Analytic control of jet substructure

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Based on a paper with Gregory Soyez and Mirinal Dasgupta

Pheno 2016 - May 10, 2016
Outline

1. Introduction: jets at the LHC
2. Jet substructure: looking inside the jet
3. Control of jet substructure: an analytical understanding
4. Conclusion and prospectives
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2. Jet substructure: looking inside the jet

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4. Conclusion and prospectives
Introduction

- QCD partons produced in collisions cannot be directly observed;
- Due to QCD collinear divergence, their final state are complex collimated structures called jets;

Jets are used very frequently in LHC analyses.
A \textit{jet definition} is how one clusters particles into jets;

Composed of a \textit{clustering algorithm} (e.g. anti-$k_t$) and its \textit{parameters} (e.g. the jet radius $R$);

Introduction: jets at the LHC

Jet substructure: looking inside the jet

Control of jet substructure: an analytical understanding

Conclusion and prospectives
Boosted heavy particles

- At the LHC $\rightarrow$ boosted heavy particles ($p_t \gg m$)
  $\rightarrow$ decay in very collimated final states
  $\rightarrow$ clustered into a single jet

$W/Z/H$

$p_t \lesssim m$

$p_t \gg m$

- Characteristic opening angle of the jet is $\theta \propto \frac{m}{p_t}$.
QCD background

- Jets originated by partons ($g$ or $q$) are collimated for any $p_t$;
QCD background

- Jets originated by partons ($g$ or $q$) are collimated for any $p_t$;

- How to discriminate QCD jets from $Z/W/H \rightarrow$ hadrons jets?
In order to identify a jet we need to access the jet substructure to look inside the jet.

Different techniques are available:

1. Find hard cores (1 for QCD, 2 for bosons);
2. Constrain the soft gluon radiation.
In order to identify a jet we need to access the jet substructure to look inside the jet;

Different techniques are available:
1. Find hard cores (1 for QCD, 2 for bosons);
2. Constrain the soft gluon radiation. ← focus of this talk

See e.g. M. Dasgupta, A. Fregoso, S. Marzani and G. Salam (1307.0007) for similar study in core finders.
Jet shapes

- Jet-shapes: observables which are functions of the jet constituents \( \nu(p_1^\mu, p_2^\mu, \ldots, p_n^\mu) \rightarrow \) measure the radiation within the jet;

- **Energy correlation**

\[
C_2 = \frac{e_3}{(e_2)^2},
\]
\[
e_2 = \frac{1}{p_t^2 R^2} \sum_{i<j \in \text{jet}} p_{t,i} p_{t,j} \theta_{ij}^2,
\]
\[
e_3 = \frac{1}{p_t^3 R^6} \sum_{i<j<k \in \text{jet}} p_{t,i} p_{t,j} p_{t,k} \theta_{ij}^2 \theta_{ik}^2 \theta_{jk}^2.
\]
**N-subjettiness** with axes $a_1, \ldots, a_N$

$$\tau_{21} = \frac{\tau_2}{\tau_1}, \quad \tau_N = \frac{1}{p_{t,jet} R^2} \sum_{i \in \text{jet}} p_{t,i} \min_{a_i \ldots a_N} (\theta_{i a_1}^2, \ldots, \theta_{i a_N}^2).$$

**Mass-drop** with subjets $j_1$ and $j_2$

$$\mu_p^2 = \max(m_{j_1}^2, m_{j_2}^2)/m_j^2.$$
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Control of jet substructure

- Understand differences/similarities from a first-principle analytical study;

- Compute physical quantities (cross-section, efficiency curves) with a cut $v_{cut}$ on jet shape;

- We assume $v_{cut} \ll 1$. 
Lund diagrams

- Lund diagram: graphical representation of emissions in $z\theta$ vs. $1/\theta^2$. 

![Diagram showing Lund diagram with axes labeled log($z\theta$) vs. log($1/\theta^2$)]
Structure of the results (QCD background)

- We consider *boosted jets* of a *given mass*, \( \rho = m^2/\rho_t^2 R^2 \ll 1; \)
- Approximation: emissions are strongly ordered in mass and angle;
- Independent emissions \( \rightarrow \) constraints as an exponential factor.

\[ \frac{\rho}{\sigma} \frac{d\sigma}{d\rho} = \int_0^1 dz_1 P_i(z_1) \frac{\alpha_s}{2\pi} e^{-R_{\text{plain}}(\rho)} \]

\[ R_{\text{plain}}(\rho) = \int_0^1 \frac{d\theta^2}{\theta^2} \int_0^1 dz P_i(z) \frac{\alpha_s}{2\pi} \Theta(z\theta^2 > \rho) \]
Structure of the results (QCD background)

For a jet of a given mass + a cut in the jet shape $v_{cut}$:

$$\frac{\rho}{\sigma} \frac{d\sigma}{d\rho} \bigg|_{v} = \int_{\rho}^{1} dz_1 P(z_1) \frac{\alpha_s}{2\pi} e^{-R_{plain}(\rho) - R_{v}(\rho, z_1)}$$

$$R_{v}(\rho, z_1) = \int_{0}^{1} \frac{d\theta_2^2}{\theta_2^2} \int_{0}^{1} dz_2 P_i(z_2) \frac{\alpha_s}{2\pi} \times \Theta(v > v_{cut}) \Theta(z_2 \theta_2^2 < \rho)$$

$$+ \int_{0}^{1} \frac{d\theta_{12}^2}{\theta_{12}^2} \int_{0}^{1} dz_2 P_g(z_2) \frac{\alpha_s}{2\pi} \times \Theta(v^{sec} > v_{cut})$$
Structure of the results (QCD background)

- For a jet of a given mass + a cut in the jet shape $v_{\text{cut}}$:

$$\frac{\rho \, d\sigma}{\sigma \, d\rho} \bigg|_{v_{\text{cut}}} = \int_{\rho}^{1} dz_1 P(z_1) \frac{\alpha_s}{2\pi} e^{-R_{\text{plain}}(\rho) - R_v(\rho, z_1)}$$

$$R_v(\rho, z_1) = \int_{0}^{1} \frac{d\theta_2^2}{\theta_2^2} \int_{0}^{1} dz_2 P_i(z_2) \frac{\alpha_s}{2\pi} \times \Theta(v > v_{\text{cut}}) \Theta(z_2 \theta_2^2 < \rho)$$

$$+ \int_{0}^{1} \frac{d\theta_{12}^2}{\theta_{12}^2} \int_{0}^{1} dz_2 P_g(z_2) \frac{\alpha_s}{2\pi} \times \Theta(v^{\text{sec}} > v_{\text{cut}})$$

Now all we need is to find $v(\rho, z_1, z_2, \theta_2)$. 
$\alpha_s \frac{C_R}{2\pi} \left[ \frac{L_{\tau}^2}{2} + L_\rho L_{\tau} \right]$

$+ \alpha_s \frac{C_A}{2\pi} \frac{L_{\tau}^2}{2}$

$L_X = \log(1/X)$

\[ R_{\tau}(z_1) = \]

\[ R_{\mu_{1/2}}(z_1) = \]

\[ R_{C2}(z_1) = \]

\[ \alpha_s \frac{C_R}{2\pi} \left[ \frac{L_{\mu}^2}{2} + L_\rho L_{\mu} \right] \]

\[ - \alpha_s \frac{C_R}{2\pi} \frac{L_1}{2} (L_\rho - L_1) \]

\[ + \alpha_s \frac{C_A}{2\pi} \frac{(L_{\mu} - L_1)^2}{2} \]

\[ \alpha_s \frac{C_R}{2\pi} \left[ \frac{L_e^2}{2} + (L_e - L_\rho + L_1)L_1 \right] \]

\[ + \alpha_s \frac{C_A}{2\pi} \frac{1}{2} (L_e - L_\rho + L_1)^2 \]


- Probability that $v_{QCD} < v_{cut}$ vs. probability that $v_{sig} < v_{cut}$;
- $C_2$ is the most efficient, and $\tau_{21}$ more efficient than $\mu^2$ (more delicate call).

ROC curves

![ROC curves graph]

better description of the order between shapes.
ROC curves

- Probability that $v_{QCD} < v_{cut}$ vs. probability that $v_{sig} < v_{cut}$;
- $C_2$ is the most efficient, and $\tau_{21}$ more efficient than $\mu^2$ (more delicate call).

- Good description of the order between shapes.
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Conclusion

- Good **qualitative description** of the shapes.

- **Efficiency** of the shapes: $C_2 > \tau_{21} \gtrsim \mu^2$.

**Next steps**

- Higher accuracy;
- Add grooming;
- Different jet shapes;
- 3-pronged jet shapes;
- Calculations for $\nu \sim 1$. 
Backup Slides
Generalized $k_t$ algorithm

- Depends on a parameter $p$;
- Cluster partons by smallest distance $d_{ij} = \min(z_i^{2p}, z_j^{2p}) \theta_{ij}^2$;
- Particular cases:
  - $p=0$: C/A algorithm, angular ordered;
  - $p=-1$: anti-$k_t$ algorithm;
  - $p=1/2$: similar to mass measure.
Non-perturbative effects

Pythia8, with SoftDrop ($\beta=2, z_c=0.1$)

Pythia8 (FSR and full simulations)
More on Lund diagram
Structure of the results (Signal)

- Decay of a boosted object into a pair $q\bar{q}$ or $gg$.
- For a signal jet (fixed mass always) + a cut in the shape $\nu$:

$$
\sum_{\text{sig}} = \int_{\rho}^{1} dz_1 P_{\text{sig}}(z_1) e^{-R_{v,\text{sig}}(z_1, \rho) - R_{v,\text{sig}}(1-z_1, \rho)}
$$

$$
R_{v,\text{sig}}(z_1) = \int_{0}^{1} \frac{d\theta_2}{\theta_2^2} \int_{0}^{1} dz_2 P_i(z_2) \frac{\alpha_s}{2\pi} \Theta(\nu^{\text{sig}} > \nu_{\text{cut}})
$$