Top-quark pole mass in the tadpole-free MSbar scheme

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Based on 1604.01134

Experimental goal: data $\longleftrightarrow$ top-quark mass

Theory goal: top-quark mass $\longleftrightarrow$ Lagrangian parameters

What do we mean by the top-quark mass?

- The "Monte Carlo mass" used by event generators
- Various other kinematic masses: 1S mass, potential subtracted mass, ...
- The complex pole squared mass $M_t^2 - i\Gamma_t M_t$
  - Formally, a gauge invariant observable
  - Renormalon ambiguities $\sim \Lambda_{\rm QCD}$
- The tree-level $\overline{\rm MS}$ mass $m_t = y_t v/\sqrt{2}$
  - Ambiguous. Either gauge-dependent, or a poor perturbative expansion

The top mass reported by experimental collaborations is not always simple to relate to the theory definitions. For some reviews and progress, see e.g. 0803.4214, 0808.0222, 1405.4781, 1408.6080, 1511.00841

I will concentrate on the relation between the top-quark pole mass and the running $\overline{\text{MS}}$ mass. State of the art:

- Pure QCD
  - 1-loop: Tarrach, 1981
  - 2-loop: Gray, Broadhurst, Grafe, Schilcher, 1990
  - 3-loop: Melnikov, van Ritbergen, 9912391
  - 4-loop: Marquard, Smirnov, Smirnov, Steinhauser, 1502.01030
- non-QCD
  - 1-loop: Bohm, Spiesberger, Hollik 1986; Hempfling, Kniehl 9408313; Jegerlehner, Kalmykov 0308216
  - partial 2-loop: Eiras, Faisst, Jegerlehner, Kalmykov, Kniehl, Kuhn, Pikelner, Veretin
  - full 2-loop: Kniehl, Pikelner, Veretin 1503.02138

I have redone the full 2-loop non-QCD calculation, but with a different way of treating the VEV.

The tree-level $\overline{\text{MS}}$ mass is proportional to the Higgs VEV. However, there are different definitions of the $\overline{\text{MS}}$ VEV.

Suppose the tree-level Higgs potential is:

$$V = m^2|H|^2 + \lambda|H|^4.$$

Can define the VEV as:

$$v_{\text{tree}} = \sqrt{-m^2/\lambda}.$$

This definition is gauge-fixing invariant. However, one must then include tadpole diagrams. The perturbative loop expansion parameter is

$$\frac{N_c y_t^4}{16\pi^2\lambda} \quad \text{rather than the usual} \quad \frac{N_c y_t^2}{16\pi^2}.$$

Perturbation theory based on the tree-level squared mass $m_t = y_t v_{\text{tree}}/\sqrt{2}$ converges slowly, because $y_t^2/\lambda \approx 7$.

The 1-loop non-QCD correction to the top pole mass is as big as the QCD correction!

I define the VEV v instead as:

$$v \;=\; \text{minimum of the full effective potential in Landau gauge.}$$

The tree-level $\overline{\text{MS}}$ top mass is then defined as

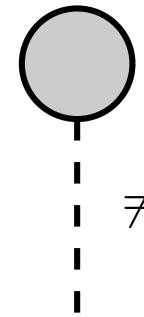
$$m_t = y_t v/\sqrt{2}$$

Pro: Tadpole diagrams vanish exactly. Perturbation theory converges more quickly, because we are expanding around the "correct" minimum of the potential.

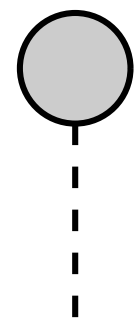
Con: The definition of v is gauge-dependent. Landau gauge is chosen because it gives the simplest (by far) effective potential.

The VEV is not a direct physical observable in any case, since both $v_{\rm tree}$ and v depend on the renormalization scale. So, it is OK that v is a Landau-gauge only quantity.

If you expand around $\langle H \rangle = v_{\rm tree}$, then tree-level tadpoles vanish, but tadpole loop diagrams don't:

$\frac{\partial V_{\rm tree}}{\partial H} = 0 \quad \leftrightarrow$  $= 0$ but  $\neq 0$

If you expand around $\langle H \rangle = v$, then tree-level tadpoles don't vanish, and tadpole loop diagrams don't vanish, but their sum vanishes:

$\frac{\partial V_{\rm tree}}{\partial H} + \frac{\partial (\Delta V)}{\partial H} = 0 \quad \leftrightarrow$  $+$  $= 0$

Standard Model effective potential

- 2-loop: Ford, Jack, Jones 0111190 (paper actually from 1992).
- 3-loop QCD and top Yukawa: SPM 1310.7553
- 4-loop leading order in QCD: SPM 1508.00912

As a result, can translate between the VEV definitions:

$$v_{\rm tree}^2 = v^2 + \frac{1}{\lambda}\sum_{L=1}^{\infty}\frac{1}{(16\pi^2)^L}\Delta_L,$$

where the Δ_L are given for $L = 1, 2, 3, 4$ in 1406.2355 and 1508.00912.

The complex top-quark pole squared mass is found as the pole in the top-quark self-energy function.

Input parameters are $\overline{\text{MS}}$ quantities:

$$v, \;\; y_t, \;\; g_3, \;\; g, \;\; g', \;\; \lambda, \quad \text{at a fixed RG scale } Q$$

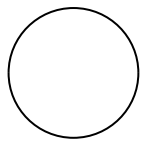
Effects of non-zero y_b are very small ($\sim$10 MeV, mostly from 2-loop QCD).

Output of the calculation:

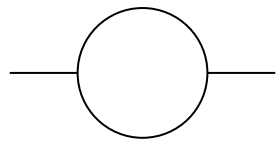
$$s_{\text{pole}} = M_t^2 - i\Gamma_t M_t$$

Other outputs include: pole masses of W, Z, h, and electroweak parameters G_F, $\alpha(0) = 1/137.036\ldots$, etc.

Do all 2-loop self-energy graphs for top quark, use Tarasov algorithm 9703319 to write in terms of basis integrals:



A



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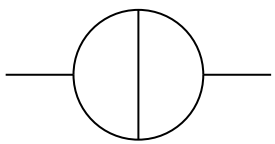

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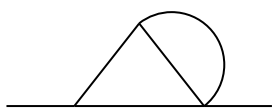

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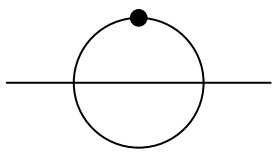

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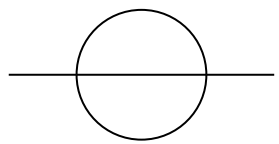

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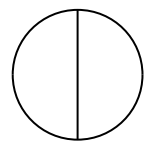


I

Most of these integrals can't be done analytically. Instead, these 2-loop basis integrals are calculated numerically using the program TSIL. (0501132, SPM and D.G Robertson). Takes $<$1 second for whole pole mass calculation on a modern laptop.

Results are then combined consistently with the existing 4-loop pure-QCD results.

The resulting contributions to the 2-loop top-quark complex pole squared mass have the form:

$$\sum_j c_j^{(2)} I_j^{(2)} + \sum_{j \le k} c_{j,k}^{(1,1)} I_j^{(1)} I_k^{(1)} + \sum_j c_j^{(1)} I_j^{(1)} + c^{(0)},$$

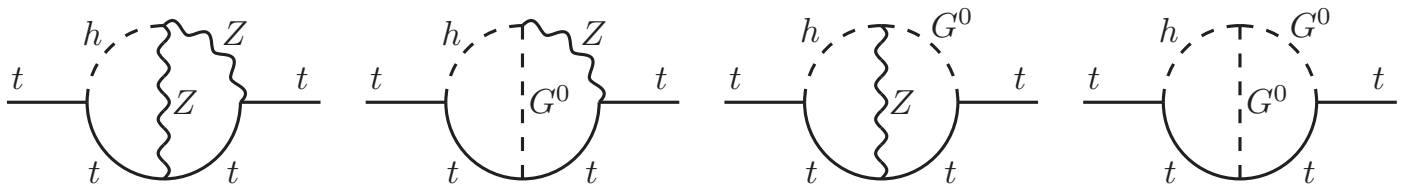
where the necessary basis integrals are:

$$I^{(1)} = \{A(h),\ A(t),\ A(W),\ A(Z),\ B(h,t),\ B(t,Z),\ B(0,W)\},$$

$$\begin{aligned} I^{(2)} = \{ & I(0,t,W),\ I(h,t,t),\ I(t,t,Z),\ M(0,0,t,W,0),\ M(0,t,t,0,t),\ M(0,t,t,h,t), \\ & M(0,t,t,Z,t),\ T(h,0,t),\ T(W,0,0),\ T(Z,0,t),\ \overline{T}(0,0,W),\ \overline{T}(0,h,t),\ \overline{T}(0,t,Z), \\ & U(0,W,0,t),\ U(t,h,t,t),\ U(t,Z,t,t),\ I(0,h,W),\ I(0,h,Z),\ I(0,W,Z),\ I(h,h,h), \\ & I(h,W,W),\ I(h,Z,Z),\ I(W,W,Z),\ M(0,0,W,W,0),\ M(0,0,W,W,Z),\ M(0,t,W,0,W), \\ & M(0,t,W,h,W),\ M(0,t,W,Z,W),\ M(0,Z,W,t,0),\ M(h,h,t,t,h),\ M(h,t,t,h,t), \\ & M(h,t,t,Z,t),\ M(h,Z,t,t,Z),\ M(t,t,Z,Z,h),\ M(t,Z,Z,t,t),\ S(0,0,0),\ S(0,h,W), \\ & T(h,0,W),\ T(h,h,t),\ T(h,t,Z),\ T(t,h,Z),\ T(W,0,h),\ T(W,0,Z),\ T(W,t,W), \\ & T(Z,0,W),\ T(Z,h,t),\ T(Z,t,Z),\ U(0,W,0,0),\ U(0,W,h,W),\ U(0,W,W,Z), \\ & U(h,t,0,W),\ U(h,t,h,t),\ U(h,t,t,Z),\ U(t,0,W,W),\ U(t,h,h,h),\ U(t,h,W,W), \\ & U(t,h,Z,Z),\ U(t,Z,0,0),\ U(t,Z,h,Z),\ U(t,Z,W,W),\ U(W,0,0,h),\ U(W,0,0,Z), \\ & U(Z,t,0,W),\ U(Z,t,h,t),\ U(Z,t,t,Z)\}. \end{aligned}$$

It would be impolite to show the results in print, even in the paper. So they're in an ancillary file available on the arXiv.

Even though Landau gauge is fixed, still get many checks from gauge invariance, through the cancellation of unphysical Goldstone boson contributions:



The physical contribution involves the 2-loop basis integral $M(h, Z, t, t, Z)$.

The individual diagrams give contributions dependent on the basis integrals with mass=0 poles: $M(h, 0, t, t, 0)$ and $M(h, Z, t, t, 0)$ and $M(h, 0, t, t, Z)$. These unphysical contributions cancel in the sum, while the physical part $M(h, Z, t, t, Z)$ survives.

There are many other consistency checks: RG invariance, cancellation of $1/\epsilon$ poles, cancellations of threshold singularities, cancellation of Goldstone singularities, . . .

For a benchmark model, consider the following inputs:

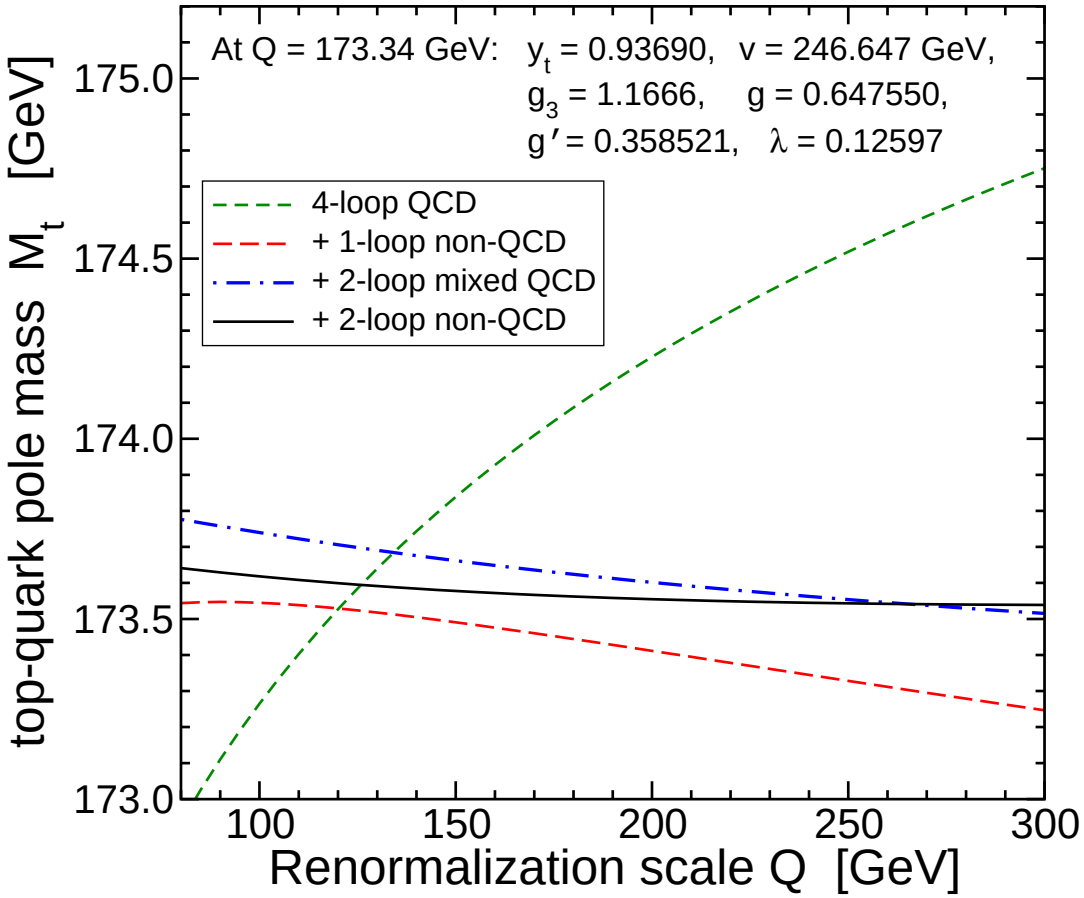
$$
\begin{aligned}
y_t(Q_0) &= 0.93690, \\
g_3(Q_0) &= 1.1666, \\
g(Q_0) &= 0.647550, \\
g'(Q_0) &= 0.358521, \\
\lambda(Q_0) &= 0.12597, \\
v(Q_0) &= 246.647 \text{ GeV},
\end{aligned}
$$

defined at the $\overline{\text{MS}}$ input renormalization scale

$$Q_0 = 173.34 \text{ GeV}.$$

These give values for M_Z, M_W, M_h in agreement with experiment.

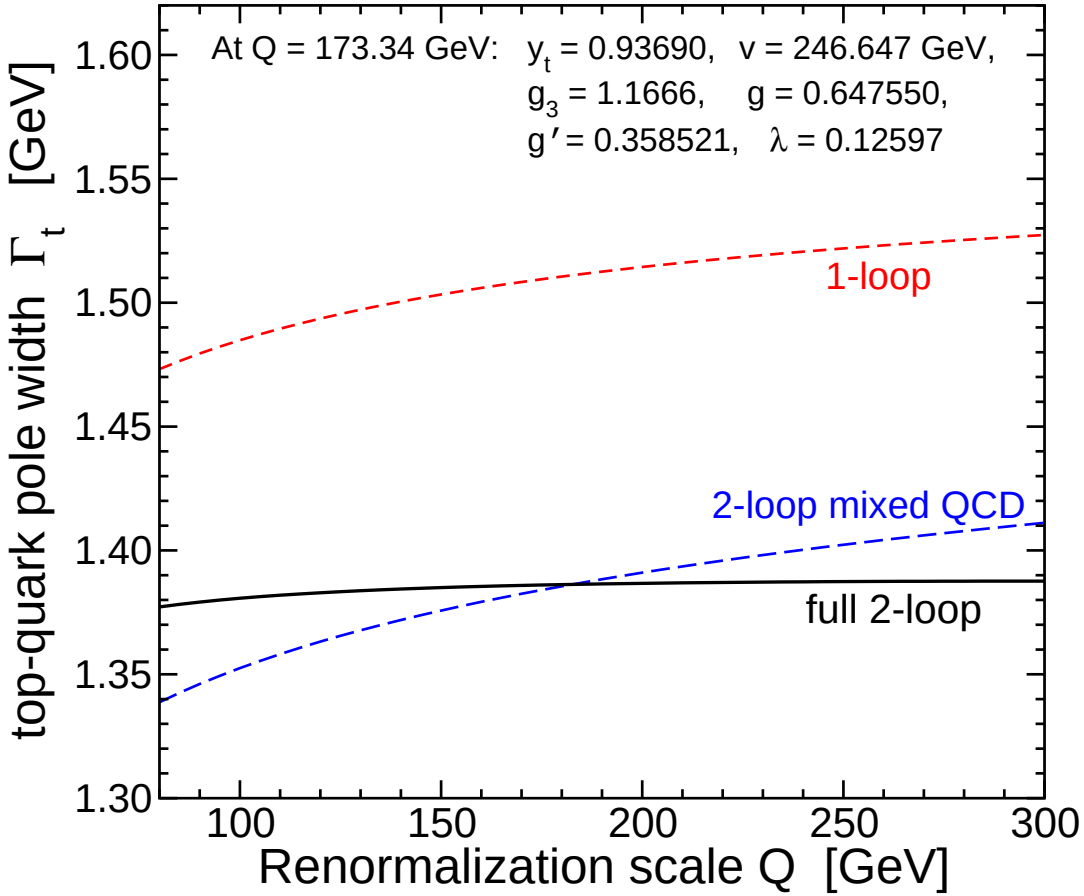
The Q dependence of the calculated M_t, in various approximations:



Scale dependence of full 2-loop + 4-loop QCD result is $<$100 MeV; very stable.

For $Q = M_t$, the full result is about 470 MeV less than the 4-loop pure QCD approximation, which has a much stronger scale dependence.

From the imaginary part of the complex pole mass, also get the width Γ_t:



Not as useful as a fully differential top-quark decay width calculation (Gao, Li, Zhu 1210.2808; Brucherseifer, Caola, Melnikov 1301.7133), but the flat scale dependence of calculated pole mass Γ_t is quite reassuring.

Outlook

- Top-quark pole mass in terms of $\overline{\text{MS}}$ Lagrangian quantities at full 2-loop order plus 4-loop pure QCD
- Scale dependence is small compared to LHC/Tevatron experimental errors
- Scale dependence is much better than pure 4-loop QCD
- Version here differs from earlier work by using VEV defined as minimum of effective potential. No tadpoles, perturbation theory formally converges faster.
- Present calculation method is consistent with methods used for earlier calculations of the pole masses of h, W, Z. Results for all will appear soon in public software code with Dave Robertson. Can input $\overline{\text{MS}}$ quantities and get pole masses, or the other way around, or any combination.