Pheno2016

Scale Invariant Magnetic Fields and the Duration of Inflation

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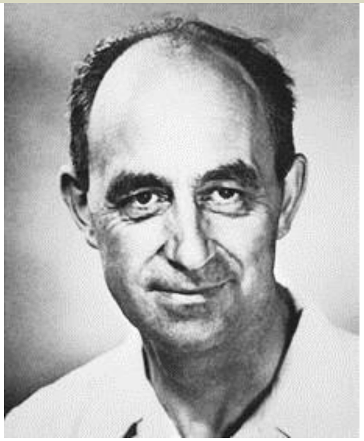
In collaboration with **Axel Brandenburg, Ruth Durrer, Alexander Tevzadze, Winston Yin**

Origin of Cosmic Magnetism



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On the Origin of the Cosmic Radiation

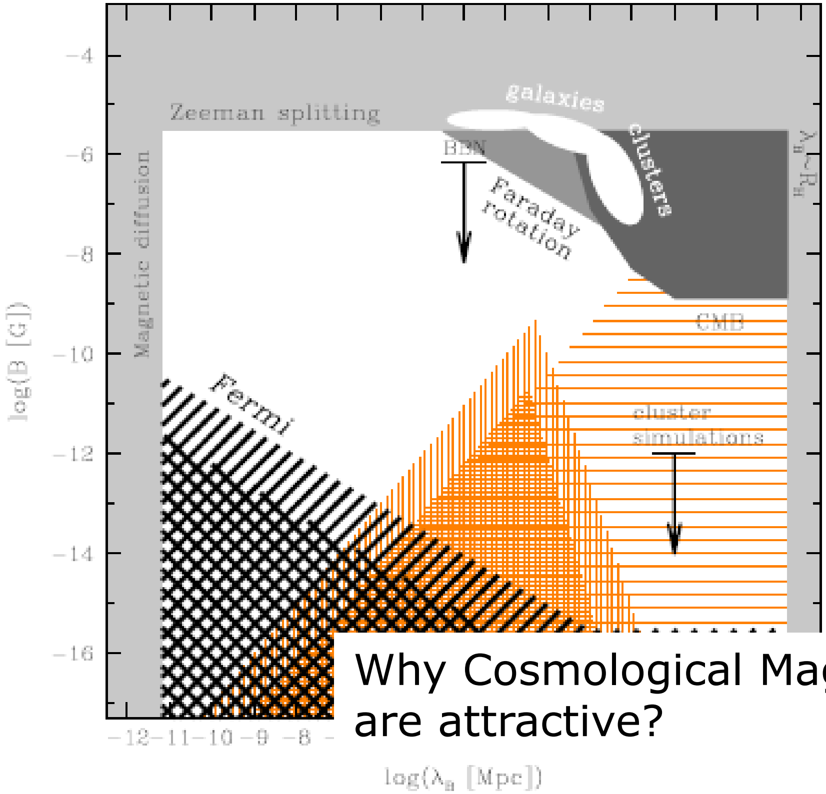
Enrico Fermi
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

Neronov and Vovk, Science 2010

Are the observed magnetic fields of a primordial origin

Why primordial magnetic fields are attractive



Why Cosmological Magnetic Fields are attractive?

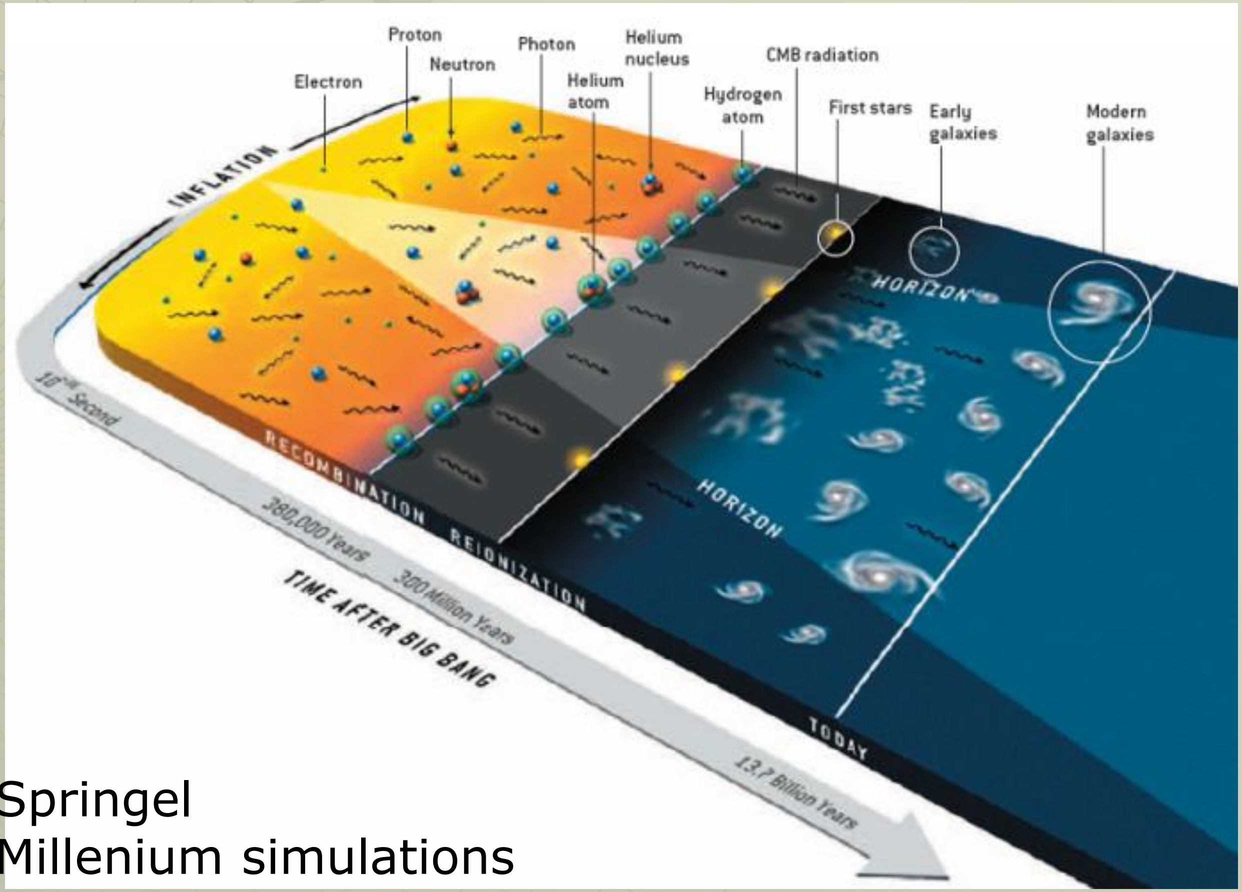
Fig. 2: Light, medium and dark grey: known observational bounds on the strength and correlation length of EGMF, summarized in the Ref. (*25*). The bound from Big Bang Nucleosynthesis marked "BBN" is from the Ref. (*2*). The black hatched region shows the lower bound on the EGMF derived in this paper. Orange hatched regions show the allowed ranges of B, λ_B for magnetic fields generated at the epoch of Inflation (horizontal hatching) the electroweak phase transition (dense vertical hatching), QCD phase transition (medium vertical hatching), epoch of recombination (rear vertical hatching) (*25*). White ellipses show the range of measured magnetic field strengths and correlation lengths in galaxies and galaxy clusters.

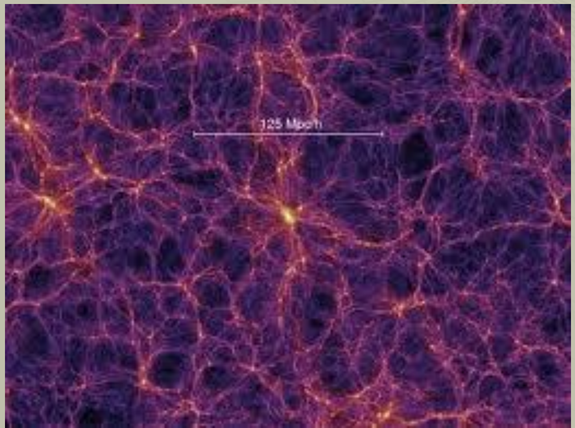
- Might serve as seeds for the observed fields in galaxies and clusters
- Might be responsible for large scale correlated magnetic fields in the voids
- Might explain some cosmological observations

Primordial Magnetic Field Hypothesis

F. Hoyle in Proc. *"La structure et l'evolution de l'Universe"* (1958)

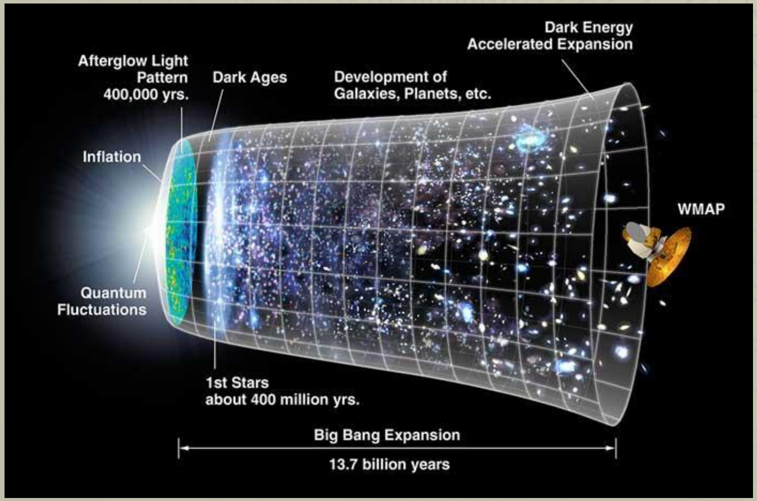
- Inflation
- Phase transitions
- Supersymmetry
- String Cosmology
- Topological defects

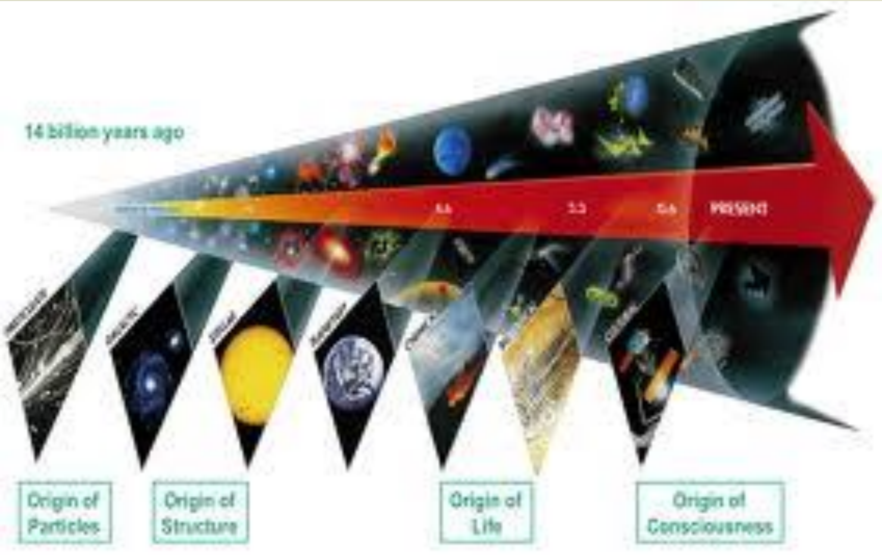




Springel
Millenium simulations

- Inflation
  - The correlation length larger than horizon
  - Scale invariant spectrum
  - Well agree with the lower bounds
- Diffuculties
  - Backreaction
  - Symmetries violation





- Phase Transitions
  - Bubble collisions – first order phase transitions
    - QCDPT
    - EWPT
  - Causal fields
  - Limitation of the correlation length
- Smoothed and effective fields approaches

Cosmological vs. Astrophysical Magnetogenesis

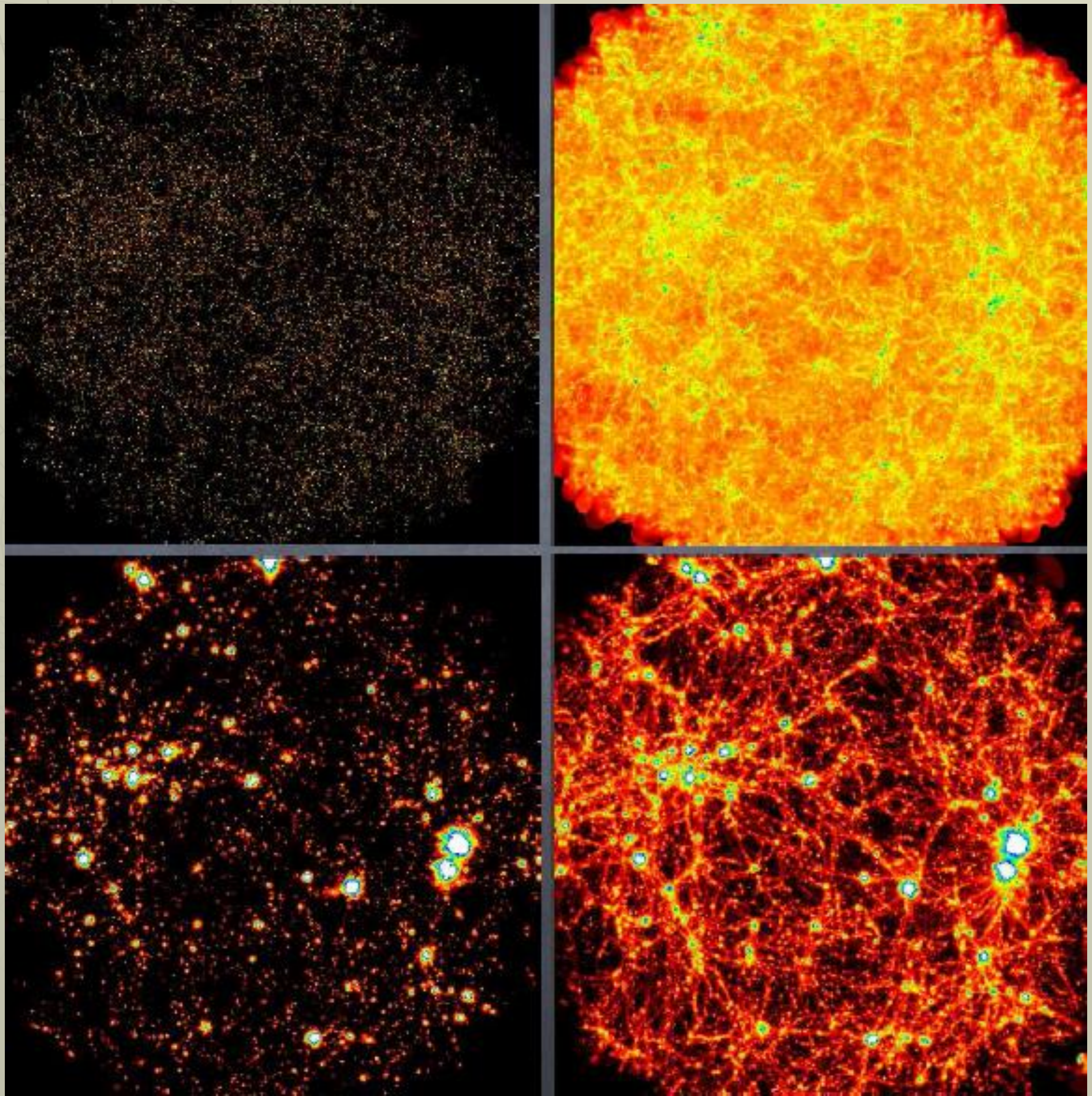
MHD Simulations by Donnert et al. 2008

Ejection

Primordial

Z=4

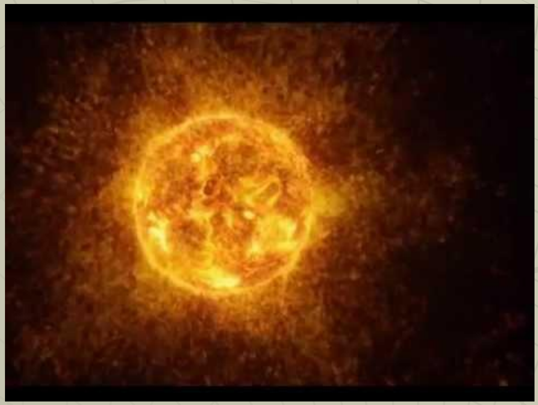
Z=4

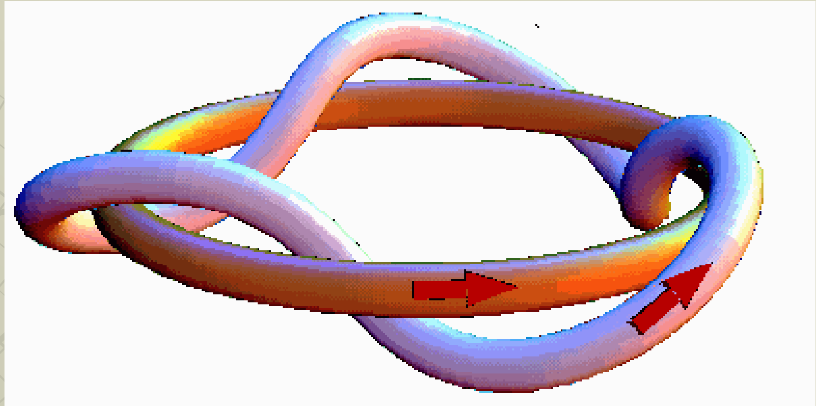


Z=0

Z=0

Magnetic Helicity





Solar activity:

- Sunspots
- Solar flares
- Coronal mass ejection
- Solar wind

$$H_B(t) = \int d^3x \mathbf{A} \cdot \nabla \times \mathbf{A},$$

Magnetic helicity reflects mirror symmetry (parity) breaking

Helical (Chiral) Magnetogenesis

- ***Phase Transitions***
  - Vachaspati, 2001
  - Sigl, 2002
  - Diaz-Gil, et al. 2008
  - Ng & Vachaspati, 2010
  - Tashiro et al., 2012
  - Long, et al., 2013

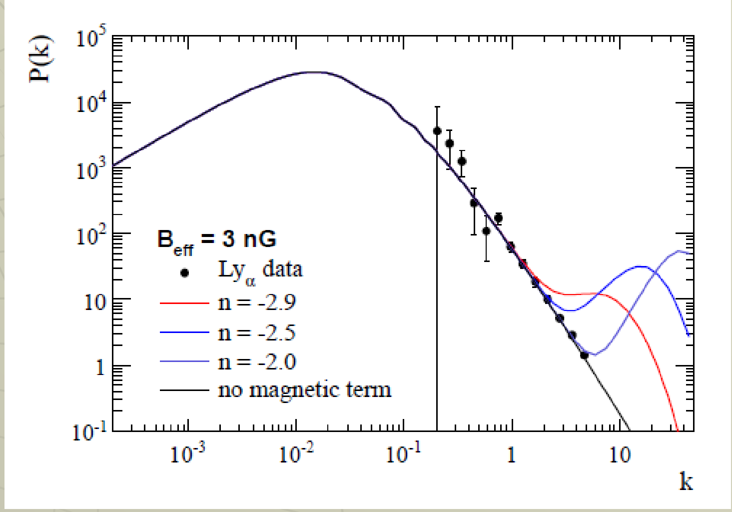
- ***Inflation***
  - Campanelli & Gianotti, 2005
  - Campanelli, 2009
  - Sorbo & Caprini, 2014
  - Campanelli, 2015

Magnetized Perturbations

- Scalar mode
  - Density perturbations – magnetosonic waves
- Vector mode
  - Vorticity perturbations- Alfven waves
- Tensor mode
  - Gravitational Waves

$$G_{ik} = 8\pi G T_{ik}$$

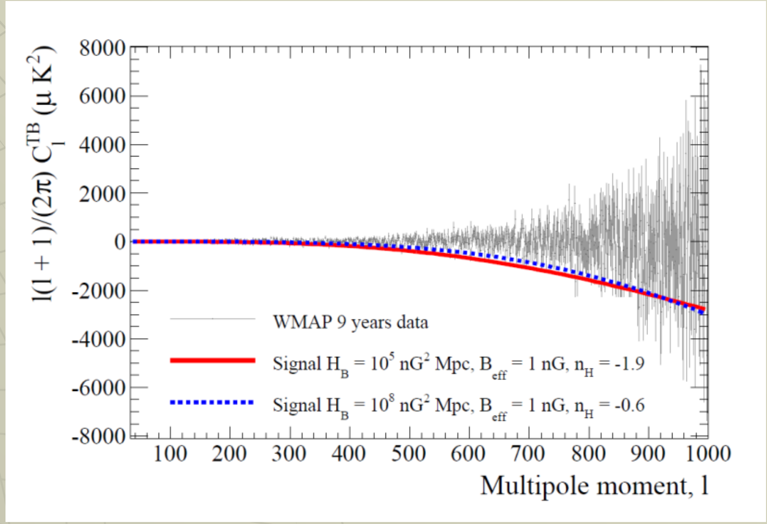
- Imprints on LSS and CMB



Kahniashvili et al. 2013

Constraints on Magnetic Helicity

- ***Tests:***
  - Cosmic microwave background: parity-odd cross correlations
  - Polarization of gravitational waves
  - Cosmic and gamma rays propagation
- ***Limits:***
  - WMAP 9yr (TB) data



Kahniashvili et al. 2014

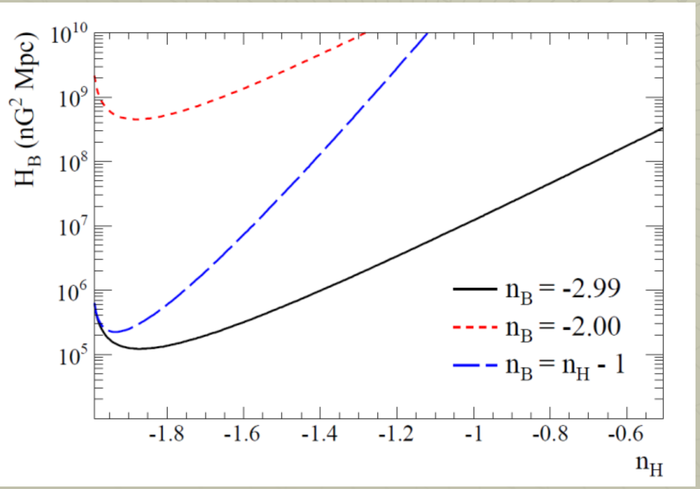
Magnetic Helicity Limits

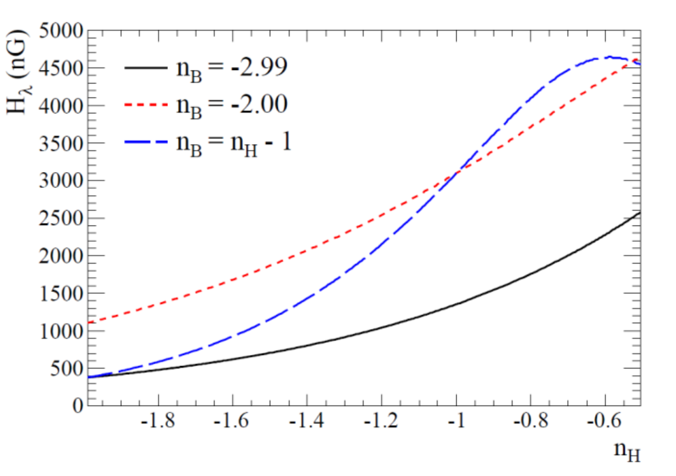
$$\frac{\Theta_l^{(\pm1)}(k,\eta_0)}{2l+1} \simeq \sqrt{\frac{l(l+1)}{2}}\,\Omega^{(\pm1)}(k,\eta_{\rm dec})\frac{j_l(k\eta_0)}{k\eta_0},$$

$$\dot{P}^{(\pm1)} - \frac{3}{10}\dot{\tau}P^{(\pm1)} \simeq \frac{k\sqrt{3}}{30}\Theta_1^{(\pm1)}$$

$$\frac{B_l^{(\pm1)}(k,\eta_0)}{2l+1} \simeq \mp\frac{\sqrt{6}}{2}\sqrt{(l-1)(l+2)}\int_0^{\eta_0} d\eta\dot{\tau}(\eta)e^{-\tau}P^{(\pm1)}(k,\eta)\frac{j_l(k\eta_0-k\eta)}{k\eta_0-k\eta}$$

$$C_l^{TB\ (V)} = \frac{2}{\pi}\int dk k^2\left[\frac{\Theta_l^{(-1)*}(k,\eta_0)}{2l+1}\frac{B_l^{(-1)}(k,\eta_0)}{2l+1} + \frac{\Theta_l^{(+1)*}(k,\eta_0)}{2l+1}\frac{B_l^{(+1)}(k,\eta_0)}{2l+1}\right]$$





Kahniashvili, et al., 2014

Inflation Generated Magnetic Helicity

Magnetic field correlation length might be as large as the Hubble horizon today or even larger (infinity)

$$\xi_M \equiv \frac{2\pi \int_0^{k_D} dk k P_B(k)}{\int_0^{k_D} dk k^2 P_B(k)}$$

$$\lambda_H = 5.8 \times 10^{-10}\ \mathrm{Mpc} \left(\frac{100\,\mathrm{GeV}}{T_\star}\right)\left(\frac{100}{g_\star}\right)^{1/6},$$

Can we see the imprints of an a-causal field?

$$|\mathcal{H}_B| \leq 2\xi_B \mathcal{E}_B\,.$$

Inflationary Magnetic Helicity

- Magnetic sources for vector or tensor modes are determined by convolutions so in causal sources all a-causal modes (k->0) contribute.

$$\tau_{ij}(\mathbf{k}) = \frac{1}{(2\pi)^3}\frac{1}{4\pi}\int d^3\mathbf{p}\,[B_i^\star(\mathbf{p})B_j(\mathbf{k}-\mathbf{p}) - \frac{1}{2}\delta_{ij}B_m^\star(\mathbf{p})B_m(\mathbf{k}-\mathbf{p})\,],$$

Minimum Energy Density

- Cauchy-Bunyakowski-Schwarz inequality

$$\left|\langle \mathbf{A}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x}) \rangle\right|^2 \le \left\langle \left|\mathbf{A}(\mathbf{x})\right|^2 \right\rangle \left\langle \left|\mathbf{B}(\mathbf{x})\right|^2 \right\rangle,$$

$$\nu_- \left|\mathbf{B}(\mathbf{x})\right|^2 \le \left(\mathrm{curl}^{-1}\mathbf{B}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x})\right) \le \nu_+ \left|\mathbf{B}(\mathbf{x})\right|^2,$$

- Minimum energy density condition

$$2\mathcal{E}_M \ge \frac{1}{\nu} \left\langle \left(\mathrm{curl}^{-1}\mathbf{B}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x})\right) \right\rangle = \frac{\mathcal{H}_M}{\nu}$$

Arnold 1974

The Realizability Condition

- Integral form:

$$\mathcal{H}_M \le 2\mathcal{E}_M \xi_M .$$

$$L_{\min} = \frac{\mathcal{H}_M}{2\mathcal{E}_M}.$$

$$\sigma = \frac{\mathcal{H}_M}{2\mathcal{E}_M \xi_M}, \qquad (\sigma \le 1).$$

- Spectral form

$$H_M(k) \le 2k^{-1} E_M(k)$$

- Can we apply the spectral form for inflation generated helicty?

Magnetic Field Correlation Length

- Causal fields – limitation

$$\lambda_H = 5.8 \times 10^{-10}\ \mathrm{Mpc} \left(\frac{100\,\mathrm{GeV}}{T_\star}\right)\left(\frac{100}{g_\star}\right)^{1/6},$$

$$k_\star \mathcal{H}_M \le 2\mathcal{E}_M\,,$$

$$L_{\min} = \frac{\mathcal{H}_M}{2\mathcal{E}_M}.$$

- Inflation field:
  - The scale invariant spectrum n_E=-1
  - Unlimited correlation length scale – cut-off scale k_C $=L_{min}^{-1}$
  - Connection to e-fold numbers

The Duration of Inflation

- Cut-off scale corresponds to the magnetic field correlation length
  - For maximally helical fields with known energy density and helicity L_{min}
- Number of e-folds before exit: $\log(H_0^{-1}/L_{min})$
- Need to determine the correlation length
- MHD processes neglected

Accounting for MHD processes

If T_{rec}=0.25 eV (recombination) temperature, T_f the temperature at which inflation ends, and n the growth rate of the correlation length,

$$L_{\min}^{-1} = \left(\frac{T_{\rm rec}}{T_f}\right)^{n_\xi} \cdot \frac{2\mathcal{E}_M}{\mathcal{H}_M}$$

Inflationary Magnetic Field Evolution (Decay)

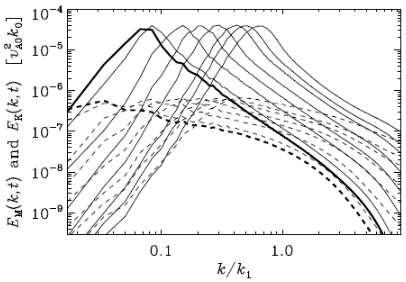


Figure 1: Magnetic and kinetic energy spectra for initial $E_K \sim k^4$, 1152^3 meshpoints, $\sigma = 1$. $\nu = \eta = 1 \times 10^{-5}$.

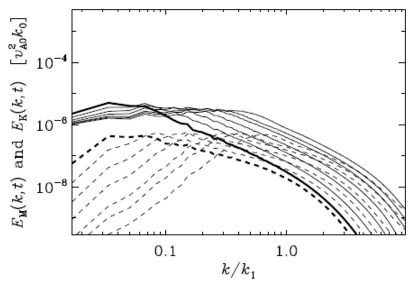


Figure 3: Magnetic and kinetic energy spectra for initial $E_K \sim k^{0.2}$, 1152^3 meshpoints, $\sigma = 1$. $\nu = \eta = 2 \times 10^{-5}$.

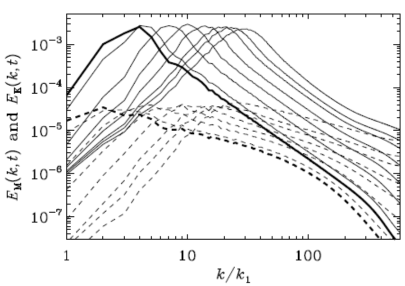


Figure 2: Magnetic and kinetic energy spectra for initial $E_K \sim k^2$. followed by a $k^{-5/3}$ subrange for $k/k_1 > 60$. 1152^3 meshpoints, $\sigma = 1$. $\nu = \eta = 5 \times 10^{-6}$

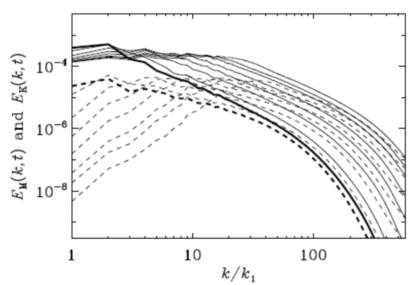


Figure 4: Magnetic and kinetic energy spectra for initial $E_K \sim k^0$, 1152^3 meshpoints, $\sigma = 1$. $\nu = \eta = 2 \times 10^{-5}$.

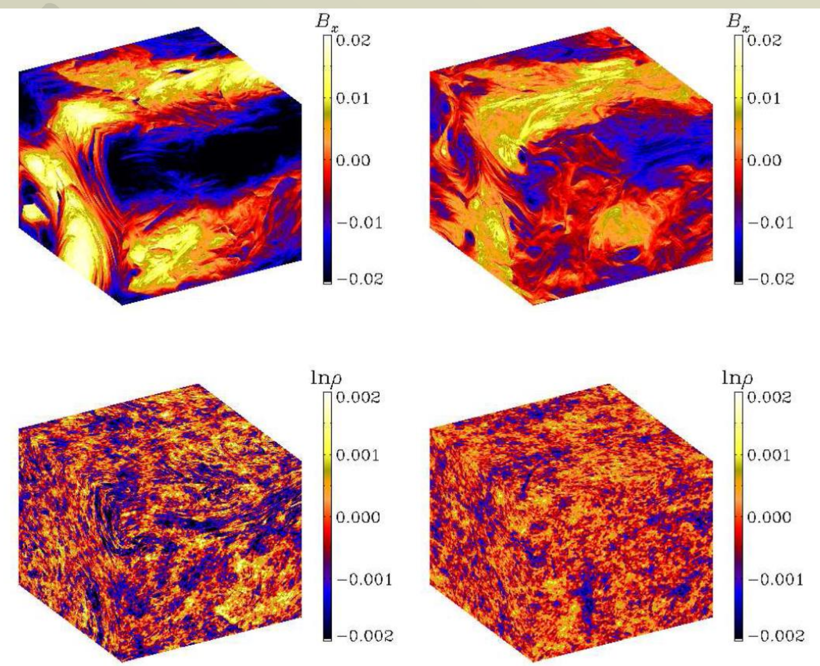
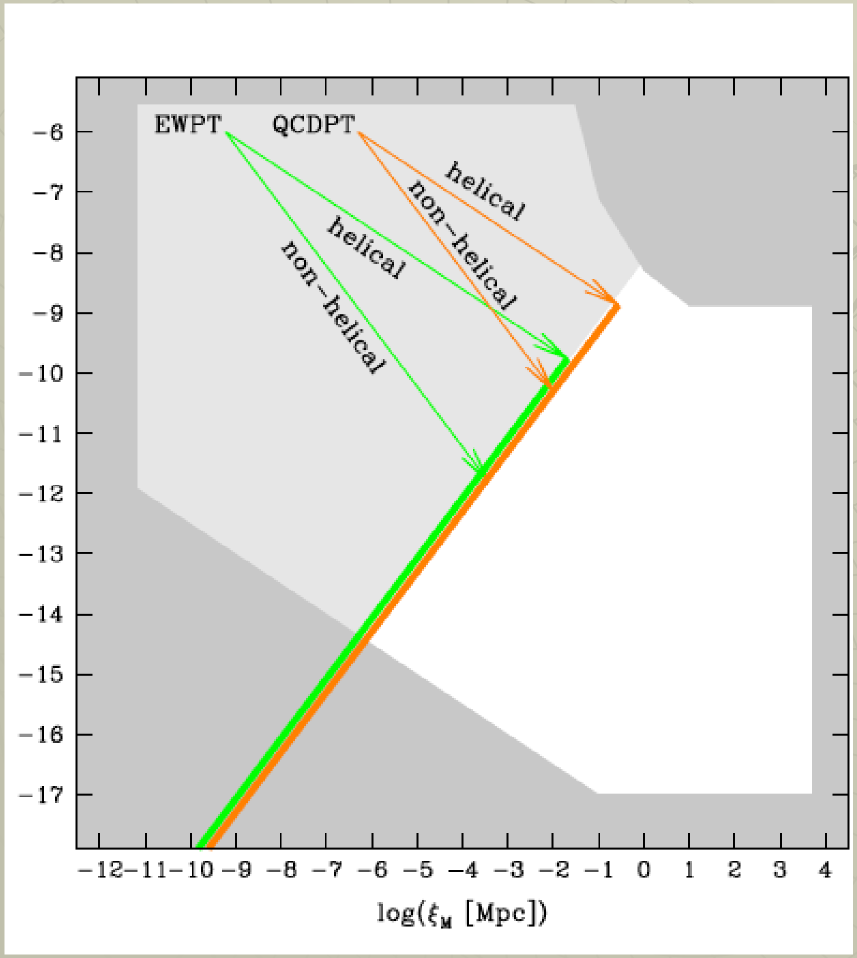


FIG. 6: Comparison of B_x (upper row) and $\ln\rho$ (lower row) for $\sigma = 1$ (left) and $\sigma = 0.03$ (right).

Brandenburg, Kahniashvili, Tevzadze, 2016

Magnetic Fields Limits



Cosmological evolution of B_{eff} and L_M for magnetic fields

Vs. Fermi data

Kahniashvili et al., 2013

Results

- No classic "inverse cascade" (with n=2/3)
- The correlation length grows extremely slowly (with n =0.2)

- The duration of inflation

$$N_e = 2.3 \log H_0 \cdot \frac{\mathcal{H}_M}{2\mathcal{E}_M} \cdot \left(\frac{T_f}{T_{\rm rec}}\right)^{n_\xi}$$

Conclusions

- Inflation generated magnetic fields can serve as seeds for the observed fields in galaxies and clusters
- Observations of fields with helicity will be a manisfestation of parity violation in the early universe.
- If detected in the CMB sky magnetic helicity can serve as probe of the inflation duration.

Thank you