Soft Wall Light Dilatons
At Finite Temperature

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Conformal Symmetry Breaking

Most general Dilaton Lagrangian invariant under dilatations:

\[ \mathcal{L}_{\text{eff}} = F \chi^4 + \frac{1}{2} (\partial_\mu \chi)^2 + \text{higher derivatives} \]

Obstruction to SBSI:
- \( F > 0 \) \( \Rightarrow \) \( \chi = 0 \) (no breaking)
- \( F < 0 \) \( \Rightarrow \) \( \chi = \infty \) (runaway)
- \( F = 0 \) \( \Rightarrow \) \( \chi = \text{anything} \) (flat direction)

Need explicit breaking...
A Holographic Model

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \]

\[ k = \kappa \sqrt{-\frac{\Lambda_{(5)}}{6}} \]

\( \mu \) Explicit breaking

\( \mu \) Spontaneous breaking

\( \mu \) Dilaton

UV brane

IR brane

Radion

AdS

CFT
A Holographic Model

Stabilizing inter-brane distance typically requires tuning $\Lambda_{IR}$ against $\Lambda_{(5)}$:

$$F \sim \Lambda_{IR} - \Lambda_{(5)}/k$$

Meanwhile, in the CFT:

Conformal Symmetry Breaking

Another possibility: \( V_{\text{eff}} = \chi^4 F(\lambda(\chi)) \)

- If quartic is mildly energy dependent, coupling can be large, but if \( \beta(\lambda) \) is small, it will slowly scan values of \( F \) and for sufficiently long running find min at \( F \sim 0 \).

\[
\frac{d\lambda}{d \log \mu} = \beta(\mu) = \epsilon b(\mu) \ll 1 \quad \text{With } \epsilon \ll 1 \quad b(\mu) = \mathcal{O}(1)
\]

- Small dilaton mass and CC for large coupling:

\[
m_{\text{dil}}^2 \sim \beta f^2 \quad \Delta \Lambda_{\text{eff}} \sim \beta f^4
\]

Meanwhile, in AdS:

Tuning on the IR brane has been relaxed.
A Near Marginal Deformation

\[ \frac{d\lambda}{d \log \mu} = \beta(\mu) = \epsilon b(\mu) \ll 1 \]

\[ V_{\text{eff}} \approx \mu^{\frac{\epsilon}{4}} \chi^{4-\epsilon} + \lambda \chi^4 \]

\[ S = \int d^5 x \sqrt{g} \left[ \frac{1}{2} (\partial_M \phi)^2 + \Lambda_{(5D)} \left( +1 + \frac{\epsilon}{3} \phi^2 \right) - \frac{1}{2\kappa^2} R \right] \]
A Near Marginal Deformation

- Goal: Probe the soft-wall in which there is a longer running region in CFT and smaller breaking scale (IR brane plays lesser role).

- A delicate balancing act:

\[ V_{\text{eff}} \approx \mu^\epsilon \chi^{4-\epsilon} + \lambda \chi^4 \]

\[ \chi_{\text{min}} \sim \frac{\mu}{\lambda^{\frac{1}{\epsilon}}} \]

\[ m_{\text{dil}} \sim \chi_{\text{min}}^2 \]

\[ V_{\text{min}} \sim \chi_{\text{min}}^4 \]

Extremely sensitive!

Light Dilaton and small CC contribution, but sensitive!
New Setup

\[ S = \int d^5 x \sqrt{g} \left[ \frac{1}{2} (\partial_M \phi)^2 - V(\phi) - \frac{1}{2\kappa^2} R \right] \quad \kappa^{-2} \equiv 2M_*^3 \]

\[ ds^2 = e^{-2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \]

Switch to coordinates: \[ ds^2 = e^{-2A} \eta_{\mu\nu} dx^\mu dx^\nu - \frac{dA^2}{G(A)} \]

\[ G(A) \sim \frac{1}{1 - \frac{\kappa^2}{12} \dot{\phi}^2} \quad \text{Parameterizes back-reaction.} \]

A drops out of EOM!

\[ \ddot{\phi} = 4 \left( \dot{\phi} - \frac{3}{2\kappa^2} \frac{\partial \log V(\phi)}{\partial \phi} \right) \left( 1 - \frac{\kappa^2}{12} \dot{\phi}^2 \right) \]
A Near Marginal Deformation

\[ \ddot{\phi} = 4 \left( \phi - \frac{1}{\kappa^2} \frac{\epsilon \phi}{1 + \kappa^2 \frac{\epsilon}{3} \phi^2} \right) \left( 1 - \frac{\kappa^2}{12} \dot{\phi}^2 \right) \]

\[ \kappa = 1 \]
\[ \phi_{UV} = 1 \]
\[ \epsilon = 1/10 \]

Large backreaction!
Calculating...

A good picture of $\phi$ insufficient for $V_{\text{eff}}$.

**Analytically:**
- Deviation from unstable trajectory give minimum:

$$\dot{\phi} \approx \frac{3}{2\kappa^2} \frac{\partial \log V(\phi)}{\partial \phi} + \delta\phi_{UV}$$

**Numerically:**
- Initial trajectory unstable: $\delta\phi_{UV} \sim e^{4A}$

**Naive $\epsilon$ expansion insufficient:**
Effective Potential

\[ V_{eff} = \kappa \phi_{IR} \phi_{UV} \epsilon \left( \frac{\Lambda_{IR}}{\Lambda_{(5)}} \right) / k \]

\[ \kappa = 1 \]
\[ \phi_{IR} = 10 \]
\[ \phi_{UV} = 1 \]
\[ \epsilon = 1/10 \]
\[ \frac{\Lambda_{IR}}{\Lambda_{(5)}} / k = 117 \]
Finite Temperature

Conformal Symmetry \hspace{1cm} \text{Phase transition} \hspace{1cm} \text{Conformal Symmetry}

High temperature conformal phase is dual to a black brane:

Hawking, Page '83
E. Witten, [arXiv:hep-th/9802150]
Finite Temperature

Ansatz for AdS-Schwarzschild:

\[ ds^2 = e^{-2A} [h(A) dt^2 + dx^2] + \frac{1}{h(A)} \frac{dA^2}{G(A)} \]

\[ \ddot{h} = 4 \dot{h} \left( 1 - \frac{1}{12} \dot{\phi}^2 \right) \]

\[ \ddot{\phi} = 4 \left( \dot{\phi} - \frac{3}{2\kappa^2} \frac{\partial \log V}{\partial \phi} \right) \left( 1 - \frac{1}{4} \frac{\dot{h}}{h} - \frac{\kappa^2}{12} \phi^2 \right) \]
Finite Temperature

Horizon criteria: $h \to 0$ in 

$$\ddot{\phi} = 4 \left( \dot{\phi} - \frac{3}{2\kappa^2} \frac{\partial \log V}{\partial \phi} \right) \left( 1 - \frac{1}{4} \frac{\dot{h}}{h} - \frac{\kappa^2}{12} \dot{\phi}^2 \right)$$

$$\dot{\phi} - \frac{3}{2\kappa^2} \frac{\partial \log V}{\partial \phi} \approx 0$$

$$\dot{\phi} \sim \epsilon \ll 1 \quad \text{Throughout}$$

$$h \approx 1 - e^{4(A-A_h)} + O(\epsilon^2)$$

Finite Temp is well approximated by AdS-S with $\phi$ on unstable trajectory.
Finite Temperature

\[ V_{eff} = k e^{-A h} \]

Analytical

Numerical

\[ \kappa = 1 \]
\[ h_{UV} = 1 \]
\[ \phi_{UV} = 1 \]
\[ \epsilon = 1/10 \]
\[ \frac{\Lambda_{IR}}{\Lambda(5) / k} = 117 \]
\[ T = 1.4 \cdot 10^{-6} \]
Conclusions

• Understanding backreaction is important, and have to be careful in order to get effective potential correct.

• With pieces falling into place, to do list:
  • Dilaton normalization.
  • Calculate transition temperature for phase transition.
  • Multiple fields more realistic and appropriate for cosmology:

\[
\ddot{\phi}_i = 4 \left( \dot{\phi}_i - \frac{3}{2\kappa^2} \frac{\partial \log V}{\partial \phi_i} \right) \left( 1 - \frac{\kappa^2}{12} \sum_i \phi_i^2 \right)
\]

• Are results distinct from Creminelli,Nicolis,Rattazzi (hep-th/0107141)?
Effective potential

\[ V_{\text{eff}} = e^{-4y_0} \left[ V_0(\phi(y_0)) - \frac{6}{\kappa^2} \sqrt{G(y_0)} \right] + e^{-4y_1} \left[ V_1(\phi(y_1)) + \frac{6}{\kappa^2} \sqrt{G(y_1)} \right] \]

UV contribution:

IR contribution:

\[ \mu_0 = k e^{-y_0} \]
\[ \mu_1 = k e^{-y_1} \]
\[ \chi = k e^{-y_c} \]

\[ V_{\text{eff}} \approx \mu \epsilon \chi^{4-\epsilon} + \lambda \chi^4 \]
Effective Potential

Preliminary results
A Holographic Model

A mildly running quartic balances large initial value:

\[ V_{\text{eff}} = 0 \]

\[ \beta \neq 0 \]
Finite Temperature

\[ T = 50 \]

\[ T = 40 \]

\[ T = 30 \]

\[ T = 20 \]

\[ T = 10 \]

Jay's Numerical
Zeroth Order Analytical
First Order Analytical
A Near Marginal Deformation

\[
\ddot{\phi} = 4 \left( \dot{\phi} - \frac{1}{\kappa^2} \frac{\epsilon \phi}{1 + \kappa^2 \frac{\epsilon}{3} \phi^2} \right) \left( 1 - \frac{\kappa^2}{12} \dot{\phi}^2 \right)
\]