Bounds on Axion Stars from Stability and Decay

Joshua Eby
with
Peter Suranyi, Cenalo Vaz, Rohana Wijewardhana

For more information, see arXiv: 1412.3430, 1512.01709

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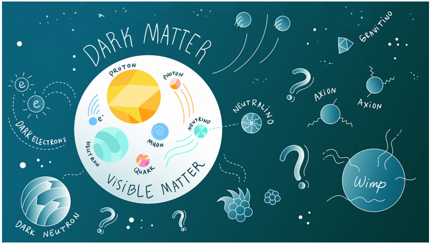
Outline

- Axions are a well-motivated scalar dark matter candidate, show up in QCD and elsewhere
- Boson Stars are gravitationally bound, macroscopic condensates of scalar fields (e.g. axions)
- Stability of axion stars can be analyzed through gravity, tunneling, and decay
- Open Questions and Future Work

Axions Can Solve Two Problems

The nature of Dark Matter (DM) remains an open question



[0] Image Credit: Symmetry Magazine

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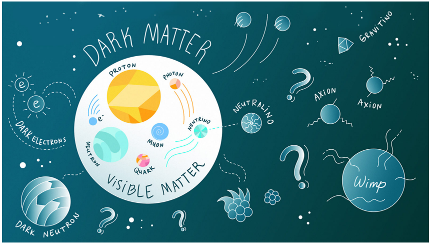

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$$d_n \lesssim 2.9 \times 10^{-26} \mathrm{e} \cdot \mathrm{cm}$$

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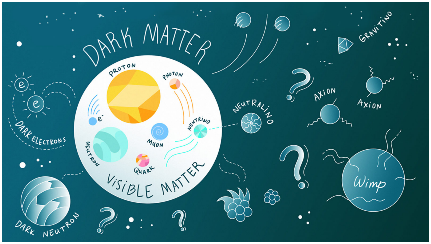

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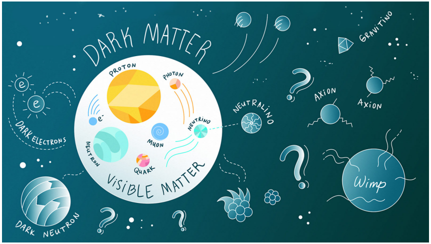

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The axion, a pseudo-goldstone boson from a high-scale symmetry breaking, can solve either one, or even both!

(Model Dependent)

(Requires $m\, f_a \approx 6 \times 10^{-3}\ \mathrm{GeV}^2$)

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- Typically noncompact: Wavefunctions extend to infinity
  - Define "size" R_{99} inside which .99 of the mass is contained
- Mass is bounded above generically by gravitational stability

Historical Interlude

- Ruffini and Bonazzola, "Systems of Self-Gravitating Particles in General Relativity..." (1969)
  - Second-quantized a free real scalar field, put all N particles in ground state $|N\rangle$
  - Took expectation value of Einstein+Klein Gordon equations in ground state
  - Maximum mass for bound states: $M_{max}^{free} = .633 M_P^2/m$

Chandrasekhar limit: $M_{max} \sim M_P^3/m^2$

Historical Interlude

- Ruffini and Bonazzola, "Systems of Self-Gravitating Particles in General Relativity..." (1969)
- Barranco and Bernal, "Self-Gravitating System Made of Axions" (2011)
  - Axion potential: $V(\mathcal{A}) = m^2 {f_a}^2 \Big(1 - \cos(\mathcal{A}/f_a)\Big) \approx \frac{m^2}{2}\mathcal{A}^2 + \ldots$
  - Used RB method to quantize field, found numerical solutions with sizes $R \sim 1 - 10$ m and masses $M \sim 10^{13} - 10^{14}$ kg.

Historical Interlude

- Ruffini and Bonazzola, "Systems of Self-Gravitating Particles in General Relativity..." (1969)
- Barranco and Bernal, "Self-Gravitating System Made of Axions" (2011)
- Eby, Suranyi, Vaz, and Wijewardhana, "Axion Stars in the Infrared Limit" (2014)
  - A simple variational calculation suggested to us that axion stars could have larger masses, of $\mathcal{O}\left(10^{19}\right)$ kg
  - Expanding on the work of RB and BB, we developed an expansion method which simplifies the equations of motion
  - We emphasize that the RB method is based on a Born approximation and is applicable only in the weak binding limit, where pair production and loop effects are negligible

Ruffini-Bonazzola Method

Consider in greater detail the RB method:

1. Begin with a canonically normalized second-quantized scalar field

$$\mathcal{A}(r,\theta,\phi) = \sum_{n,l,m} R_{n,l}(r)\Big[e^{iE_{n,l}t}Y_l^m(\theta,\phi)a_{n,l,m} + h.c.\Big]$$

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$$|N\rangle = \frac{\left(a^\dagger_{1,0,0}\right)^N}{\sqrt{N!}}|0\rangle$$

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3. Evaluate expectation value of EKG equations

$$\langle N|G^{\mu\nu}|N\rangle = \frac{1}{M_P^2}\langle N|T^{\mu\nu}|N\rangle$$

$$\langle N-1|\left[\Box\mathcal{A} - \frac{1}{2}W'(\mathcal{A})\right]|N\rangle = 0$$

Axion Stars in the Weak Binding Limit

The axion field theory can be defined by a low-energy potential, whose expectation value has the form

$$\begin{aligned} V(\mathcal{A}) &= m^2\, {f_a}^2 \langle N|1 - cos\Big(\mathcal{A}/f_a\Big)|N\rangle \\ &= m^2\, {f_a}^2 \Big[1 - J_0\Big(2\sqrt{N}R(r)/f_a\Big)\Big] \end{aligned}$$

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We also require weak gravity and weak binding, parameterized by

$$\delta \equiv \frac{{f_a}^2}{M_P^2} \ll 1 \qquad \Delta \equiv \sqrt{1 - \frac{E_0^2}{m^2}} \ll 1$$

We scale the axion wavefunction, radial coordinate, and metric functions accordingly:

$$x = \Delta\, m\, r,\ Y(x) = \frac{2\sqrt{N} R(r)}{f_a\, \Delta},$$
$$A(r) = 1 + \delta a(x),\ B(r) = 1 + \delta b(x)$$

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Leading Order Equations

To leading order in δ and Δ we find simplified set of equations for the axion wavefunction $Y(x)$ and metric functions $a(x)$ and $b(x)$:

$$a'(x) = \frac{x}{2}\,Y(x)^2 - \frac{a(x)}{x}$$

$$b'(x) = \frac{a(x)}{x}$$

$$Y''(x) = [1 + \kappa\, b(x)]\,Y(x) - \frac{2}{x}Y'(x) - \frac{1}{8}\,Y(x)^3$$

Depends on only one free parameter, $\kappa \equiv \delta/\Delta^2$.

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<u>Note:</u> Leading corrections are $\mathcal{O}(\delta)$ and $\mathcal{O}(\kappa\delta)$.

Requiring $\Delta \ll 1$ and $\kappa\delta \ll 1$ gives the range of validity:

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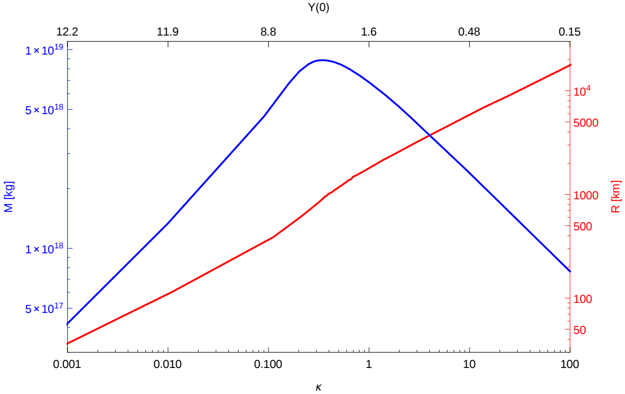
$$10^{-14} \sim \frac{{f_a}^2}{M_P^2} \ll \kappa \ll \frac{M_P^2}{{f_a}^2} \sim 10^{14}$$

for QCD axions with $m = 10^{-5}$ eV

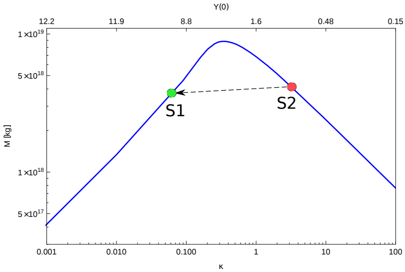
Gravitational Stability

- Maximum mass of weakly bound QCD axion stars with $m_a = 10^{-5}$ eV is $\mathcal{O}(10^{19})$ kg with radius $\mathcal{O}(100 - 1000)$ km
- Heavier stars are gravitationally unstable and must either shed excess axions ("bosenova"), or go into some other state (black hole, strongly bound state, etc.)



Tunneling Effects

- Each state on the right branch is matched by one on the left with equal N but lower total energy



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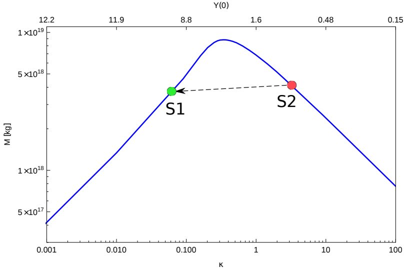

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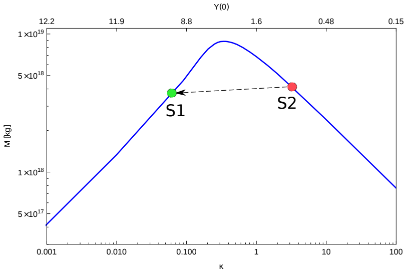

- The transition rate is suppressed by the very small factor e^{-N} (using WKB), but could potentially be catalyzed by collisions with e.g. neutron stars
- Using $M(\Delta) = Nm\sqrt{1-\Delta^2}$,

$$\frac{M_2 - M_1}{M_2} \approx \frac{1}{2}\left(\Delta_1^2 - \Delta_2^2\right)$$
$$= \frac{\delta}{2}\left(\kappa_1^{-1} - \kappa_2^{-1}\right)$$

Even a small mass fraction $\mathcal{O}(\delta \cdot M) = $ (“many” kg), and could have *significant observational effects!*

Axion Star Decay

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- In the nonrelativistic limit of real scalar fields, number changing operators are (justifiably) neglected, but they should be present in the full field theory
- We extend our expansion in ground-state operators to include a free axion term,

$$\mathcal{A}(r) = R(r)\, e^{i\, E_0\, t}\, a_0 + \int \frac{d^3 p}{\sqrt{2\, E_p}}\, a_p\, e^{i\, p\cdot r - i\, E_p\, t} + \text{h.}c.$$

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Axion stars can decay through self-interactions because:

1. Axions are Hermitian fields: their number is not conserved
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$$V_{int} \sim -\frac{m^2}{{f_a}^2}\mathcal{A}^4 + \mathcal{O}\left(\mathcal{A}^6\right)$$

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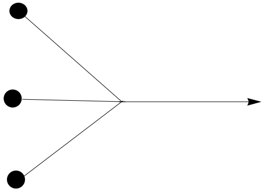
- Wavefunctions are localized on a radius $R \sim 1/(m\,\Delta)$
- Momentum uncertainty is thus $\delta p \sim m\,\Delta$



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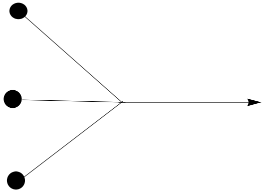

The process $3\,a \to a$ leads to a nonzero matrix element for

$$[N \text{ condensed}] \to [(N-3) \text{ condensed} + 1 \text{ emitted}]$$

giving axion stars a finite lifetime.

Lifetime of Axion Stars

We have calculated the leading-order decay rate (through $3\,a \to a$) for axion stars in the weak binding limit:

$$\Gamma_3 = \frac{{f_a}^2}{4\sqrt{2}\pi\, m}\Big|\mathcal{I}_3(\Delta)\Big|^2$$

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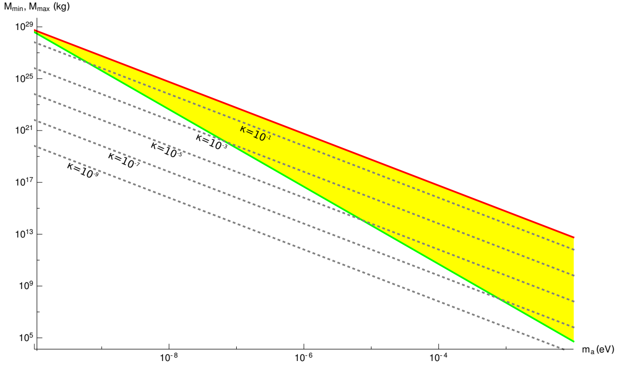
- Note: The other leading-order processes, $n\,a \to \mu a$ with $n \geq 4$ and/or $\mu \geq 2$, are suppressed by either $\Delta \ll 1$ or the ratio $m^2/{f_a}^2 \ll 1$.

Combined Bounds

Red line: Upper bound for gravitational stability
Green line: Lower bound from decay analysis



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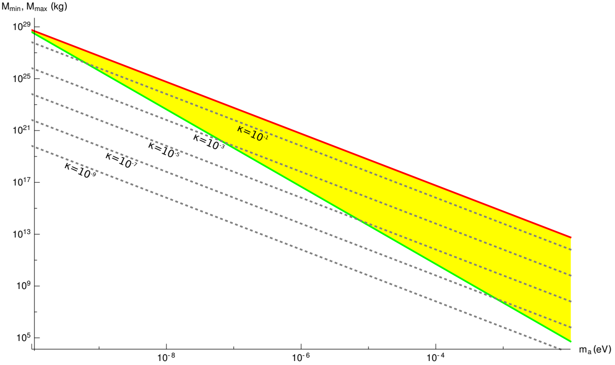
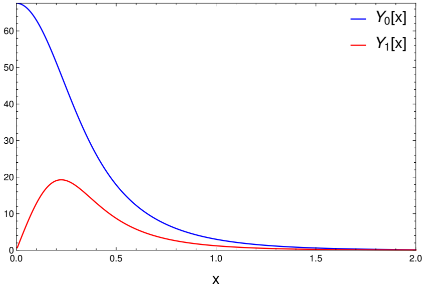


- Over a large region of the important parameter space, $\kappa \ll 1$
  - In this region, the gravitational interaction completely decouples from axion field
  - Attractive self-interaction balanced by kinetic pressure

Current and Future Work

- We are currently working to extend our method to the case of rotating stars, requiring an expansion in spherical harmonics

(Preliminary)



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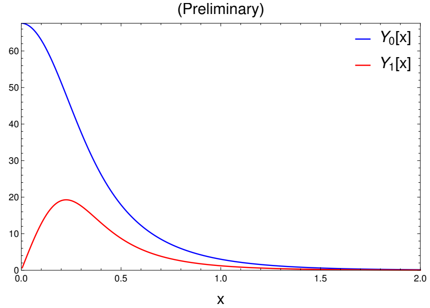

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- All of our results so far apply only to weakly bound stars. The case of strong binding requires an entirely new method

Conclusions

- Our expansion in δ and Δ simplifies the equations and elucidates important properties of axion stars
- The maximum mass of weakly bound QCD axion stars is $\mathcal{O}(10^{19})$ kg with $R_{99} \sim 1000$ km for $m = 10^{-5}$ eV
- Through stability analysis, we find significant constraints on the masses of weakly bound axion stars surviving from the early universe
- The cases of rotating or strongly bound axion stars are of great interest for the development of a realistic DM model and to obtain a full picture of the parameter space of axion stars
- Thanks!

Backup Slides

Backup: Strong CP Problem

The θ-term in the QCD Lagrangian violates CP:

$$\mathcal{L}_{QCD} \ni \theta \frac{g_s^2}{32\pi^2} G^{a\mu\nu} \tilde{G}^a_{\mu\nu} \tag{1}$$

But the lack of detection of a neutron EDM constrains the free parameter θ severely:

$$\begin{aligned} d_n \approx & 5 \times 10^{-16} \theta \text{ e} \cdot \text{cm} \lesssim 10^{-25} \text{ e} \cdot \text{cm} \\ & \Rightarrow \theta \lesssim 10^{-10} \end{aligned} \tag{2}$$

In principle, $\theta \sim \mathcal{O}(1)$. Why should it be so small?

Backup: Peccei-Quinn Mechanism

Solving the Strong-CP Problem:

- Promote $\theta \to a(x)/f$, a dynamical field, whose potential is minimized at $a(x) = 0$. Naturalness saved!
- The Lagrangian for $a(x)$ has a symmetry, $U(1)_{PQ}$, which is broken at the energy scale f
- The physical axion field is the Goldstone boson of $U(1)_{PQ}$-breaking

ϕ is initially massless, but acquires a mass during the QCD phase transition due to nonperturbative effects:

$$m_a = \frac{\Lambda_{QCD}^2}{f} \tag{3}$$

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- Suppose a scalar field Φ is complex and has total energy E_0. In the non-relativistic limit, $|E_0 - m| = \delta E \ll m$.

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- Let $\Phi = e^{-imt}\phi$. Then $|\dot{\phi}| \approx \delta E\,\phi \ll m\,\phi$, so

$$\ddot{\Phi} = m^2 \left(\underbrace{\frac{\ddot{\phi}}{m^2}}_{\mathcal{O}(\Delta^2)} \underbrace{-2\,i\,\frac{\dot{\phi}}{m}}_{\mathcal{O}(\Delta)} + \underbrace{\phi}_{\mathcal{O}(1)} \right) e^{-imt}$$

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Thus

$$0 = \left(\ddot{\Phi} - \nabla^2\Phi - m^2\,\Phi\right) \approx \left(-2\,i\,m\,\dot{\phi} - \nabla^2\phi\right) e^{-imt}$$

or

$$i\,\dot{\phi} = -\frac{1}{2m}\nabla^2\phi$$

Non-Relativistic Axions

Note that axions are real scalar fields, and the above procedure does not quite apply.

A similar method reduces the Klein-Gordon equation by expanding the axion field ϕ in terms of a complex one:[1]

$$\phi = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi + e^{imt}\psi^*\right]$$

[1] Guth, Hertzberg, and Prescod-Weinstein. Phys. Rev. D **92**, 103513 (2015). arXiv: 1412.5930.

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$$\phi = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi + e^{imt}\psi^*\right]$$

The resulting equation of motion for the new field ψ at $\Delta \ll 1$ is known as the Gross-Pitäevskii equation, coupled to gravity:

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - G\,m^2\,\psi\int d^3x'\,\frac{|\psi(r')|^2}{|r-r'|}$$

[1] Guth, Hertzberg, and Prescod-Weinstein. Phys. Rev. D **92**, 103513 (2015). arXiv: 1412.5930.

Non-Relativistic Axions

Note that axions are real scalar fields, and the above procedure does not quite apply.

A similar method reduces the Klein-Gordon equation by expanding the axion field ϕ in terms of a complex one:[1]

$$\phi = \frac{1}{\sqrt{2m}}\left[e^{-imt}\psi + e^{imt}\psi^*\right]$$

The resulting equation of motion for the new field ψ at $\Delta \ll 1$ is known as the Gross-Pitäevskii equation, coupled to gravity:

$$i\,\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + \frac{\lambda}{8m^2}|\psi|^2\psi - G\,m^2\,\psi\int d^3x'\,\frac{|\psi(r')|^2}{|r-r'|}$$

Note: Completely equivalent to our system at leading Δ.

[1] Guth, Hertzberg, and Prescod-Weinstein. Phys. Rev. D **92**, 103513 (2015). arXiv: 1412.5930.

Gross-Pitäevskii-Poisson System

In the non-relativistic limit, scalar field dynamics can be described by the Gross-Pitäevskii, or nonlinear Schrödinger, equation coupled to Poisson gravity:

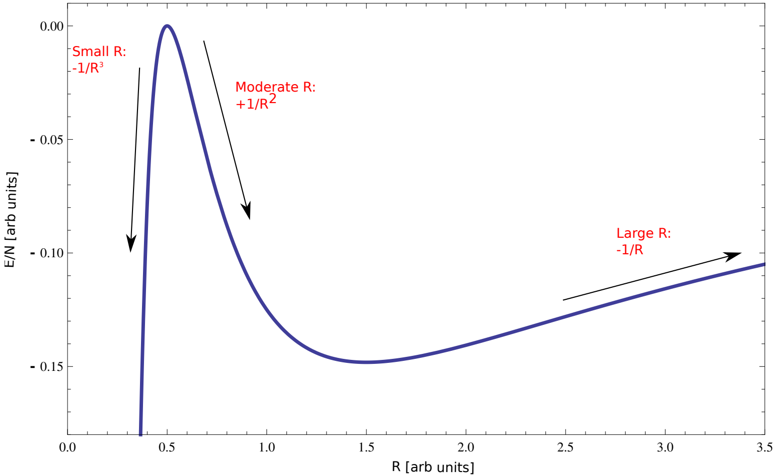
$$i\,\dot{\psi}(r) = -\frac{1}{2m}\nabla^2\psi(r) + m\,V(r)\psi(r) + \frac{\lambda}{8m^2}|\psi(r)|^2\psi(r)$$
$$\nabla^2 V(r) = 4\pi G\,m\,|\psi(r)|^2$$

The GPP energy is

$$E[\psi] = \int d^3r\,\Big[-\frac{1}{2m}|\nabla\psi|^2 + V\,\psi + \frac{\lambda}{16m^2}|\psi|^2\psi\Big]$$

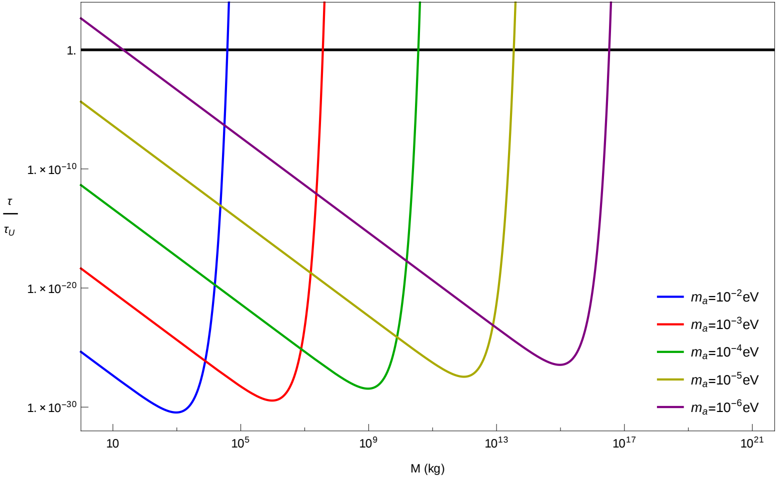
Variational Method

$$E(R) = \frac{A}{R^2} - \frac{B\,N}{R} - \frac{C\,N}{R^3}$$



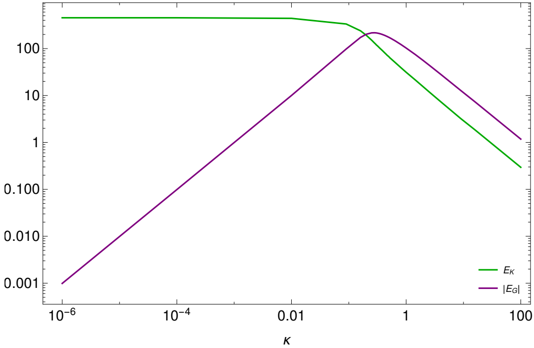
Lifetime Change with Mass

We required $\tau > \tau_U \approx 13.6$ billion years, but softening this requirement does not change much the lower bound on axion star masses



Decoupling of Gravity

When $\kappa \ll 1$, the total gravitational energy is significantly smaller than the kinetic or interaction energies



When $\kappa = \mathcal{O}(1)$, the terms are of roughly the same order

Chiral vs Instanton Potential

The instanton potential $V(\phi) = m^2 f_a{}^2\left(1 - J_0(\phi/f_a)\right)$ is really only an approximation. Axions are more correctly described by a chiral potential which takes into account QCD effects (see e.g. 1604.00669).

In fact, updating the potential in this way does not change our results much at all: