A Bottom-Up Approach to Lepton Flavor and Generalized CP Symmetries

Alexander J. Stuart

May 9, 2016

Pheno 2016

Based on: L.L. Everett, T. Garon, and AS, JHEP **1504**, 069 (2015) [arXiv:1501.04336], and work in progress with L.L. Everett

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Leptonic Dirac CP Violation Predictions from Residual Discrete Symmetries

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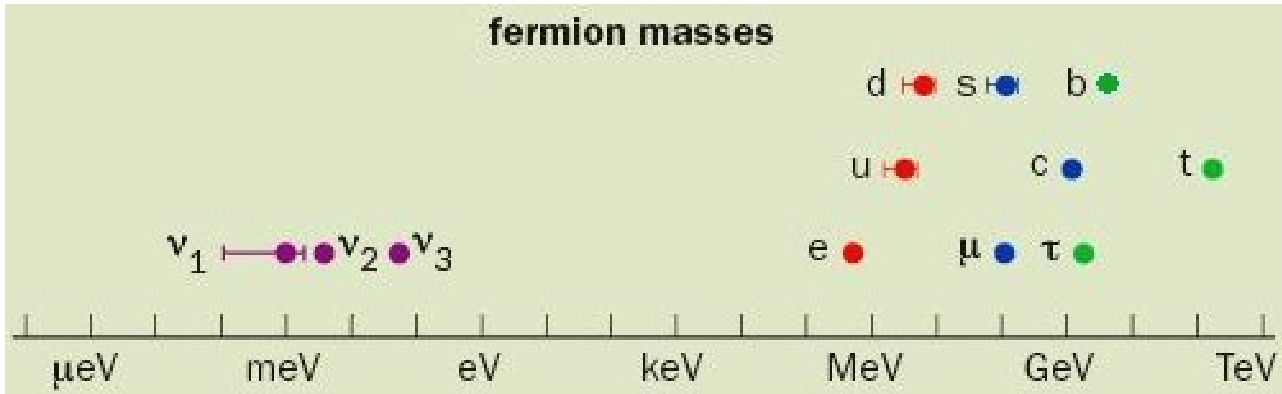
Based on: I. Girardi, S.T. Petcov, A.V. Titov, and AS, Nucl. Phys. B **902**, 1 (2016)[arXiv:1509.02502]

The Standard Model

Triumph of modern science, but incomplete-
Fails to predict the measured fermion masses and mixings.

http://www.particleadventure.org/standard_model.html

What We Taste



Quark Mixing

$$U_{CKM} = R_1(\theta_{23}^{CKM}) R_2(\theta_{13}^{CKM}, \delta_{CKM}) R_3(\theta_{12}^{CKM})$$

$$\theta_{13}^{CKM} = 0.2^\circ \pm 0.1^\circ$$

$$\theta_{23}^{CKM} = 2.4^\circ \pm 0.1^\circ$$

$$\theta_{12}^{CKM} = 13.0^\circ \pm 0.1^\circ$$

$$\delta_{CKM} = 60^\circ \pm 14^\circ$$

Lepton Mixing

$$U_{PMNS} = R_1(\theta_{23}) R_2(\theta_{13}, \delta_{CP}) R_3(\theta_{12}) P$$

$$\theta_{13} = 8.50^\circ \left({}^{+0.20^\circ}_{-0.21^\circ}\right)$$

$$\theta_{23} = 42.3^\circ \left({}^{+3.0^\circ}_{-1.6^\circ}\right)$$

$$\theta_{12} = 33.48^\circ \left({}^{+0.78^\circ}_{-0.75^\circ}\right)$$

$$\delta_{CP} = 306^\circ \left({}^{+39^\circ}_{-70^\circ}\right)$$

M.C. Gonzalez-Garcia et al: 1409.5439

http://lbne-old.fnal.gov/how-work.shtml

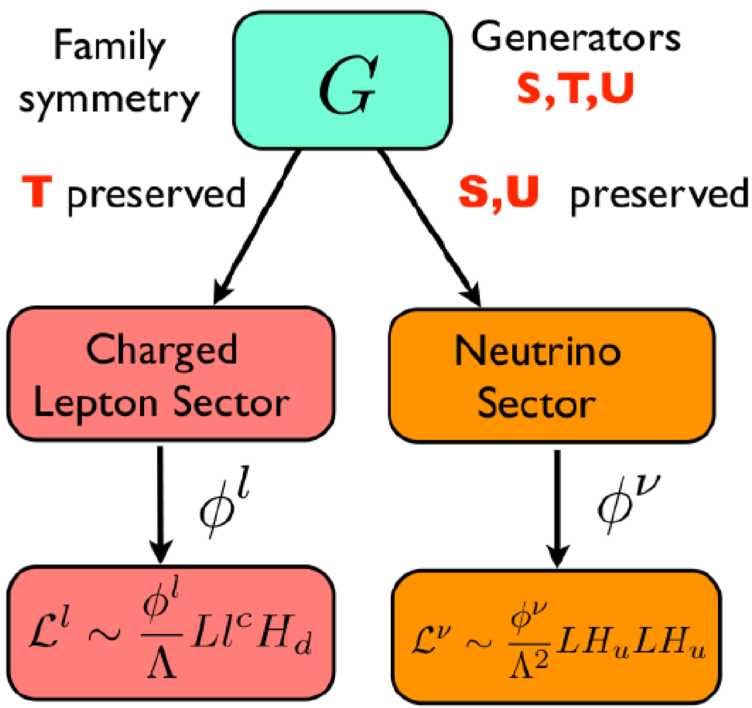
Focus on leptons.

Motivated by Symmetry

Introduce set of flavon fields (e.g. ϕ^{ν} and ϕ^{ℓ}) whose vevs break G_f to G_ν in the neutrino sector and G_e in the charged lepton sector.

$$T\langle\phi^{\ell}\rangle \approx \langle\phi^{\ell}\rangle \qquad S\langle\phi^{\nu}\rangle = U\langle\phi^{\nu}\rangle = \langle\phi^{\nu}\rangle$$

Non-renormalizable couplings of flavons to mass terms can be used to explain the smallness of Yukawa Couplings.



S.F. King, C. Luhn (2013)

Now that we better understand the framework, what can these symmetries be?

Residual Charged Lepton Symmetry

Since charged leptons are Dirac particles, consider $M_e = m_e m_e^\dagger$.
When **diagonal**, this combination is left invariant by a phase matrix

$$Q_e = \mathrm{Diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})$$

Because $Q_e^\dagger M_e Q_e = M_e$

$$\mathrm{Det}(Q_e) = +1 \implies \beta_1 = -\beta_2 - \beta_3$$

$$\text{Assume } \beta_{2,3} = 2\pi k_{2,3}/n_{2,3} \text{ with } k_{2,3} = 0, \ldots, n_{2,3} - 1$$

Supposed we keep all $T = Q_e$, then

$$G_e \cong Z_{n_2} \times Z_{n_3} = Z_n \times Z_m$$

Can apply same logic to neutrino sector if neutrinos are Dirac particles, but what if they are Majorana?

Residual Neutrino Flavor Symmetry

Key: Assume neutrinos are Majorana particles

$$U_\nu^T M_\nu U_\nu = M_\nu^{\rm Diag} = {\rm Diag}(m_1, m_2, m_3) = {\rm Diag}(|m_1|e^{-i\alpha_1}, |m_2|e^{-i\alpha_2}, |m_3|e^{-i\alpha_3})$$

Notice $U_\nu \to U_\nu Q_\nu$ with $Q_\nu = {\rm Diag}(\pm 1, \pm 1, \pm 1)$ also diagonalizes the neutrino mass matrix. Restrict to ${\rm Det}(Q_\nu) = 1$ and define $G_0^{\rm Diag} = 1$

$$G_1^{\rm Diag} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad G_2^{\rm Diag} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, G_3^{\rm Diag} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Observe non-trivial relations:

$$(G_i^{\rm Diag})^2 = 1, \text{ for } i\text{=}1,\ 2, \text{ and } 3,$$
$$G_i^{\rm Diag} G_j^{\rm Diag} = G_k^{\rm Diag}, \text{ for } i \neq j \neq k$$

Sometimes called *SU, S,* and *U*

Therefore, these form a $Z_2 \times Z_2$ residual Klein symmetry!

In non-diagonal basis: $M_\nu = G_i^T M_\nu G_i$ with $G_i = U_\nu G_i^{\rm Diag} U_\nu^\dagger$

What if we do not keep the whole Klein symmetry?

$G_\nu = Z_2^U$

Dictated by Simplicity

$$U = -\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$U = U_e^\dagger U_\nu = (\tilde{U}_e)^\dagger \Psi \tilde{U}_\nu Q_0$$

Frampton, Petcov, Rodejohann (2004)

$\tilde{U}_e$ and $\tilde{U}_\nu$ are CKM like matrices.

$$\Psi = \mathrm{diag}\left(1, \mathrm{e}^{-i\psi}, \mathrm{e}^{-i\omega}\right) \qquad Q_0 = \mathrm{diag}\left(1, \mathrm{e}^{i\frac{\xi_{21}}{2}}, \mathrm{e}^{i\frac{\xi_{31}}{2}}\right)$$

$$\tilde{U}_\nu = R_{23}\left(\theta_{23}^\nu\right) R_{12}\left(\theta_{12}^\nu\right) = \begin{pmatrix} \cos\theta_{12}^\nu & \sin\theta_{12}^\nu & 0 \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Marzocca, et al. (2013)
Petcov (2014)
Girardi, Petcov, Titov (2015)

Has this particular form been used in the literature? Said again, can we fix θ_{12}^ν by symmetry?

$G_\nu = Z_2^S \times Z_2^U$

Popular Forms

$$\tilde{U}_\nu = R_{23}\left(\theta_{23}^\nu\right) R_{12}\left(\theta_{12}^\nu\right) = \begin{pmatrix} \cos\theta_{12}^\nu & \sin\theta_{12}^\nu & 0 \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{\sin\theta_{12}^\nu}{\sqrt{2}} & \frac{\cos\theta_{12}^\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Marzocca, et al. (2013)
Petcov (2014)
Girardi, Petcov, Titov (2015)

TriBiMaximal (TBM) Mixing: $\sin^2\theta_{12}^\nu = 1/3$

BiMaximal (BM) Mixing: $\sin^2\theta_{12}^\nu = 1/2$

Golden Ratio A (GRA) Mixing: $\sin^2\theta_{12}^\nu = (2+r)^{-1} \cong 0.276$

$r = (1+\sqrt{5})/2$

Golden Ratio B (GRB) Mixing: $\sin^2\theta_{12}^\nu = (3-r)/4 \cong 0.345$

HexaGonal (HG) Mixing: $\sin^2\theta_{12}^\nu = 1/4$

These forms 'naturally' arise from simpler symmetries, e.g., A_4, S_4, A_5
However, they all have a vanishing reactor mixing angle θ_{13}.

Could assume a larger flavor symmetry, e.g., Δ(600)(Lam (2013); Holthausen et al. (2012)),
To better match reactor mixing data (U_{13}=0.170), but then what's the point?

Charged Lepton Corrections

For example, assume the theoretically motivated form:

(Marzocca, Petcov, Romanino, Sevilla (2013); Petcov(2014))

$$\tilde{U}_e = R_{23}^{-1}(\theta_{23}^e)\, R_{12}^{-1}(\theta_{12}^e)$$

This form gives rise to a nonzero reactor mixing angle when applied to $\tilde{U}_\nu$, i.e.,

$$\sin\theta_{13} = |U_{e3}| = \sin\theta_{12}^e \sin\hat{\theta}_{23} \quad \sin^2\hat{\theta}_{23} = \frac{1}{2}\left(1 - 2\sin\theta_{23}^e\cos\theta_{23}^e\cos(\omega-\psi)\right)$$

It also give rise to a "sum rule" relating the Dirac CP-violating phase to the mixing parameters, i.e.,

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12}\sin\theta_{13}}\left[\cos 2\theta_{12}^\nu + \left(\sin^2\theta_{12} - \cos^2\theta_{12}^\nu\right)\left(1 - \cot^2\theta_{23}\,\sin^2\theta_{13}\right)\right]$$

Notice that the free parameter from 'lack of symmetry' enters sum rule.

Use sum rule to analyze complete parameter space of Dirac phase associated with specific breaking patterns yielding viable phenomenological predictions for the groups A_4, S_4, A_5 !

Procedure

$$G_f = A_4, T', S_4, A_5$$

Although results are generic until restricted to one of these groups.

1) $G_e = Z_2$ and $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$;

2) $G_e = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$ and $G_\nu = Z_2$;

3) $G_e = Z_2$ and $G_\nu = Z_2$;

4) G_e is fully broken and $G_\nu = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$;

5) $G_e = Z_n$, $n > 2$ or $Z_n \times Z_m$, $n, m \geq 2$ and G_ν is fully broken.

Perhaps an example might help?

Example - Case 2

$$U^\circ(\theta_{12}^\circ, \theta_{13}^\circ, \theta_{23}^\circ, \delta_{13}^\circ) = R_{23}(\theta_{23}^\circ) R_{12}(\theta_{12}^\circ) U_{13}(\theta_{13}^\circ, \delta_{13}^\circ)$$

$$U = R_{23}(\theta_{23}^\circ) R_{12}(\theta_{12}^\circ) U_{13}(\theta_{13}^\circ, \delta_{13}^\circ) U_{13}(\theta_{13}^\nu, \delta_{13}^\nu) Q_0$$

By comparing MNSP elements in PDG convention to above form yields the sum rule:

$$\cos\delta = -\frac{\cos^2\theta_{13}(\cos^2\theta_{12}^\circ \cos^2\theta_{23}^\circ - \cos^2\theta_{23}) + \sin^2\theta_{12}^\circ(\cos^2\theta_{23} - \sin^2\theta_{13}\sin^2\theta_{23})}{\sin 2\theta_{23} \sin\theta_{13} |\sin\theta_{12}^\circ| (\cos^2\theta_{13} - \sin^2\theta_{12}^\circ)^{\frac{1}{2}}}$$

where

$$\sin^2\theta_{23} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2} = \frac{1}{\cos^2\theta_{13}} \left[\cos^2\theta_{23}^\circ \sin^2\hat{\theta}_{13} \sin^2\theta_{12}^\circ + \cos^2\hat{\theta}_{13} \sin^2\theta_{23}^\circ \right.$$
$$\left. - \frac{1}{2} \sin 2\hat{\theta}_{13} \sin 2\theta_{23}^\circ \sin\theta_{12}^\circ \cos\hat{\delta}_{13} \right],$$

$$\sin^2\theta_{13} = |U_{e3}|^2 = \cos^2\theta_{12}^\circ \sin^2\hat{\theta}_{13}$$

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} = \frac{\sin^2\theta_{12}^\circ}{\cos^2\theta_{13}}$$

Application to A_4

$(G_e, G_\nu) = (Z_3, Z_2)$	$\cos\delta$	$\sin^2\theta_{12}$
B1 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	0.570	0.341

$$(\sin^2\theta_{12})_{\rm BF} = 0.308\,, \qquad 0.259 \leq \sin^2\theta_{12} \leq 0.359$$

$$(\sin^2\theta_{23})_{\rm BF} = 0.437\ (0.455)\,, \qquad 0.374\ (0.380) \leq \sin^2\theta_{23} \leq 0.626\ (0.641)$$

$$(\sin^2\theta_{13})_{\rm BF} = 0.0234\ (0.0240)\,, \qquad 0.0176\ (0.0178) \leq \sin^2\theta_{13} \leq 0.0295\ (0.0298)$$

Best fit and 3σ of Capozzi et al, (2014) of Normal (Inverted) Ordering

See 1509.02502 for different cases involving various charged lepton rotations as well as the groups $G_f = A_4, T', S_4, A_5$

Conclusion

- It is possible to understand the observed pattern of 3-neutrino oscillation with discrete family symmetries.
- The residual symmetries can constrain the forms of U_e and U_ν and hence constrain the form of the MNSP mixing matrix.
- This can lead to a correlation between the neutrino mixing angles and the Dirac CP-violating phase, i.e., a 'sum rule'.
- In 1509.02502, we have considered all possible breaking patterns of an arbitrary discrete symmetry group G_f and applied these results to $G_f = S_4, A_4, T', A_5$.
- The results of this study show that with the accumulation of more precise data on the mixing angles and the measurement of the Dirac CP-violating phase, it is possible to critically test currently phenomenologically viable theories based on discrete family symmetries.

Backup Slides

Patterns Considered

$G_e \subset G_f$	$G_\nu \subset G_f$	U_e d.o.f.	U_ν d.o.f.	U d.o.f.
fully broken	fully broken	$9 \to 6$	$9 \to 8$	$12 \to 4\ (+2)$
Z_2	fully broken	$4 \to 2$	$9 \to 8$	$10 \to 4\ (+2)$
$\left\{ \begin{array}{c} Z_n,\ n > 2 \text{ or} \\ Z_n \times Z_m,\ n,m \geq 2 \end{array} \right\}$	fully broken	0	$9 \to 8$	$8 \to 4\ (+2)$
fully broken	Z_2	$9 \to 6$	4	$10 \to 4\ (+2)$
fully broken	$\left\{ \begin{array}{c} Z_n,\ n > 2 \text{ or} \\ Z_n \times Z_m,\ n,m \geq 2 \end{array} \right\}$	$9 \to 6$	2	$8 \to 4\ (+2)$
Z_2	Z_2	$4 \to 2$	4	$4\ (+2)$
$\left\{ \begin{array}{c} Z_n,\ n > 2 \text{ or} \\ Z_n \times Z_m,\ n,m \geq 2 \end{array} \right\}$	Z_2	0	4	$2\ (+2)$
Z_2	$\left\{ \begin{array}{c} Z_n,\ n > 2 \text{ or} \\ Z_n \times Z_m,\ n,m \geq 2 \end{array} \right\}$	$4 \to 2$	2	$2\ (+2)$
$\left\{ \begin{array}{c} Z_n,\ n > 2 \text{ or} \\ Z_n \times Z_m,\ n,m \geq 2 \end{array} \right\}$	$\left\{ \begin{array}{c} Z_n,\ n > 2 \text{ or} \\ Z_n \times Z_m,\ n,m \geq 2 \end{array} \right\}$	0	2	$0\ (+2)$

Degrees of Freedom relevant for MNSP angles and phases. Arrows indicate reduction of parameters by redefinition of charged lepton fields. Parentheses indicate inclusion of Majorana phases.

Results (S_4)

$(G_e, G_\nu) = (Z_2, Z_2)$	$\cos\delta$	$\sin^2\theta_{ij}$
C1 $\sin^2\theta^\circ_{23} = 1/4$	-0.806	not fixed
C2 $\sin^2\theta^\circ_{23} = 1/2$	not fixed	$\sin^2\theta_{23} = 0.512$
C3 $\sin^2\theta^\circ_{13} = 1/4$	-1^*	not fixed
C4 $\sin^2\theta^\circ_{12} = 1/4$	0.992	not fixed
C5 $\sin^2\theta^\circ_{12} = 1/4$	not fixed	$\sin^2\theta_{12} = 0.256$
C7 $\sin^2\theta^\circ_{23} = 1/2$	not fixed	$\sin^2\theta_{23} = 0.488$
C8 $\sin^2\theta^\circ_{23} = \{1/2, 3/4\}$	$\{-1^*, 1^*\}$	not fixed
$(G_e, G_\nu) = (Z_3, Z_2)$	$\cos\delta$	$\sin^2\theta_{12}$
B1 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	0.570	0.341
B2 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (1/6, 1/5)$	-0.269	0.317
$(G_e, G_\nu) = (Z_4, Z_2),\ (Z_2 \times Z_2, Z_2)$	$\cos\delta$	$\sin^2\theta_{12}$
B1 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/4, 1/3)$	-1^*	0.256
$(G_e, G_\nu) = (Z_2, Z_4),\ (Z_2, Z_2 \times Z_2)$	$\cos\delta$	$\sin^2\theta_{23}$
A1 $(\sin^2\theta^\circ_{13}, \sin^2\theta^\circ_{23}) = (1/3, 1/4)$	-1^*	0.488
A2 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/2, 1/2)$	1^*	0.512

Results (A_5)

$(G_e, G_\nu) = (Z_2, Z_2)$	$\cos\delta$	$\sin^2\theta_{ij}$
C1 $\sin^2\theta^\circ_{23} = 1/4$	-0.806	not fixed
C3 $\sin^2\theta^\circ_{13} = 0.0955,\ 1/4$	$0.688,\ -1^*$	not fixed
C4 $\sin^2\theta^\circ_{12} = 0.0955,\ 1/4$	$-1^*,\ 0.992$	not fixed
C5 $\sin^2\theta^\circ_{12} = 1/4$	not fixed	$\sin^2\theta_{12} = 0.256$
C8 $\sin^2\theta^\circ_{23} = 3/4$	1^*	not fixed
C9 $\sin^2\theta^\circ_{12} = 0.3455$	not fixed	$\sin^2\theta_{12} = 0.330$
$(G_e, G_\nu) = (Z_3, Z_2)$	$\cos\delta$	$\sin^2\theta_{12}$
B1 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (1/3, 1/2)$	0.570	0.341
$(G_e, G_\nu) = (Z_5, Z_2)$	$\cos\delta$	$\sin^2\theta_{12}$
B1 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (0.2764, 1/2)$	0.655	0.283
B2 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (0.1382, 0.1604)$	-0.229	0.259
$(G_e, G_\nu) = (Z_2 \times Z_2, Z_2)$	$\cos\delta$	$\sin^2\theta_{12}$
B2 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{13}) = (0.0955, 0.2764)$	-1^*	0.330
$(1/4, 0.1273)$	0.805	0.330
$(G_e, G_\nu) = (Z_2, Z_3)$	$\cos\delta$	$\sin^2\theta_{23}$
A1 $(\sin^2\theta^\circ_{13}, \sin^2\theta^\circ_{23}) = (0.2259, 0.4363)$	0.716	0.553
A2 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (0.2259, 0.4363)$	-0.716	0.447
$(G_e, G_\nu) = (Z_2, Z_5)$	$\cos\delta$	$\sin^2\theta_{23}$
A1 $(\sin^2\theta^\circ_{13}, \sin^2\theta^\circ_{23}) = (0.4331, 0.3618)$	-1^*	0.630
A2 $(\sin^2\theta^\circ_{12}, \sin^2\theta^\circ_{23}) = (0.4331, 0.3618)$	1^*	0.370