Energy Resolution estimation

with non linear calorimeter response

Sven Menke and Paola Giovannini

ATLAS

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

MARIE CURIE ACTIONS

INTRODUCTION

- applying ANY energy correction that is NOT linear in energy, changes the energy resolution
- the way the resolution changes depends on the function used to correct
- this happens simply because of error propagation

- this effect has to be washed out when comparing resolutions after different calibration schemes
- in order to do this a simple tool has been developed:
  - ✓ it works for every scheme, for every cut (et, isolation…) because this approach stems from the actual response plot, we want to derive the resolution from
  - ✓ it can be used via the JetPerformance package (Thanks Kristin!!) or implemented as 5 additional code lines in your performance checks..

ERROR PROPAGATION

General rule for error propagation

{valid for "small" errors,
see [Barlow, ATATISTICS]}

$$\text{if} \quad y = f(x)$$
$$\text{then} \quad \sigma_y = \frac{df(x)}{dx}\Big|_{\bar{x}} \cdot \sigma_x$$

the resolution σ is the error on the energy measurement ➔

LET'S DEFINE ::

$$\begin{array}{lll}
 & E_T & \rightarrow \text{true en} \\
 & E_M & \rightarrow \text{measured energy} \quad E_M = s(E_T) \cdot E_T \\
 & E_C & \rightarrow \text{corrected energy} \quad E_C = w(E_M) \cdot E_M \\
\text{if} & \sigma_M & \rightarrow \text{resolution on the measured energy} \\
\text{then} & \sigma_C & \rightarrow \text{resolution on the corrected energy} \\
\text{is} & \sigma_C & = \frac{d(w(E_M) \cdot E_M)}{dE_M} \cdot \sigma_M \\
 & & = (w(E_M) + E_M \cdot \frac{dw(E_M)}{dE_M}) \cdot \sigma_M
\end{array}$$

DIVIDING BY $E_C = w(E_M) * E_M$

ERROR PROPAGATION

$$\boxed{\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M} \cdot \left(1 + \frac{E_M}{w(E_M)} \cdot \frac{dw(E_M)}{dE_M}\right)}$$

This formula contains the whole relation between energy resolution before and after energy correction are applied… BUT we study the resolution as a function of the Truth Energy

how to derive this is explained in my June talk (committed to the wiki)

ANALYTICAL FORMULA

$$\boxed{\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M}(E_T) \cdot \frac{s(E_T)}{s(E_T) + \frac{ds(E_T)}{dE_T} \cdot E_T}}$$

IMPLEMENTATION

when dealing with histograms a discrete approach can be preferred to consider the local behavior and to avoid fit uncertainties …

if one writes the derivative:: $$\frac{df(x)}{dx} \approx \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

DISCRETE FORMULA

$$\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M}(E_T) \cdot \frac{s(E_T)\cdot \Delta E_T}{s(E_T+\Delta E_T)\cdot(E_T+\Delta E_T) - s(E_T)\cdot E_T}$$

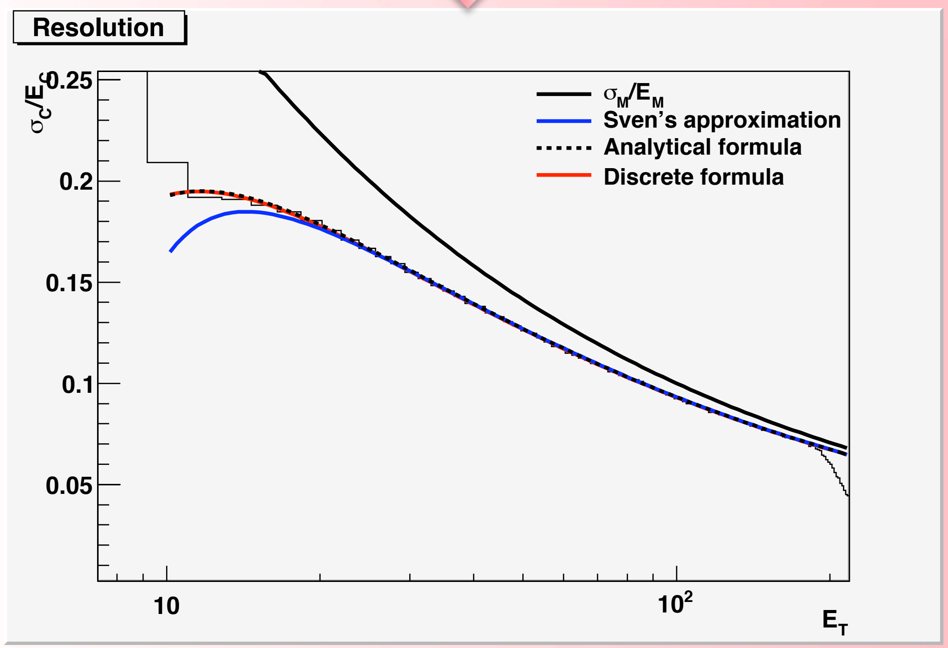
IF ONE ASSUMES THAT ONEσ DISTANCE IS A GOOD MEASURE FOR ΔE_T → $$\Delta E_T = \frac{\sigma_M}{E_M}(E_T)\cdot E_T$$

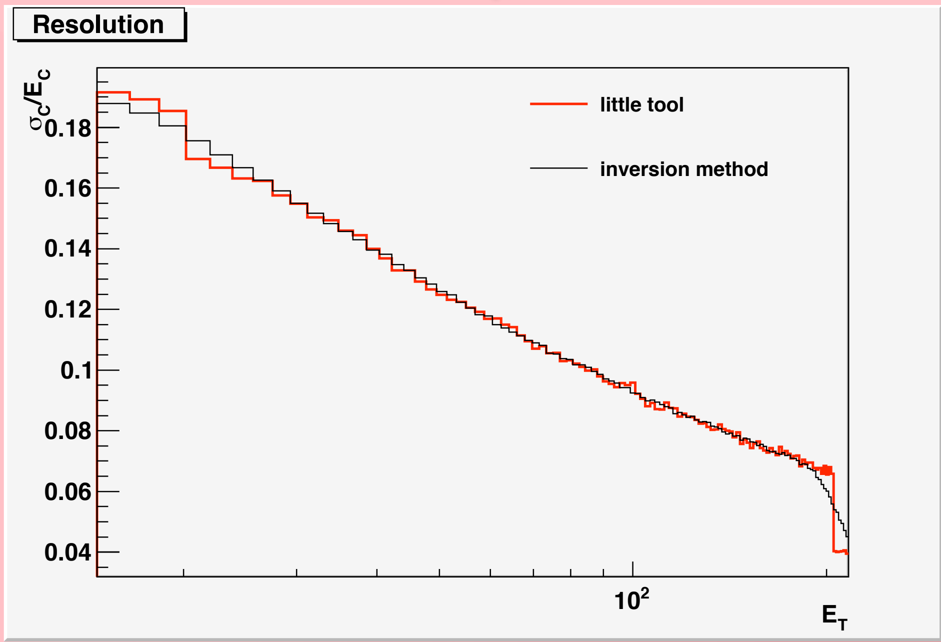
… then the discrete formula can be easily implemented without the analytical knowledge of the function $s(E_T)$…!

IMPLEMENTATION

In the Toy MC, s(E_T) is a known function,
so the formula can be implemented
as a function and plotted on the histogram..
THE FORMULA WORKS!!

resolution with inversion method
and with discrete formula
implementation (LittleTool)
they are compatible!!





CONCLUSIONS

- having a **non linear** response function from our calorimeter puts us into troubles when estimating energy resolution
- any non linear correction to the energy causes a change in resolution that can be understood in terms of **error propagation**
- this effect has to be washed out before comparing resolution obtained with different calibration methods
- error propagation provides a very simple tool to do this
- the tool boils down when the **binning** in energy is wider than the resolution → our calibration run into troubles if this happens!
- it could be a **general** tool to compare resolution from different calibration approach (non isolated jets, different cone matching ecc..) for which inversion method constants are not calculated