Energy Resolution

with non-linear calorimeter response

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OUTLINE

Using a simple Toy MC it was shown that:

- ✓ applying any energy correction, that is not linear in energy, changes resolution;
- ✓ the way resolution changes depends on the function that is applied;
- ✓ this effect has to be washed out, to be able to compare resolution after different calibrations.

http://www.mppmu.mpg.de/~menke/pdf/calononlincorr19122008.pdf

In this talk:

- ✓ why is resolution changing?
- ✓ error propagation provides a general formula to calculate the resolution change due to non-linearity;
- ✓ how this formula works and can be implemented.

Error propagation

for "small" errors
(if the derivative does not change too rapidly over few sigma)
[STATISTICS, Barlow]

$$\text{if} \qquad y = f(x)$$

$$\text{then} \qquad \sigma_y = \frac{df(x)}{dx}\Big|_{\bar{x}} \cdot \sigma_x$$

The energy resolution σ is our estimation of the overall error on energy measurement ➔ the rules of error propagation can be applied to energy resolution as well

LET'S DEFINE

$$\begin{cases} E_T & \rightarrow \text{true en} \\ E_M & \rightarrow \text{measured energy} \quad E_M = s(E_T) \cdot E_T \\ E_C & \rightarrow \text{corrected energy} \quad E_C = w(E_M) \cdot E_M \end{cases}$$

Error propagation

$$\begin{aligned} &\text{if} && \sigma_M && \rightarrow \text{resolution on the measured energy} \\ &\text{then} && \sigma_C && \rightarrow \text{resolution on the corrected energy} \\ &\text{is} && \sigma_C && = \frac{d(w(E_M)\cdot E_M)}{dE_M}\cdot\sigma_M \\ &&&&& = (w(E_M) + E_M \cdot \frac{dw(E_M)}{dE_M})\cdot\sigma_M \end{aligned}$$

$$\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M}\cdot(1 + \frac{E_M}{w(E_M)}\cdot\frac{dw(E_M)}{dE_M})$$

if the response is linear in energy the derivative is 0 and the RELATIVE resolution remains unchanged

$$\begin{aligned} &\text{if} && \Rightarrow E_M = a\cdot E_T \\ &&& \Rightarrow w(E_M) = a \\ &&& \Rightarrow \frac{dw(E_M)}{dE_M} == 0 \\ &\text{then} && \Rightarrow \frac{\sigma_C}{E_C} = \frac{w(E_M)\cdot\sigma_M}{w(E_M)\cdot E_M} = \frac{\sigma_M}{E_M} \end{aligned}$$

Error propagation

$$\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M} \cdot (1 + \frac{E_M}{w(E_M)} \cdot \frac{dw(E_M)}{dE_M})$$

- this formula contains the full relation between the relative resolution before and after non linear corrections
- it is not easy to be used in energy resolution studies because it is not expressed as a function of the truth energy
- this can be done with a little bit of math!!!

Let's consider the derivative of the inverse of a function :: (see back-up for theorem)

$$\text{if} \quad y = f(x) \quad \text{and} \quad x = f^{-1}(y)$$

then

$$\frac{df^{-1}(y)}{dy} = \frac{1}{\frac{df(x)}{dx}}$$

Error propagation

$$E_T \quad \rightarrow \text{true en}$$

$$E_M \quad \rightarrow \text{measured en} \quad E_M = s(E_T) \cdot E_T == f(E_T)$$

$$E_C \quad \rightarrow \text{corrected en} \quad E_C = w(E_M) \cdot E_M == f^{-1}(E_M)$$

$$\frac{\sigma_C}{E_C} = \frac{1}{f^{-1}(E_M)} \cdot \frac{df^{-1}(E_M)}{dE_M} \cdot \sigma_M$$

$$= \frac{1}{f^{-1}(E_M)} \cdot \frac{df^{-1}(E_M)}{dE_M} \cdot \frac{\sigma_M}{E_M} \cdot E_M$$

$$\frac{1}{E_T} \qquad \frac{1}{\frac{df(E_T)}{dE_T}} \qquad \frac{\sigma_M}{E_M}(E_T) \qquad f(E_T)$$

Error propagation

$$\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M}(E_T) \cdot \frac{f(E_T)}{E_T} \cdot \frac{1}{\frac{df(E_T)}{dE_T}}$$

DEFINITION OF RELATIVE RESOLUTION

in terms of the response function $s(E_T)$

ANALYTICAL FORMULA

$$\frac{df(E_T)}{dE_T} = s(E_T) + \frac{ds(E_T)}{dE_T} \cdot E_T$$

$$\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M}(E_T) \cdot \frac{s(E_T)}{s(E_T) + \frac{ds(E_T)}{dE_T} \cdot E_T}$$

Error propagation

$$\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M}(E_T) \cdot \frac{f(E_T)}{E_T} \cdot \frac{1}{\frac{df(E_T)}{dE_T}}$$

DISCRETE FORMULA

$$\frac{\sigma_C}{E_C} = \frac{\sigma_M}{E_M}(E_T) \cdot \frac{s(E_T) \cdot \Delta E_T}{s(E_T + \Delta E_T) \cdot (E_T + \Delta E_T) - s(E_T) \cdot E_T}$$

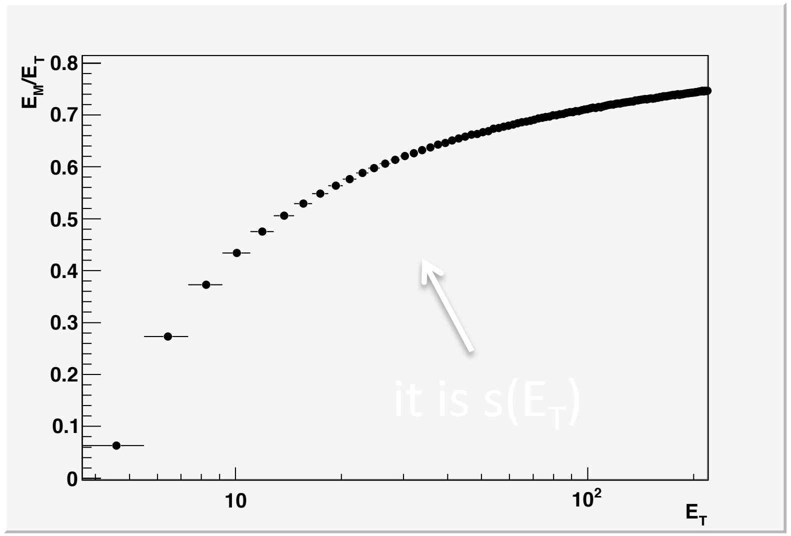
IF ONE ASSUMES
THAT ONEσDISTANCE IS A
GOOD MEASURE FOR ΔE_T

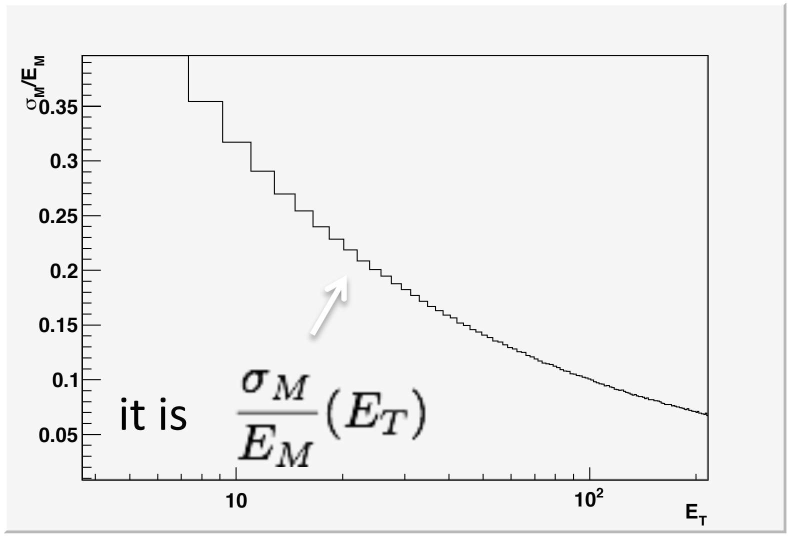
→

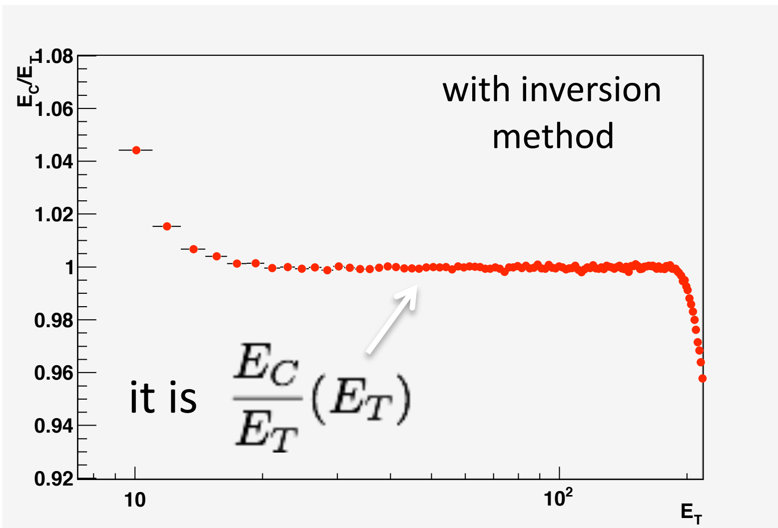
$$\Delta E_T = \frac{\sigma_M}{E_M}(E_T) \cdot E_T$$

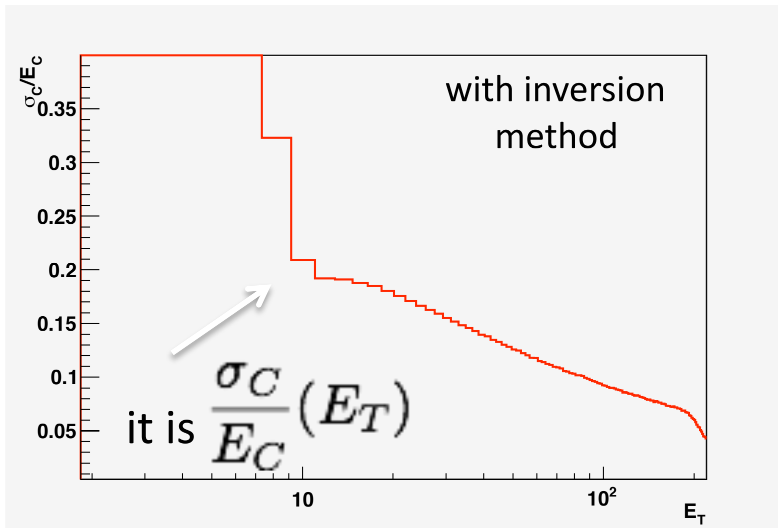
… then the discrete formula can be easily implemented without the analytical knowledge of the function $s(E_T)$…!

Toy Monte Carlo Example



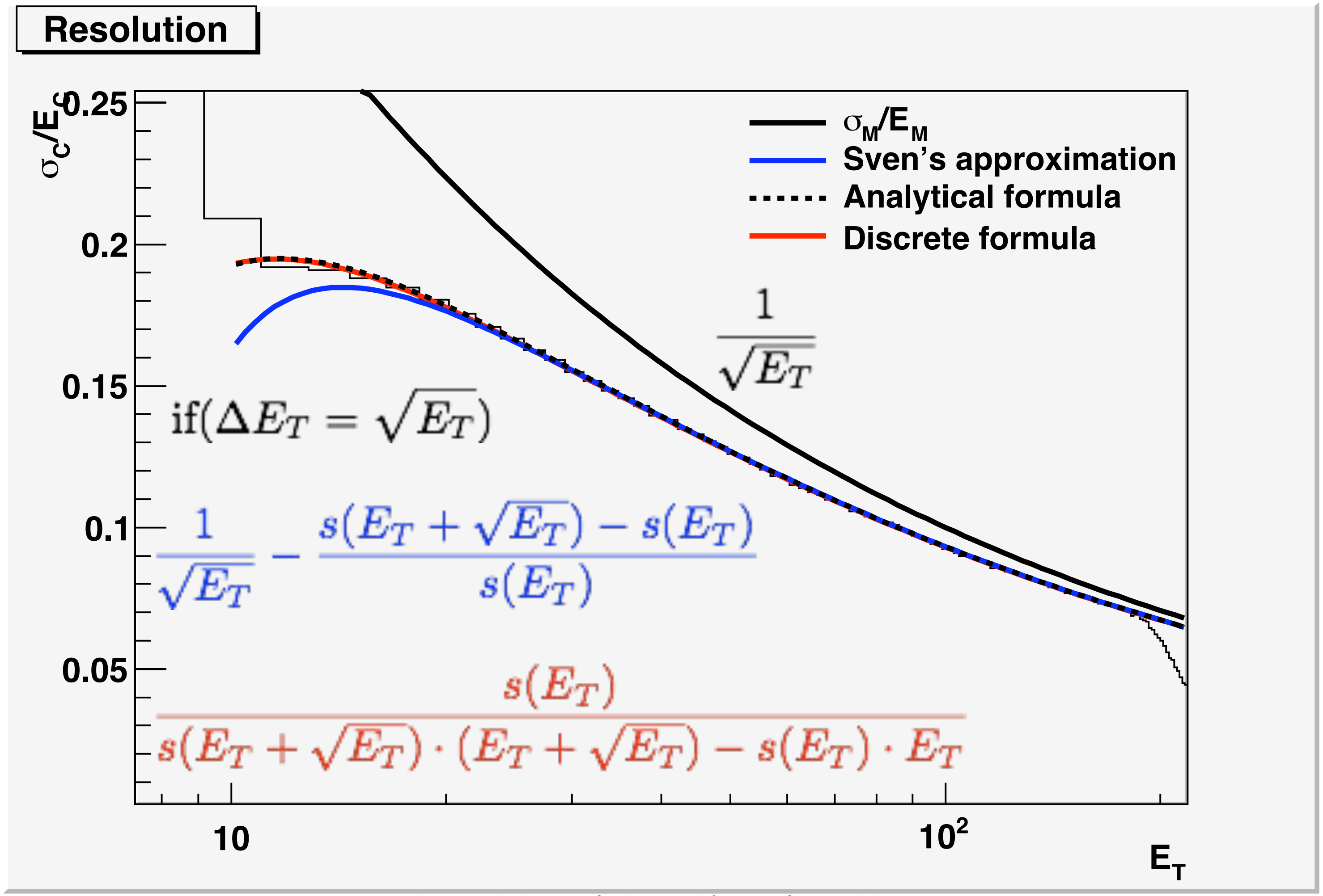






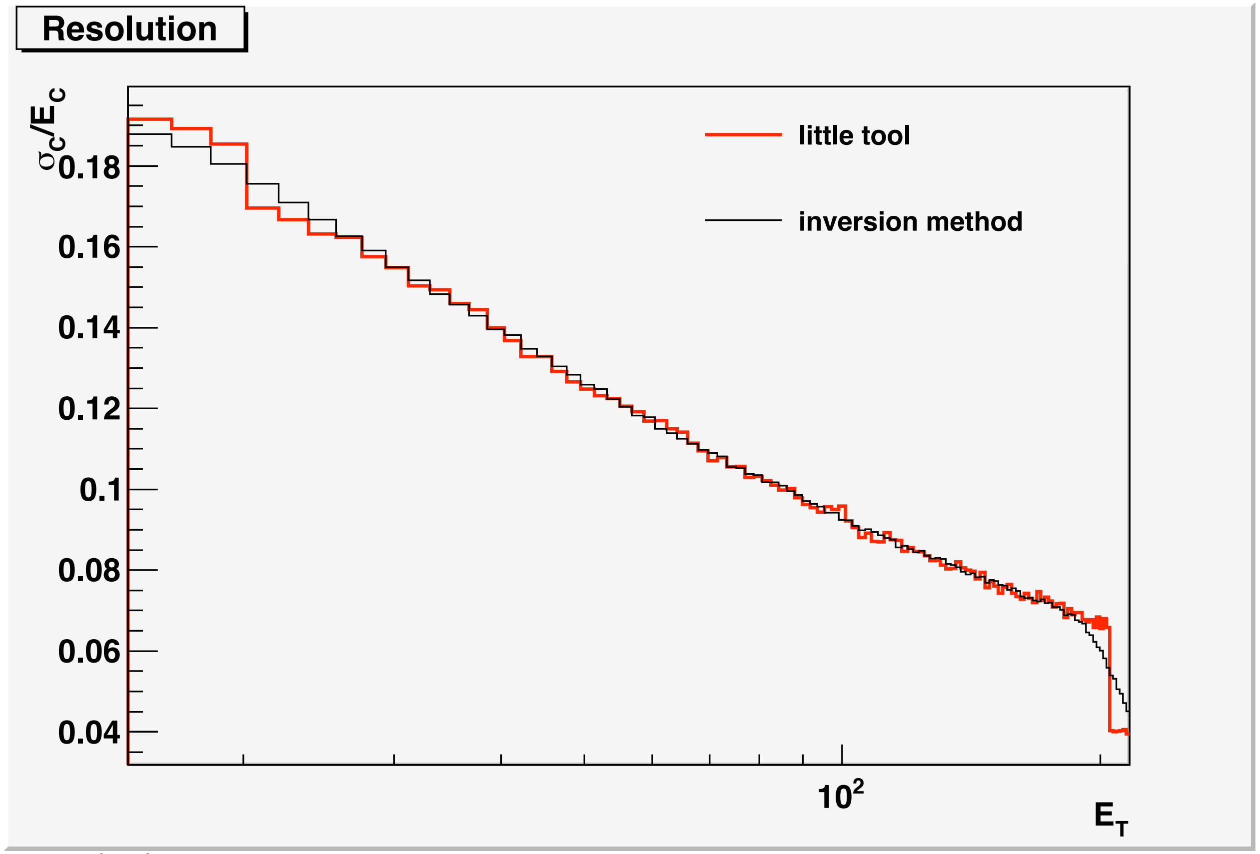
Toy Monte Carlo Example

In the Toy MC, $s(E_T)$ is a known function, so the formula can be implemented as a function and plotted on the histogram.. IT WORKS!!



Toy Monte Carlo Example

In "real" Monte Carlo, it is better to avoid the use of a fitting function, to be able to take into account the local behavior. If the linearity histogram is used as a function, results seem to be still valid::



TH1F * LittleTool (TProfile* liny,TH1F* mres) given a linearity histogram returns the corresponding resolution as if the response had been made linear.

ROOT macro which location in CVS?

Conclusions

- having a non linear response function from our calorimeter puts us into troubles when estimating energy resolution
- any non linear correction to the energy causes a change in resolution that can be understood in terms of error propagation
- this effect has to be washed out before comparing resolution obtained with different calibration methods
- it was shown that it is not needed to make the response linear because the resolution change can be calculated from the linearity spectrum
- a little tool has been written to implement this idea
- it has to be applied to "real" Monte Carlo data, in order to validate it and to get further insights into the "real" physics implications

BACK-UP SLIDES

Derivative of the inverse

Consider a function f, defined in a interval I. f being continuous and strictly monotonic, thus invertible. Consider a point c, belonging to the interval I. If f is derivable in c, with derivative f'(c)≠0, then the inverse function is derivable in the point d=f(c) and :

$$D(f^{-1}(f(c))) = \frac{1}{Df(c)}$$

or, as a function of d

$$D(f^{-1}(d)) = \frac{1}{Df(f^{-1}(d))}$$

[http://www.batmath.it/index.asp]