

# Determining the Local Dark Matter Density

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Based on:

Silverwood et al., MNRAS 469, 2016,  
arXiv:1507.08581

Sivertsson et al., in preparation

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GRavitation AstroParticle Physics Amsterdam



APS Paris, 2016.



# Why do we care about local DM density?

Direct Detection (e.g. XenonIT, LUX...)

$$\frac{dR}{dE} = \frac{\rho_{\odot}}{m_{\text{DM}} m_{\mathcal{N}}} \int_{v > v_{\text{min}}} d^3v \frac{d\sigma}{dE}(E, v) v f(\vec{v}(t))$$

Indirect Detection through Solar Capture and annihilation to neutrinos (IceCube, Antares, KM3NeT)

$$C^{\odot} \approx 1.3 \times 10^{21} \text{ s}^{-1} \left( \frac{\rho_{\text{local}}}{0.3 \text{ GeV cm}^{-3}} \right) \left( \frac{270 \text{ km s}^{-1}}{v_{\text{local}}} \right) \times \left( \frac{100 \text{ GeV}}{m_{\chi}} \right) \sum_i \left( \frac{A_i (\sigma_{\chi i, SD} + \sigma_{\chi i, SI}) S(m_{\chi}/m_i)}{10^{-6} \text{ pb}} \right)$$

Scans of theoretical parameter space, eg Supersymmetry



# How do we measure local DM density?

- **Global measurements (rotation curves):**

powerful, but have to assume global properties of the halo.

e.g. Dehnen & Binney 1998; Weber & de Boer 2010; Catena & Ullio 2010; Salucci et al. 2010; McMillan 2011; Nesti & Salucci 2013; Piffl et al. 2014; Pato & Iocco 2015; Pato et al. 2015

- **Local measurements:**

larger uncertainties but fewer assumptions

e.g. Jeans 1922; Oort 1932; Bahcall 1984; Kuijken & Gilmore 1989b, 1991; Creze et al. 1998; Garbari et al. 2012; Bovy & Tremaine 2012; Smith et al. 2012; Zhang et al. 2013; Bienaymé et al. 2014



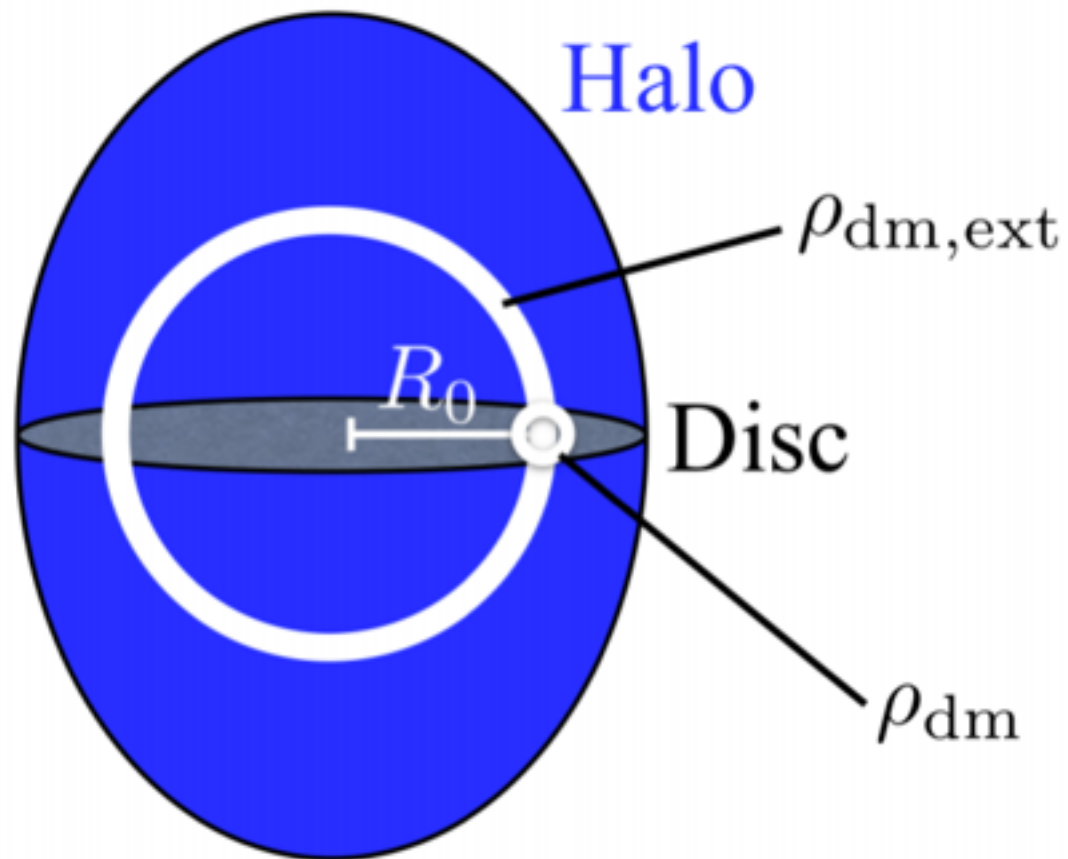
# Combination of Local and Global Measurements

Justin Read,  
*The Local Dark Matter Density*, 2014.  
J. Phys. G: Nucl. Part. Phys. 41 063101.  
arXiv: 1404.1938

Local

Global

a)  $\rho_{\text{dm}} < \rho_{\text{dm,ext}}$

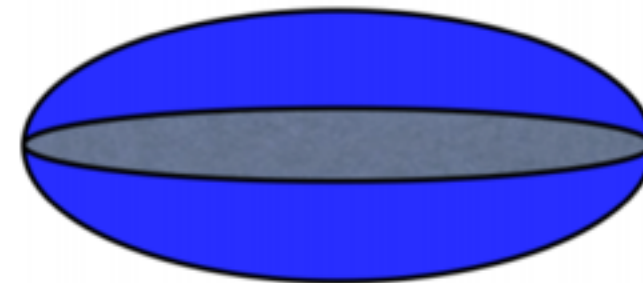


Prolate Halo

Local

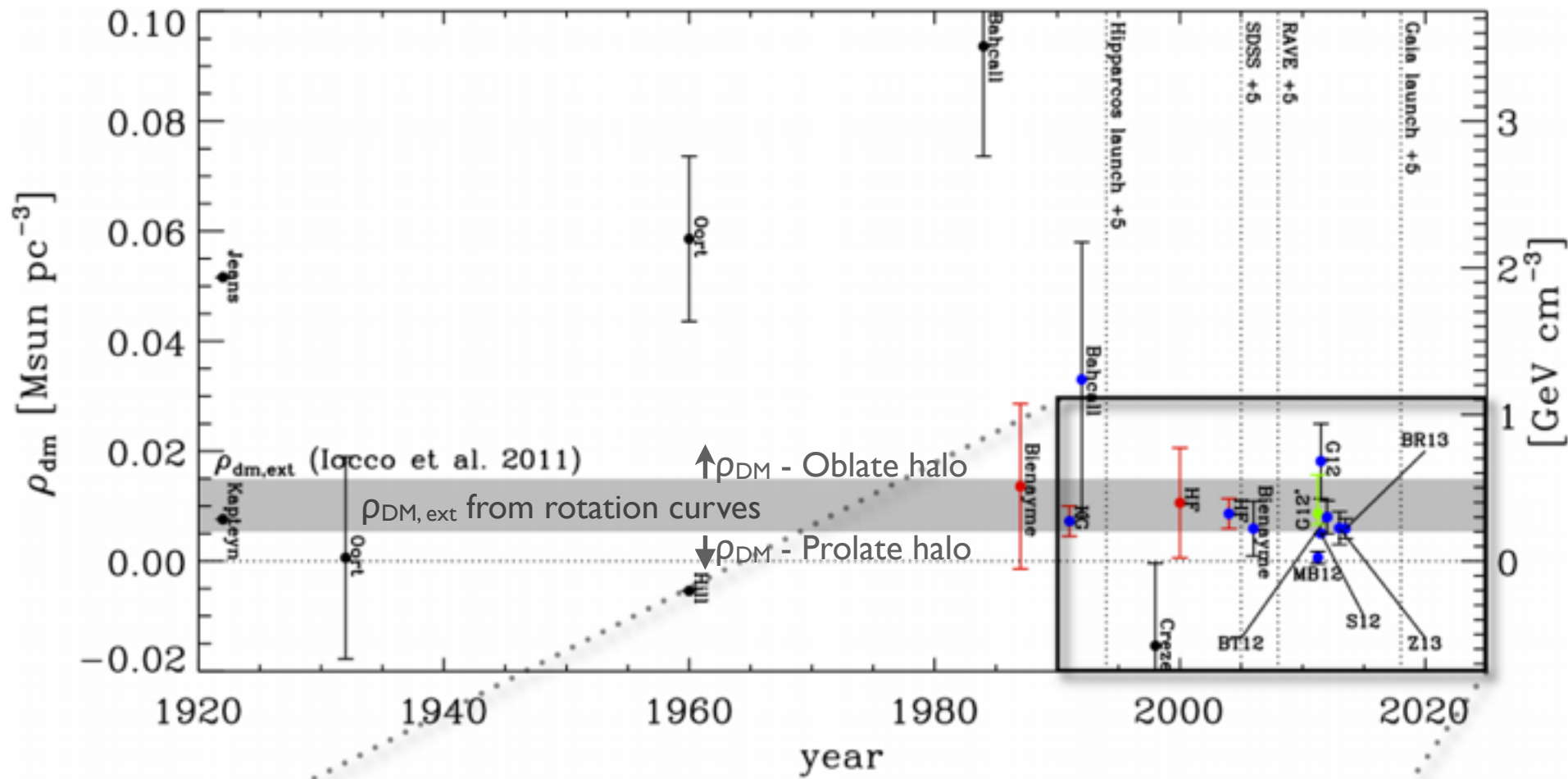
Global

b)  $\rho_{\text{dm}} > \rho_{\text{dm,ext}}$



Oblate Halo



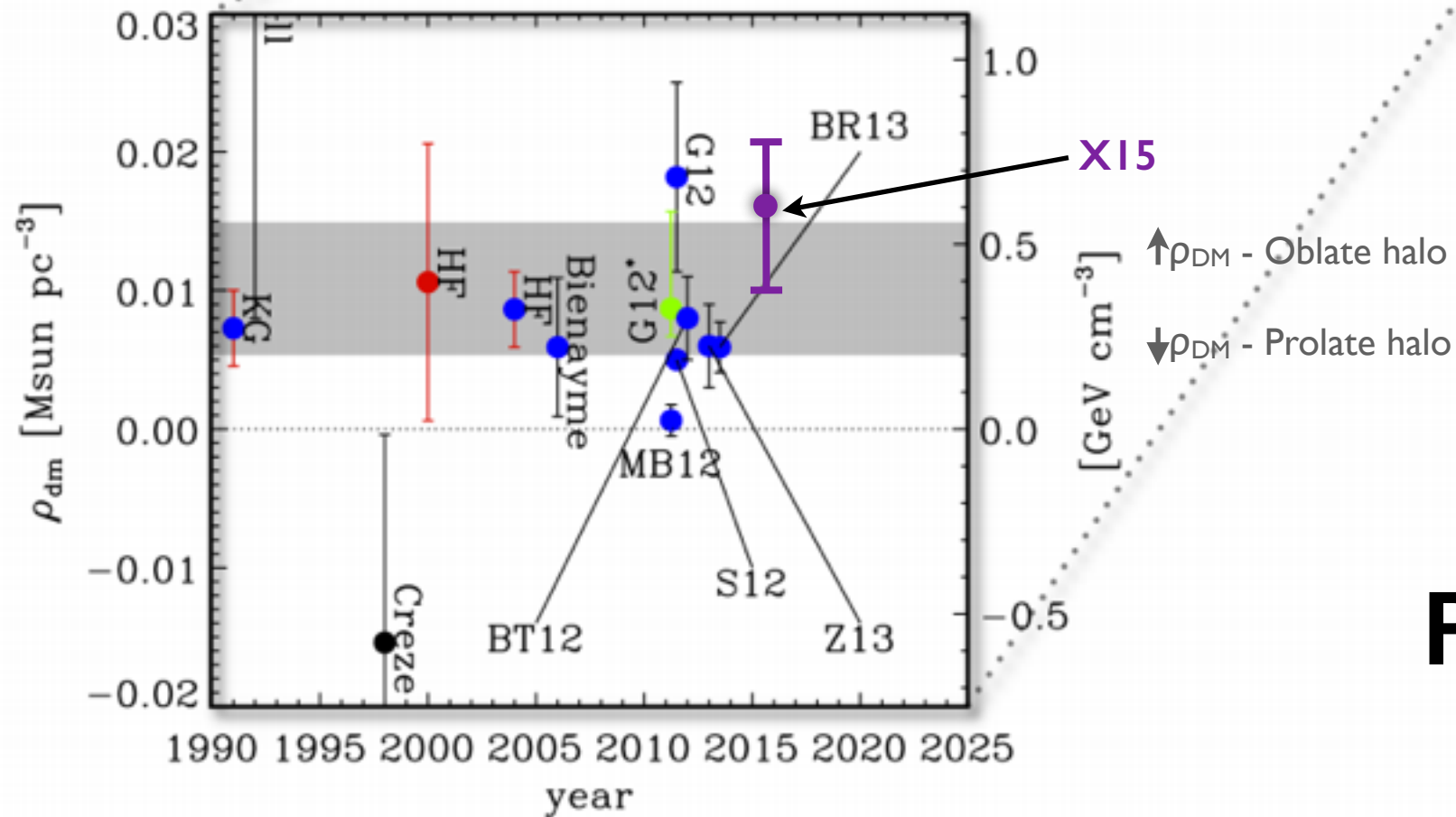


**S12** - Smith et al., SDSS  
**Z13** - Zhang et al., SDSS  
**BR13** - Bovy & Rix, SDSS

**MB12** - Moni Bidin et al., 412 red  
 giants towards South Galactic Pole  
**BT12** - Bovy & Tremaine,  
 reanalysis of MB12 data set

**G12** - Garbari et al., ~2000 K-  
 dwarfs from Kuijken & Gilmore  
 1989

**X15** - Xia et al., released last  
 week, 1427 G & K type MS stars  
 from LAMOST survey



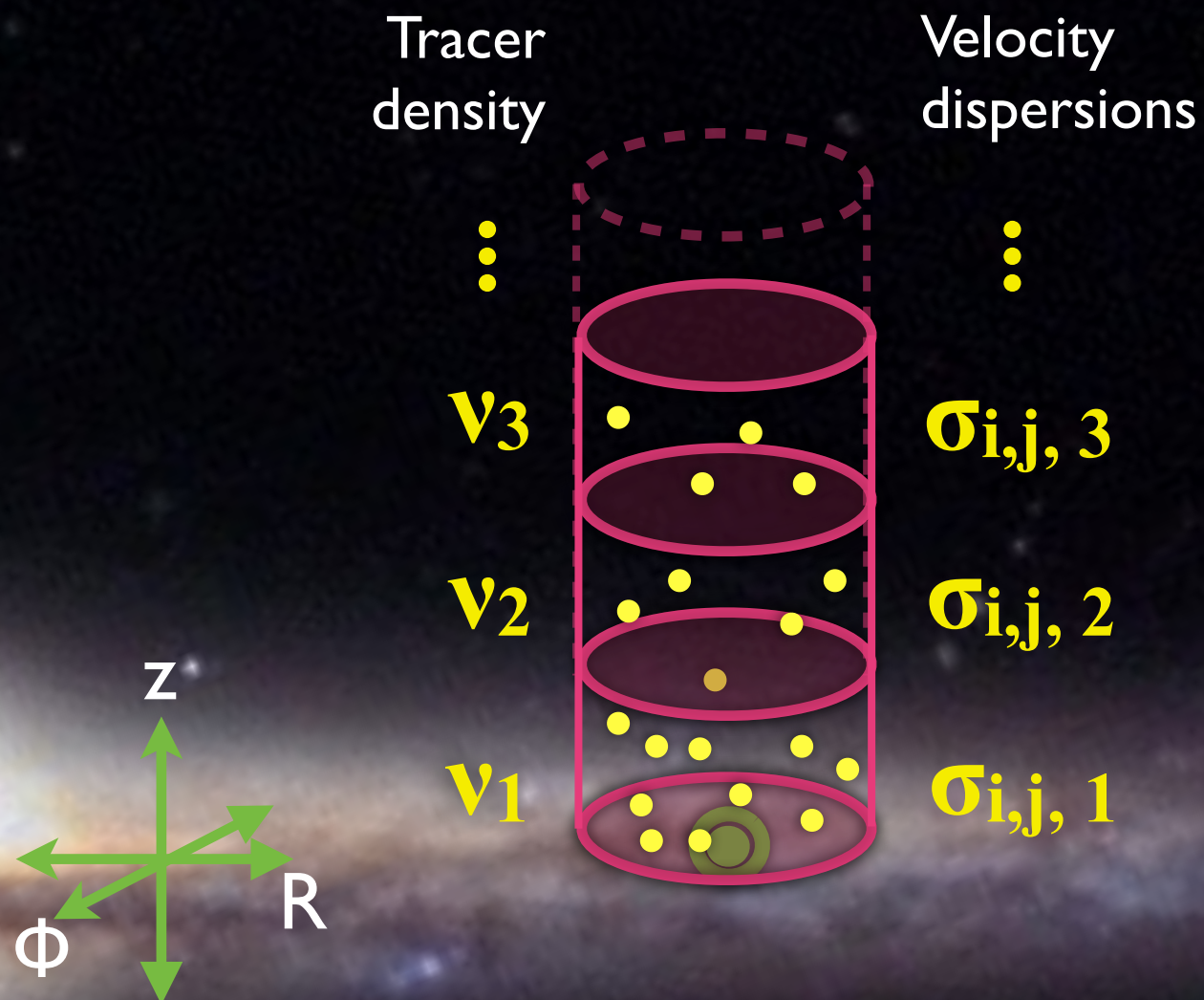
# Previous Local DM Measurements



# Our Method - Basics

- Local measurements in z-direction and R-direction
- Data points are **positions** and **velocities** for a set of tracer stars in a cylindrical volume.
- data is binned to get **tracer density** and **velocity dispersions**

$$\sigma_{ij}^2(\mathbf{x}) = \overline{v_i v_j} - \bar{v}_i \bar{v}_j$$





# Our Method - Integrated Jeans Equations

- We need to link positions and velocities to the mass distribution
- Tracer stars follow the Collisionless Boltzman Equation:

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \nabla_x f \cdot \mathbf{v} - \nabla_v f \cdot \nabla_x \Phi = 0$$

- $f(\mathbf{x}, \mathbf{v})$  - stellar distribution function, positions  $\mathbf{x}$ , velocities  $\mathbf{v}$ , gravitational potential  $\Phi$
- Integrate over velocities, switch to spherical-polar co-ordinates, and get the **Jeans Equation in  $z$** .

$$\underbrace{\frac{1}{R\nu} \frac{\partial}{\partial R} (R\nu\sigma_{Rz})}_{\text{'tilt' term: } \mathcal{T}} + \underbrace{\frac{1}{R\nu} \frac{\partial}{\partial \phi} (\nu\sigma_{\phi z})}_{\text{'axial' term: } \mathcal{A}} + \frac{1}{\nu} \frac{d}{dz} (\nu\sigma_z^2) = \underbrace{-\frac{d\Phi}{dz}}_{K_z}$$

Surface Density  $\Sigma_z(z) = \frac{|K_z|}{2\pi G}$



$$\underbrace{\frac{1}{R\nu} \frac{\partial}{\partial R} (R\nu\sigma_{Rz})}_{\text{'tilt' term: } \mathcal{T}} + \underbrace{\frac{1}{R\nu} \frac{\partial}{\partial \phi} (\nu\sigma_{\phi z})}_{\text{'axial' term: } \mathcal{A}} + \frac{1}{\nu} \frac{d}{dz} (\nu\sigma_z^2) = \underbrace{-\frac{d\Phi}{dz}}_{K_z}$$

↓  
**Integrate to avoid noise**  
↓

$$\sigma_z^2(z) = \frac{1}{\nu(z)} \int_0^z \nu(z') [K_z(z') - \mathcal{T}(z') - \mathcal{A}(z')] dz' + \frac{C}{\nu(z)}$$

= 0 from axisymmetry

Construct model for

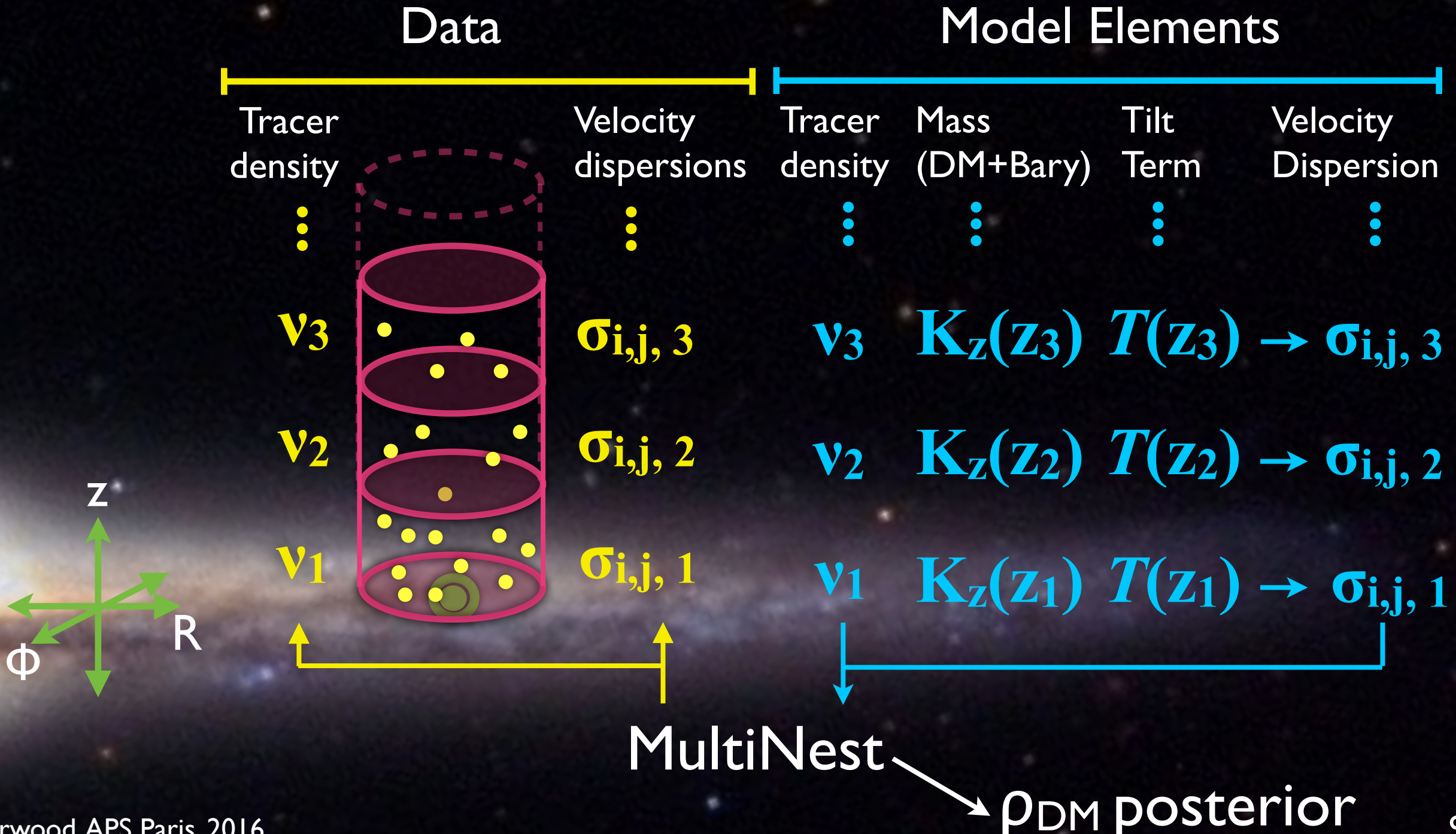
- tracer density  $\nu$ ,
- Dark Matter + Baryon density  $\rightarrow K_z$ ,
- tilt term  $\mathcal{T}(z)$ .

Calculate velocity dispersion  $\sigma_z$ , then fit the model to velocity dispersion, tracer density & tilt term to data. Use **MultiNest** to derive posterior distribution on DM.



# Our Method - Modeling and MultiNest

- Construct models for the tracer density, baryon+DM mass, tilt term
- Calculate z velocity dispersion
- Fit tracer density and z-velocity dispersion to data with MultiNest





Modeling the Components:

# Mass profile - $K_z$ term

$$K_z = -\frac{d\Phi}{dz}$$

- We assume constant DM density going up in  $z$
- Simplified two-parameter baryon profile for mock data testing.
- Poisson Equation in Cylindrical Coordinates picks up a Rotation Curve term

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial z^2} + \underbrace{\frac{1}{R} \frac{\partial V_c^2(R)}{\partial R}}_{\text{'rotation curve' term: } \mathcal{R}} = 4\pi G\rho$$

- Flat rotation curve makes rotation curve term disappear.
- Rotation curve term becomes a shift in the density.

$$\frac{\partial^2\Phi}{\partial z^2} = 4\pi G\rho(z)_{\text{eff}} \quad \rho(z)_{\text{eff}} = \rho(z) - \frac{1}{4\pi GR} \frac{\partial V_c^2(R)}{\partial R}$$

- We assume a locally flat RC, but from Oort constants we can estimate the systematic uncertainty from this to be on the order of  $0.1 \text{ GeV/cm}^3$ .



# Tilt Term

$$\underbrace{\frac{1}{R\nu} \frac{\partial}{\partial R} (R\nu\sigma_{Rz}^2)}_{\text{'tilt' term: } \mathcal{T}}$$

$$\mathcal{T}(R_{\odot}, z) =$$

- Tilt term links vertical and radial motion of a set of stars.
- Tilt becomes larger and thus more important at higher  $z$ .
- Require information about the radial variation of  $\sigma_{Rz}^2$  which we currently do not have.
- Thus we assume it has the same dependence as the tracer density  $\nu$
- Traditionally (e.g. Binney & Tremaine) tracer density  $\nu$  is an exponential falling with radius, eg:

$$\nu(R, z) = \nu(z)|_{R_{\odot}} \exp\left(-\frac{R - R_{\odot}}{R_0}\right),$$

$$\Rightarrow \sigma_{Rz}^2(R, z) = \sigma_{Rz}^2(z)|_{R_{\odot}} \exp\left(-\frac{R - R_{\odot}}{R_1}\right)$$

$$R_0 = R_1$$

- Model  $\sigma_{Rz}^2$  as a power law:

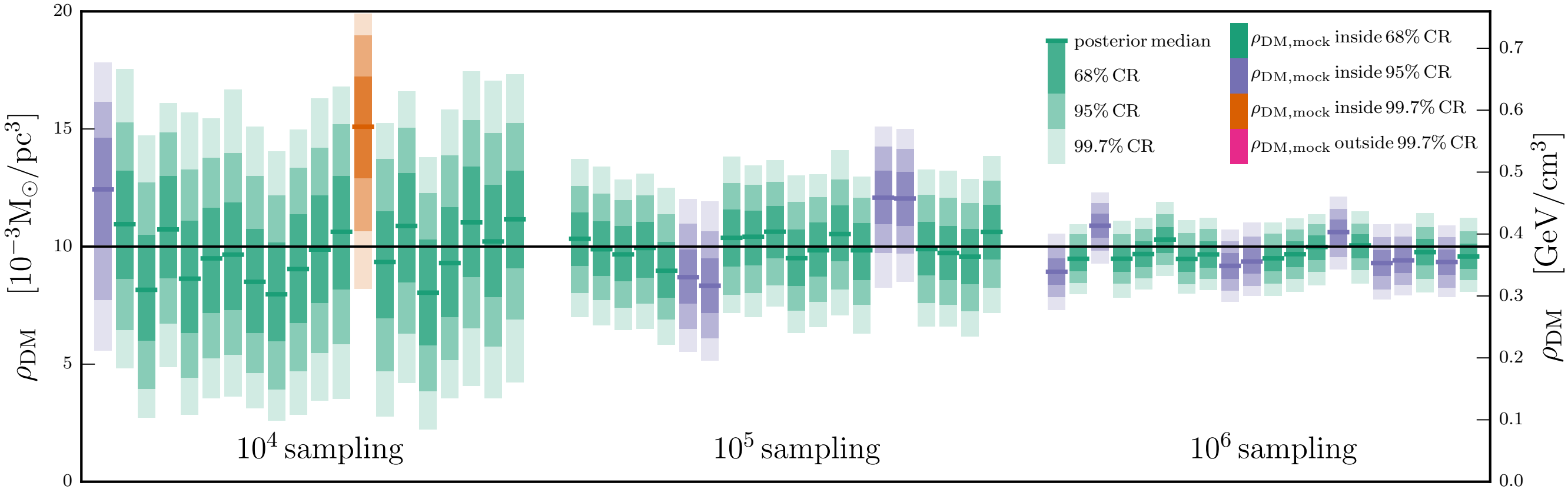
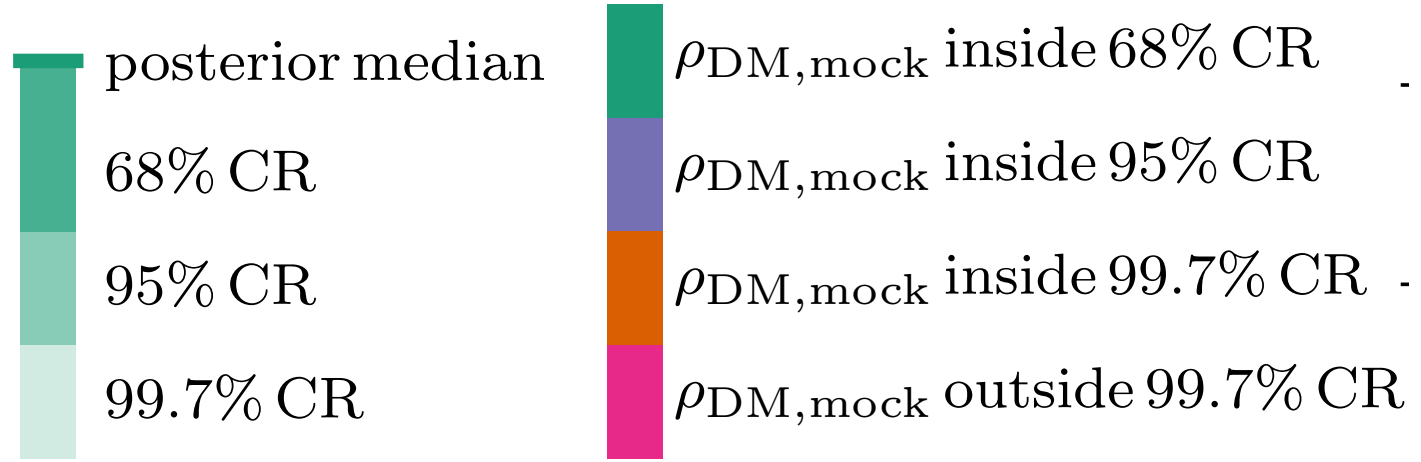
$$\sigma_{Rz}^2(z)|_R = A \left(\frac{z}{\text{kpc}}\right)^n \Big|_R$$

Silverwood et al.  
arXiv:1507.08581

$$\Rightarrow \mathcal{T}(R_{\odot}, z) = A \left(\frac{z}{\text{kpc}}\right)^n \Big|_{R_{\odot}} \left[ \frac{1}{R_{\odot}} - \frac{2}{R_0} \right]$$

# Testing with 20 Simple Mock Data Sets

Sampling: More data points (stars) = better result.



$10^4$  tracer stars

$10^5$  tracer stars

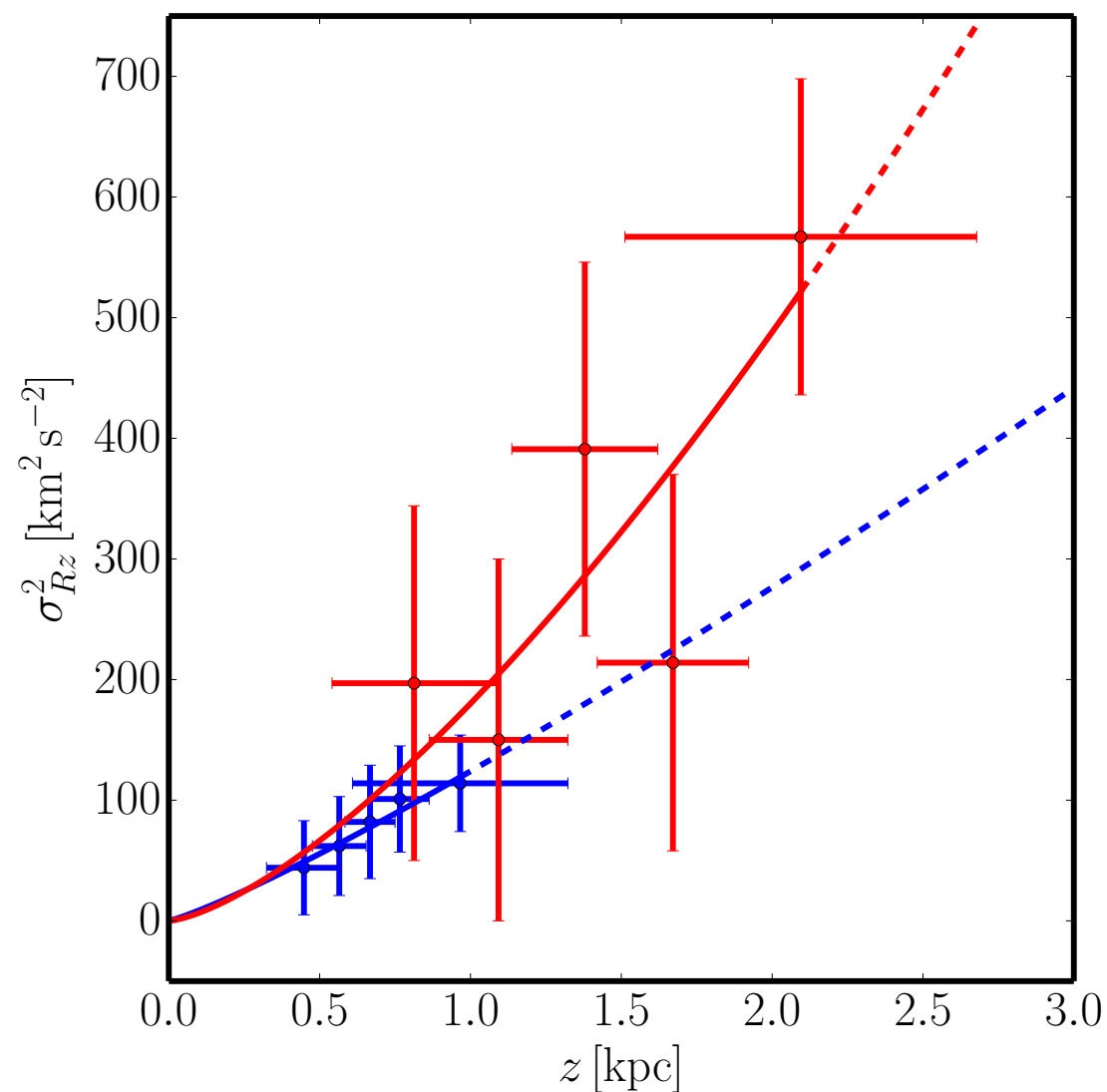
$10^6$  tracer stars



# Testing with 20 Simple Mock Data Sets

## The Importance of the Tilt Term

We generate our tilt mock data by fitting our tilt model to  $\sigma_{Rz}^2$  data from Budenbender et al. 2014.



$$\sigma_{Rz}^2(z)|_R = A \left( \frac{z}{\text{kpc}} \right)^n \Big|_R \quad \text{Positive}$$

$$\mathcal{T}(R_\odot, z) = A \left( \frac{z}{\text{kpc}} \right)^n \Big|_{R_\odot} \left[ \frac{1}{R_\odot} - \frac{2}{R_0} \right]$$

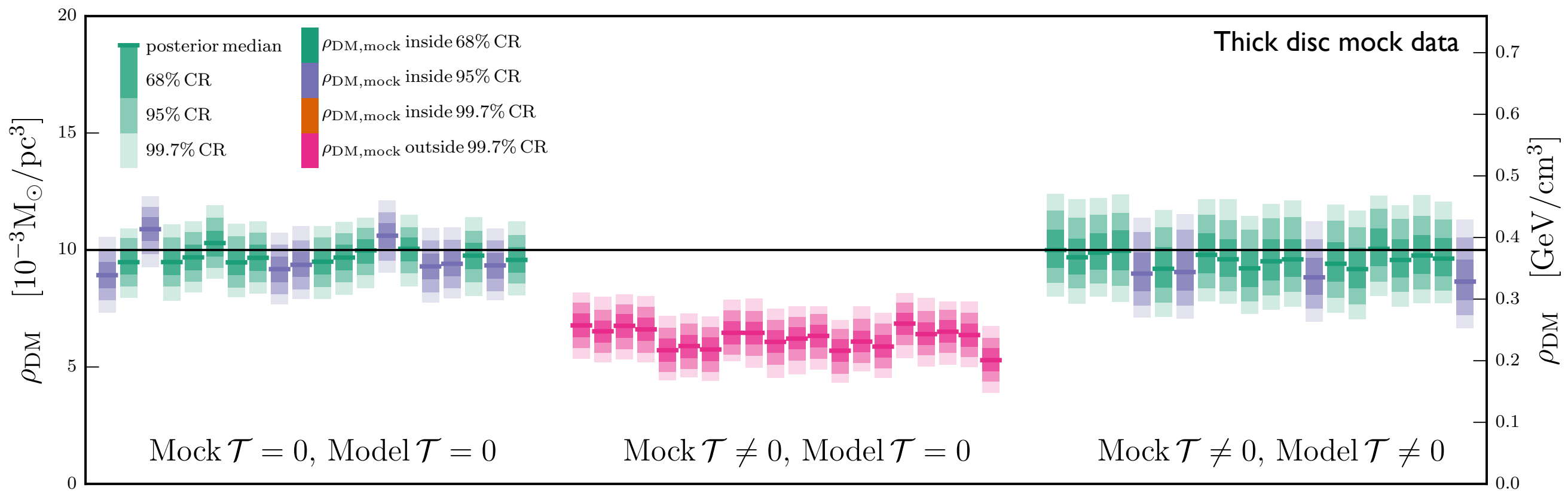
Negative

Positive

Negative

# Testing with 20 Simple Mock Data Sets

## The Importance of the Tilt Term



Mock: No tilt  
Recon: No tilt

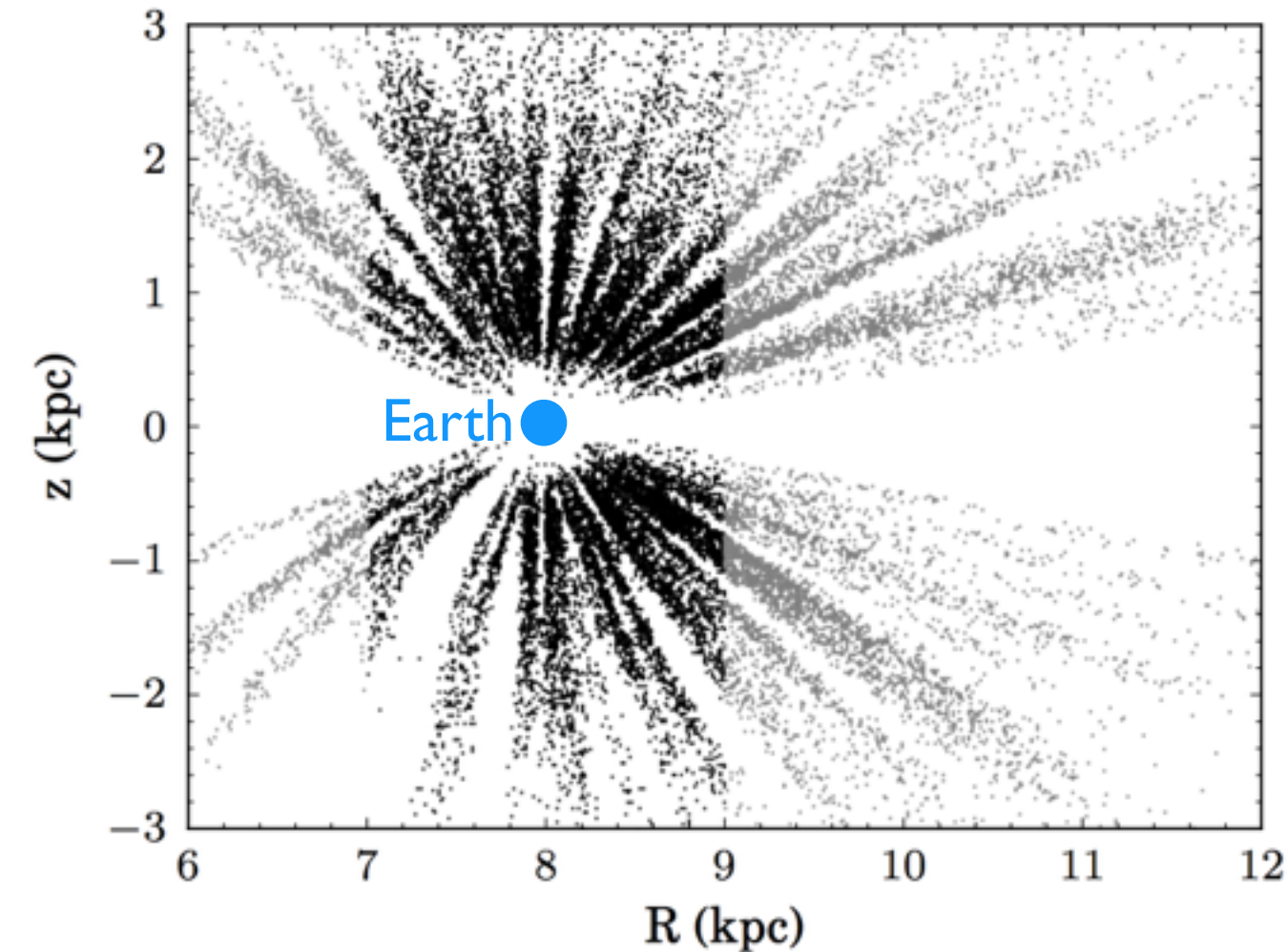
Mock: Tilt  
Recon: No tilt

Mock: Tilt  
Recon: Tilt

Tilt is the coupling between Radial and Vertical motions  
Neglecting tilt leads to a systematic **underestimation** of the dark matter density when the tilt term is **negative**.



# Initial Tests with SDSS Data from Budenbender et al.



- Stellar kinematics data from SDSS G-dwarfs from Budenbender et al., MNRAS 452 (2015) 956–968, arXiv:1407.4808.
- Observational baryon profile derived from McKee et al., ApJ 814 (2015) 13, arXiv:1509.05334

# Tilt Term Redux

$$\underbrace{\frac{1}{R\nu} \frac{\partial}{\partial R} (R\nu\sigma_{Rz}^2)}_{\text{'tilt' term: } \mathcal{T}}$$

- We assume  $\sigma_{Rz}^2$  has the same radial dependence as the tracer density  $\nu$
- **Traditionally** (e.g. Binney & Tremaine) tracer density  $\nu$  is an exponential falling with radius, eg:

$$\nu(R, z) = \nu(z)|_{R_\odot} \exp\left(-\frac{R - R_\odot}{R_0}\right), \quad R_0 = R_1$$

$$\Rightarrow \sigma_{Rz}^2(R, z) = \sigma_{Rz}^2(z)|_{R_\odot} \exp\left(-\frac{R - R_\odot}{R_1}\right)$$

$$\sigma_{Rz}^2(z)|_R = A \left(\frac{z}{\text{kpc}}\right)^n \Big|_R$$

$$\mathcal{T}(R_\odot, z) = A \left(\frac{z}{\text{kpc}}\right)^n \Big|_{R_\odot} \left[ \frac{1}{R_\odot} - \frac{2}{R_0} \right]$$

**Negative**

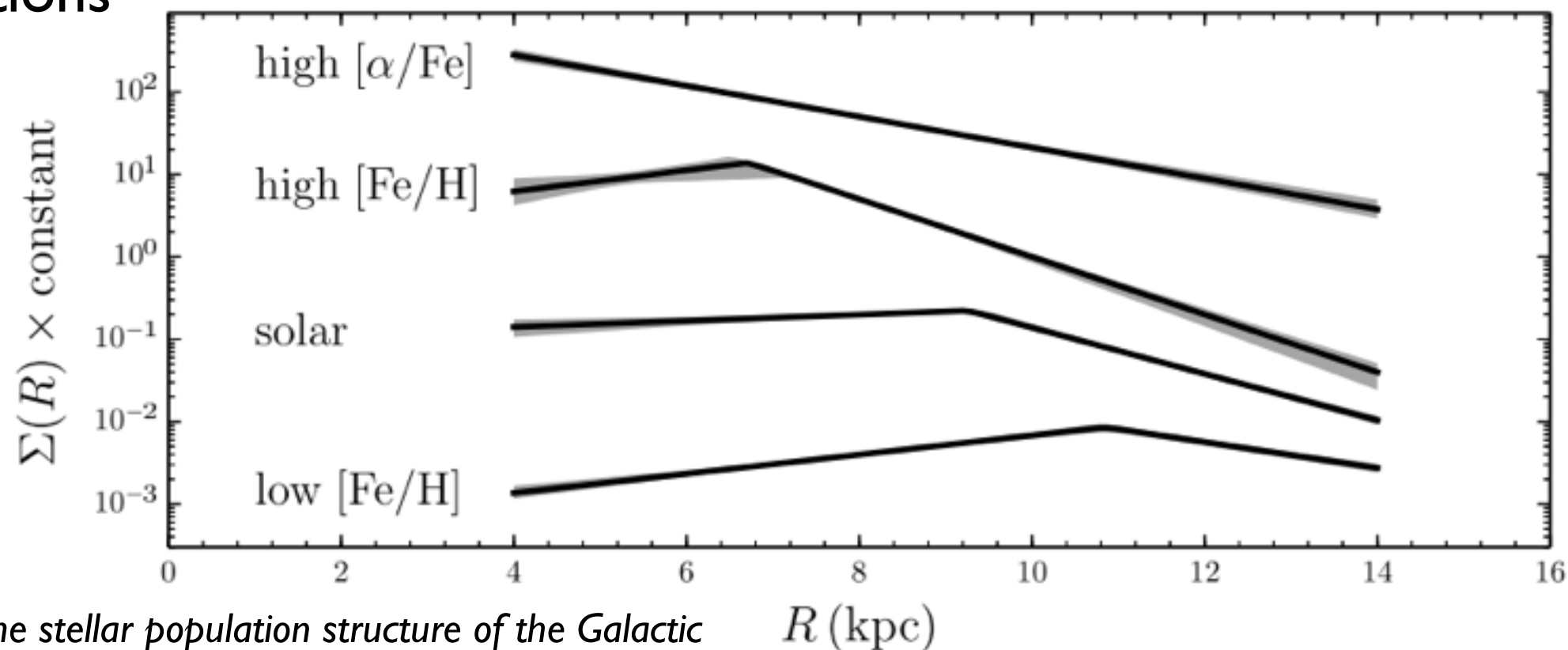
**Positive**

**Negative**



# Tilt Term Redux

- But recent SDSS results show a surface density rising with radius for some populations



Bovy et al., *The stellar population structure of the Galactic disk*, *Astrophys.J.*823:30, 2016, arXiv: 1509.05796

- Thus we model the tilt term as the following, with a flat prior on  $k$  that ranges from negative to positive values.

$$\mathcal{T}(R_{\odot}, z) = \sigma_{Rz}^2(R_{\odot}, z) \left[ \frac{1}{R_{\odot}} - 2k \right]$$

alpha-young  $k = [-1.3, 1.0]$   
alpha-old  $k = [-0.5, 1.5]$

Positive or Negative

Positive

Positive or Negative

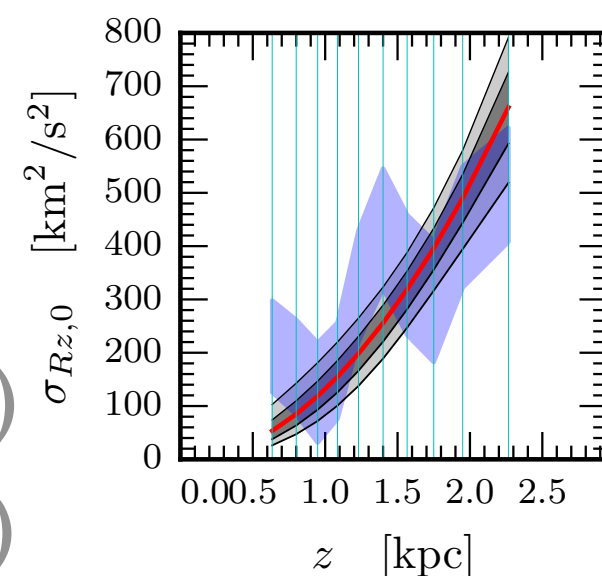
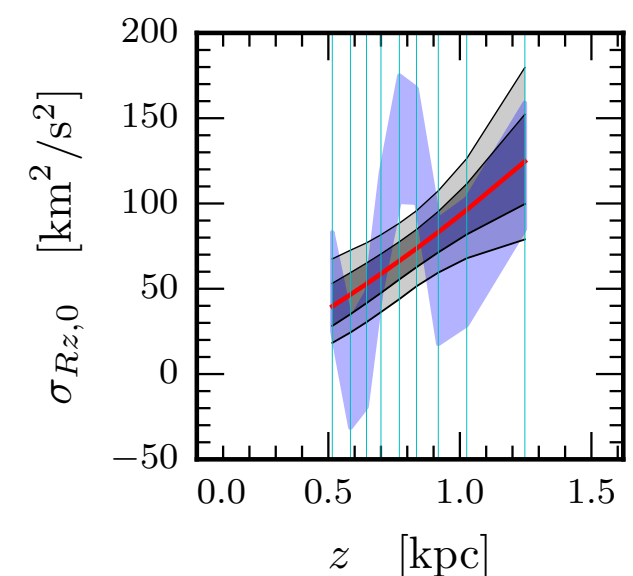
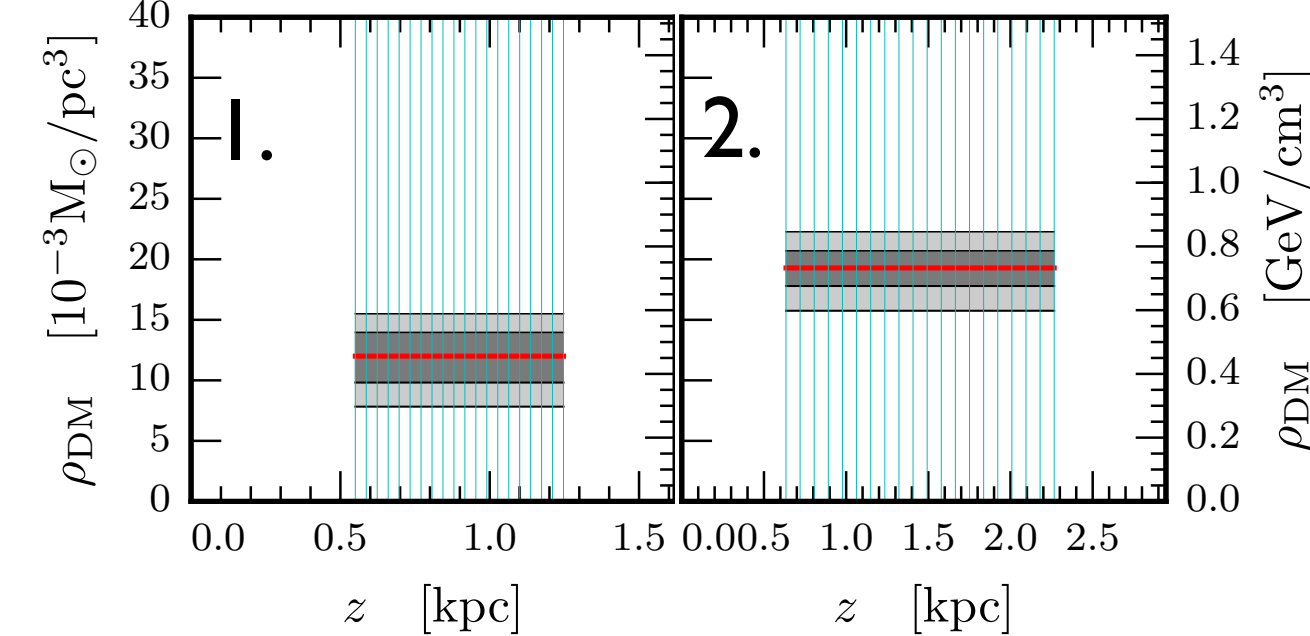
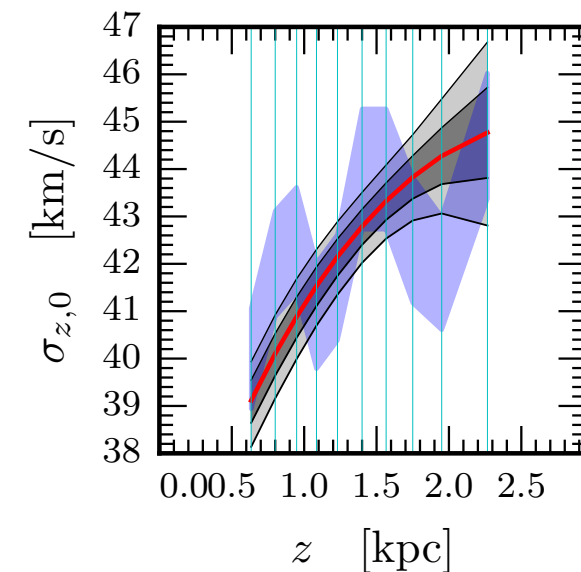
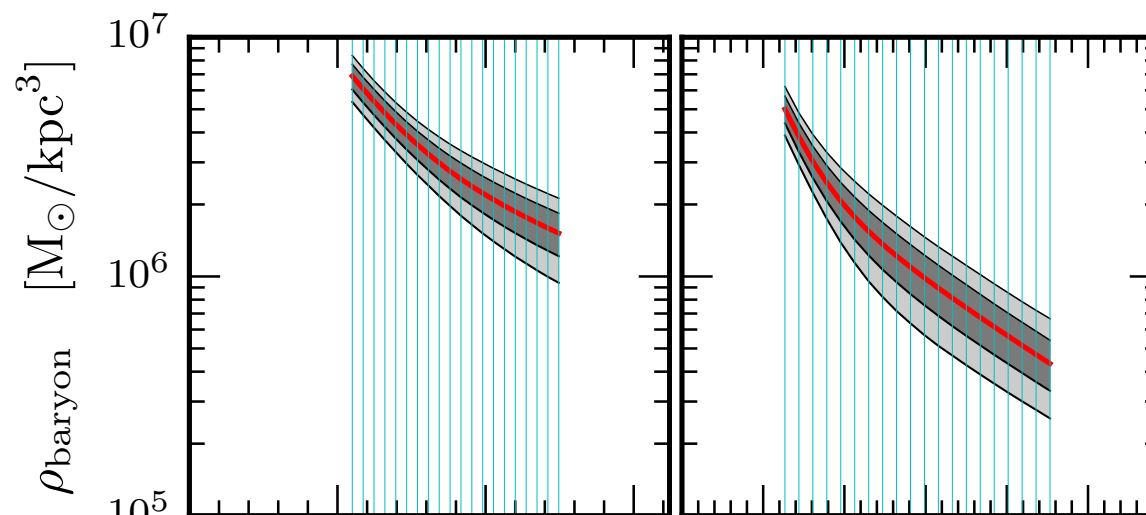
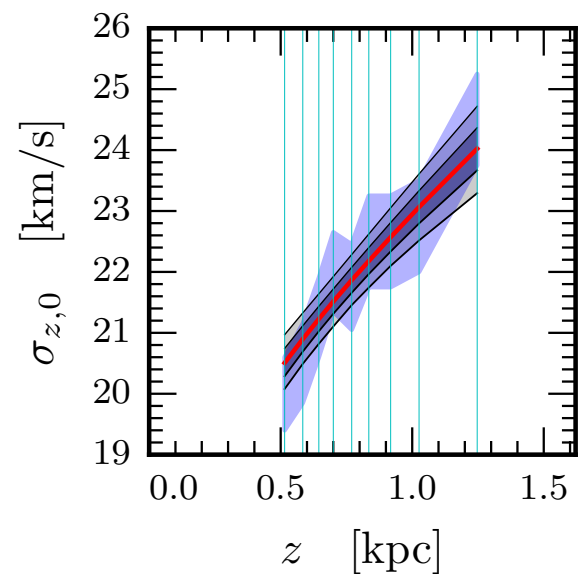
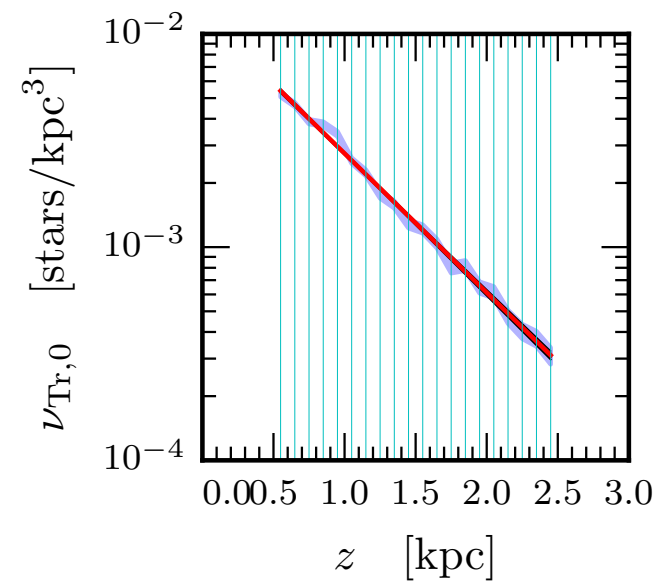
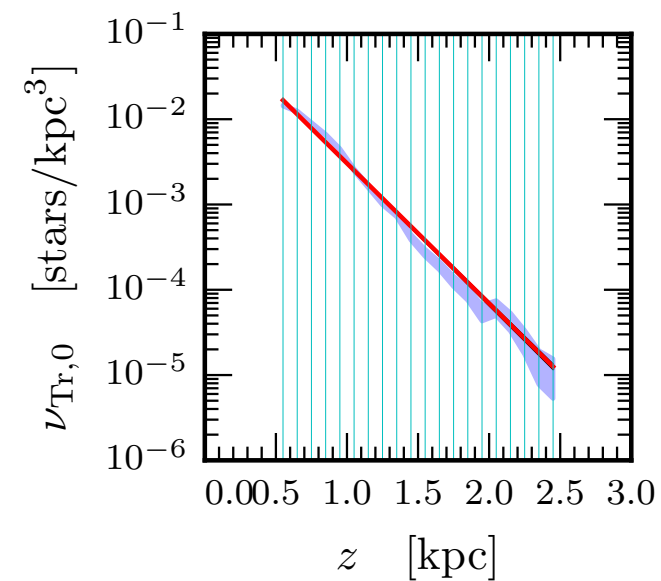
Alpha-young population  
(‘thin disc’)

# Preliminary Results.

Alpha-old population  
(‘thick disc’)

SDSS-SEGUE G-dwarf data from Budenbender et al. 2014  
I 407.4808v2. Tilt priors informed by data from SDSS-  
APOGEE, Bovy et al. I 509.05796.

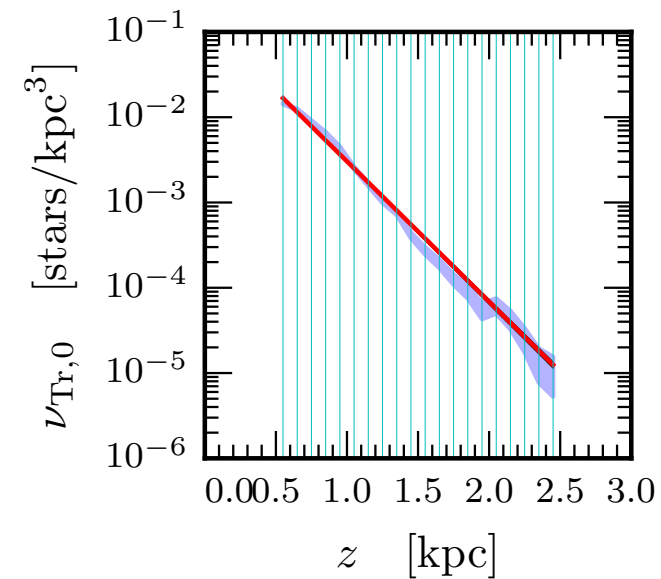
Analyzed separately,  $2\sigma$  uncertainties quoted.



1.  $\rho_{\text{DM}} = 0.46^{+0.13}_{-0.16} \text{ GeV/cm}^3$  (tilt: 0.48)  
 2.  $\rho_{\text{DM}} = 0.73^{+0.13}_{-0.13} \text{ GeV/cm}^3$  (tilt: 0.42)



Alpha-young population  
(‘thin disc’)

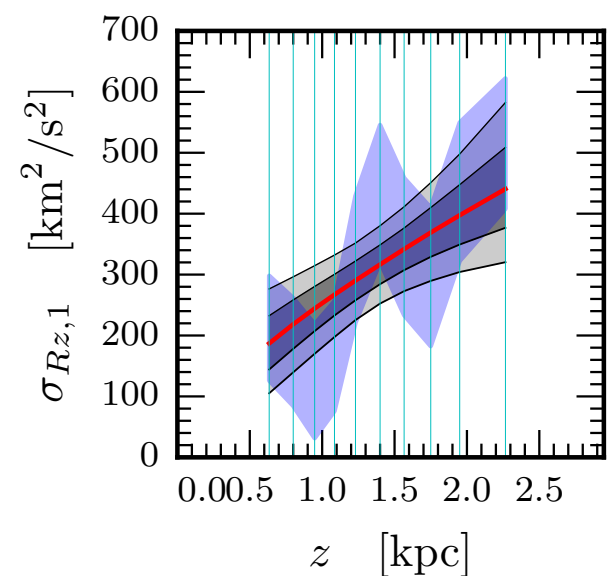
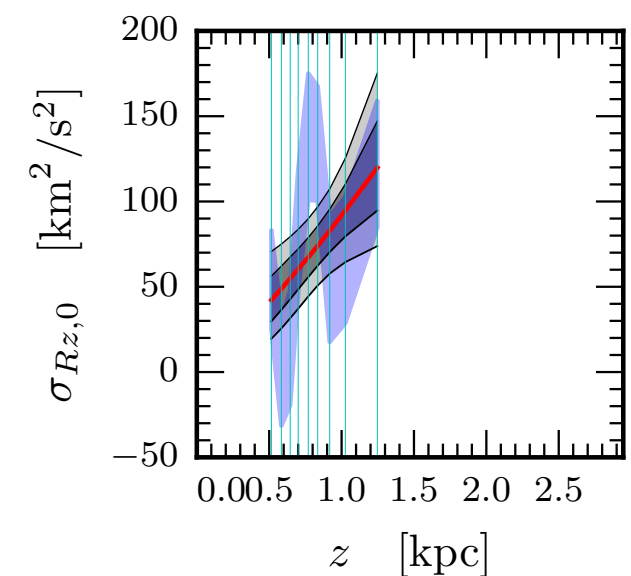
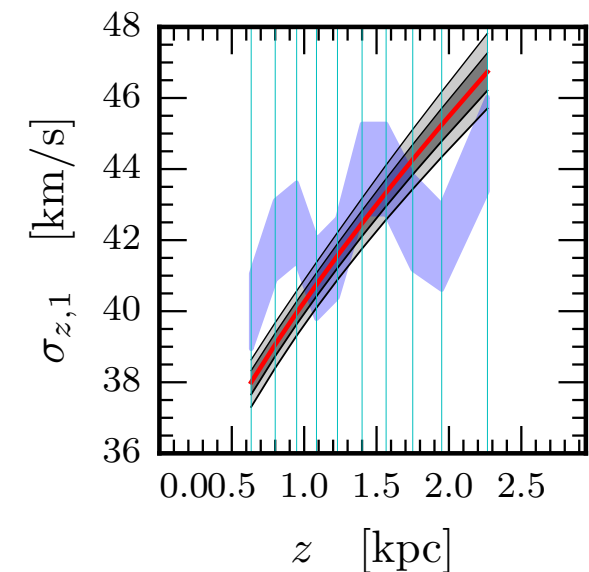
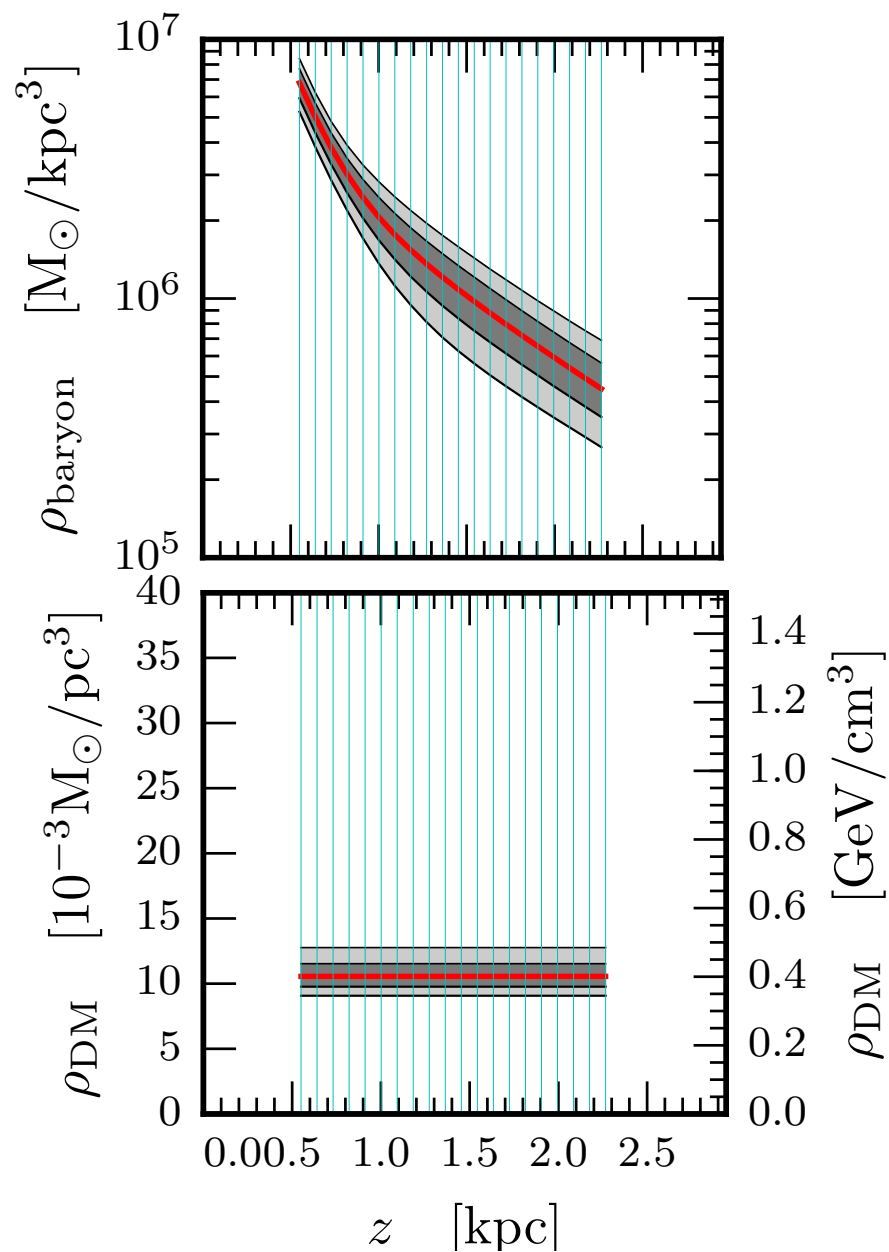
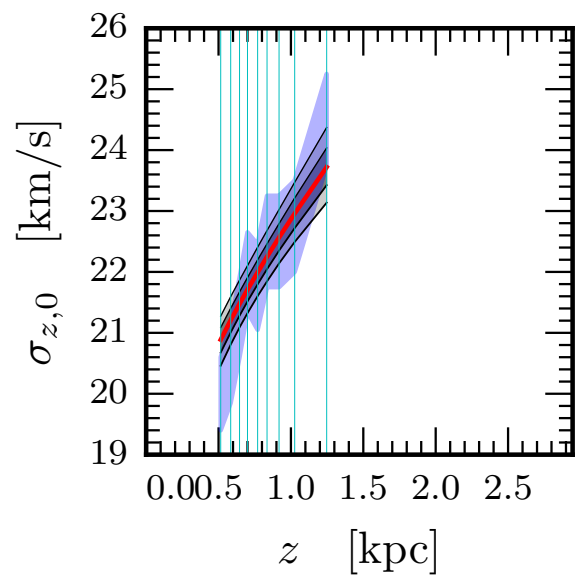
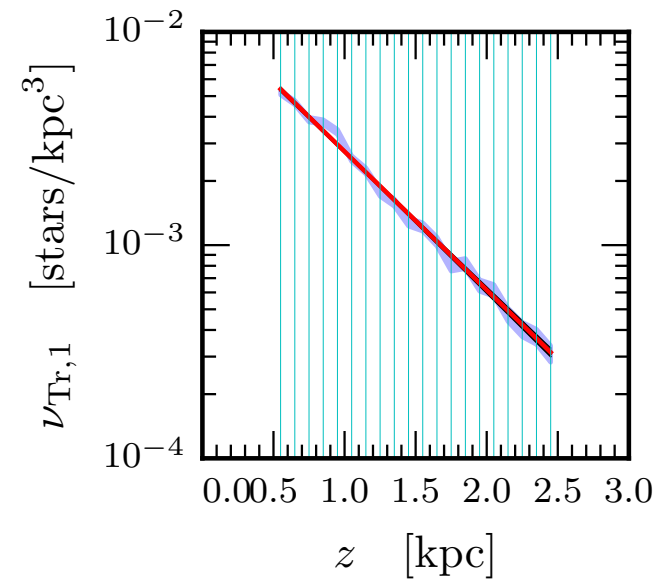


# Preliminary Results.

SDSS-SEGUE G-dwarf data from Budenbender et al. 2014 I407.4808v2. Tilt priors informed by data from SDSS-APOGEE, Bovy et al. I509.05796.

Combined Analysis, 2 $\sigma$  uncertainties quoted.

Alpha-old population  
(‘thick disc’)



$$\rho_{\text{DM}} = 0.40^{+0.08}_{-0.06} \text{ GeV}/\text{cm}^3$$

# SDSS Preliminary Results: Summary

Thin Disk only:  $\rho_{\text{DM}} = 0.46^{+0.13}_{-0.16} \text{ GeV/cm}^3 (2\sigma)$  (0.48 w/out tilt)

Thick Disc only:  $\rho_{\text{DM}} = 0.73^{+0.13}_{-0.13} \text{ GeV/cm}^3 (2\sigma)$  (0.42 w/out tilt)

Thin+Thick Disc:  $\rho_{\text{DM}} = 0.40^{+0.08}_{-0.06} \text{ GeV/cm}^3 (2\sigma)$

1. Thin disk result less sensitive to tilt term than the thick disc
2. Combining thick and thin gives a result that is lower than either separate result - still under investigation.
3. Statistical uncertainty is now less than the systematic uncertainty arising from the rotation curve term - this needs to be tackled.
4. We assume the radial variation of  $\sigma_{Rz}^2$  matches that of the tracer density - we need to measure the  $\sigma_{Rz}^2$  radial variation.
5. Tilt term can now be negative or positive, giving a systematic under- or over-estimation of the local DM density if ignored.



# Gaia Satellite, 2013-

- Astrometrics mission, successor to Hipparcos (1989-1993)
- $10^4$  times more stars with factor 50-100 higher accuracy compared to Hipparcos.
- Full data set will include 5D data for  $\sim 1$  billion stars
  - sky positions ( $\alpha, \delta$ ),
  - parallaxes ( $\omega$ ),
  - proper motions ( $\mu_\alpha, \mu_\delta$ )
- Radial velocities  $\mu_r$  for  $\sim 150$  million stars.

## Data Release 1: 14 September

- Observations taken between July 2014 and September 2015
- Sky positions ( $\alpha, \delta$ ) and G-magnitude for  $\sim 1$  billion stars
- TGAS solution for 2.5 million stars...





# Tycho-Gaia Astrometric Solution (TGAS)

- Hipparcos astrometric satellite produced the Tycho catalogue of 2.5 million stars.
- TGAS combines sky position ( $\alpha$ ,  $\delta$ ) from Tycho with initial 5D data from Gaia to produce improved 5D astrometric data.





# Radial Measurements

- Ideally we need full 6D information.
- Both TGAS and final Gaia data release have a radial velocity deficit:
  - TGAS: No radial data
  - Full Gaia data release: radial data for only 150m of 1b stars
- Near term: TGAS + RAVE radial data
- Long term: Gaia + WEAVE + 4MOST spectrographic surveys

## RAVE, 2003-13

UK Schmidt Telescope,  
Australia



## WEAVE, 2018-

William Herschel Telescope,  
La Palma



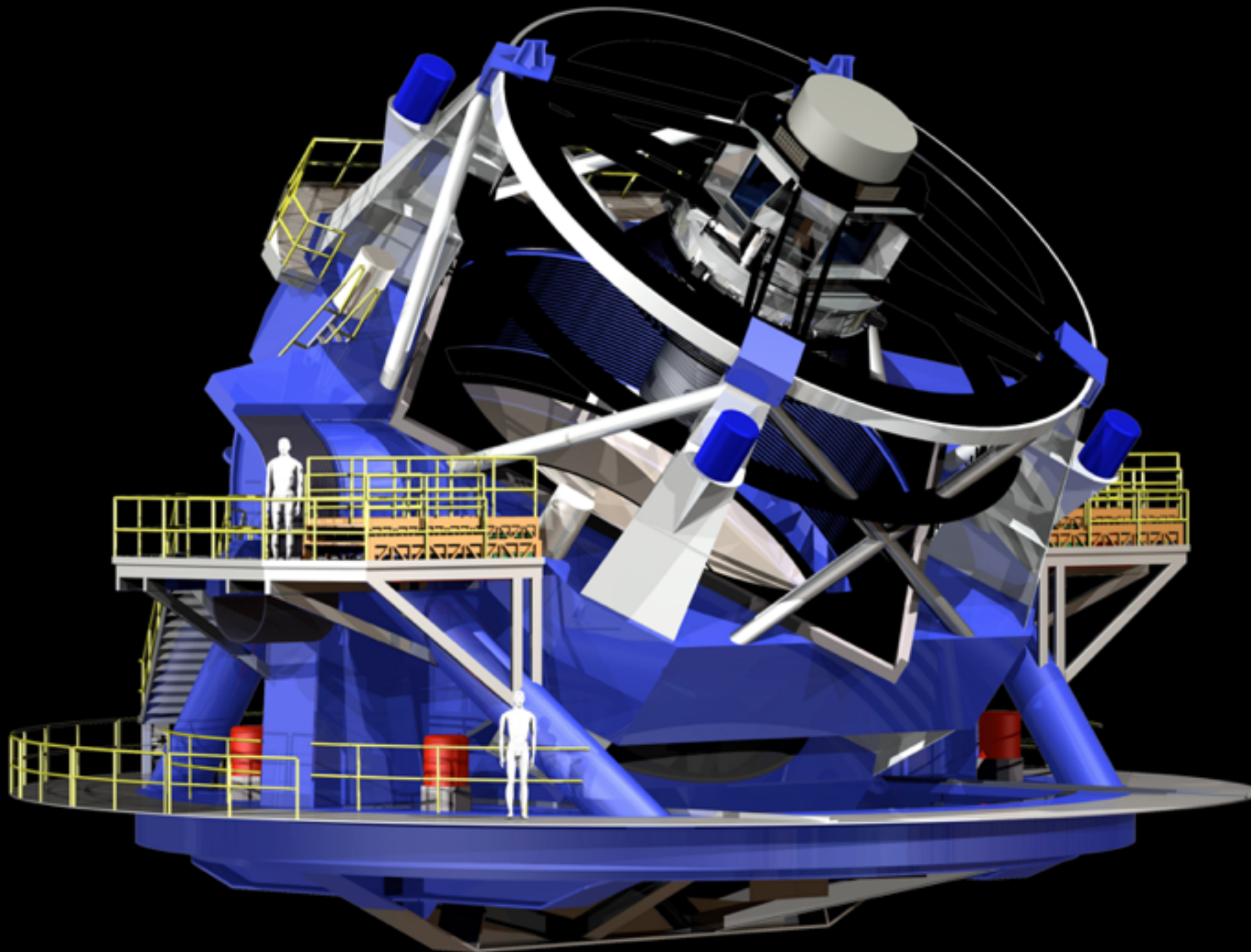
## 4MOST, 2021-

VISTA Telescope,  
Paranal, Chile



# LSST 2019-

Deep complement to Gaia survey, seeing dimmer stars and reaching further out into the halo.





# Conclusions

- Tilt term is important (ignore at your peril!), and can now potentially be positive or negative.
- We still need more data on the tilt term - namely radial variation of  $\sigma_{Rz}^2$
- Preliminary analysis of thin disc and thin+thick disc Budenbender SDSS data yield a local dark matter density inline with previous estimates, but analysis is ongoing.
- Statistical uncertainty is now less than the systematic uncertainty arising from the rotation curve term.
- Gaia Data Release 1: **14 September**
- TGAS + RAVE 6D will be very exciting.

