Sphalerons far from equilibrium and associated phenomena

Mark Mace
Stony Brook/BNL

Based on MM, S. Schlichting, R. Venugopalan
Phys.Rev. D93 (2016) 074036 (arXiv:1601.07342 [hep-ph])

BEST Workshop - Indiana University, Bloomington - May 10, 2016

Stony Brook University

BROOKHAVEN NATIONAL LABORATORY

Outline

- Motivation- CME
- Topology in real-time lattice gauge theory
- Far from equilibrium sphalerons
- Outlook to future work

Motivation-CME

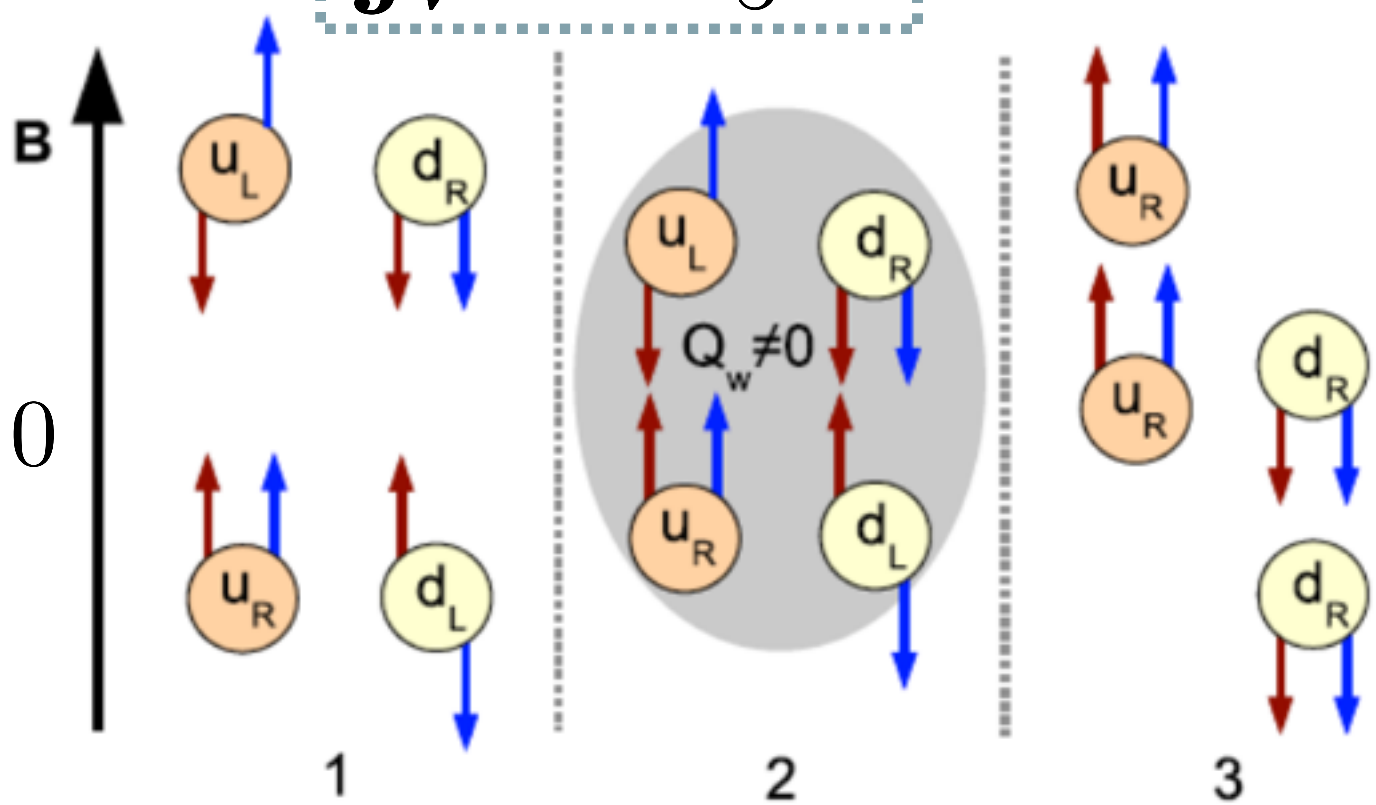
- Chiral Magnetic Effect

(Kharzeev, McLerran, Warringa NPA803, 227 (2008); Fukushima, Kharzeev, Warringa PRD78 (2008) 074033)

  - Vector Current= Axial charge imbalance + U(1) B-field

$$\mathbf{j}_V \sim n_5 \mathbf{B}$$

$$\mathbf{B} \gg 0$$



(Kharzeev Prog. Part. Nucl. Phys. 75 (2014) 133-151)

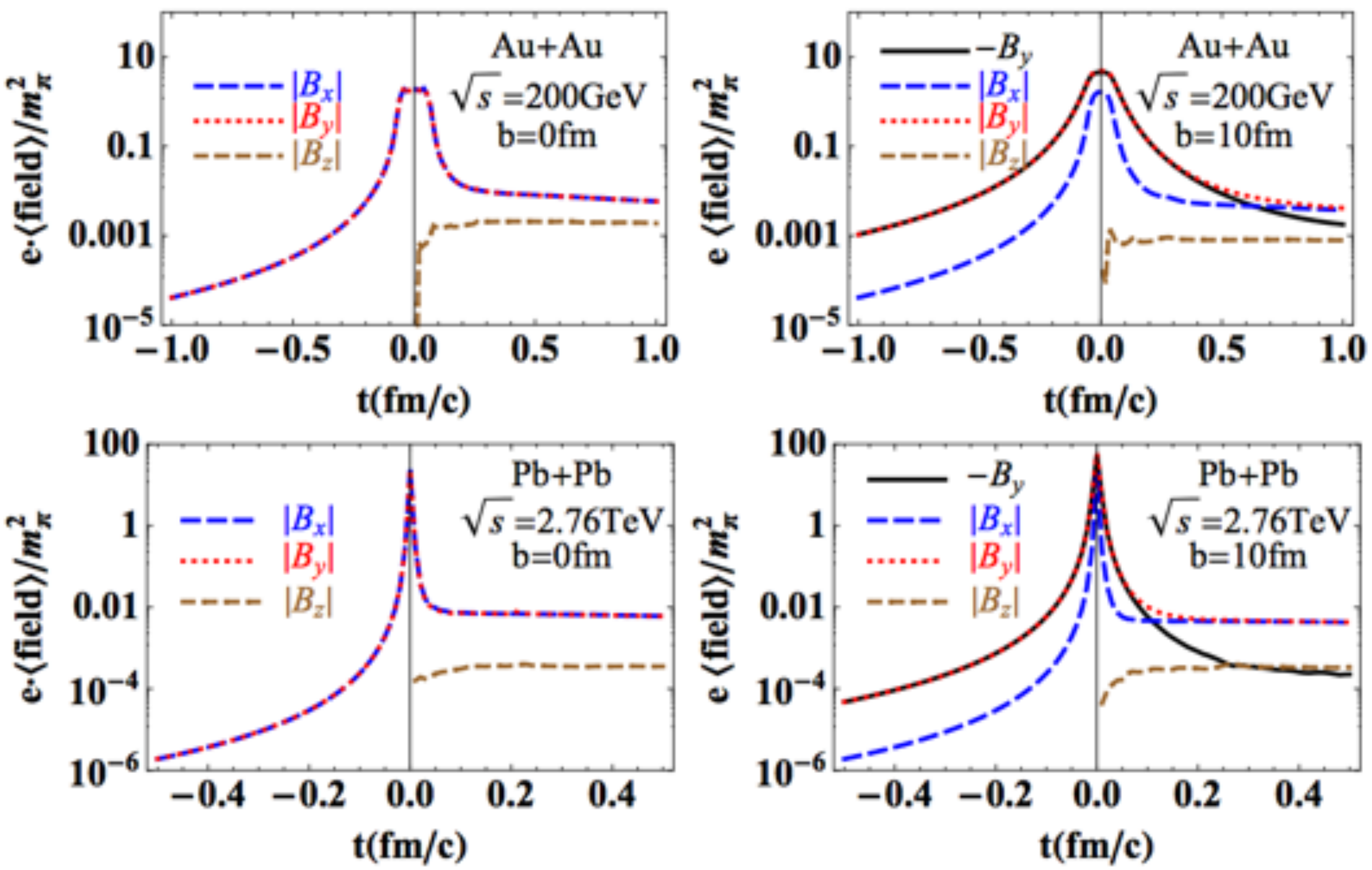
Red=momentum; Blue=spin;

Motivation-CME

$$\mathbf{j}_V \sim n_5 \mathbf{B}$$

- Magnetic field strongest at early times
  - $e\mathbf{B} \equiv (m_\pi)^2 \sim 10^{18}\mathrm{G}$
- Dependent on impact parameter and beam energy
- Very short lifetime
  - $t \sim 0.1 - 0.2 \frac{\mathrm{fm}}{\mathrm{c}}$
  - may be increase by conductivity

Theoretical model calculations



(Deng and Huang, PRC85, 044907)

- Much recent progress
  (Deng and Huang, PRC85, 044907; McLerran and Skokov NPA929 (2014) 184-190; Tuchin et al PRC91 (2015) 064902, arXiv:1604.04572)

Motivation-CME

- In order to fully understand CME, need to understand initial axial charge

$$\mathbf{j}_V \sim \boxed{n_5}\mathbf{B}$$

- Axial charge production governed by anomaly (t'Hooft; Adler, Bell, Jackiw)

$$\partial_\mu j^\mu_{5\ f} = 2m_f \bar{q} i \gamma_5 q - \frac{g^2}{16\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu}_a$$

Axial current
$j^\mu_5 = (n_5, \mathbf{j}_5)$

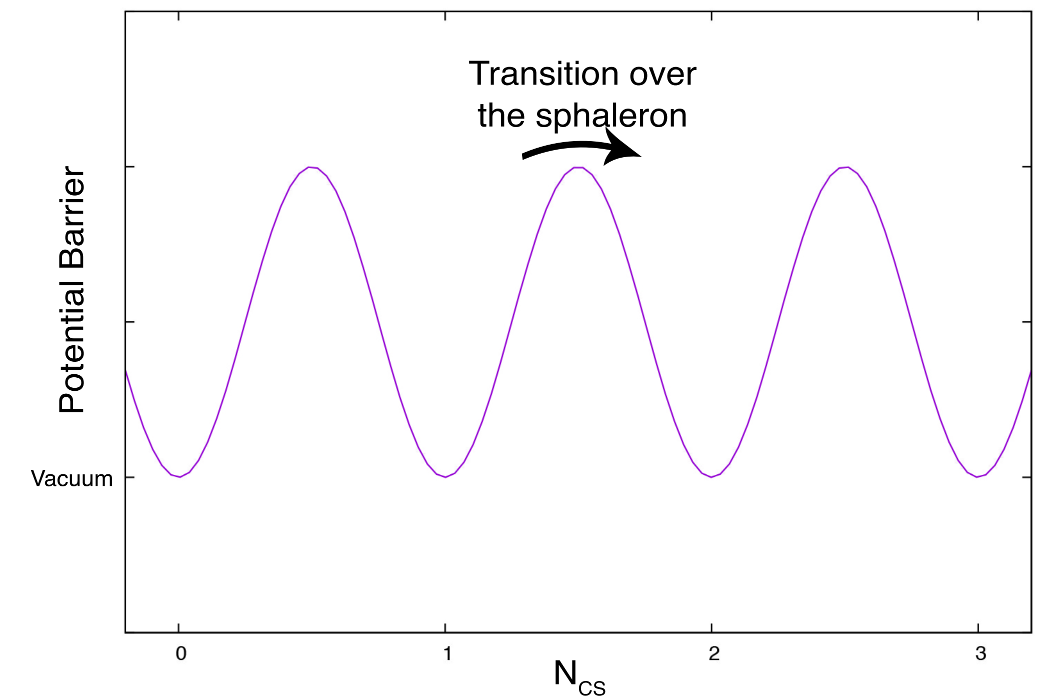
Fermion field contribution: ~fermion mass

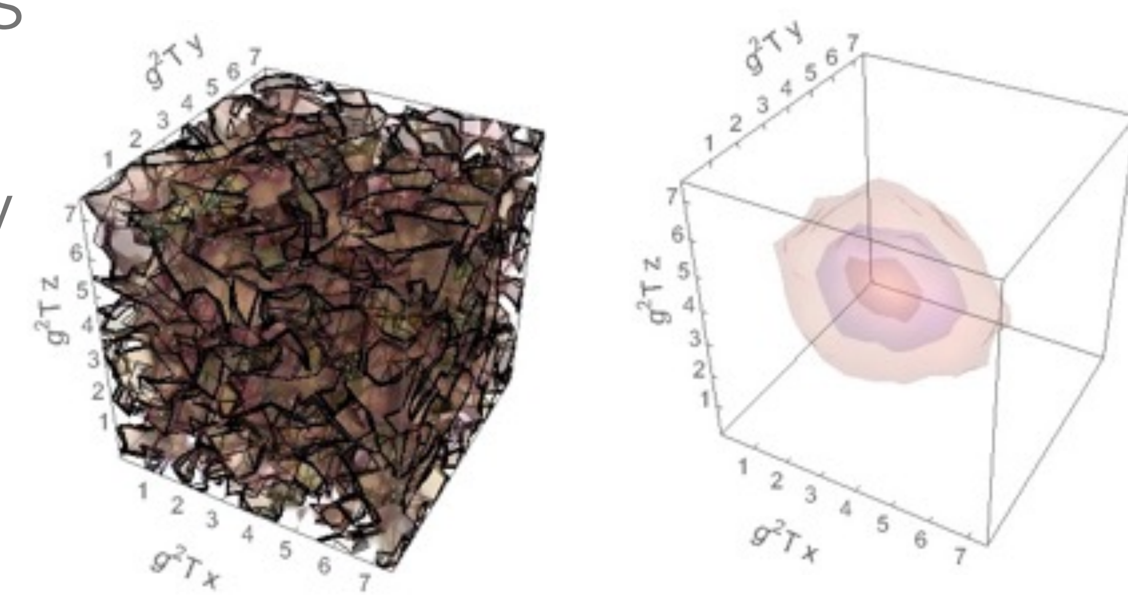
Gauge field contribution: receives contributions from topological transitions, field strength fluctuations,…

Axial charge

$$\partial_\mu j^{\mu}_{5\,f} \sim F^a_{\mu\nu}\tilde{F}^{\mu\nu}_a \propto \mathbf{E}^a\mathbf{B}^a$$

- Axial charge receives contributions
  - Topological transitions - present focus
    - Real time: sphaleron transitions (Klinkhamer, Manton PRD30 (1984) 2212)
      - Sphaleron= Greek for ‘ready to fall’
  - Field strength fluctuations





(MM, Schlichting, Venugopalan PRD 074036)

Axial charge

- Sphaleron transitions correspond to integer change in Chern-Simons number

$$\Delta N_{CS} = \frac{g^2}{8\pi}\int d^4x \mathbf{E}^a \cdot \mathbf{B}^a \qquad N_{CS}(t) = -2N_f \int d^3x j_5^0$$

- Determine global to axial charge production via the anomaly by tracking change in Chern-Simons number

$$\Delta N_5 = \int d^4x j_5^0 = -2N_f \Delta N_{CS}$$

Sphalerons in equilibrium

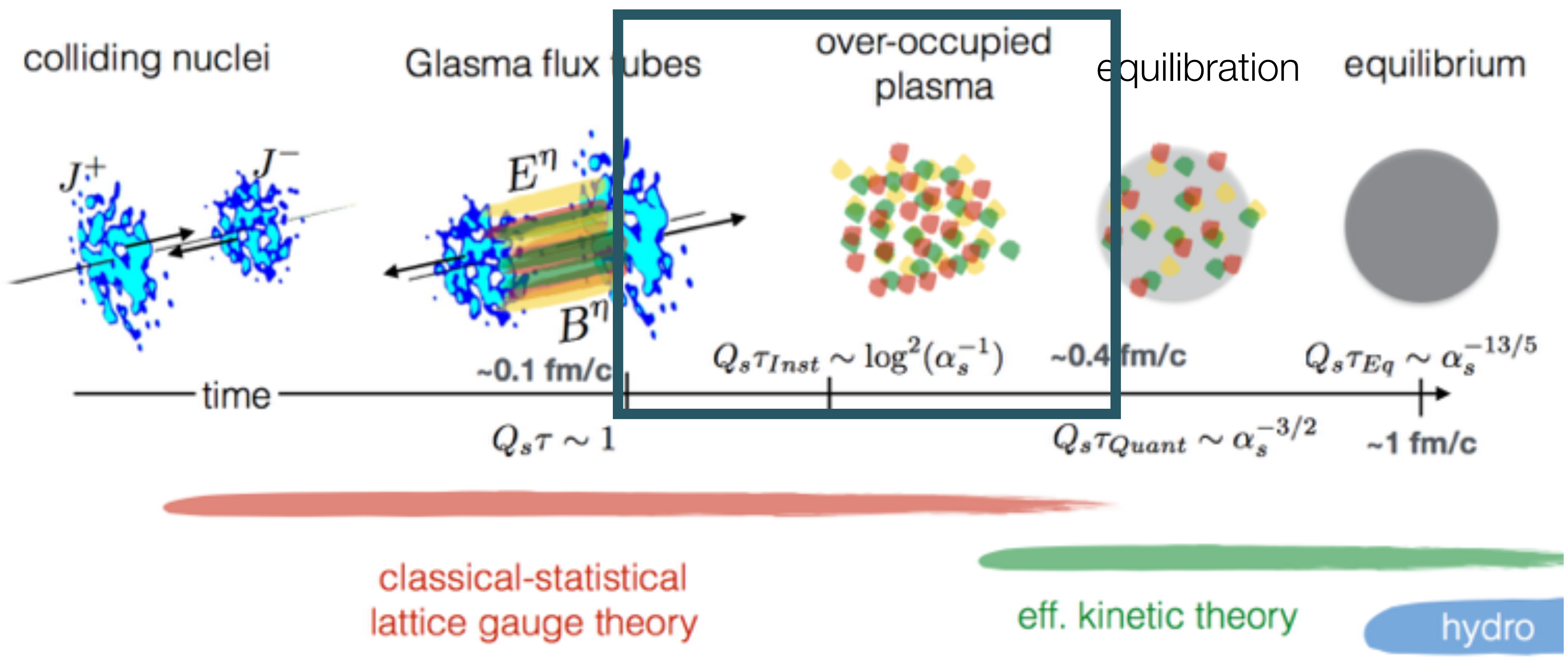
- Sphaleron transitions are given by modes on the order of the magnetic screening length $\Lambda_s \sim \frac{1}{g^2 T}$
  (Arnold and McLerran, PRD37, 1020 (1988))
- In equilibrium, sphaleron transitions dominate long time Chern-Simons number diffusion- probabilistic
  - Phase space: $(\alpha_S T)^3$ Typical freq: $(\alpha_S^2 T)$
    (Arnold, Son, and Yaffe, PRD55, 6264 (1997); Bodeker PLB426 (1998) 351-360)
  - Integer random walk with known diffusion constant

$$\Gamma^{eq}_{sph} = \lim_{\delta t \to \infty} \frac{\langle (N_{CS}(t+\delta t) - N_{CS}(t))^2 \rangle}{V \delta t} = \kappa \alpha_S^5 T^4$$

  - Most recent calculations κ~25
    (Moore and M. Tassler, JHEP 02, 105 (2011))

The glasma

- Magnetic field is understood to be strongest at $\leq 1\mathrm{fm/c}$
- This is when system is far from equilibrium, gluon dominated
  - Described by the 'glasma'
    (McLerran and Lappi, NPA772 (2006), Krasnitz and Venugopalan, NPB557 (1999) 237)
  - Necessary to study axial charge generation at this time



(Adapted from S.Schichtling Bielefeld 2016)

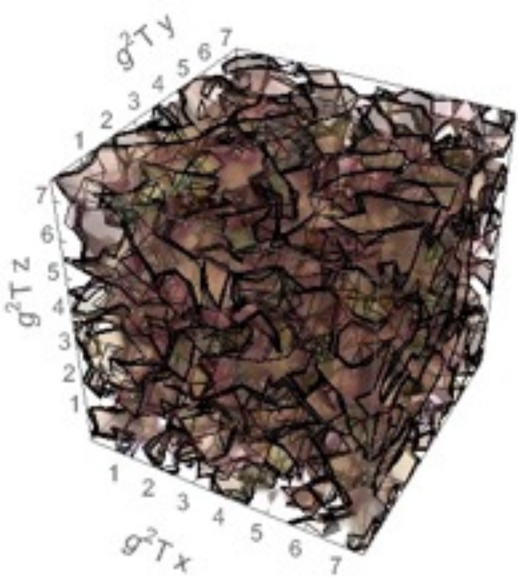
Early times after HIC

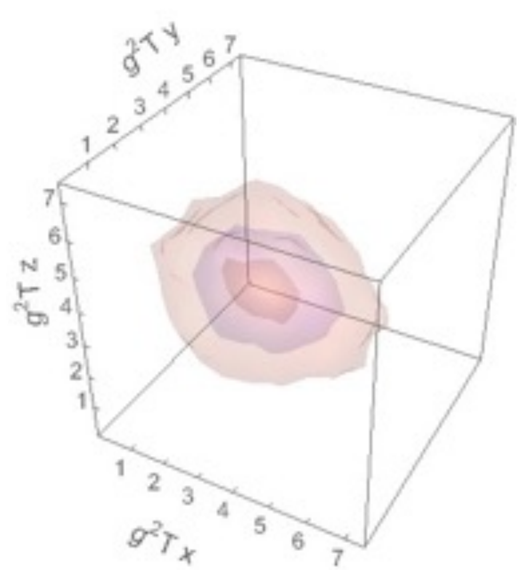
- Initially gluon occupation much larger than quark
  - Study classical Yang-Mills
    (McLerran, Venugopalan PRD49 2233 (1994))
- Non-perturbative large phase-space density of gluon
  - $f(p \sim Q_s) \sim \dfrac{1}{\alpha_S}$
  - Amenable to classical-statistical lattice description
    (Berges, et al PLB 681 2009; Berges et al PRD89, 074011 (2014))
- 2D boost-invariant case previously studied
  (Kharzeev, Krasnitz, Venugopalan PLB545, 298 (2002))
- Neglect longitudinal expansion and consider SU(2) for simplicity

Topology on the lattice

- Solve classical Yang-Mills in real time 3D spatial lattice
- Calculate $$\frac{dN_{CS}}{dt} = \frac{1}{8\pi^2}\int d^3x E_i^a B_i^a$$
- Lattice $E_i^a B_i^a$ not a total derivative, plagued by UV noise
- Need gradient flow (cooling) to see topological contribution
  (Moore NPB480 (1996) 657-688; Ambjorn and Krasnitz, NPB506, 387 (1997))

$$E_i^a B_i^a(\vec{x})$$



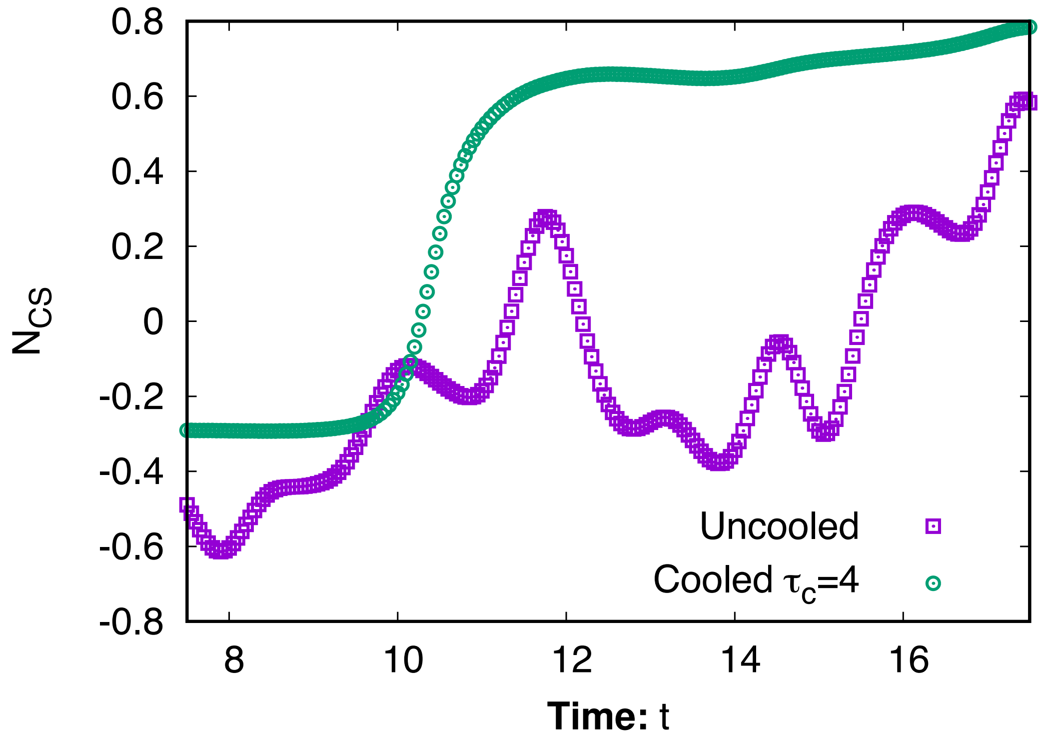


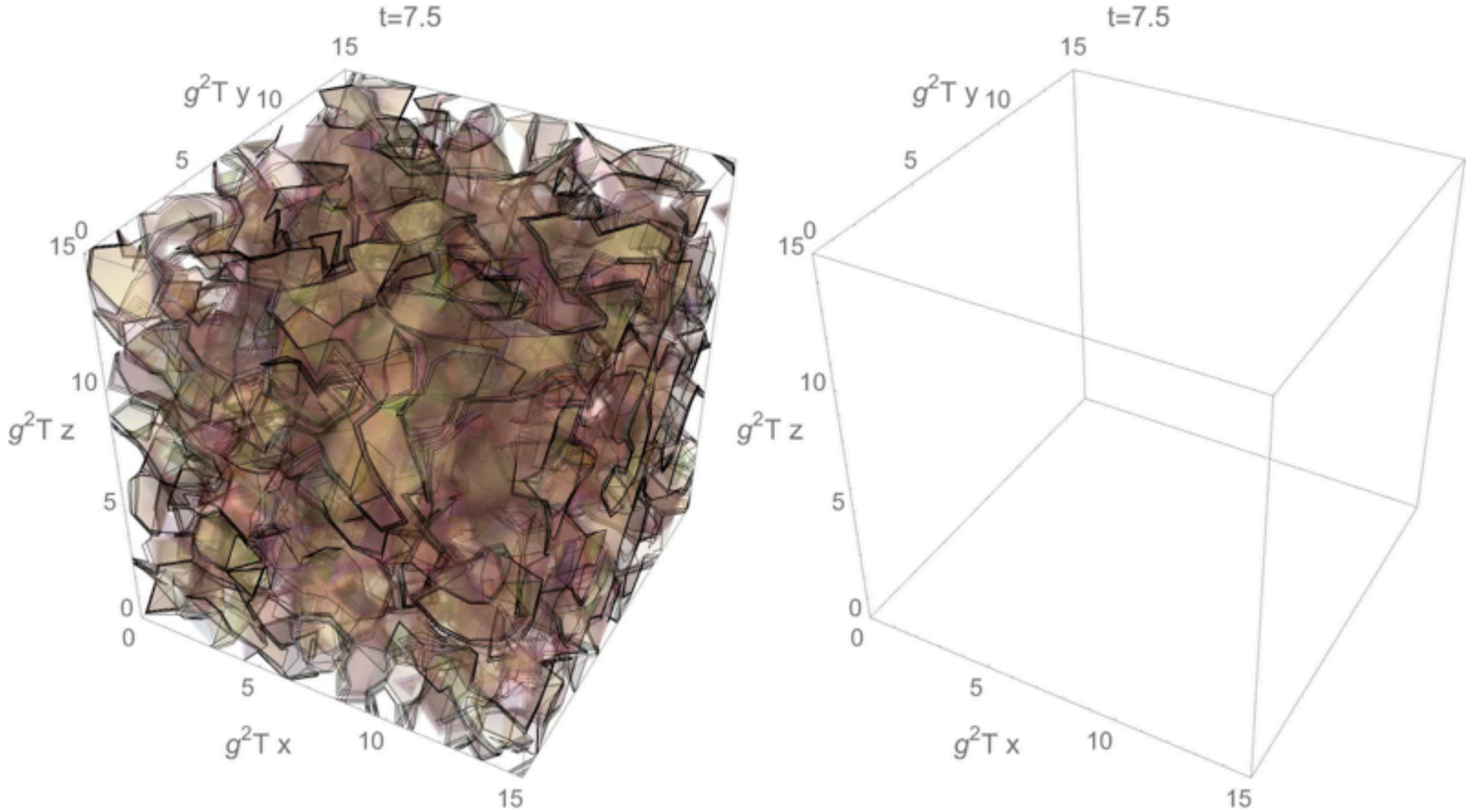
$$\frac{\partial A_i^a(x)}{\partial \tau_c} = -\frac{\partial H}{\partial A_i^a(x)}$$

Cooling →

(MM, Schlichting, Venugopalan PRD 074036)

Sphaleron in action





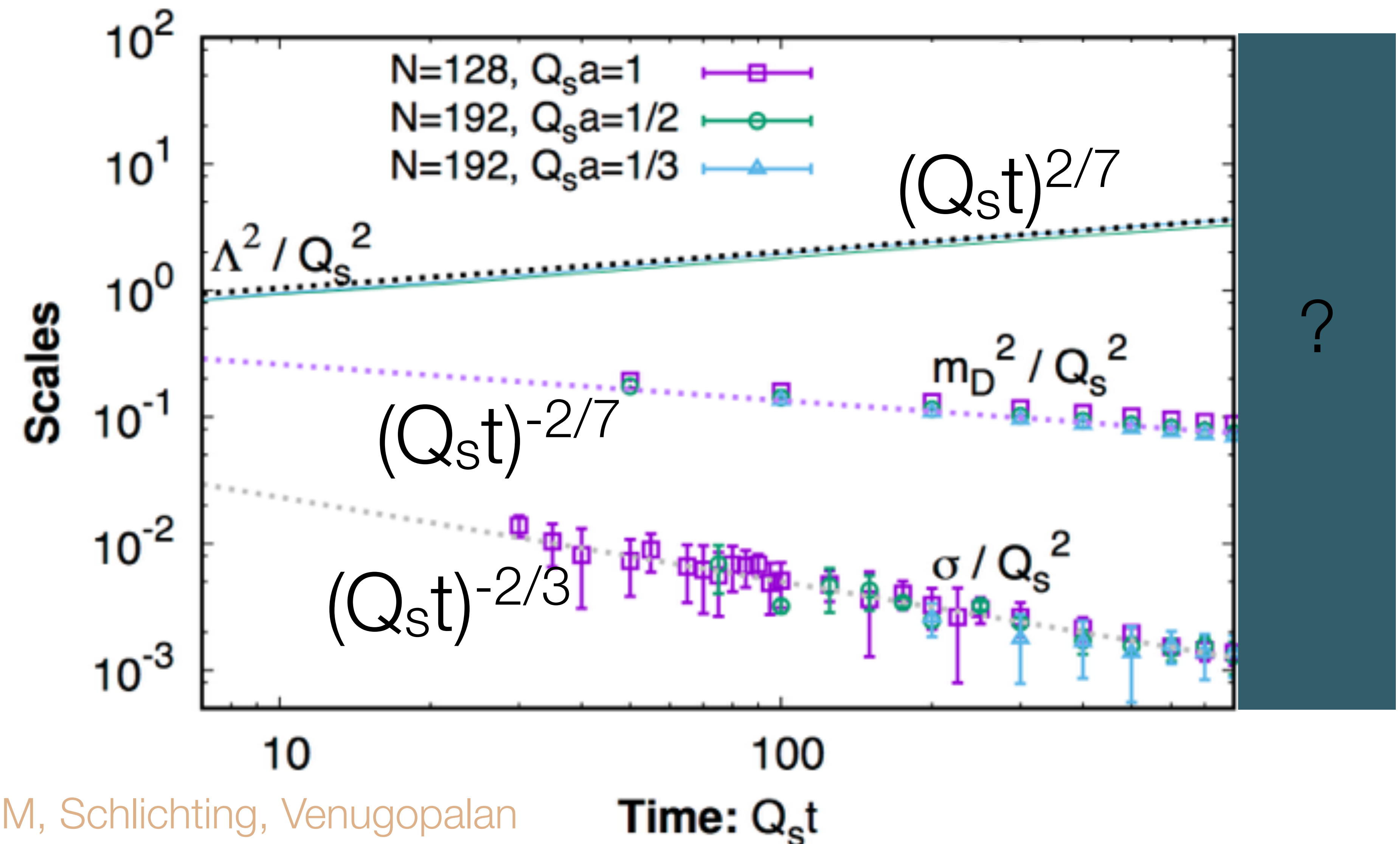
Dynamical scales at a glance

- Hard scale $Q_s \longrightarrow \Lambda^{eq} \sim T$
  - Average typical momentum of hard modes $\Lambda(t)^2 = \frac{2}{3}\frac{\int d^3p\, p^3\ f(p)}{\int d^3p\ p\ f(p)}$
    (Kurkela and Moore, JHEP 1112, 044 (2011); Schlichting PRD86, 065008 (2012))
- Electric screening scale $Q_s \longrightarrow m_D^{eq} \sim gT$
  - Appeal to perturbative formula $m_D^2 = 4g^2 N_c \int \frac{d^3p}{(2\pi)^3}\frac{f(p)}{p}$
    (Kurkela and Moore, PRD86, 056008 (2012))
- Magnetic screening scale $Q_s \longrightarrow \Lambda_s^{eq} \sim \sqrt{\sigma} \sim g^2 T$
  - Measure spatial string tension
    (Dumitru, Lappi, and Nara, PLB734, 7 (2014))
  - Large area fall limit of spatial Wilson loop

$$\langle W(A,t)\rangle \approx \exp(-\sigma A)$$

Dynamical separation of scales

- Kinetic theory predictions exist (Blaizot et al, NPA873, 68 (2012))
  - 2/3 in agreement



At equilibrium:

Hard Scale
$\Lambda^{eq} \sim T$

Electric Screening
$m_D^{eq} \sim gT$

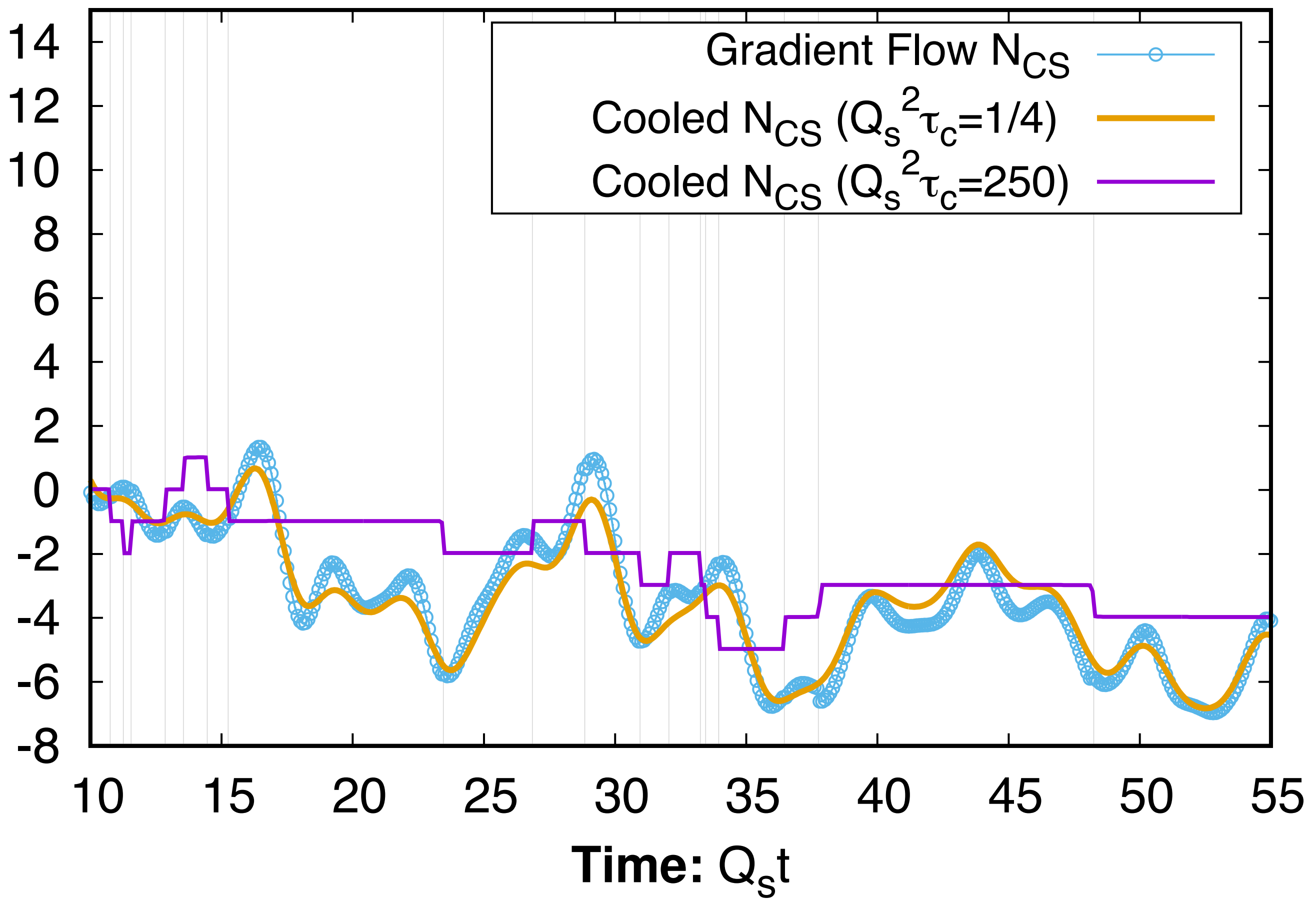
Magnetic Screening
$\Lambda_s^{eq} \sim \sqrt{\sigma} \sim g^2 T$

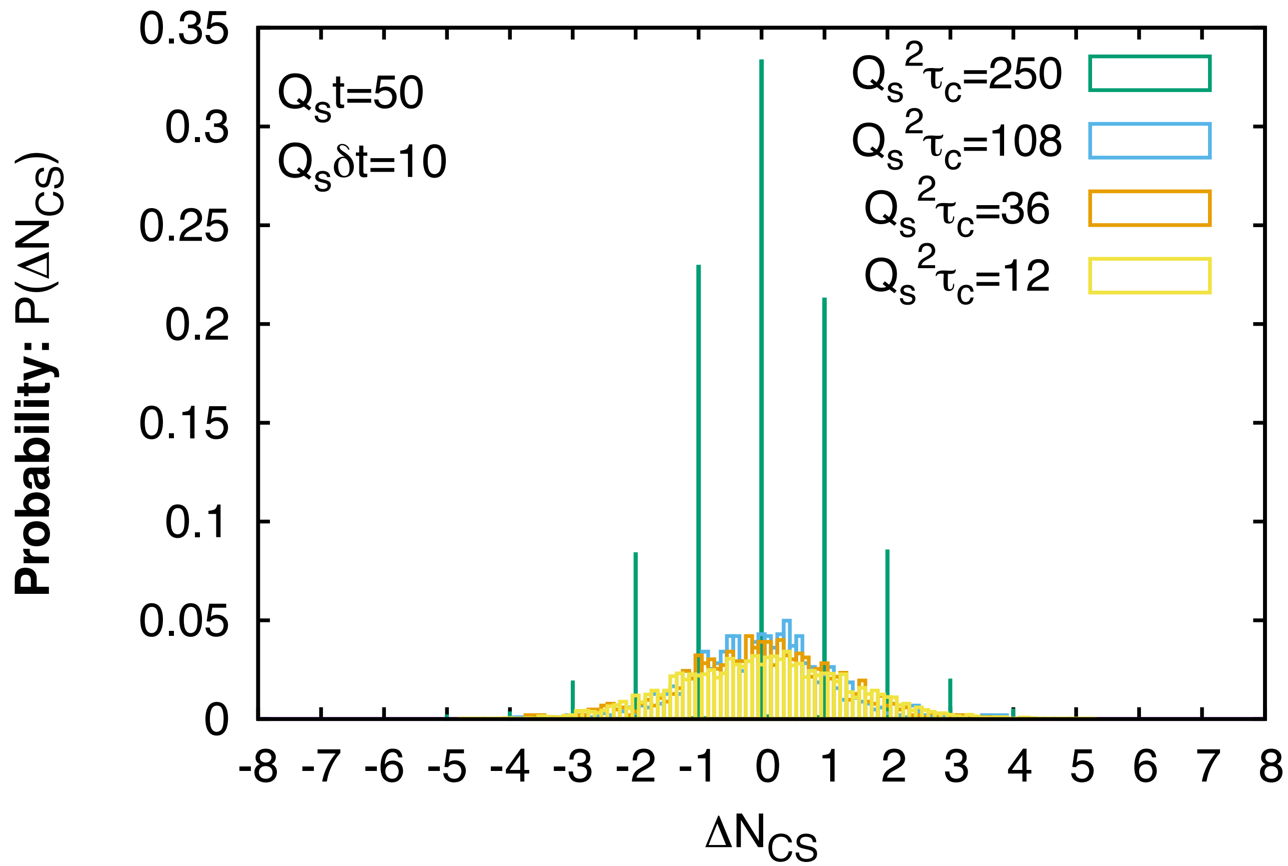
(MM, Schlichting, Venugopalan PRD 074036)

Sphalerons in the glasma

- Detect integer changes in Chern-Simons number for single configuration
- Histogram Chern-Simons diffusion of many configurations

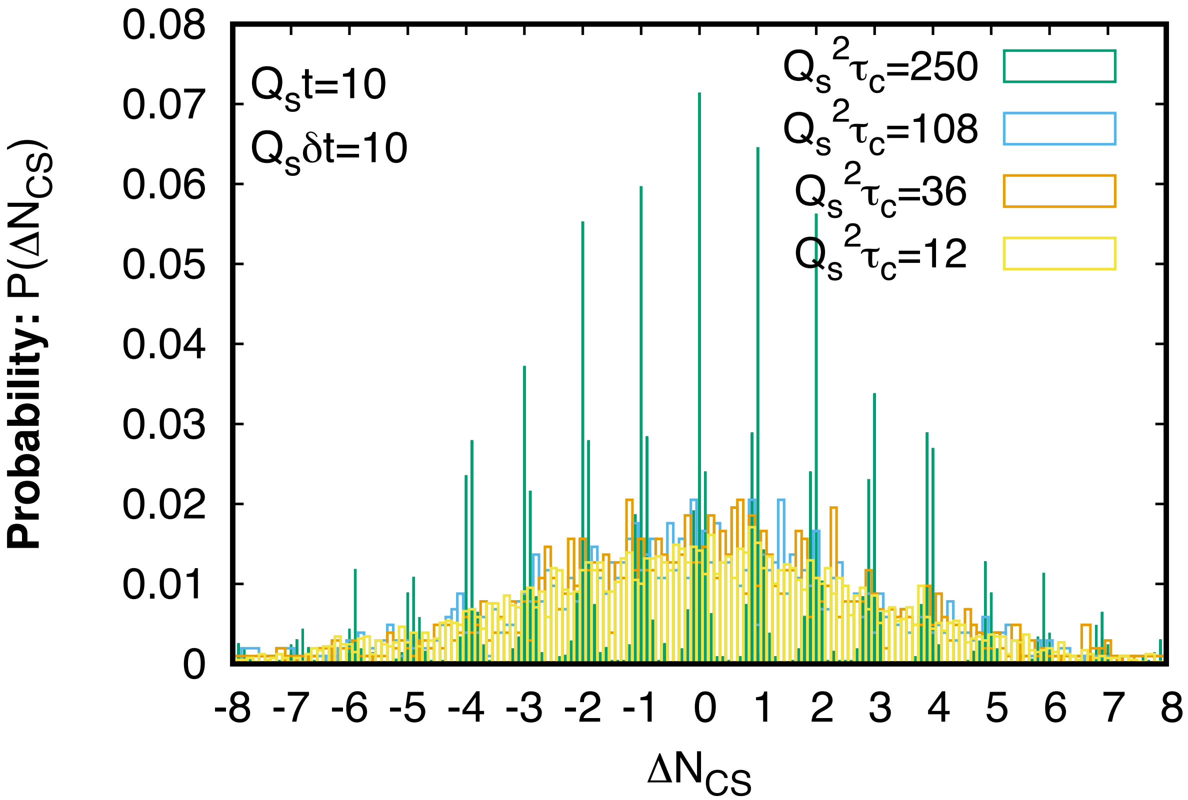
(MM, Schlichting, Venugopalan PRD 074036)



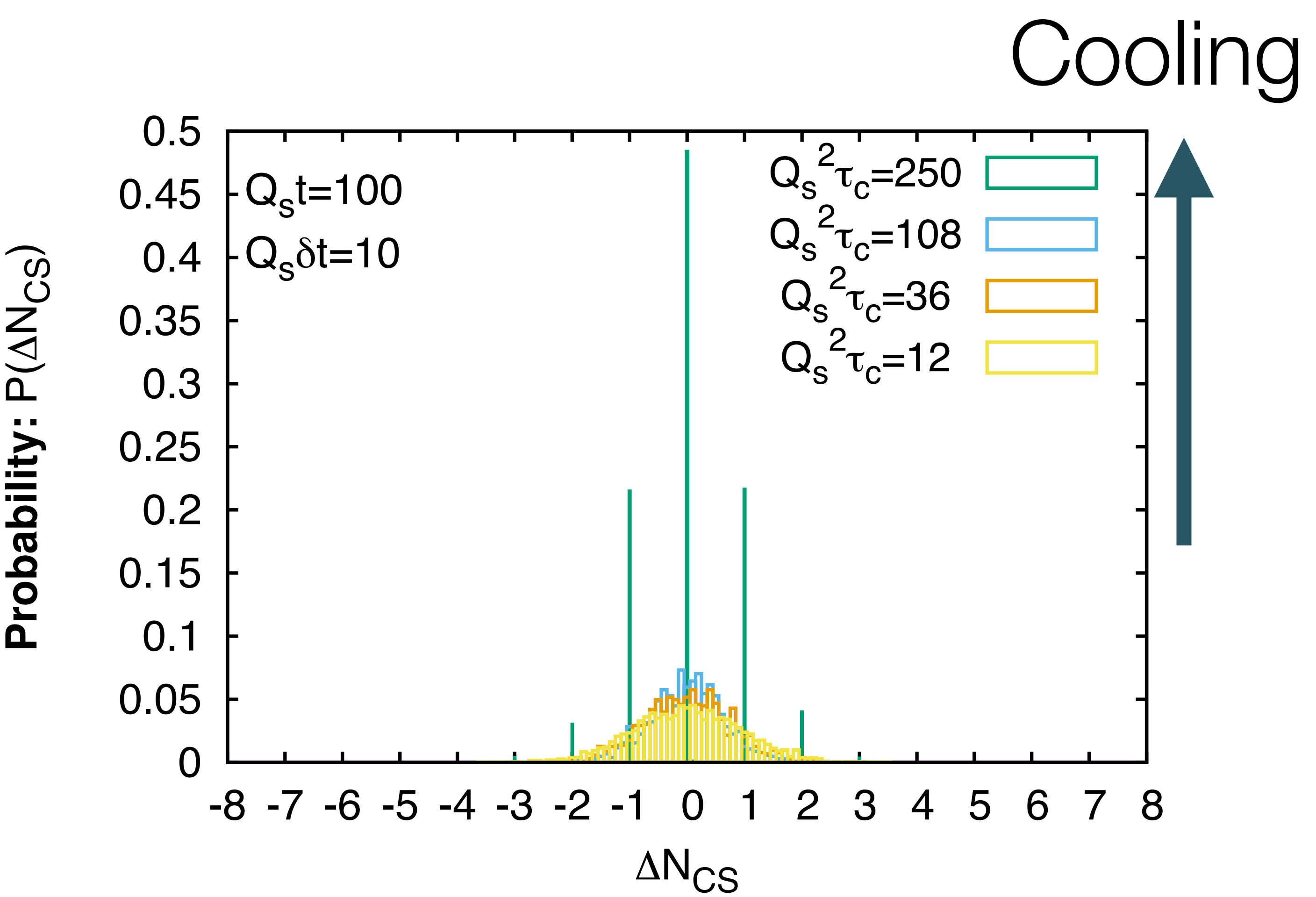


Can isolate transitions from background

Sphalerons in the glasma



Early: $Q_S t$=10-20



Late: $Q_S t$=100-110

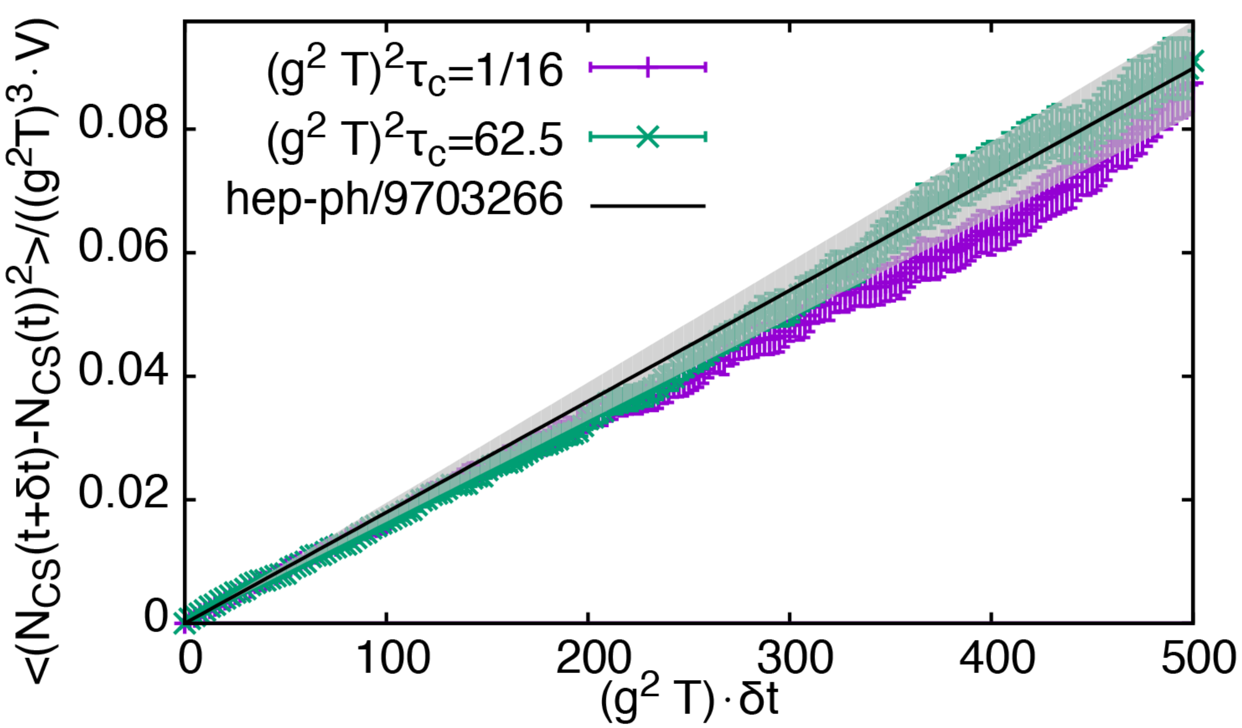
(MM, Schlichting, Venugopalan PRD 074036)

Most transitions happen at early times

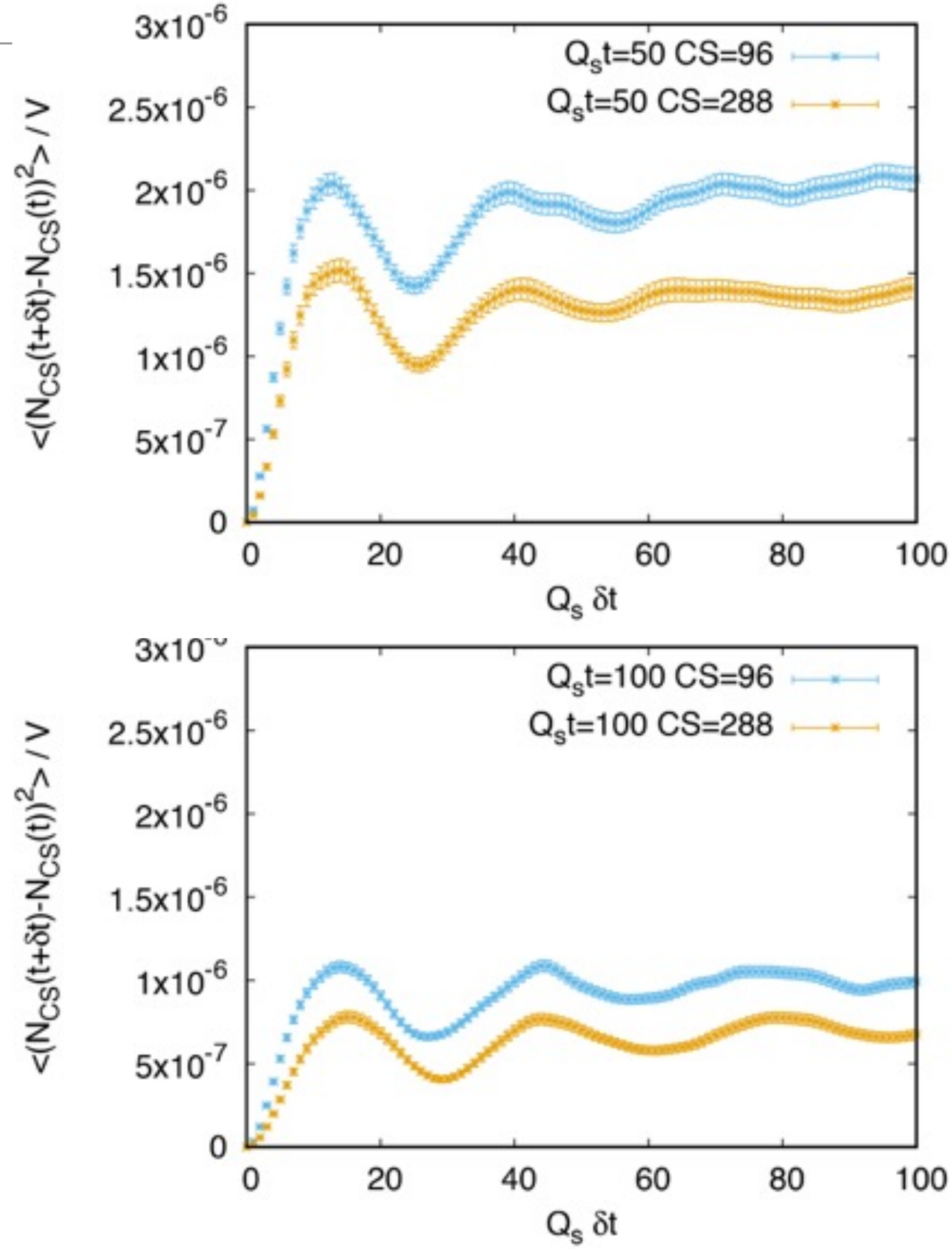
Chern-Simons number correlations

(MM, Schlichting, Venugopalan PRD 074036)

- In equilibrium: random walk
- Out of equilibrium: Non-Markovian
  - Simple probabilistic picture breaks down



Thermal Equilibrium

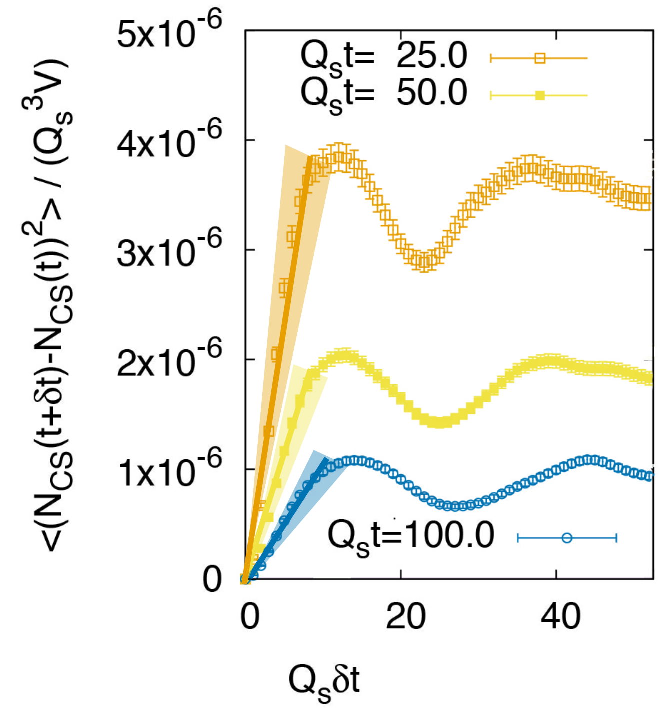


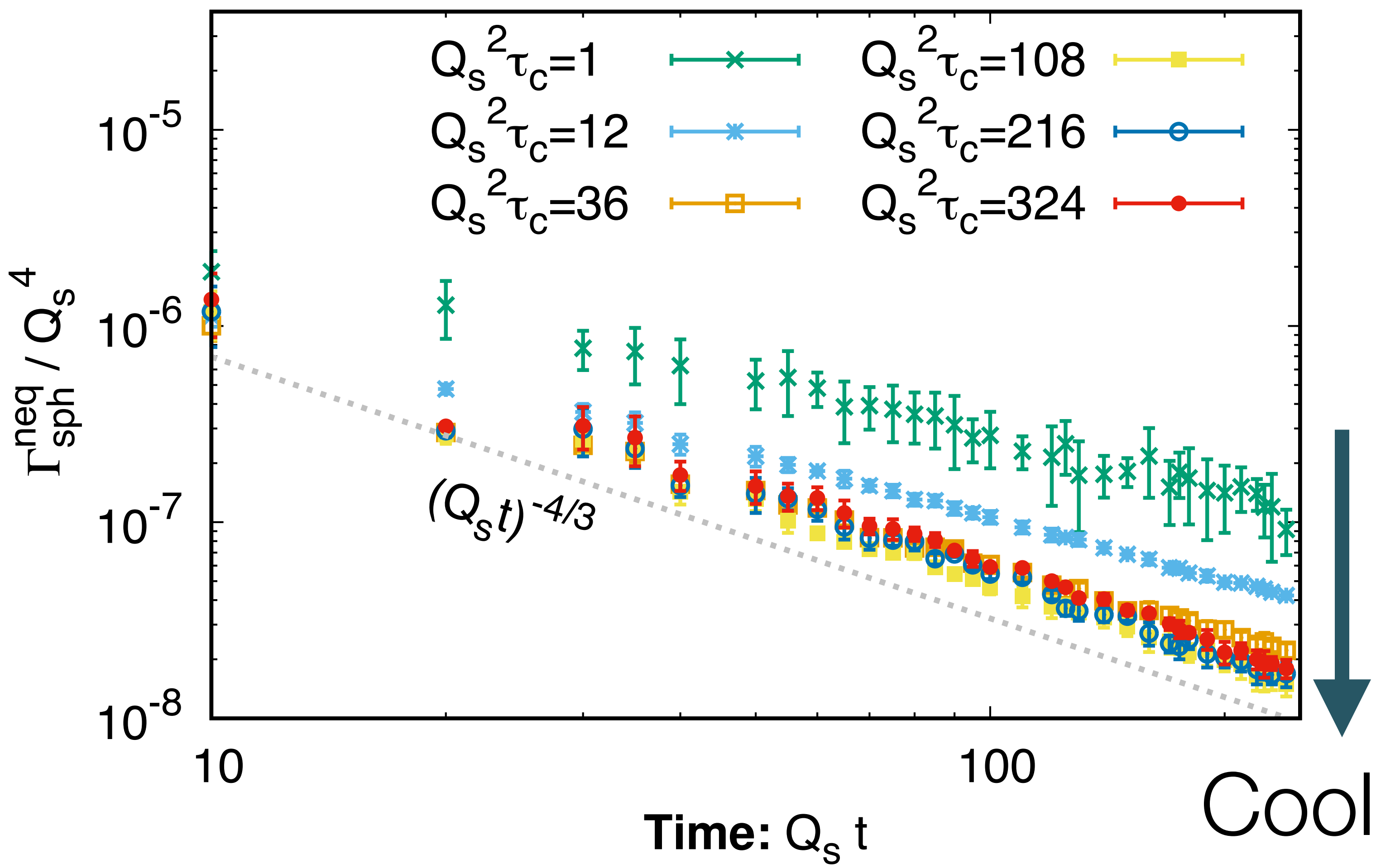
Glasma

Non-equilibrium sphaleron transition rate

- Define non-equilibrium transition rate as initial rise in autocorrelation function

$$\Gamma^{neq}_{sph}(t) = \left\langle \frac{(N_{CS}(t+\delta t) - N_{CS}(t))^2}{V\delta t} \right\rangle_{Q_s\delta t<10}$$





Cooling

- Strongly time dependent
- Contributions from field strength fluctuations

(MM, Schlichting, Venugopalan PRD 074036)

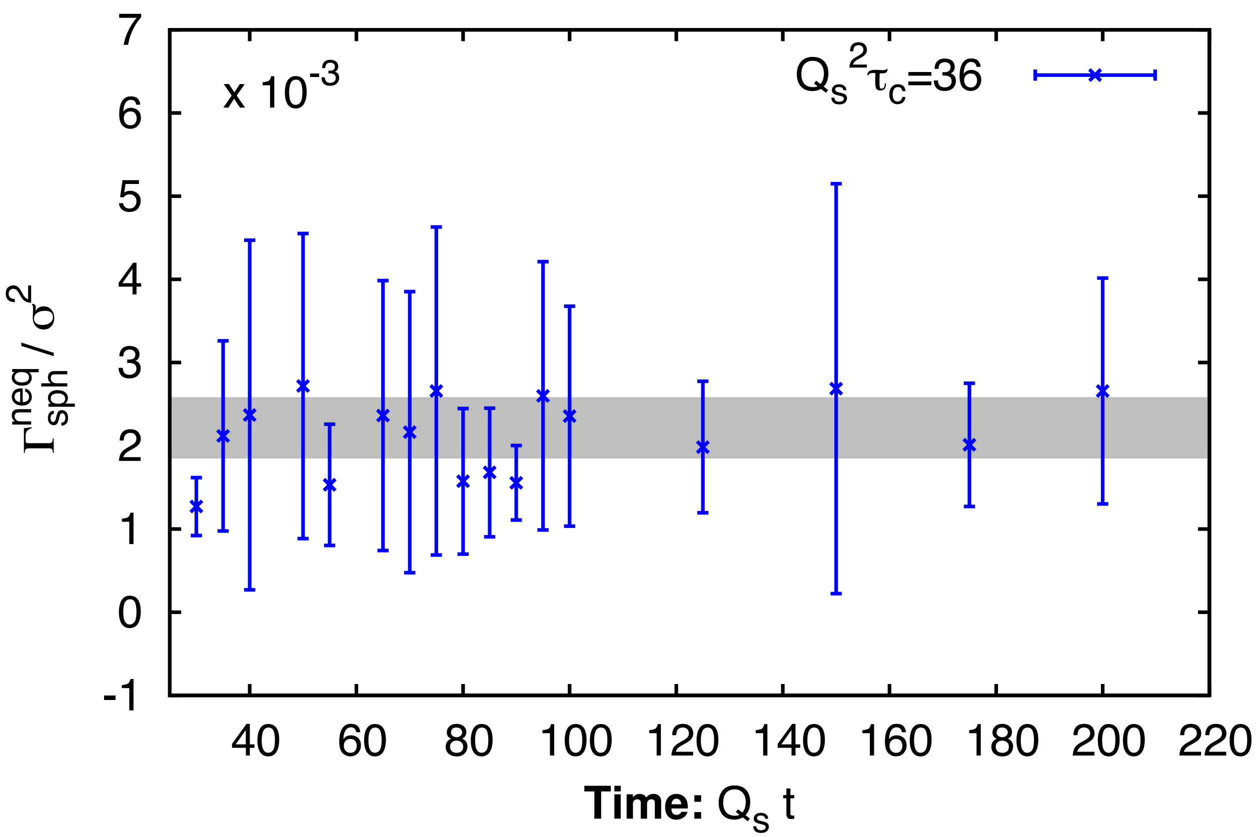
Non-equilibrium sphaleron transition rate

- In equilibrium, sphaleron rate controlled by magnetic modes
- From dynamical separation of scales, non-equilibrium sphaleron rate controlled by modes of order magnetic screening

$$\Gamma_{sph}^{neq}(t) = 2 \times 10^{-2} \sigma^2(t)$$

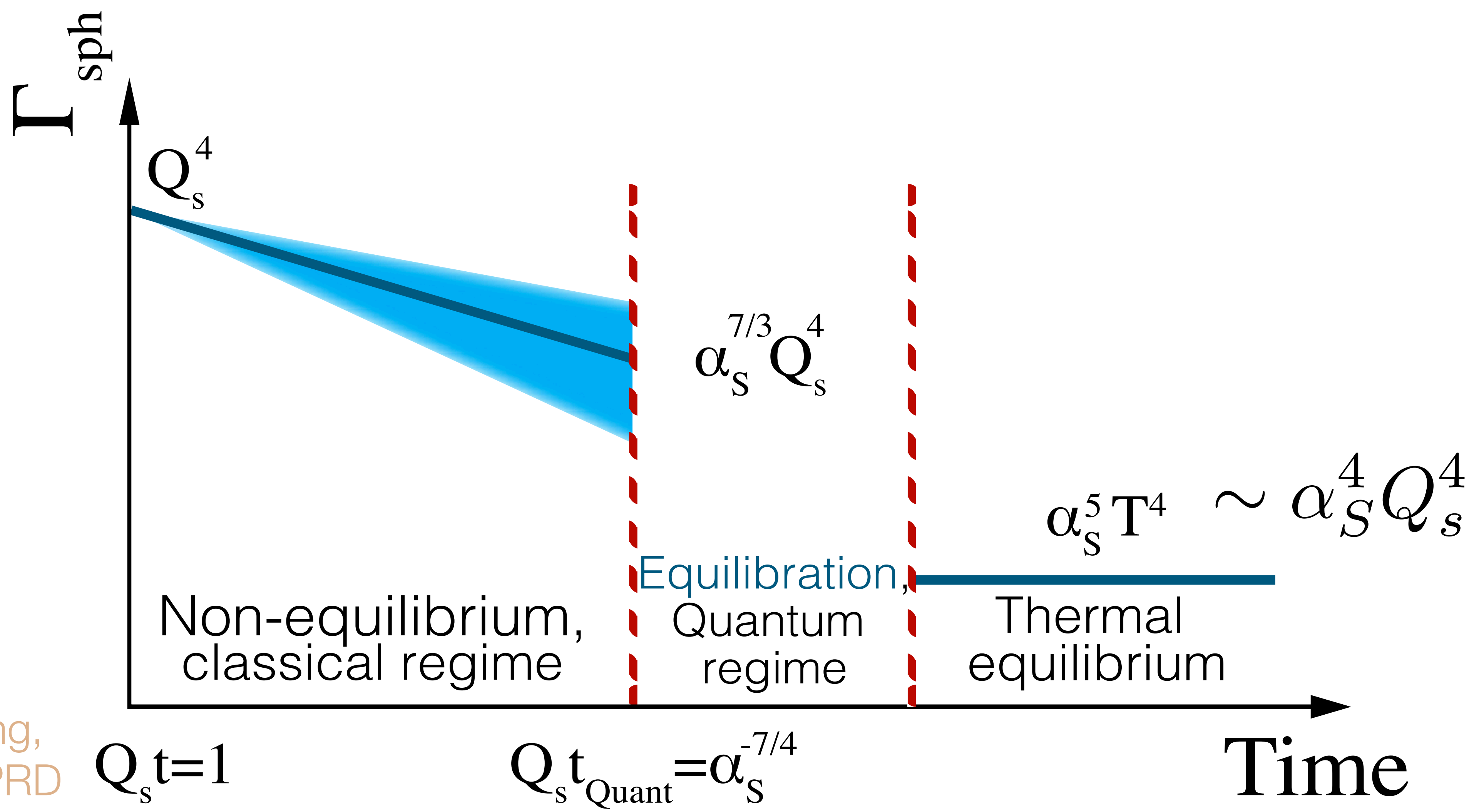
$$\sigma^2(t) \approx Q_s^2 (Q_s t)^{-2/3}$$

(MM, Schlichting, Venugopalan PRD 074036)



Sphalerons in non-Abelian plasmas

- Glasma (early times): $\Gamma^{neq}_{sph} \approx Q_s^4 \rightarrow \Gamma^{neq}_{sph}(t) \approx Q_s^4 (Q_s t)^{-4/3}$
- Equilibrium (late times): $\Gamma^{eq}_{sph} \approx \alpha_S^5 T^4$



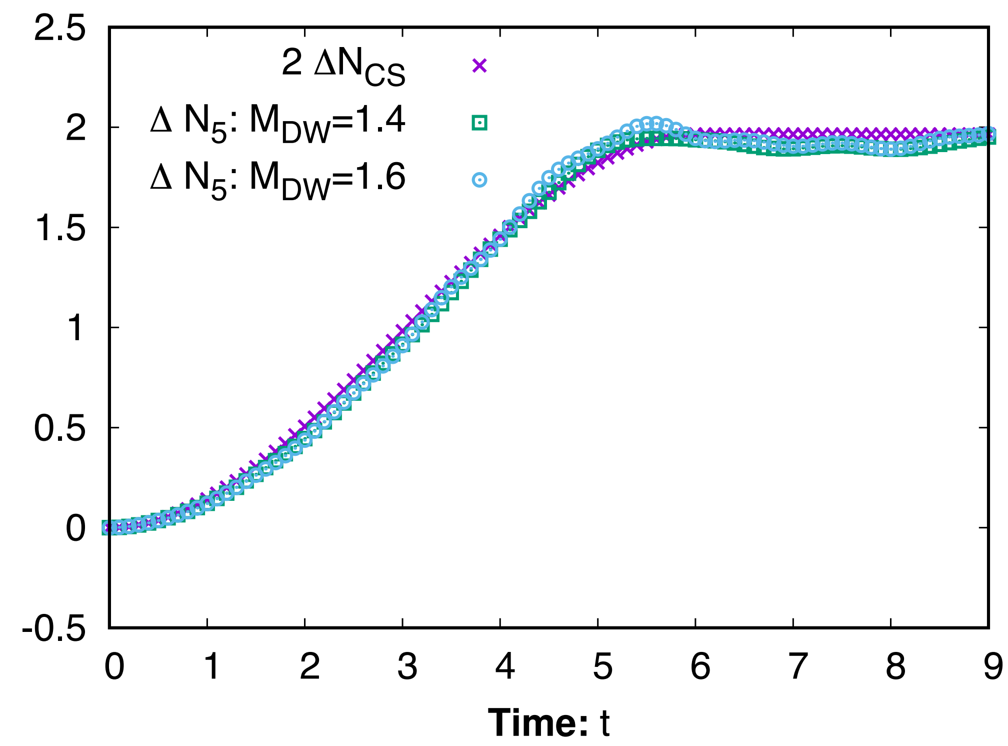
(MM, Schlichting, Venugopalan PRD 074036)

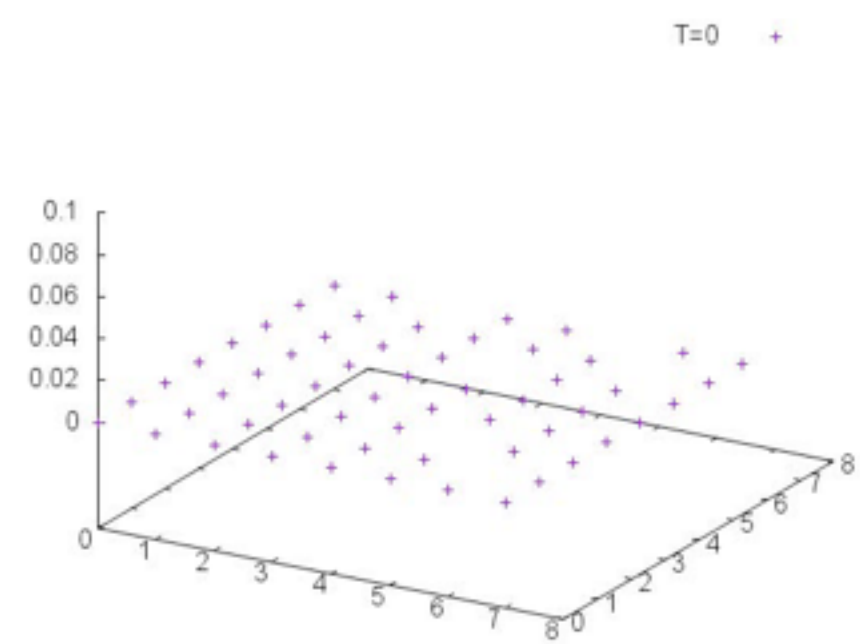
Implications for the CME

- Generalizing for longitudinal expansion in the glasma
  - Initial times appear to be dominant in all respects
    - Greatest magnetic field
    - Dominant amount of axial charge (should be) generated
      - Glasma: $\Gamma_{sph}^{neq} \sim {Q_s}^4$ Equilibrium: $\Gamma_{sph}^{eq} \sim \alpha_S^5 T^4 \sim \alpha_S^4 Q_s^4$
- However, need longitudinal expansion to address definitively and be more quantitative
  - Need to understand topological transitions on expanding lattice
  - Need to understand magnetic screening in anisotropic plasmas

Future outlook

- Add lattice fermions, study real-time generation of axial charge and formation of CME/CMW with same non-equilibrium gauge configurations
  - Overlap fermions with background magnetic field
    - Work started with S. Schlichting, S. Sharma
- Sphaleron inspired initial condition for anomalous hydrodynamics
  - Discussions begun with B. Schenke, D. Kharzeev, Y. Hirono,





Thanks!