Classification of 4d rank-1 $\mathcal{N} = 2$ superconformal field theories

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in collaboration with

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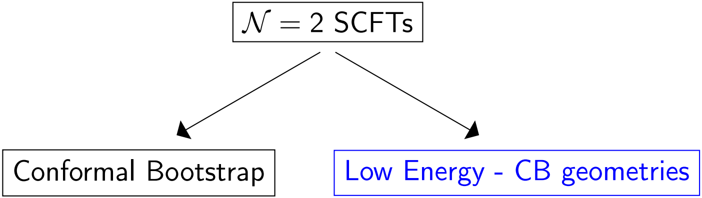
Why $\mathcal{N} = 2$ SCFTs?

- CFTs are great venues to study exact properties of QFTs, independent of perturbative expansions;
- $\mathcal{N} = 2$ is an Ideal amount of SUSY: provides enough constraints, at the same time theories have rich properties to be determined.
- Approaches to the problem:



- But what does it mean to study these theories?
  What we really want to do is to extract information on the matter content and the flavor structure.

Lagrnagian vs. Geometrical data

- The most obvious approach would be:
  write down the $\mathcal{N} = 2$ Lagrangian: analyze its field content and properties.

Problem

We do not have a Lagrangian description for numerous $\mathcal{N} = 2$ SCFTs.

- It turns out in $\mathcal{N} = 2$ SUSY we have more powerful techniques: we can study the geometry of the *moduli spaces of vacua*.
- Analyzing the geometry of the Coulomb Branch (CB) provides us with the structure of the underlying SCFT.

Moduli spaces of vacua

In general, supersymmetric theories have extended moduli spaces of vacua:

- Coulomb Branch (CB);
- Higgs Branch

We will be focusing on Coulomb Branches, parametrized by vevs coming from the vector multiplets:

$$\begin{array}{ccccccc} & \swarrow\!\!\nearrow & \lambda_\alpha & \leftrightarrow & A_\mu & & \\ \phi & \leftrightarrow & \tilde{\lambda}_\alpha & \swarrow\!\!\nearrow & & \rightarrow & \langle\phi\rangle = a \end{array}$$

On CBs there is a residual gauge symmetry (unbroken by the vev(s)) that lets us put some constraints on their geometries.

The coordinate used to describe the Coulomb Branch will be the gauge invariant parameter $u = \frac{1}{2}a^2$.

Electric-Magnetic Duality (1)

We are interested in rank-1 Coulomb Branches, one complex dimensional spaces. In this case the residual gauge symmetry is just a $U(1)$.

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The low energy theory has an $\mathcal{N} = 2$ SUSY version of QED. Maxwell theory can be written in a symmetric way between electric and magnetic charges and fields.

The gauge coupling can be restated as:

$$\tau \equiv \frac{4\pi i}{g^2} + \frac{\theta}{2\pi},$$

that combines the two two real components g and θ, coming from:

$$\mathcal{L} = \frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{16\pi} \epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

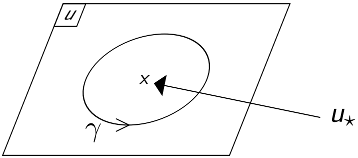
It will turn out that τ is the the main object of study in our construction.

Gauge coupling and monodromies

- Now, we obtain equivalent theories by redefining electric and magnetic charges/fields, this is encoded in transformations of τ of the from:

$$\tau(u) \to \frac{a\tau + b}{c\tau + d} \qquad M_\gamma = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

- On the Coulomb Branch singular points encode these transformations of the gauge coupling:

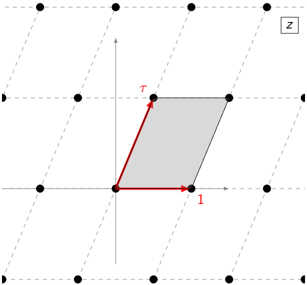


The monodromy γ uniquely characterizes the singularity at $u_\star$.

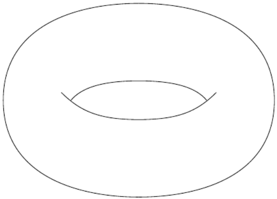
Description of $\tau(u)$

There are several ways to describe the gauge coupling $\tau(u)$:

- Complex torus:



$\leftrightarrow$



- Elliptic curve

$$y^2 = x^3 + f(u)x + g(u)$$

Coulomb Branches - scale invariant case

We now have the ingredients to determine the geometric features of our Coulomb Branches.

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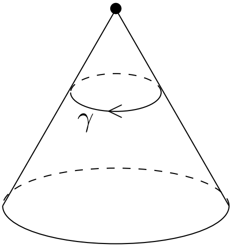
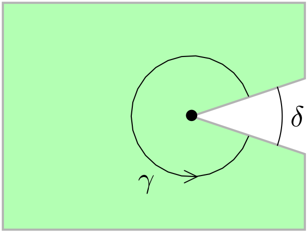
From constraints of $\mathcal{N} = 2$ SUSY and EM Duality,
the allowed CB scale invariant geometries are complex cones:





with a discrete set of allowed deficit angles δ.

$$\left\{ \begin{array}{c|ccccccc} \delta & \pi/3 & \pi/2 & 2\pi/3 & \pi & 4\pi/3 & 3\pi/2 & 5\pi/3 \\ \Delta(u) & 6 & 4 & 3 & 2 & 3/2 & 4/3 & 6/5 \end{array} \right\}$$

Scale invariant CB geometries

Allowed geometries correspond to Kodaira classification of elliptic curves:

Singularity	Curve $y^2 =$	$D(u)$	$\mathrm{ord}_0(\Delta)$	M_0	δ
II^*	$x^3 + u^5$	6	10	ST	$\pi/3$
III^*	$x^3 + u^3 x$	4	9	S	$\pi/2$
IV^*	$x^3 + u^4$	3	8	$-(ST)^{-1}$	$2\pi/3$
I_0^*	$x^3 + \tau u^2 x + u^3$	2	6	$-I$	π
IV	$x^3 + u^2$	3/2	4	$-ST$	$4\pi/3$
III	$x^3 + ux$	4/3	3	S^{-1}	$3\pi/2$
II	$x^3 + u$	6/5	2	$(ST)^{-1}$	$5\pi/3$
$I_{n\geq 1}$	$(x-1)(x^2 + (u/\Lambda)^n)$	1	n	T^n	2π (cusp)
$I^*_{n>1}$	$x^3 + ux^2 + u^{n+3}\Lambda^{-2n}$	2	$n+6$	$-T^n$	2π (cusp)

Table: Col 1: Kodaira type of singularity. Col 2: Scale invariant SW curve. Col. 3: Mass dimension of u. Col.4: Order of the discriminant. Col.5: Monodromy around singularity. Col.6: Deficit angle.

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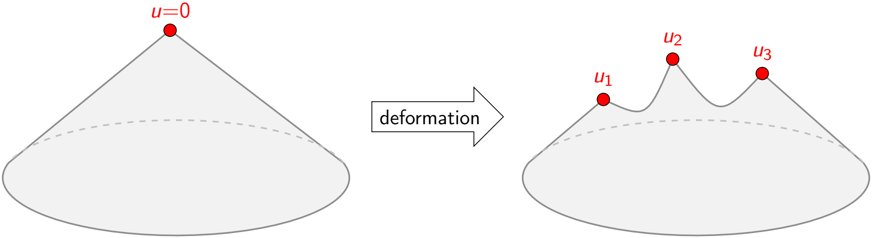
However, it can't be the end of the story, there are multiple examples of **SCFTs with the same Kodaira singularity**. $\rightarrow$ We need something more.

Introducing (mass) deformations

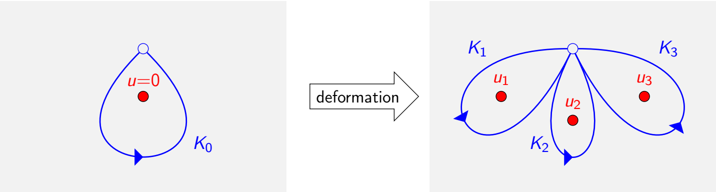
From the Lagrangian perspective it is clear how to introduce "deformations" $\rightarrow$ add mass terms for the hypermultiplets in the theory.

What do we do in general though, in cases where we don't have necessarily have a Lagrangian?
From the CFT point of view, introduce *relevant* $\mathcal{N}=2$ *preserving operators*:



Constraints on allowed deformations

We see that introducing deformations splits the singularities on the CB



We impose some constraints on the allowed ones:

- Special Kähler condition: overall monodromy is the product of the monodromies around each singularity (see figure below);
- Dirac quantization condition: charges of the matter multiplets becoming massless have to be commensurate.

Determining the flavor structure

- For the explicit calculation, deformations are introduced on the elliptic curves:

$$y^2 = x^3 + f(u)x + g(u) \to y^2 = x^3 + f(u, M_\alpha)x + g(u, M_\alpha)$$

- The position of the singularities is determined by the zeros of the discriminant of the curve:

$$\Delta_x(u) = 4f^3(u, M_\alpha) + 27g^2(u, M_\alpha)$$

- M_α are Weyl invariants of the flavor algebra. By introducing linear masses which reconstruct these polynomials, we identify the flavor structure of each deformed geometry.

General set of allowed deformations

Safe deformations of regular, rank 1, scale-invariant CBs							
Kodaira singularity	deformation pattern	flavor symmetry	central charges			Higgs branches	
			k_F	$12 \cdot c$	$24 \cdot a$	h_1	h_0
II^*	$\{{I_1}^{10}\}$	E_8	12	62	95	0	29
	$\{{I_1}^6, I_4\}$	$\mathrm{sp}(10)$	7	49	82	5	16
III^*	$\{{I_1}^9\}$	E_7	8	38	59	0	17
	$\{{I_1}^5, I_4\}$	$\mathrm{sp}(6) \oplus \mathrm{sp}(2)$	$5 \oplus 8$	29	50	3	8
IV^*	$\{{I_1}^8\}$	E_6	6	26	41	0	11
	$\{{I_1}^4, I_4\}$	$\mathrm{sp}(4) \oplus \mathrm{u}(1)$	$4 \oplus ?$	19	34	2	4
	$\{I_1, I_1^*\}$	$\mathrm{u}(1)$	?	$14+h$	$29+h$	h	?
I_0^*	$\{{I_1}^6\}$	$\mathrm{so}(8)$	4	14	23	0	5
	$\{{I_1}^2, I_4\} \simeq \{{I_2}^3\}$	$\mathrm{sp}(2)$	3	9	18	1	1
IV	$\{{I_1}^4\}$	$\mathrm{su}(3)$	3	8	14	0	2
III	$\{{I_1}^3\}$	$\mathrm{su}(2)$	8/3	6	11	0	1
II	$\{{I_1}^2\}$	–	–	22/5	43/5	0	0
with the assumption of a frozen IV* SCFT with central charge c'							
II^*	$\{I_2, IV^*\}$	$\mathrm{su}(2)$	?	$24c'+h+4$	$24c'+h+37$	h	?
or							
II^*	$\{{I_1}^2, IV^*\}$	G_2	?	$24c'+h+10$	$24c'+h+43$	h	?
III^*	$\{I_1, IV^*\}$	$\mathrm{su}(2)$	?	$16c'+h+\frac{10}{3}$	$16c'+h+\frac{73}{3}$	h	?
with the assumption of a frozen III* SCFT with central charge c''							
II^*	$\{I_1, III^*\}$	$\mathrm{su}(2)$	?	$18c''+h+5$	$18c''+h+38$	h	?

Mismatch in the Weyl groups? (1)

Alternative approaches:

- I. Garcìa-Etxebarria and D. Regalado, *arXiv:1512.06434*;
- O. Chacaltana, J. Distler, and A. Trimm, *arXiv:1601.02077*.

constructed theories that didn't match our classification

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*h*owever

our original assumption was that the symmetry group of the CB geometries $\Gamma = Weyl(F)$.
We can instead interpret the symmetries as:

$$\mathfrak{f} \simeq \Gamma'' \ltimes F_{\Gamma'},$$

a subgroup $\Gamma' \subset \Gamma$ is the Weyl group of the flavor algebra F and there are additional factors Γ''.

Revised flavor assignments

Some rank 1 $\mathcal{N}=2$ SCFTs

Kodaira singularity	deformation pattern	flavor symmetry	central charges k_F	$12\cdot c$	$24\cdot a$	Higgs branches h_1	h_0
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	$\{{I_1}^6, I_4\}$	C_5	7	49	82	5	16
	$\{{I_1}^3, I_1^*\}$	$A_3 \rtimes \mathbb{Z}_2$	14	42	75	4	9
	$\{{I_1}^2, IV^*_{Q=1}\}$	$A_2 \rtimes \mathbb{Z}_2$	14	38	71	3	?
	$\{I_2, IV^*_{Q=\sqrt{2}}\}$	$\mathrm{u}(1) \rtimes \mathbb{Z}_2$	?	33	66	1	1
	$\{I_1, III^*\}$	$\mathrm{u}(1) \rtimes \mathbb{Z}_2$	?	33	66	1	1
III^*	$\{{I_1}^9\}$	E_7	8	38	59	0	17
	$\{{I_1}^5, I_4\}$	$C_3 \oplus A_1$	$5 \oplus 8$	29	50	3	8
	$\{{I_1}^2, I_1^*\}$	$A_1 \oplus (\mathrm{u}(1) \rtimes \mathbb{Z}_2)$	$10 \oplus ?$	24	45	2	?
	$\{I_1, IV^*_{Q=1}\}$	$\mathrm{u}(1) \rtimes \mathbb{Z}_2$	?	21	42	1	1
	III^*	$\varnothing$	–	18	39	0	0
IV^*	$\{{I_1}^8\}$	E_6	6	26	41	0	11
	$\{{I_1}^4, I_4\}$	$C_2 \oplus \mathrm{u}(1)$	$4 \oplus ?$	19	34	2	4
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Thank you!