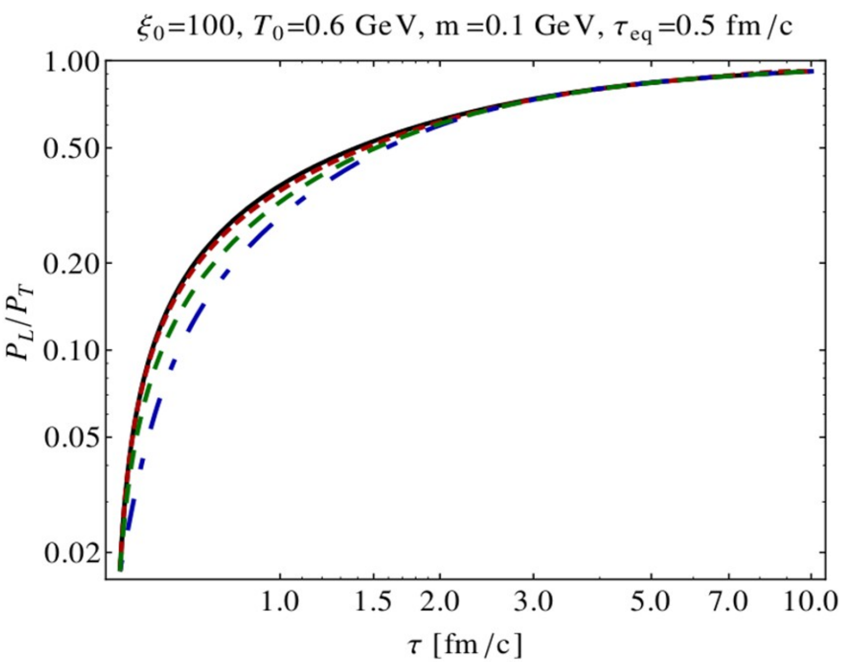
$$f_{\rm aniso} \equiv f_{\rm iso}\left(\frac{1}{\Lambda}\sqrt{m^2 + (1+\Phi)p_\perp^2 + (1+\Phi+\xi)p_z^2}\right)$$

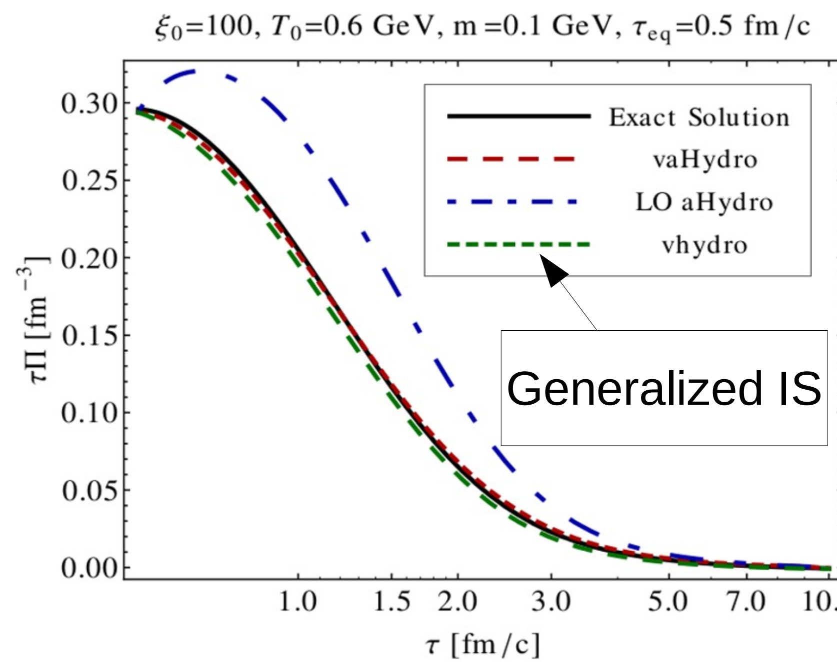
- Slight change to equations of motion
- Get additional equation of motion from second moment of BE



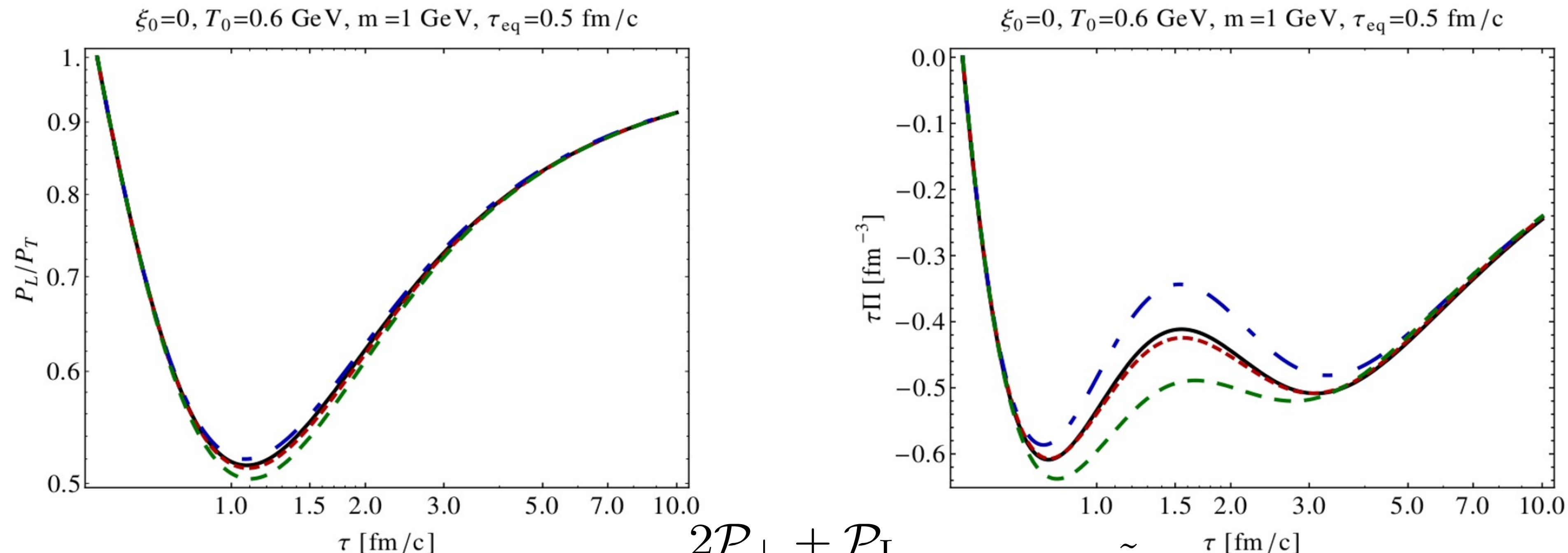


$$\Pi \equiv \frac{2\mathcal{P}_\perp + \mathcal{P}_\mathrm{L}}{3} - \mathcal{P}_\mathrm{eq} + \tilde{\Pi}$$

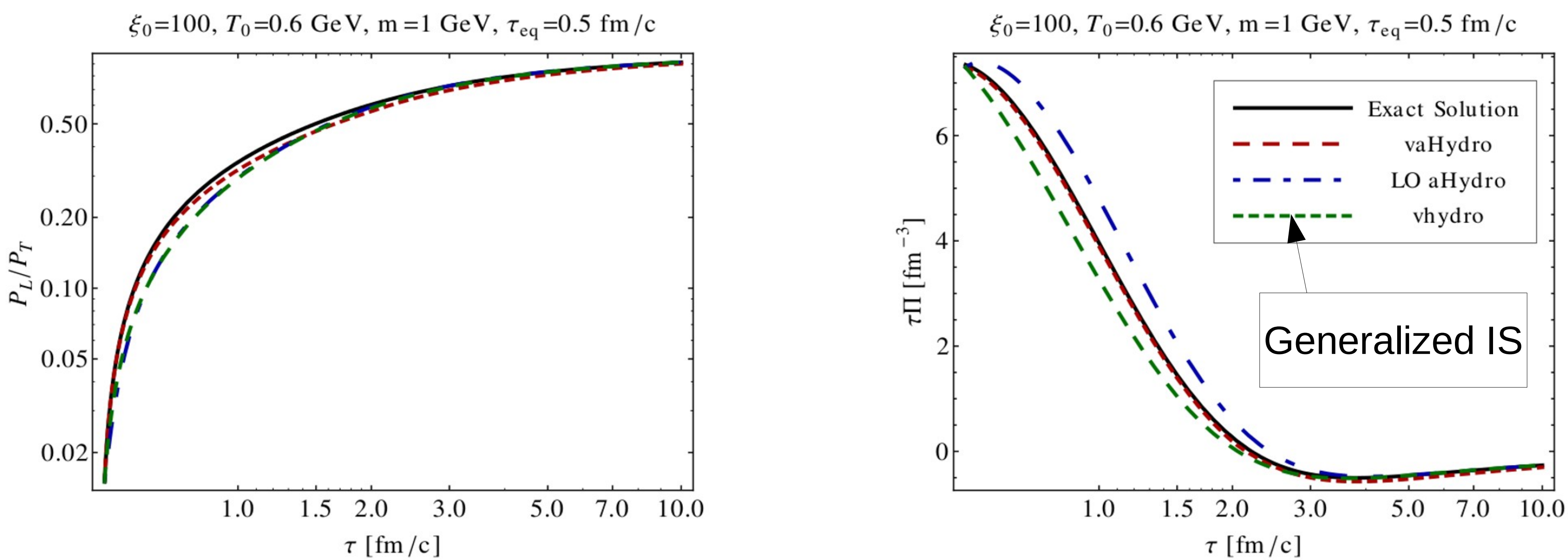




DB&Heinz&Martinez, arxiv:1503.07443



$$\Pi \equiv \frac{2\mathcal{P}_\perp + \mathcal{P}_\mathrm{L}}{3} - \mathcal{P}_\mathrm{eq} + \tilde{\Pi}$$



DB&Heinz&Martinez, arxiv:1503.07443

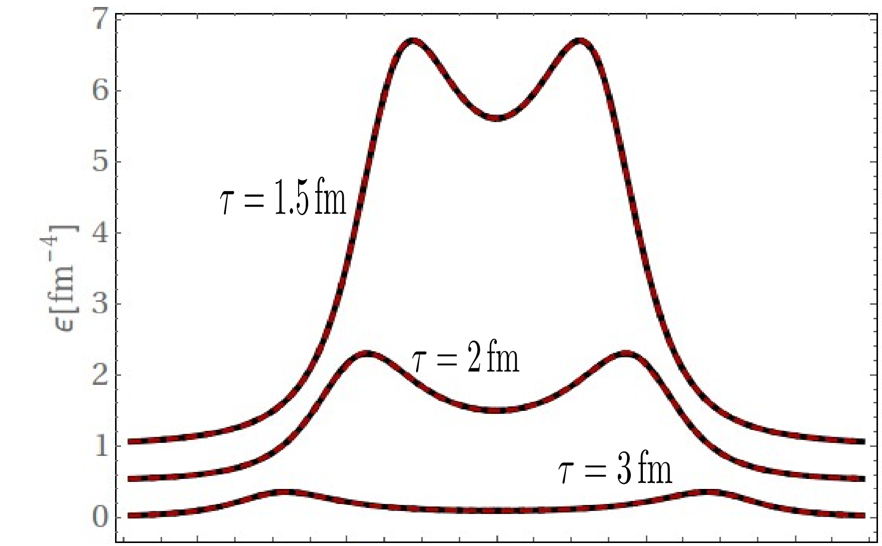
3+1d viscous hydro code

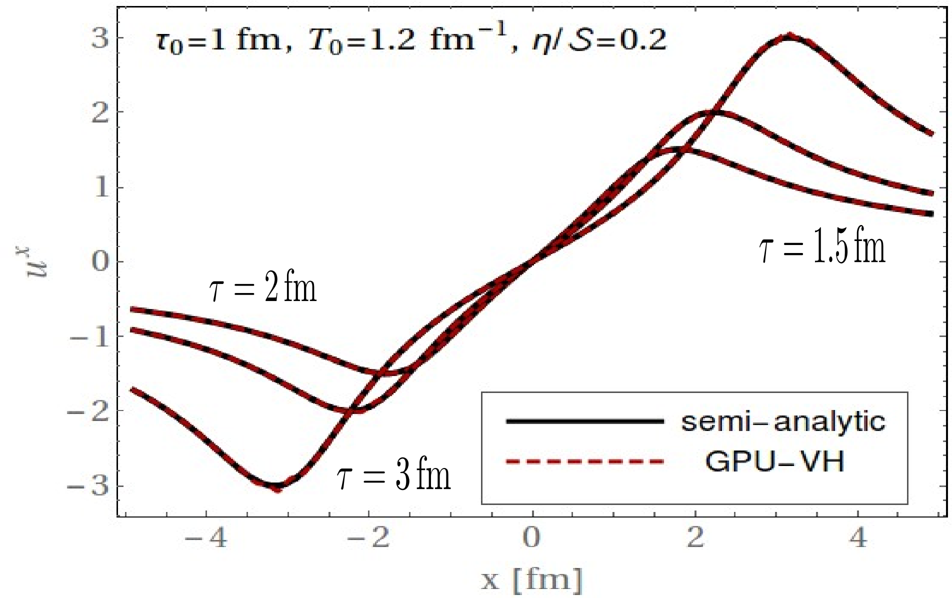
- Use KT algorithm like MUSIC
  - Schenke&Jeon&Gale, arxiv:1004.1408 (ideal)
  - Schenke&Jeon&Gale, arxiv:1109.6289 (viscous)
- But evolve dissipative quantities with KT
- On GPUs using CUDA C, ~88x speedup from highly optimized serial C code
  - Run on Tesla K20M (2500 cores)
- Fluctuating ICs, shear and bulk, QCD EoS
- Solve generalized IS equations from Denicol et al.

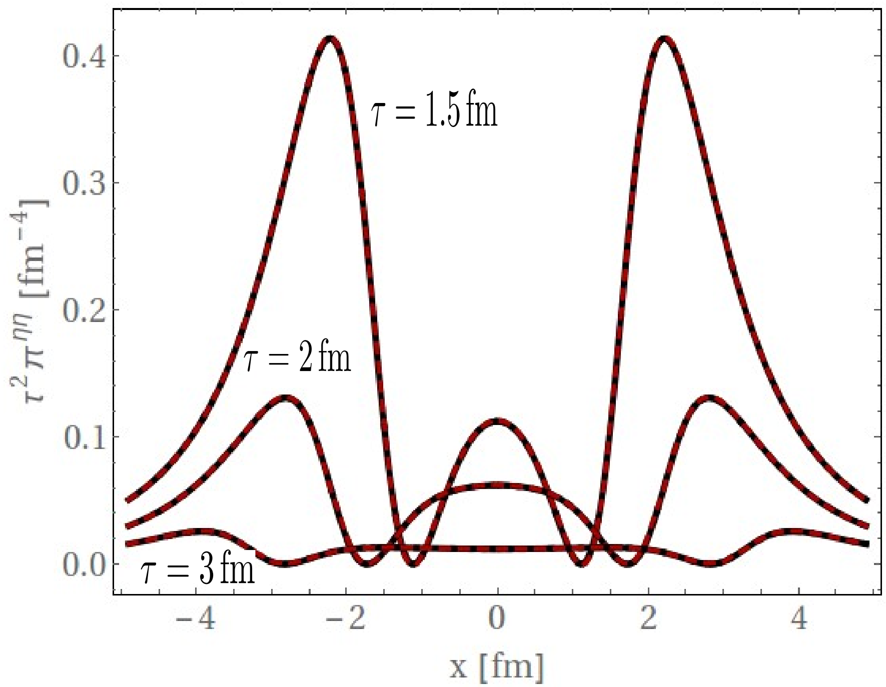
DB&Heinz&Strickland, in preparation

DB&Heinz&Strickland, in preparation

Gubser test







Gubser flow embodies key feature of HICs: very different longitudinal and transverse expansion rates.

IS equations for Gubser flow, results in semi-analytic solution: Marrochio&Noronha&Denicol&Luzum&Jeon&Gale, arxiv:1307.6130

Compare to viscous hydro codes--”gold standard” code test