Dissipative Properties of Chiral Media

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Chiral Effects

- Dynamics of chiral media is pretty different because of the presence of an extra classically conserved charge - chirality.
- This charge is anomalous meaning that at the quantum level there is a violation of the chiral symmetry

$$\partial \cdot J_5 = C_a E \cdot B \,.$$

- Despite of the quantum nature of the anomaly **it has macroscopic manifestations** through new transport phenomena:

$$J_\mu = \xi B_\mu + \xi_\omega \omega_\mu$$

where ξ, ξ_B are fixed by C_a (Note: counterparts in s^μ and $T^{0\mu}$).

It appears that these effects are

- pretty universal - the same form of anomalous kinetic coefficients **for both weak and strong coupling limits**;
- even more universal - the same form of anomalous kinetic coefficients **for both weak and strong fields** (at least for CME);
- anomalous, as mentioned above, so they **may posses the same features as the anomaly does** (at least in specific limits): no further perturbative corrections, no dependence on T and μ, etc;
- proportional to the axial vectors and thus they could be expected to be **equilibrium currents** in the full similarity to the textbook reasoning why B doesn't produce work;

- Let's compare CME current and the usual electric conductivity

$$\vec{J} = \sigma_B \vec{B} \quad , \quad \vec{J} = \sigma_E \vec{E}.$$

- Under the time reversal symmetry

$$\sigma_B^T = \sigma_B \quad , \quad \sigma_E^T = -\sigma_E$$

  while it is obvious that σ_E must be positive.
- The even time-parity of σ_B indicates that it could be described by a Hamiltonian formalism[1] and thus the anomalous transport could be considered as **dissipation-free.**
- However there is a question how robust the transport and consequently this result are.

[1] D. Kharzeev, H.-U. Yee, PRD (2011)

Dragging a Heavy Quark

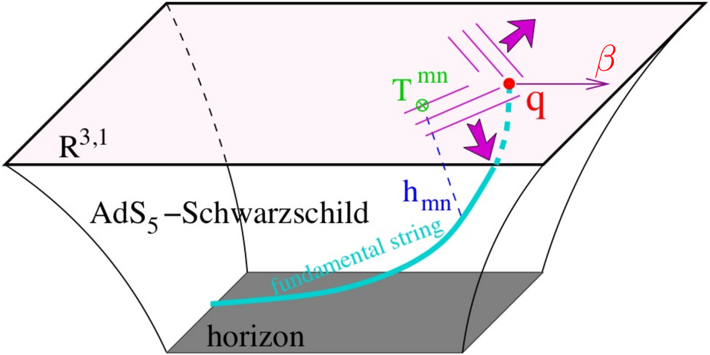
This problem could be related to the known study of a holographic plasma:

- Indeed, to probe dissipative properties of strongly coupled thermal plasma one could calculate a drag force for a heavy quark, $M \to \infty$.
- Considering a probe string in a dual model it was shown[2] that to drag a heavy quark with constant velocity $\vec{\beta}$ through a **static, homogeneous, equilibrium, $\mu = 0$** plasma of $\mathcal{N} = 4$ SYM theory at temperature T one has to exert a force:

$$\vec{f} = -\frac{\sqrt{\lambda}}{2\pi}\pi^2 T^2 \vec{\beta} \propto \frac{\vec{p}}{M}$$

  with $\lambda \equiv g^2 N_c$.
- However the fluid of interest is **not static, not homogeneous, not at equilibrium and it may have non-zero chemical potentials**

[2] L. Yaffe et al, JHEP (2006); S. Gubser, PRD (2006)


T mn
q
β
R3,1
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fundamental string
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Chiral Drag Force

The drag force gains anomalous contributions[3] and to the lowest nonzero order in μ, for a heavy quark moving through a thermal strongly interacting plasma (we suppose the mixed anomaly to be turned off) they are

$$f_\mu^B = -\frac{\sqrt{\lambda}}{2\pi^3\gamma}\left(\frac{\mu}{T}\right)^2 s^2\kappa\ C_B(\pi T\sqrt{-s})\left(B_\mu + (B\cdot w)w_\mu\right)$$

$$f_\mu^l = \frac{\kappa\sqrt{\lambda}\mu^3}{3\gamma\pi^3 T^2}\frac{l_\mu + (l\cdot w)w_\mu}{s}.$$

where $l_\mu \equiv \epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha u^\beta$, $B_\mu \equiv \epsilon_{\mu\nu\alpha\beta}u^\nu\partial^\alpha A^\beta$, $C_B(r)$ is a known function and parity could be restored by $\mu^2 \to 4\mu_A\mu_V$ and $\mu^3 \to 6\mu_A\mu_V^2 + 2\mu_A^3$.
Therefore every heavy quark or antiquark (no matter the charge) feels the same force!

[3] K. Rajagopal, AS, JHEP (2015)

Chiral Effects and Dissipation

- One should note that the chiral drag force is **nonzero even for a quark at rest** with the respect to the medium:

$$\vec{f} = -\frac{\kappa\sqrt{\lambda}}{2\pi^3}\frac{\mu^2}{T^2}\vec{B} - \frac{2\kappa\sqrt{\lambda}}{3\pi^3}\frac{\mu^3}{T^2}\vec{\Omega}.$$

- Comparing with the zero order drag force $\vec{f} = \frac{\sqrt{\lambda}}{2\pi}\pi^2 T^2 \vec{v}$ we conclude that there is **a terminal velocity**:

$$\vec{v}_{terminal} = \kappa\frac{\mu^2\vec{B}}{(\pi T)^4} + \frac{4\kappa}{3}\frac{\mu^3\vec{\Omega}}{(\pi T)^4}$$

- This result looks surprising since a massive quark shouldn't probe the anomalous dynamics

By a direct calculation one may check that the boost to the entropy rest frame coincides with the terminal velocity:

$$\vec{v}_{boost} = -\frac{1}{2}\frac{C}{\epsilon + P}\left(\mu^2\vec{B} + \frac{4}{3}\mu^3\vec{\Omega}\right)$$

Thus in the "no drag frame"' the heavy quark at rest:

- Feels no force.
- Sees the charge and momentum flows around it, with both $\propto \kappa$ along the $\vec{B}$ or $\vec{\Omega}$ direction.
- Sees no entropy current which appears to be zero in this frame. (Note: it is no the case in the presence of the mixed gauge-gravitational CS.)

Thus a heavy ($M \to \infty$) quark **may be considered as a "defect"** in the anomalous fluid flow and the absence of the drag force is **another consequence of its non-dissipative nature**.

Phenomenological consequences

- Let's make an estimate of the momentum gained by b quark:

$$\vec{p}_{\mathrm{terminal}} \simeq -3\ \mathrm{MeV}\ \frac{m_b}{4.2\ \mathrm{GeV}}\ \frac{\mu_V}{0.1\ \mathrm{GeV}}\ \frac{\mu_A}{0.1\ \mathrm{GeV}}\ \frac{\vec{B}}{(0.1\ \mathrm{GeV})^2}\left(\frac{0.5\ \mathrm{GeV}}{\pi T}\right)^4$$

  Thus at least in principle **there is a unique correlation observable**: net out-of-plane momentum *for all D* and *B* mesons in a given event, *in the direction opposite to the CME current in that event*.

- Also massive probes **would feel this"anomalous wind"** in the presence of the chiral asymmetry and external fields whenever they are considered in the Landau rest frame:

$$v_{anom} = -\frac{C_a \mu_A \mu B}{\epsilon + P}$$

Now we could turn back to the discussion above and try to address new questions:

- Is this non-dissipativity **a sign of** some quantum behavior similar to **the superfluidity or** it is **just an artifact** of our description (the transport couldn't be measured/probed, disappears in any realistic setup, etc.)?
- If it does have further similarity to the superfluidity **is there any restrictions** required to be satisfied to gain the non-dissipative dynamics?
- What is **the microscopic origin** of the phenomenon. Indeed for other similar situations one would expect an excitation responsible for the non-dissipativity (an edge mode for Hall current, goldstones for superfluidity, etc.).

We could try to answer these questions turning to **the drag force study in the weak coupling limit**.

Landau Criteria

- For a heavy impurity in an equilibrium fluid the energy conservation is

$$\epsilon = E_f - E_i = v \cdot q - \frac{q^2}{2M} \quad , \quad q = P_f - P_i$$

- For a superfluid $\epsilon(q)$ is a dispersion of low-lying excitations and **there is a kinematic constraint** $\left[\frac{\epsilon(q)}{q}\right]_{\text{min}} < v$.
- In the CME transport HLL contributions cancel each others for $\pm k_z$ and it is **realized just through the LLL**. Thus we can identify it with the anomalous component of the system.
- If we consider the motion of a heavy impurity along $\hat{B}$ and a chiral fermion at LLL gains momentum q_B then its energy increases linearly and **the kinematic constraint won't be satisfied for $v < 1$** (Note that the transition from LLL to HLL is also forbidden).

Drag force calculation

Let's consider a chiral medium at finite $\mu_{(R,L)}$ and T with $B \gg \mu^2, T^2$ (we suppose existence of a chiral zero mode(s) tied with the anomaly).

- In the P-even case with conserved chirality and weak medium-impurity coupling the dominant processes are $HR \to HR$ and $HL \to HL$.
- In general the chirality flipping rate Γ_χ is non-zero but typically small and consequently $HL \to HR$ is suppressed.

Thus one can try to push the previous non-dissipative ideal case further to investigate whether these properties survive in more realistic systems.

Here I'd bring results for two complementary examples[4] of a weakly coupled chiral fermion gas and a strongly interacting chiral liquid in the limit of weak impurity-medium coupling.

[4] AS, Yi Yin, 2015

We start with a generic Hamiltonian for the medium-impurity interaction

$$H_I = \int d^3x \int d^3x' n_v(x,t) U(x-x') n_{imp}(x',t')$$

and the LO contribution to the medium-impurity momentum transfer could be obtained using Fermi's golden rule[5]

$$(2\pi)^3 \frac{d\mathcal{R}}{d^3q} = |U_q|^2 \frac{\rho(\omega,q)}{1-e^{-\omega/T}}$$

where $\rho(\omega,q) = -2\mathrm{Im}\, G^R(\omega,q)$ with $G^R \sim \langle n_V n_V \rangle$ and one can use a textbook formula for the drag force

$$F = \frac{dP}{dt} = -\int q\mathcal{R}(v\cdot q, q) d^3q = -\int q|U_q|^2 \rho(v\cdot q, q) d^3q$$

[5] see e.g. M. Bellac, 2011; P. Nozieres and D, Pines, 1999

Now let's calculate the leading contribution to the drag force in powers of the medium-impurity coupling:

- For a weakly coupled chiral fermions $\rho(\omega, q) \sim e^{-q_\perp^2/(\sqrt{2}B)^2} \rho_{2D}(\omega, q_z)$ in the large B limit, where

$$\rho_{2D}(\omega, q_z) = \omega \left[\delta(\omega - q_z) + \delta(\omega + q_z)\right]$$

and the delta function **results in the zero drag force** reproducing the kinematic constraint, indeed

$$F \sim \int q|U_q|^2 e^{-\frac{q_\perp^2}{2B^2}} \delta((1 \pm v)q_z) d^3q = 0$$

- In the case of strongly interacting medium $G^R(\omega, q)$ could be read off constitute relations

$$\begin{aligned} J_V^l &= C_a \mu_A B^l + \sigma^{lm} E_m - D^{lm} \nabla_m n_V \\ J_A^l &= C_a \mu_V B^l - D^{lm} \nabla_m n_A, \;\; l, m = 1, 2, 3 \end{aligned}$$

where $D = \chi\sigma$ is the diffusion coefficient tensor. Perturbing the system by $\delta A_0 \sim e^{-i\omega t + ik\cdot x}$ and substituting currents in

$$\partial \cdot J_V = 0 \;\; , \;\; \partial \cdot J_A = C_a E \cdot B - \Gamma_\chi n_A$$

we get after some algebra

$$\begin{aligned} G_R(\omega, q_z) &= \chi \left[1 - \frac{\omega(\omega + i\Gamma_{q_z} + i\Gamma_\chi}{\Delta(\omega, q_z)} \right] \\ \Delta(\omega, q_z) &= (\omega + i\Gamma_{q_z})(\omega + i\Gamma_\chi) - v_\chi^2 q_z^2 \; , \;\; v_\chi = \frac{C_a B}{\chi} \end{aligned}$$

where we used the Drude approximation for large B

$$v_\chi \to 1, \;\; \chi \to C_a B, \;\; D_L \sim B^{-1}, \;\; D_T \sim B^{-3}$$

One can separate two physical limits

- $\Gamma_{q_z} \gg \Gamma_\chi$ where

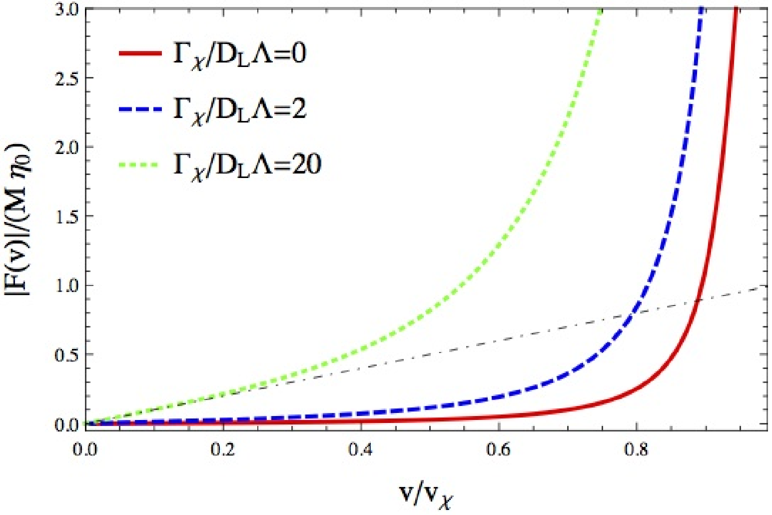
$$\rho_{hyd} = \sum_{s=\pm} \frac{\omega\chi\Gamma_{q_z}}{(\omega - s v_\chi q_z)^2 + \Gamma_{q_z}^2} \xrightarrow[B\to\infty]{} \sum_{s=\pm} \omega\chi\pi\delta(\omega - s v_\chi q_z)$$

  and we reproduce the result of the ideal case (note that $v_\chi \to 1$ in this limit);

- $\Gamma_{q_z} \ll \Gamma_\chi$ where the dispersion is reduced to anomalous diffusion $\omega = -i v_\chi^2 q_z^2/\Gamma_\chi$ and small velocities $v \ll v_\chi$

$$\rho(v q_z, q_z) = \frac{2 v\chi\Gamma_\chi}{v_\chi^2 q_z} \quad , \quad F \sim \Gamma_\chi v$$

Thus **the drag force is suppressed for small Γ_χ** but it grows linearly in the opposite limit.



Drag force for a heavy impurity moving along $\hat{B}$ with velocity v in an anomalous chiral fluid with strong magnetic field (here $U(q) = U_\perp(q_\perp)e^{-q_z^2/\Lambda^2}$ and $D_L\Lambda^2 = 10^{-4}$).

Summary

- There is an **anomalous contribution to the drag force** on a heavy quark (or other heavy impurity).
- **Heavy probes** could be also influenced by chiral effects (anomalous contribution to the dissociation of a quarkonium);
- A heavy impurity moving along $\hat{B}$ in the weak medium-impurity coupling limit and at $B \gg \mu^2_{(R)}, T^2$ **feels no drag force** for any velocity of the impurity;
- **The chirality flipping results in suppressed corrections** to the drag force while it is kept small (comparing with $D_L \Lambda^2$ where Λ is a characteristic momentum).