Resolving the Proton Radius Puzzle Using QED-NRQED Effective Field Theory

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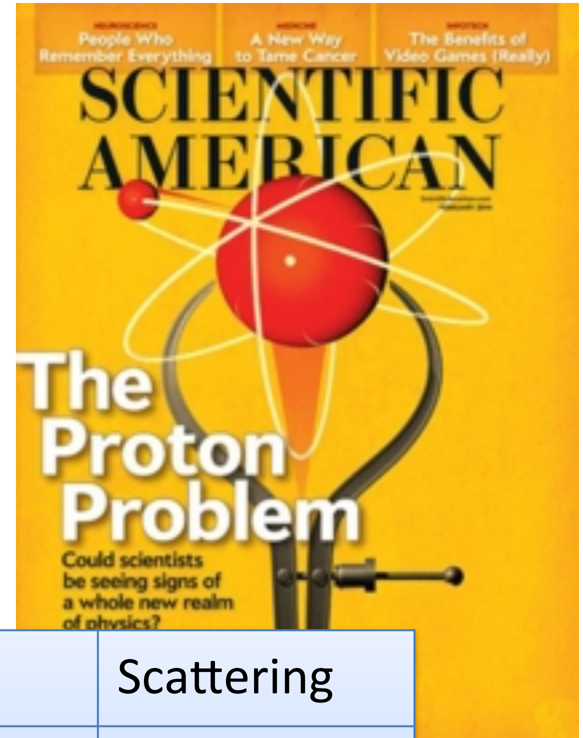
Based on paper by: Dye, Gonderinger, and Paz
Arxiv:1602.07770 Submitted to Physical Review D

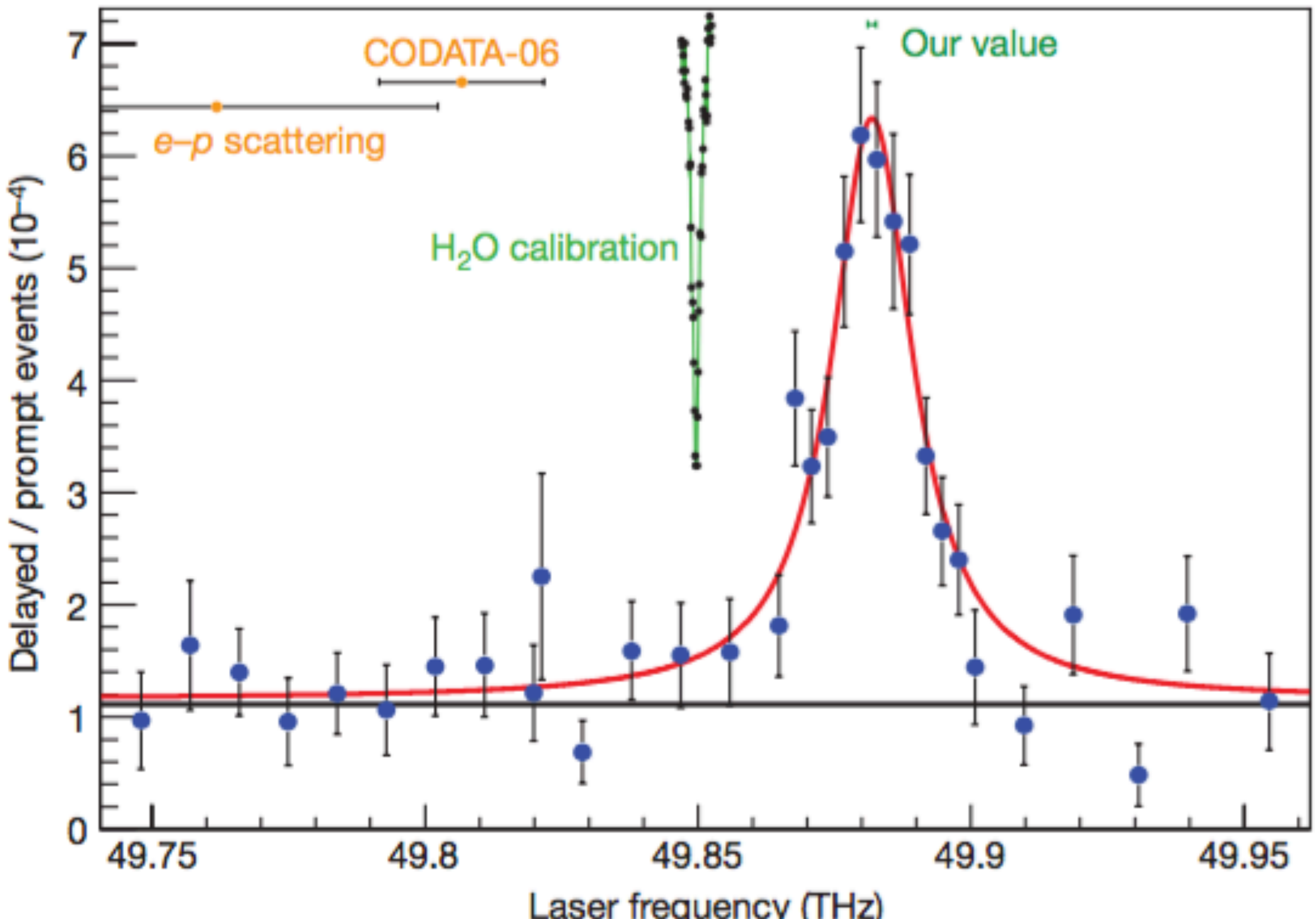
Proton Radius Puzzle



Proton Radius	Spectroscopy	Scattering
Electron	0.8758(77) fm[1]	0.8-0.9 fm[2]
Muon	0.84087(39) fm[3]	???

[1] P.J. Mohr, B.N. Taylor, D.B. Newell, Rev. Modern Phys. 84 (2012)
[2] K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update
[3] Antognini, Aldo *et al.* Science 339 (2013) 417-420

Muon Spectroscopy Data



Pohl et al. Nature 466, 213 (2010)

Outline

- Experimental Background
- Why QED-NRQED?
- One photon exchange at power m^2/M^2
- Two photon exchange at leading power
- Future endeavors

Muonic Spectroscopy

- Experimental precision requires separation of one and two photon exchange

- E in meV

$$\Delta E_L = 206.0336(15) - 5.2275(10) r_E^2 + \Delta E_{TPE}$$ [1]

Muon QED One Photon Two Photon

- ΔE_{TPE} is the contribution from the Two Photon Exchange

[1] Antongnini et al. Science **339,** 417 (2013)

Muonic Scattering Experiment (MUSE)

- Experiment at Paul Scherrer Institute in Switzerland

MUSE

Quantity	Coverage
Beam momenta	0.115, 0.153, 0.210 GeV/c
Scattering angle range	20° - 100°
Azimuthal coverage	30% of 2π typical
Q^2 range for electrons	0.0016 GeV^2 - 0.0820 GeV^2
Q^2 range for muons	0.0016 GeV^2 - 0.0799 GeV^2

- $d\sigma_{MUSE}$ ➔QED-NRQED ➔Spectroscopy

MUSE Collaboration Technical Design Report January 8, 2016

Why QED-NRQED?

QED-NRQED

- Effective Field Theories describe physics within a certain energy scale
- QED-NRQED combines
  - Quantum Electrodynamics (QED)
  - Non-Relativistic Quantum Electrodynamics (NRQED)
- Relativistic particles use QED
- Non-Relativistic particles use NRQED

- m=muon mass ~ 100 MeV/c^2
- M=proton mass ~ 1000 MeV/c^2
- Muonic Hydrogen: p ~mcα ~ 1 MeV/c
  - Muon is non-relativistic
- MUSE: p ~ mc ~ 100 MeV/c
  - Muon is relativistic: Use QED
  - Proton is non-relativistic: Use NRQED

Overview of NRQED

- To Order $1/M^2$ [1]

$$\mathcal{L} = \psi^\dagger \left\{ iD_t + c_2 \frac{\boldsymbol{D}^2}{2M} + c_F e \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}}{2M} + c_D e \frac{[\boldsymbol{\nabla} \cdot \boldsymbol{E}]}{8M^2} + i c_S e \frac{\boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D})}{8M^2} \right\} \psi + \cdots$$

- Schrödinger’s Equation
- Spin Orbit Coupling
- Darwin Term

[1] W.E. Caswell and G.P. Lepage Phys.Lett. B167 (1986) 437

Overview of NRQED

- To Order $1/M^2$ [1]

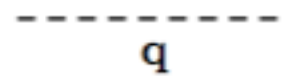
$$\mathcal{L} = \psi^\dagger \left\{ iD_t + c_2 \frac{\boldsymbol{D}^2}{2M} + c_F e \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2M} + c_D e \frac{[\boldsymbol{\nabla}\cdot\boldsymbol{E}]}{8M^2} + ic_S e \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E}-\boldsymbol{E}\times\boldsymbol{D})}{8M^2} \right\} \psi + \cdots$$

- c_F is the magnetic moment ~2.79[2]
- c_D is equivalent to the proton radius

[1] W.E. Caswell and G.P. Lepage Phys.Lett. B167 (1986) 437
[2] K.A. Olive et al. (Particle Data Group), Chin. Phys. C, 38, 090001 (2014) and 2015 update

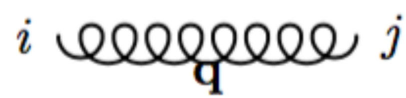
NRQED Feynman Rules[1]

- Coulomb Photon Propagator



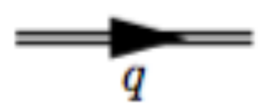
$$\frac{i}{\mathbf{q}^2 + \lambda^2}$$

- Space-like Photon Propagator



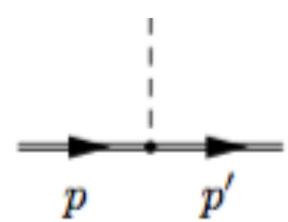
$$\frac{i(\delta^{ij} - \frac{q^i q^j}{\mathbf{q}^2 + \lambda^2})}{(q^0)^2 - \mathbf{q}^2 - \lambda^2 + i\epsilon}$$

- NR Fermion Propagator



$$\frac{i}{E - \frac{\mathbf{q}^2}{2M} + i\epsilon}$$

- Coulomb Vertex

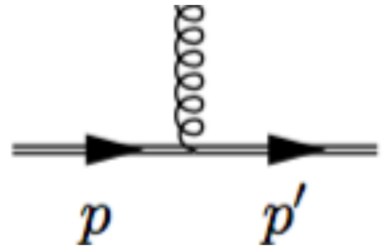


$$-ie$$

[1] T. Kinoshita and M. Nio, Phys. Rev. D **53**, 4909 (1996) [hep-ph/9512327]

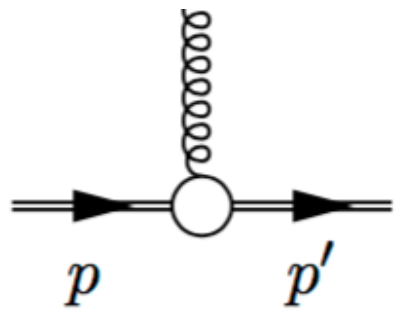
NRQED Feynman Rules[1]

- Dipole Vertex



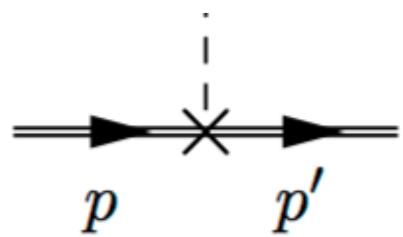
$$\frac{iec_2}{2M}(\mathbf{p}' + \mathbf{p})$$

- Fermi Vertex



$$\frac{ec_F}{2M}(\mathbf{p}' - \mathbf{p}) \times \boldsymbol{\sigma}$$

- Darwin Vertex



$$\frac{iec_D}{8M^2}|\mathbf{p}' - \mathbf{p}|^2$$

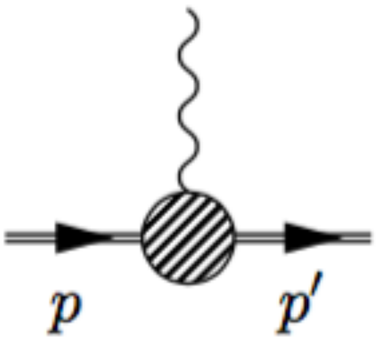
[1] T. Kinoshita and M. Nio, Phys. Rev. D **53**, 4909 (1996) [hep-ph/9512327]

Form Factors

- Arise from matrix element of electromagnetic current

$$\langle N(p_f)|J_\mu^{em}|N(p_i)\rangle = \bar{u}(p_f)\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m}F_2(q^2)q_\nu\right]u(p_i)$$

- Where $q = p_f - p_i$



- FF's describe interactions between particles without going into detail about the interaction

From Wilson Coefficients to the Charge Radius

- Wilson Coefficients related to Form Factors[1][2]

$$c_D = F_1(0) + 2F_2(0) + 8M^2 F_1'(0) \qquad c_F = F_1(0) + F_2(0)$$

$$F_1' = dF_1(q^2)/dq^2 \qquad G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

- Relation between form factors and charge radius[3]

$$\langle r_E^2 \rangle^{\frac{1}{2}} = -\frac{6\hbar^2}{G_E(0)} \frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0}$$

[1] A. V. Manohar, Phys. Rev. D **56**, 230 (1997)
[2] R. J. Hill and G. Paz, Phys Rev. Lett. **107**, 160402 (2011) [arXiv:1103.4617]
[3] J. C. Bernauer *et al.* [A1 Collaboration], Phys. Rev. Lett **105**, 241001 (2010)

Why Not QED-QED?

- QED makes the assumption that all particles are fundamental particles (point like)

- Can't make this assumption for the proton!

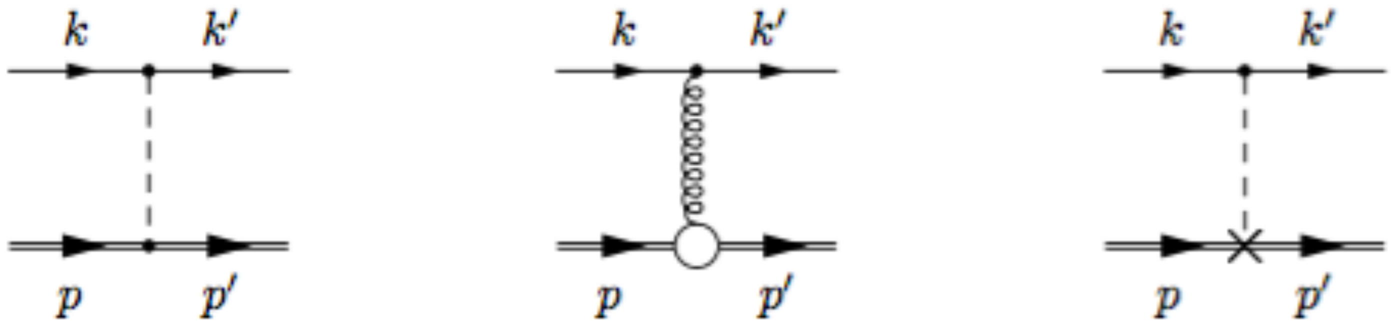
Why Not NRQED-NRQED?

- NRQED is power expanded in p/m (or v)
- At relativistic momentum (p ~ m), series does not converge

QED-NRQED Scattering of One Photon Exchange to power m^2/M^2

Dye, Gonderinger, and Paz Arxiv:1602.07770

- Lepton-Proton elastic scattering $\ell(k) + p(p) \to \ell(k') + p(p')$



- At power m^2/M^2

$$\mathcal{M}_{\rm QN} = -e^2 Z Q_\ell \left[\left(1 - c_D \frac{\vec{q}^{\,2}}{8M^2}\right) \frac{1}{\vec{q}^{\,2}} \xi^\dagger_{p'} \xi_p \bar{u}(k') \gamma^0 u(k) + i \frac{c_F}{2M} \frac{1}{q^2} \epsilon^{ijk} q^j \xi^\dagger_{p'} \sigma^k \xi_p u(k') \gamma^i u(k) \right]$$

- Z =1 for a proton

- Q_l is the lepton charge (±1)

Compare to Rosenbluth Scattering

- To power m^2/M^2

$$\overline{|\mathcal{M}|}^2_{\mathrm{QN}} = \frac{e^4 Z^2 Q_\ell^2}{\vec{q}^{\,2}} \left[\frac{1}{\vec{q}^{\,2}} \left(4E^2 - \vec{q}^{\,2}\right) - \frac{2E}{M} + \frac{\vec{q}^{\,2} + c_F^2 \left(\vec{q}^{\,2} + 4E^2 - 4m^2\right) + c_D \left(\vec{q}^{\,2} - 4E^2\right)}{4M^2} \right]$$

- Replacing WC's with FF's reproduces Rosenbluth scattering to power m^2/M^2 [1]

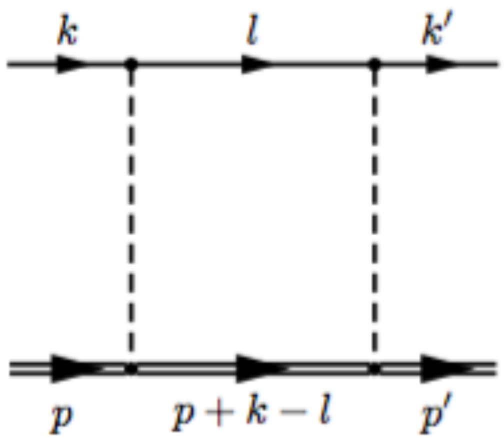
$$\frac{d\sigma}{d\Omega'} = \frac{\alpha^2}{q^4} \frac{p'/p}{1 + (E - (pE'/p')\cos\theta)/M} \Big[G_E^2 \frac{(4EE' + q^2)}{1 - q^2/4M^2} + G_M^2 \Big((4EE' + q^2)\big(1 - \frac{1}{1 - q^2/4M^2}\big) + \frac{q^4}{2M^2} + \frac{q^2 m^2}{M^2} \Big) \Big]$$

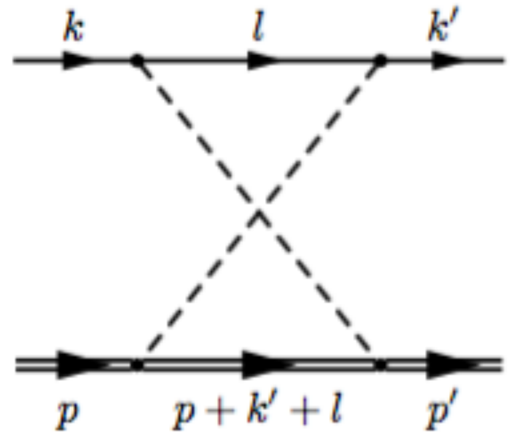
[1] E. Borie arXiv:1207.6651

QED-NRQED Scattering of Two Photon Exchange at Leading Power

Dye, Gonderinger, and Paz Arxiv:1602.07770

QED-NRQED Amplitude

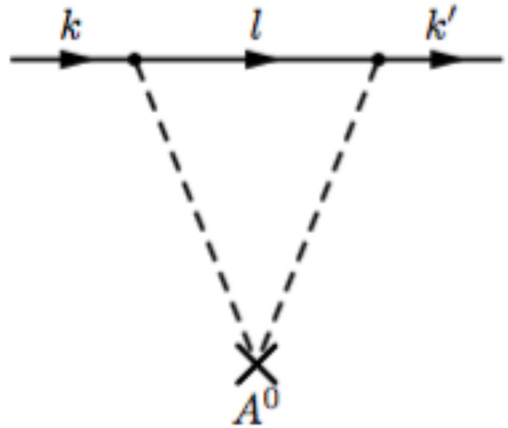




To leading power m/M

$$i\mathcal{M}\,(2\pi)^4\delta^4(k'+p'-k-p) = \int \frac{d^4l}{(2\pi)^4}\frac{2\pi\delta(l^0-k^0)}{(l-k)^2-\lambda^2}\frac{2\pi\delta(l^0-k'^0)}{(l-k')^2-\lambda^2}\frac{\bar{u}(k')\gamma^0\,(\not{l}+m)\,\gamma^0 u(k)}{l^2-m^2}$$
$$\times\,(-)iZ^2Q_\ell^2e^4(2\pi)^3\delta^3(\vec{k}'+\vec{p}'-\vec{k})$$

Static Potential Amplitude



Using a screened coulomb potential:

$$\vec{A} = 0 \qquad A^0 = \frac{Ze\, e^{-\lambda r}}{4\pi r} = -Ze \int \frac{d^4q}{(2\pi)^4} \frac{2\pi\delta(q^0)}{q^2 - \lambda^2} e^{iqx}$$

$$i\mathcal{M}\,(2\pi)\delta(k'^0 - k^0) = -iZ^2 Q_\ell^2 e^4 \int \frac{d^4l}{(2\pi)^4} \frac{2\pi\delta(l^0 - k^0)}{(l-k)^2 - \lambda^2} \cdot \frac{2\pi\delta(l^0 - k'^0)}{(l-k')^2 - \lambda^2} \cdot \frac{\bar{u}(k')\gamma^0\,(\not{l} + m)\,\gamma^0 u(k)}{l^2 - m^2}$$

R. H. Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)

Comparing Results

- Both methods give the same result

QED-NRQED:

$$i\mathcal{M}\,(2\pi)^4\delta^4(k'+p'-k-p) = \int \frac{d^4l}{(2\pi)^4}\frac{2\pi\delta(l^0-k^0)}{(l-k)^2-\lambda^2}\frac{2\pi\delta(l^0-k'^0)}{(l-k')^2-\lambda^2}\frac{\bar{u}(k')\gamma^0\,(\not{l}+m)\,\gamma^0 u(k)}{l^2-m^2}$$

$$\times\,(-)iZ^2Q_\ell^2e^4(2\pi)^3\delta^3(\vec{k}'+\vec{p}'-\vec{k})$$

Static Potential:

$$i\mathcal{M}\,(2\pi)\delta(k'^0-k^0) = -iZ^2Q_\ell^2e^4\int \frac{d^4l}{(2\pi)^4}\frac{2\pi\delta(l^0-k^0)}{(l-k)^2-\lambda^2}\cdot\frac{2\pi\delta(l^0-k'^0)}{(l-k')^2-\lambda^2}\cdot\frac{\bar{u}(k')\gamma^0\,(\not{l}+m)\,\gamma^0 u(k)}{l^2-m^2}$$

- Same Amplitude

Cross Section

- Both amplitudes result in the same cross section

$$\frac{d\sigma}{d\Omega} = \boxed{\frac{4Z^2\alpha^2 Q_\ell^2 E^2\left(1 - v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}}\left[1 - \alpha Z Q_\ell \frac{\pi v \sin\frac{\theta}{2}\left(1 - \sin\frac{\theta}{2}\right)}{1 - v^2\sin^2\frac{\theta}{2}}\right]$$

- Mott Scattering with α correction

- v=p/E

R. H. Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)

Future Work

- Establish a direct relation between μ-p scattering and muonic Hydrogen
  - TPE contributes to b_1 constant in nucleon-relativistic lepton effective lagrangian[1]

$$\mathcal{L}_{\ell\psi} = \frac{b_1}{M^2}\psi^\dagger\psi\bar{\ell}\gamma^0\ell + \ldots \qquad b_1 \sim \mathcal{O}(\alpha^2)$$

- Look at Two Photon Exchange up to power m^2/M^2 [2]
  - Include $1/M$ and $1/M^2$ power vertices

[1] R. J. Hill, G. Lee, G. Paz and M. P. Solon, Phys. Rev. D **87**, no. 5, 053017 (2013) [arXiv:1212.4508 [hep-ph]]
[2] Dye, Gonderinger, Paz *In Progress*

Summary

- Looked at One Photon Exchanges
  - To leading power ✓
  - To power m/M ✓
  - To power m^2/M^2 ✓

- Looked at Two Photon Exchanges
  - To leading power ✓
  - To power m/M
  - To power m^2/M^2

- Reproduced known results with QED-NRQED Effective Field Theory ✓

- Establish a direct comparison between spectroscopy and scattering data

END

Extra Slides

Proton Charge Radius Data

VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
0.8775	**±0.0051**		MOHR	12	RVUE	2010 CODATA, $e p$ data
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.879	±0.005	±0.006	BERNAUER	14	SPEC	$e p \rightarrow e p$ form factor
0.879	±0.005	±0.006	BERNAUER	10	SPEC	See BERNAUER 14
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old $e p$ data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
0.895	±0.010	±0.013	SICK	03		$e p \rightarrow e p$ reanalysis

Spectroscopy

- Measure Lamb shift
- Difference between $2S_{1/2}$ and $2P_{1/2}$ energy states

Textbook Example

- Assume the charge density ρ is spherically symmetric

$$F(\mathbf{q}) = \int d^3r \rho(|\mathbf{r}|) e^{i\mathbf{q}\cdot\mathbf{r}} \quad \text{or} \quad \rho(|\mathbf{r}|) = \frac{1}{(2\pi)^3} \int F(\mathbf{q}) e^{-i\mathbf{q}\cdot\mathbf{r}} d^3q$$

- For small q

$$F(\mathbf{q}) = 1 - \frac{\mathbf{q}^2 \langle r^2 \rangle}{6} + \cdots$$

- From Gauss's Law

$$V(\mathbf{r}) = \frac{|e|}{(2\pi)^3}\int d^3q e^{-i\mathbf{q}\cdot\mathbf{r}}\frac{F(\mathbf{q})}{\mathbf{q}^2}$$

- Potential can be expanded to be

$$V(r) = \frac{|e|}{4\pi r} - \frac{\langle r^2\rangle}{6}|e|\delta^3(\mathbf{r}) + \cdots$$

- From perturbation theory

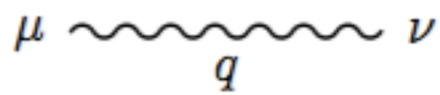
$$\Delta E_{\langle r^2\rangle} = \langle\psi|\frac{\langle r^2\rangle}{6}|e|\delta^3(\mathbf{r})|\psi\rangle = \frac{2\alpha^4}{3n^3}m_r^3\langle r^2\rangle\delta_{\ell 0},$$

- Muon is ~200 times more massive then the electron
- Effect is ~200^3 times larger
- Smaller error for the radius
- Muonic Hydrogen: r = 0.84087(39) fm
- Atomic Hydrogen: r = 0.8758(77) fm

Overview of QED

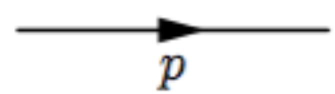
- QED Lagrangian: $\mathcal{L} = \bar{\ell}\,\gamma^{\mu}\, i\,(\partial_{\mu} + ieQ_{\ell}A_{\mu})\,\ell - m\bar{\ell}\ell$

- Photon Propagator:



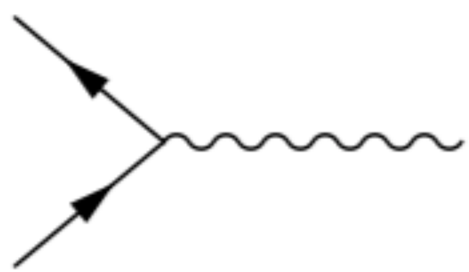
$$\frac{-ig_{\mu\nu}}{q^2 - \lambda^2}$$

- Fermion Propagator:



$$\frac{i(\not{p} + m_p)}{p^2 - m_p^2 + i\epsilon}$$

- Vertex:



$$-ie\gamma^{\mu}$$

Wilson Coefficients

$$c_D = F_1(0) + 2F_2(0) + 8M^2F_1'(0) \qquad c_F = F_1(0) + F_2(0)$$

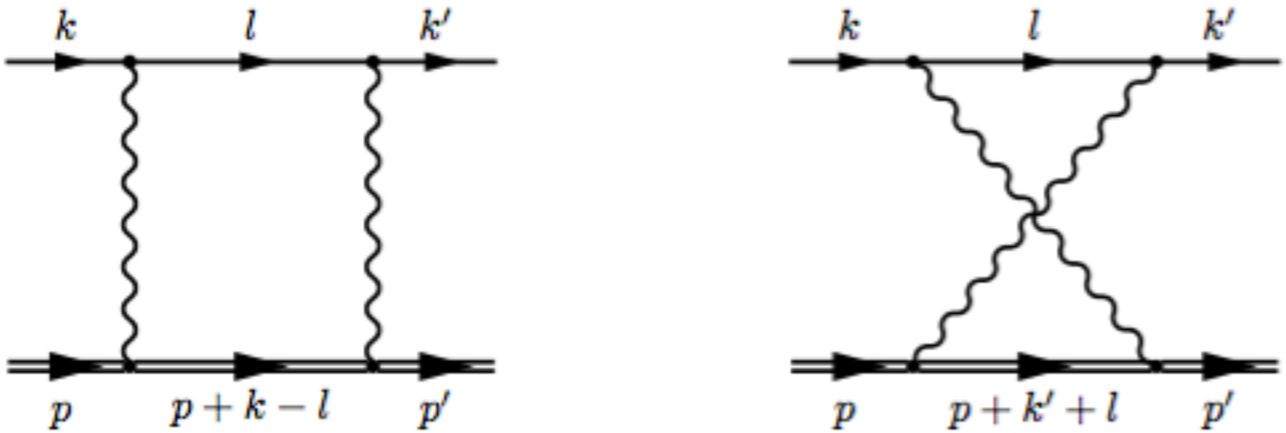
$$F_1' = dF_1(q^2)/dq^2 \qquad G_E(q^2) = F_1(q^2) + \frac{q^2}{4M^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$\langle r_E^2 \rangle^{\frac{1}{2}} = -\frac{6\hbar^2}{G_E(0)} \frac{dG_E(q^2)}{dq^2}\bigg|_{q^2=0}$$

Where r_E is the charge radius

R. J. Hill, G. Lee, G. Paz and M. P. Solon, Phys. Rev. D **87**, no. 5, 053017 (2013) [arXiv:1212.4508 [hep-ph]]

Point Particle QED Amplitude



- Taken to the limit of M➔∞

$$i\mathcal{M} = Z^2 Q_\ell^2 e^4 \int \frac{d^4 l}{(2\pi)^4} \frac{\bar{u}(k')\gamma^0 (\not{l} + m)\gamma^0 u(k) \xi_{p'}^\dagger \xi_p}{(l-k)^2 (l-k')^2 (l^2 - m^2)} \left(\frac{1}{k^0 - l^0 + i\epsilon} + \frac{1}{l^0 - k'^0 + i\epsilon} \right)$$

- Same result as QED-NRQED

Differences in Scatterings

- Coulomb Scattering: NR and massless lepton
- Mott Scattering: R and massless lepton
- Rosenbluth Scattering: R and massive lepton[1]

$$\frac{d\sigma}{d\Omega'} = \frac{m^2 M}{4\pi^2} \frac{p'/p}{M + E - (pE'/p')cos\theta} |\mathfrak{M}_{fi}|^2$$

- All hit an infinitively massive, point like proton

[1] J.D. Bjorken, S.D. Drell, *Relativistic Quantum Mechanics,* McGraw-Hill, New York,1964