Soft Wall Light Dilatons At Finite Temperature

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Phenomenology 2016 Symposium

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SYRACUSE UNIVERSITY
SUOS CULTORES SCIENTIA CORONAT
FOUNDED AD 1870

Hamilton

KIAS KOREA INSTITUTE FOR ADVANCED STUDY

Conformal Symmetry Breaking

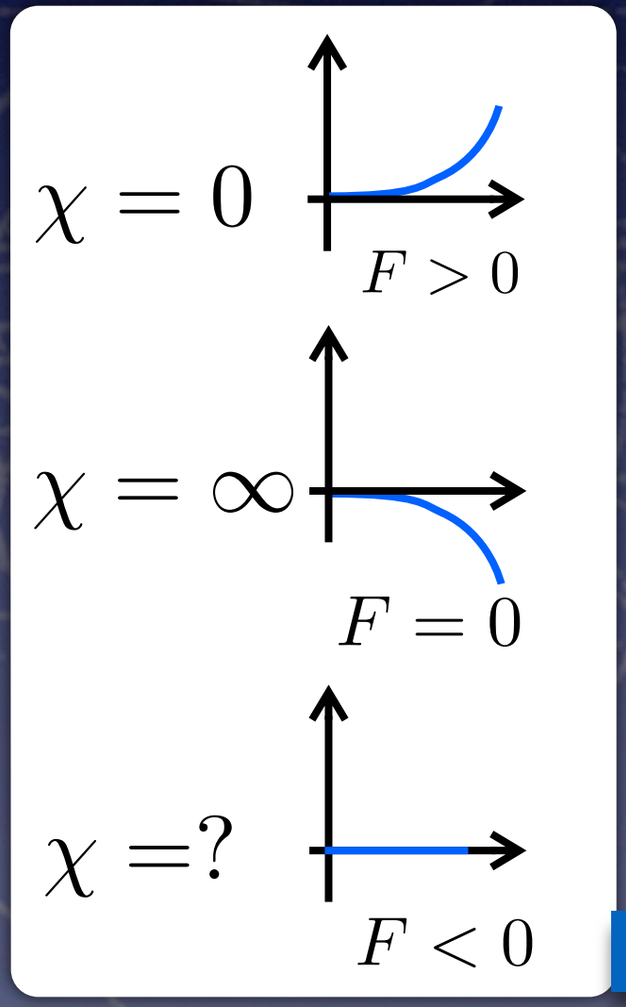
Most general Dilaton Lagrangian invariant under dilatations:

$$\mathcal{L}_{\text{eff}} = F\chi^4 + \frac{1}{2}(\partial_\mu \chi)^2 + \text{higher derivatives}$$

Obstruction to SBSI:

- F > 0 → χ = 0 (no breaking)
- F < 0 → χ =∞ (runaway)
- F = 0 → χ = anything (flat direction)

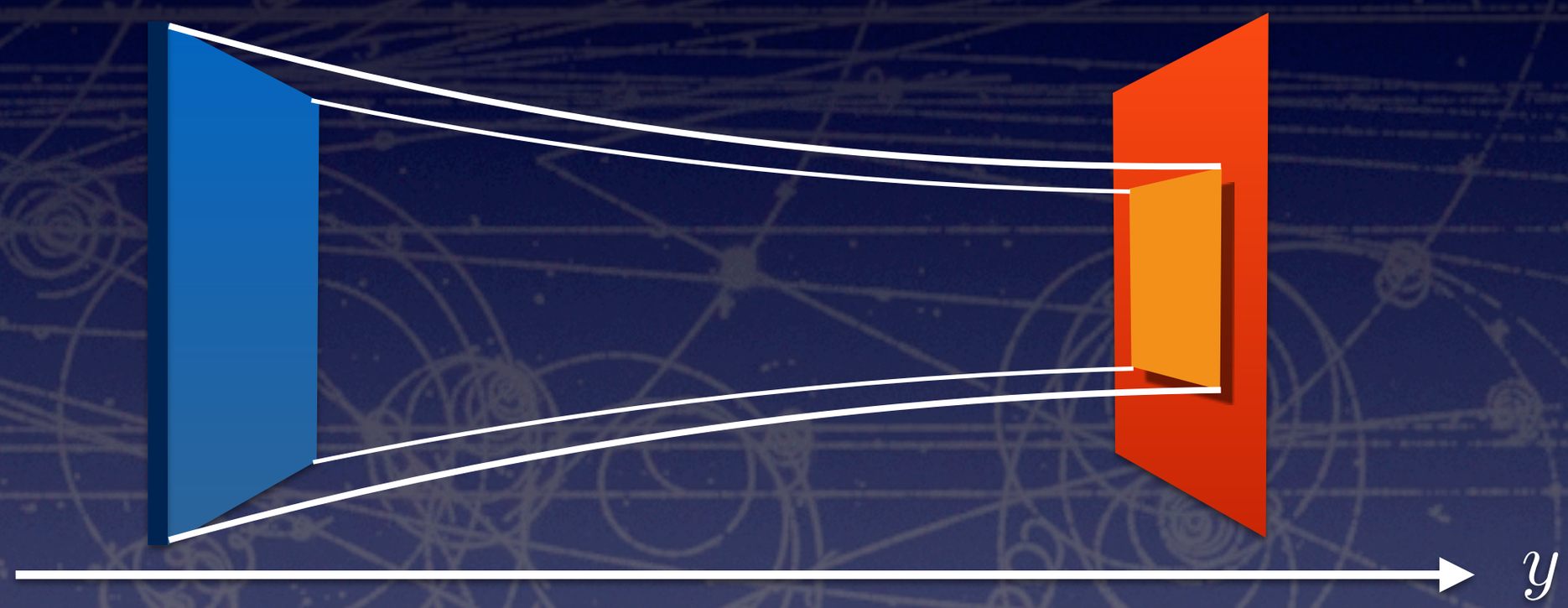
Need explicit breaking…



Fubini '76

A Holographic Model

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \qquad k = \kappa\sqrt{\frac{-\Lambda_{(5)}}{6}}$$



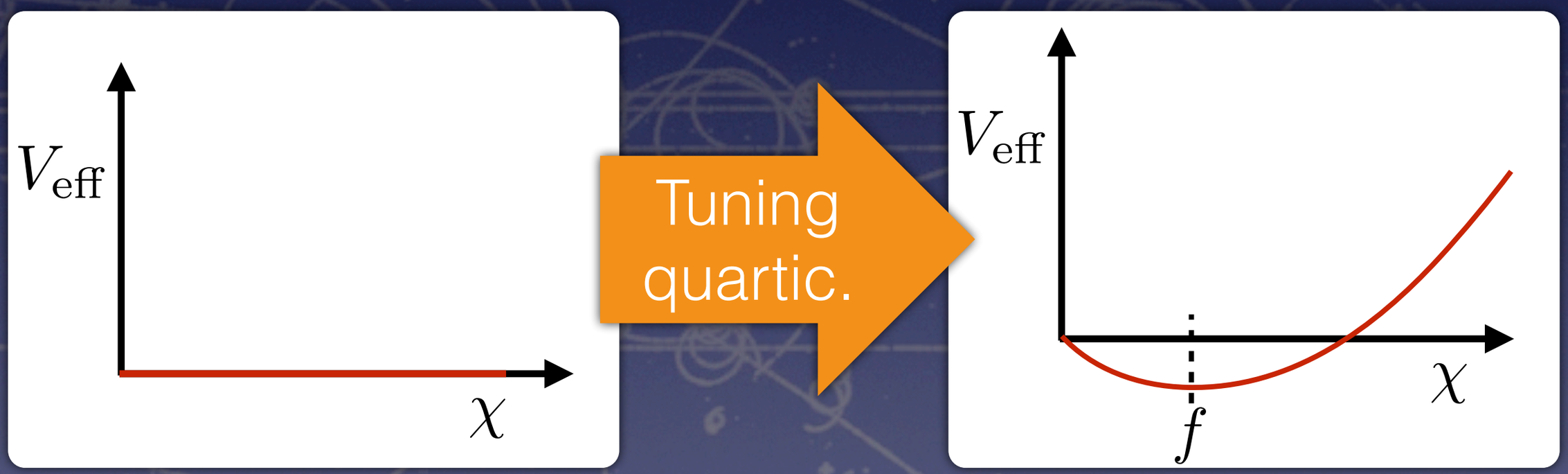
AdS		CFT
ke^{-ky}	⟷	μ
UV brane	⟷	Explicit breaking
IR brane	⟷	Spontaneous breaking
Radion	⟷	Dilaton

A Holographic Model

Stabilizing inter-brane distance typically requires tuning Λ_{IR} against $\Lambda_{(5)}$:

$$F \sim \Lambda_{IR} - \Lambda_{(5)}/k$$

Meanwhile, in the CFT:



W. D. Goldberger and M. B. Wise, hep-ph/9907447.

Conformal Symmetry Breaking

Another possibility: $V_{\rm eff} = \chi^4 F(\lambda(\chi))$

- If quartic is mildly energy dependent, coupling can be large, but if $\beta(\lambda)$ is small, it will slowly scan values of F and for sufficiently long running find min at F~0.

Contino, Pomarol, Rattazzi (Planck 2010)

$$\frac{d\lambda}{d\log\mu} = \beta(\mu) = \epsilon b(\mu) \ll 1 \qquad \text{With } \epsilon \ll 1 \quad b(\mu) = \mathcal{O}(1)$$

- Small dilaton mass and CC for large coupling:

$$m_{dil}^2 \sim \beta f^2 \qquad \Delta\Lambda_{\rm eff} \sim \beta f^4$$

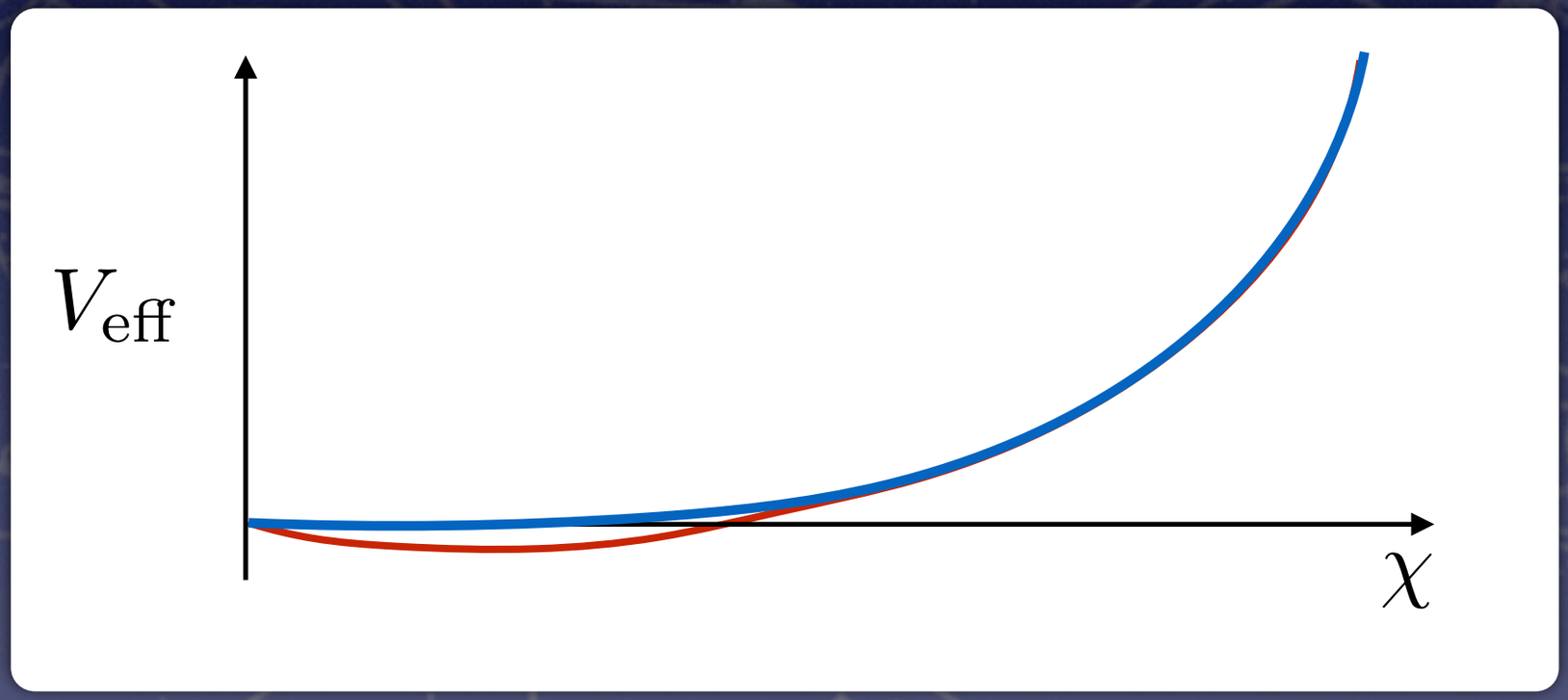
Meanwhile, in AdS:

Tuning on the IR brane has been *relaxed.*

A Near Marginal Deformation

$$\frac{d\lambda}{d\log\mu} = \beta(\mu) = \epsilon\, b(\mu) \ll 1 \quad \Rightarrow \quad V_{\text{eff}} \approx \mu^{\epsilon}\chi^{4-\epsilon} + \lambda\chi^4$$

$$S = \int d^5x\sqrt{g}\left[\frac{1}{2}(\partial_M\phi)^2 + \Lambda_{(5D)}\left(+1 + \frac{\epsilon}{3}\phi^2\right) - \frac{1}{2\kappa^2}R\right]$$



$$\phi_{5D} \longleftrightarrow \lambda_{\text{CFT}}$$

B. Bellazzini, et al. arXiv:1209.3299 [hep-ph]

Z. Chacko, et al. arXiv:1304.1795. [hep-ph]

A Near Marginal Deformation

- Goal: Probe the soft-wall in which there is a longer running region in CFT and smaller breaking scale (IR brane plays lesser role).
- A delicate balancing act:

$$V_{\rm eff} \approx \mu^{\epsilon}\chi^{4-\epsilon} + \lambda\chi^4$$

$$\chi_{min} \sim \frac{\mu}{\lambda^{\frac{1}{\epsilon}}}$$

Extremely sensitive!

$$m_{\rm dil} \sim \chi_{\rm min}^2$$

$$V_{min} \sim \chi_{\rm min}^4$$

Light Dilaton and small CC contribution, but sensitive!

New Setup

$$S = \int d^5x\sqrt{g}\left[\frac{1}{2}\left(\partial_M\phi\right)^2 - V(\phi) - \frac{1}{2\kappa^2}R\right] \qquad \kappa^{-2} \equiv 2M_*^3$$

$$ds^2 = e^{-2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$$

Switch to coordinates: $ds^2 = e^{-2A}\eta_{\mu\nu}dx^\mu dx^\nu - \frac{dA^2}{G(A)}$

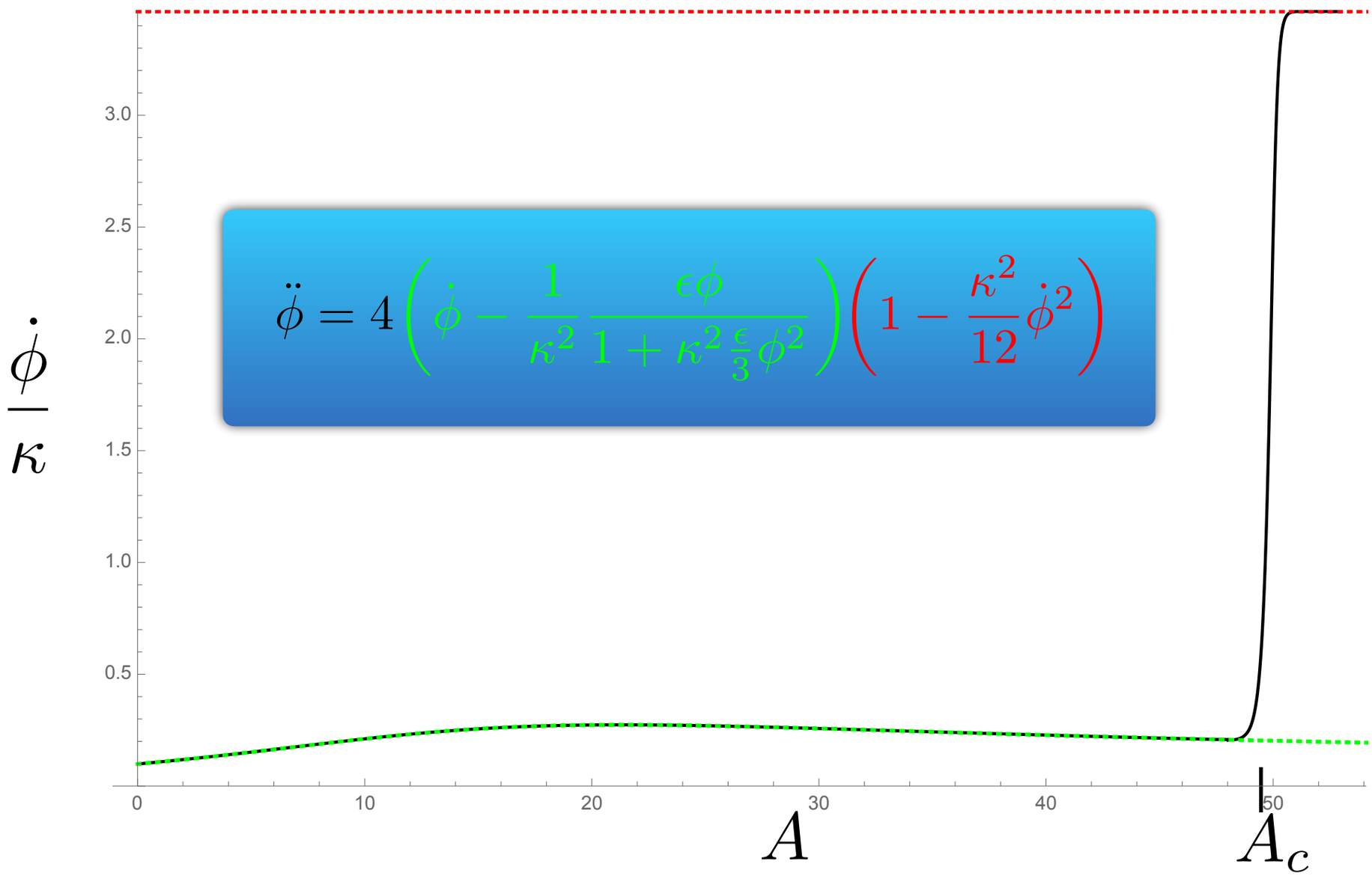
$$G(A) \sim \frac{1}{1 - \frac{\kappa^2}{12}\dot{\phi}^2}$$

Parameterizes back-reaction.

A drops out of EOM!

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial\phi}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

A Near Marginal Deformation



$\kappa = 1$
$\phi_{UV} = 1$
$\epsilon = 1/10$

$\dot{\phi} \longleftrightarrow \beta(\lambda)$

Large backreaction!

Calculating…

A good picture of ϕ insufficient for $V_{\rm eff}$.

Analytically:

- Deviation from unstable trajectory give minimum:

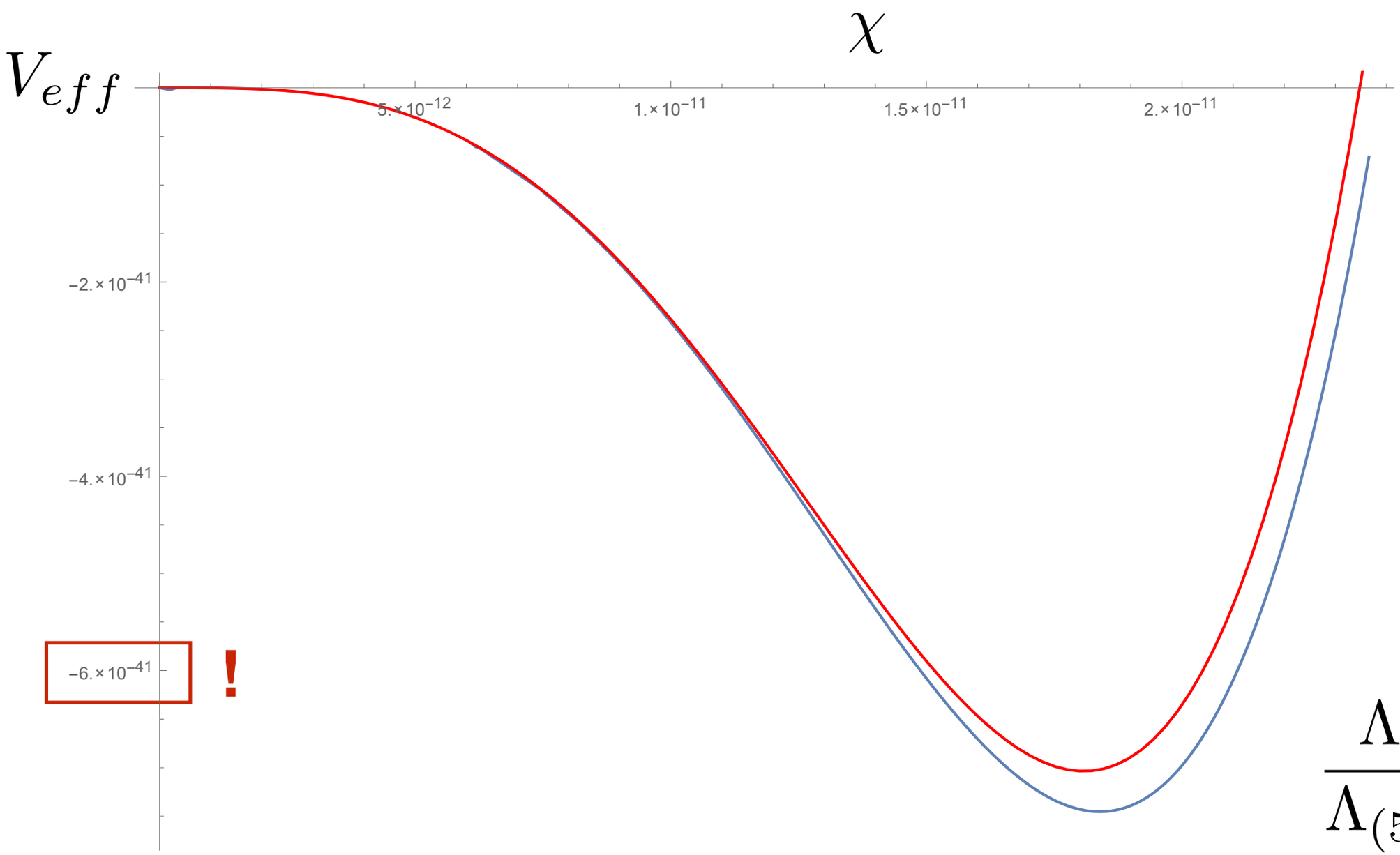
$$\dot{\phi} \approx \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi} + \delta\phi_{UV}$$

- Naive ϵ expansion insufficient:

$$\dot{\phi} \approx \frac{3}{2\kappa^2}\frac{\partial \log V(\phi)}{\partial \phi} + \delta\phi_{UV}(A, \epsilon A) \Longleftrightarrow \chi^{4-\epsilon}$$

Numerically: Initial trajectory unstable: $\delta\phi_{UV} \sim e^{4A}$

Effective Potential



Analytical

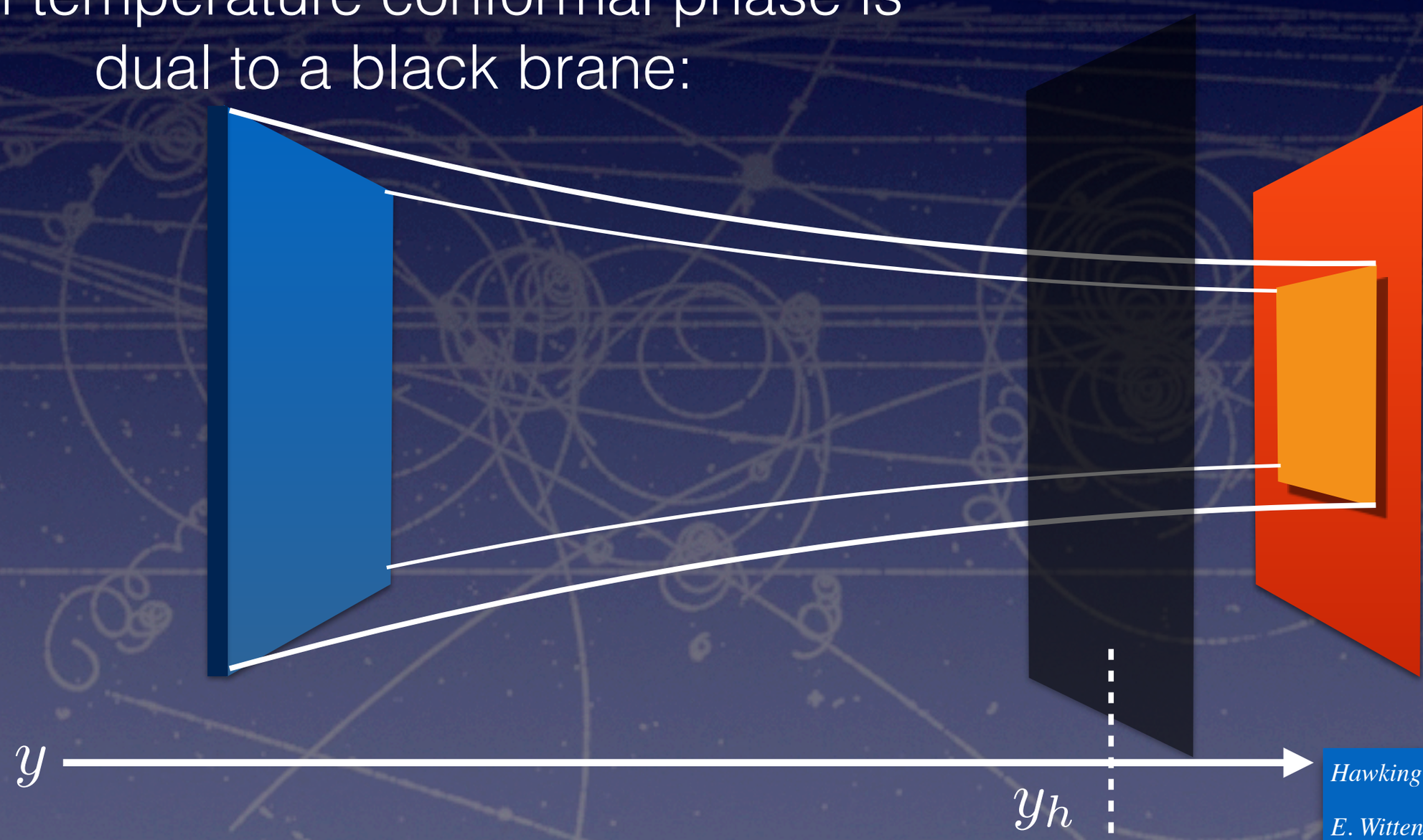
Numerical

$$\kappa = 1$$
$$\phi_{IR} = 10$$
$$\phi_{UV} = 1$$
$$\epsilon = 1/10$$
$$\frac{\Lambda_{IR}}{\Lambda_{(5)}/k} = 117$$

Finite Temperature

Conformal Symmetry → Phase transition → ~~Conformal Symmetry~~

High temperature conformal phase is dual to a black brane:



Hawking, Page '83

E. Witten, [arXiv:hep-th/9802150

Finite Temperature

Ansatz for AdS-Schwarzschild:

$$ds^2 = e^{-2A}[h(A)dt^2 + d\vec{x}^2] + \frac{1}{h(A)}\frac{dA^2}{G(A)}$$

$$\ddot{h} = 4\dot{h}\left(1 - \frac{1}{12}\dot{\phi}^2\right)$$

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi}\right)\left(1 - \frac{1}{4}\frac{\dot{h}}{h} - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$

Finite Temperature

Horizon criteria: $h \to 0$ in $\ddot{\phi} = 4\left(\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi}\right)\left(1 - \frac{1}{4}\frac{\dot{h}}{h} - \frac{\kappa^2}{12}\dot{\phi}^2\right)$

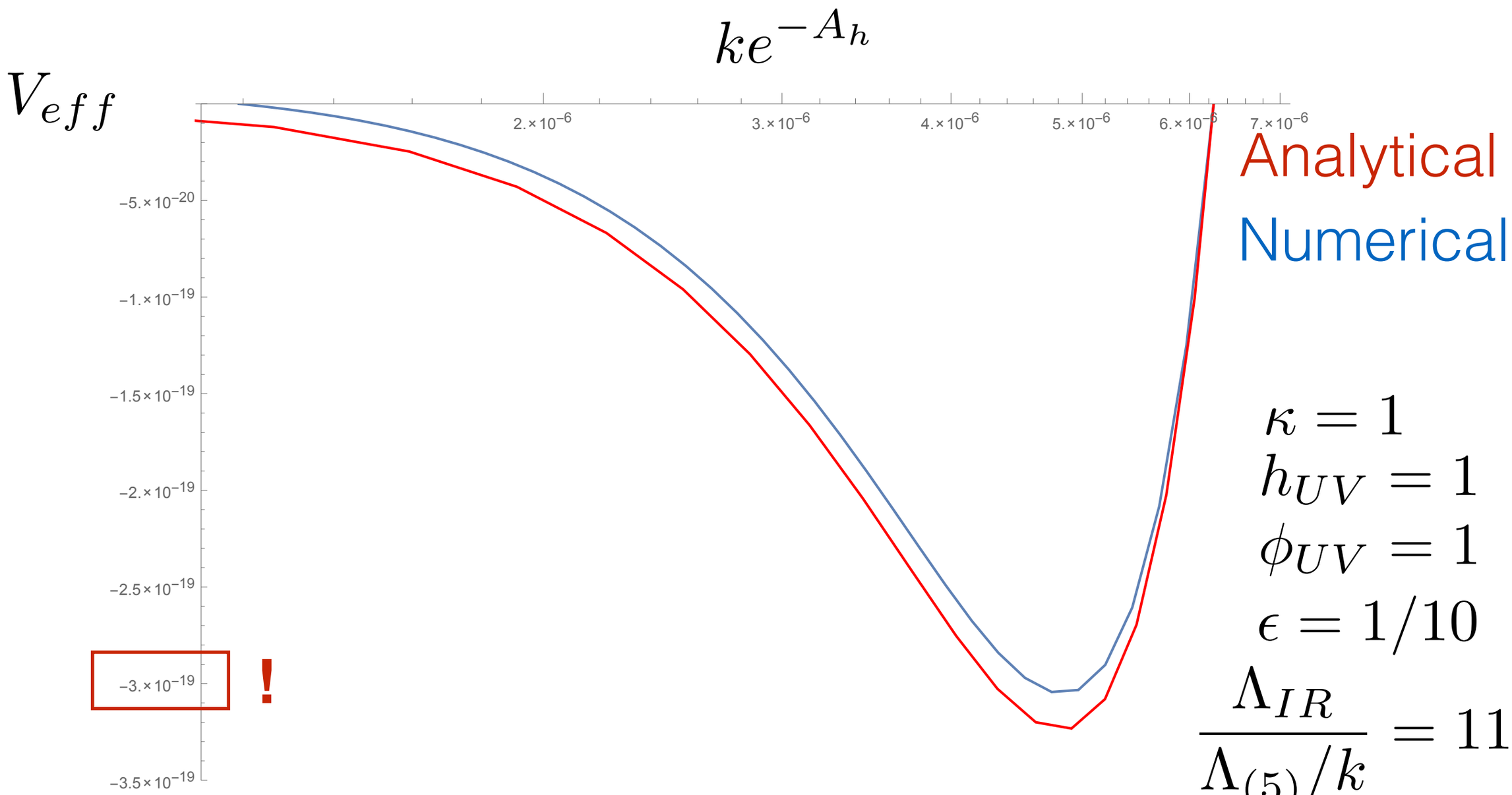
$$\dot{\phi} - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi} \approx 0 \qquad \dot{\phi} \sim \epsilon \ll 1$$

Throughout

$$h \approx 1 - e^{4(A - A_h)} + \mathcal{O}(\epsilon^2)$$

Finite Temp is well approximated by AdS-S with ϕ on unstable trajectory.

Finite Temperature



Conclusions

- Understanding backreaction is important, and have to be careful in order to get effective potential correct.
- With pieces falling into place, to do list:
  - Dilaton normalization.
  - Calculate transition temperature for phase transition.
  - Multiple fields more realistic and appropriate for cosmology:

$$\ddot{\phi}_i = 4\left(\dot{\phi}_i - \frac{3}{2\kappa^2}\frac{\partial \log V}{\partial \phi_i}\right)\left(1 - \frac{\kappa^2}{12}\sum_i \dot{\phi}_i^2\right)$$

- Are results distinct from Creminelli,Nicolis,Rattazzi (hep-th/0107141)?

Effective potential

UV contribution: IR contribution:

$$V_{\text{eff}} = e^{-4y_0}\left[V_0(\phi(y_0)) - \frac{6}{\kappa^2}\sqrt{G(y_0)}\right] + e^{-4y_1}\left[V_1(\phi(y_1)) + \frac{6}{\kappa^2}\sqrt{G(y_1)}\right]$$

$$\mu_0 = ke^{-y_0}$$

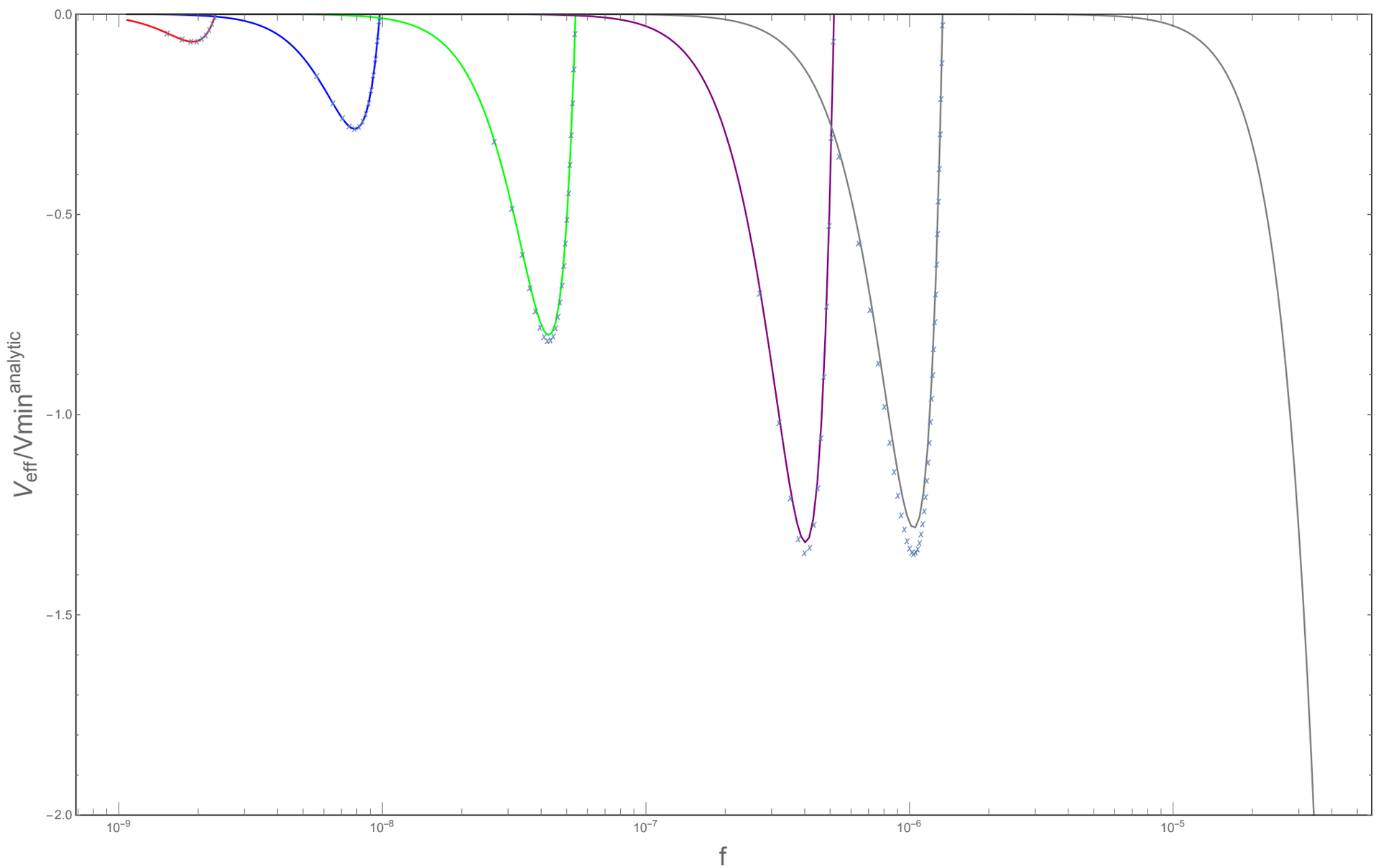
$$\mu_1 = ke^{-y_1}$$

$$\chi = ke^{-y_c}$$

$$V_{\text{eff}} \approx \mu^{\epsilon}\chi^{4-\epsilon} + \lambda\chi^4$$

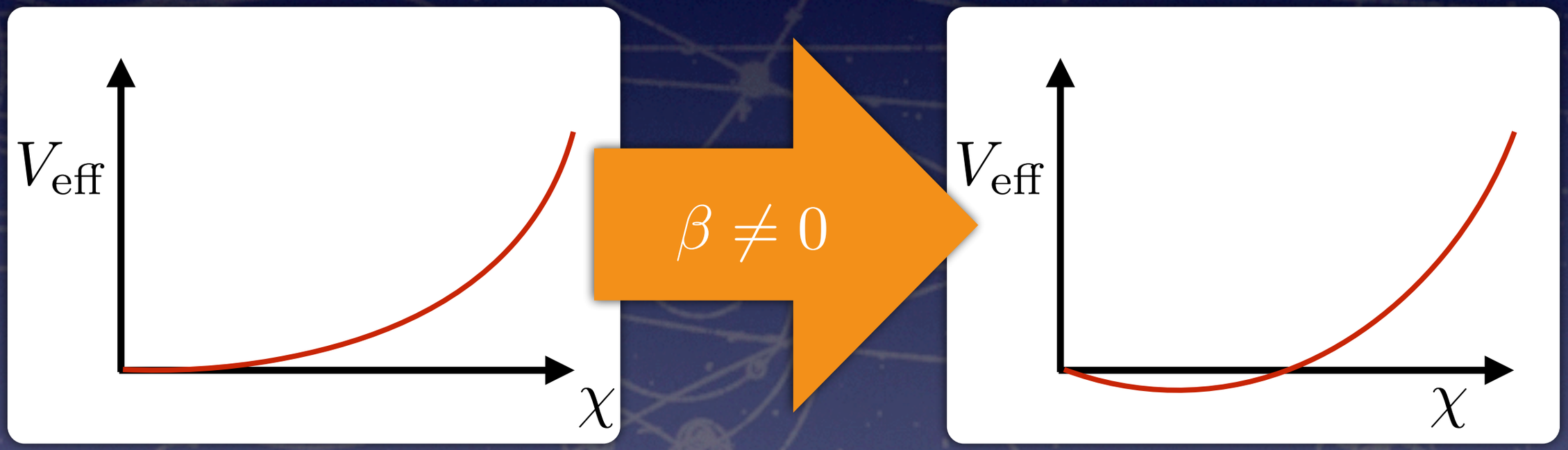
Effective Potential

Preliminary results

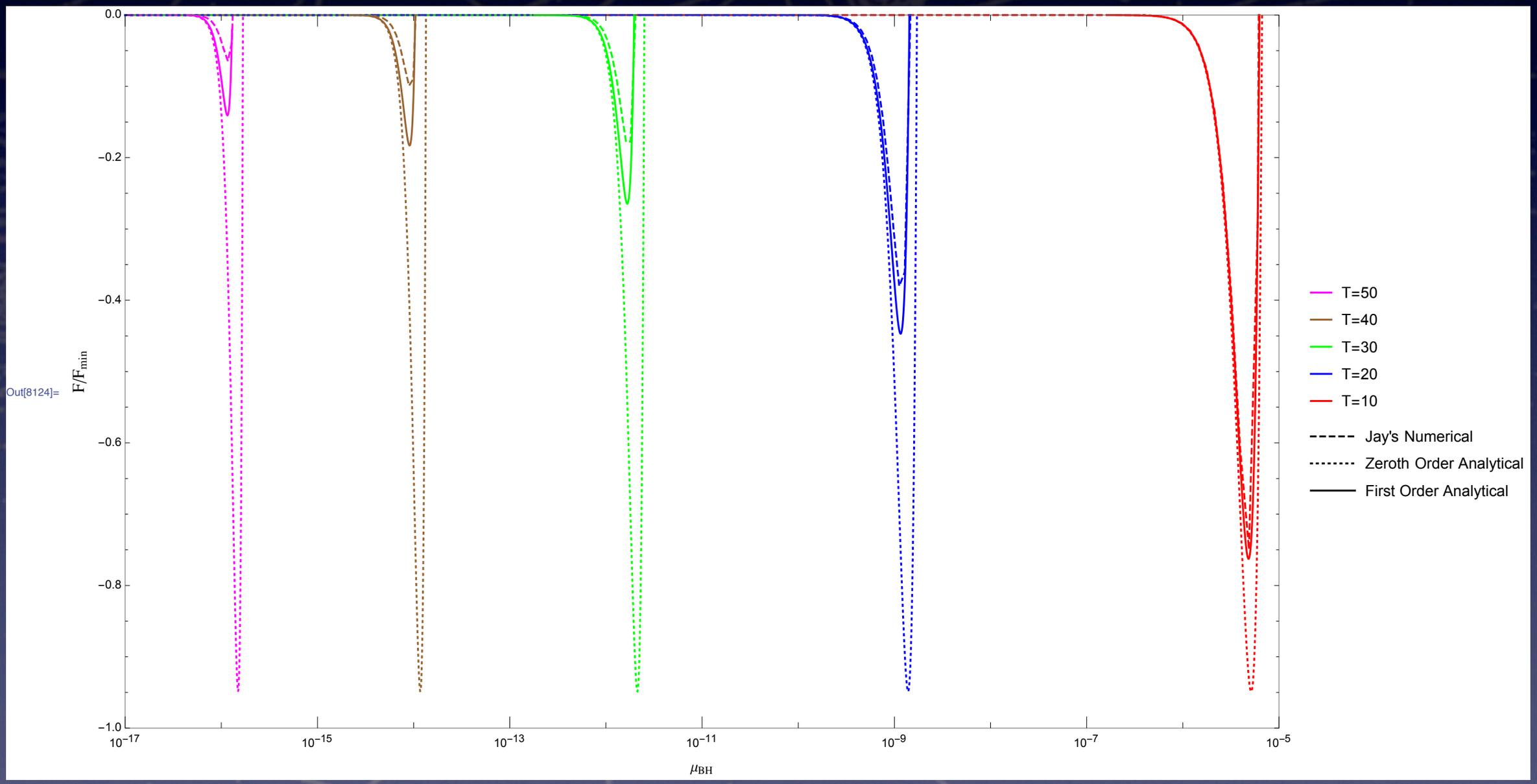


A Holographic Model

A mildly running quartic balances large initial value:

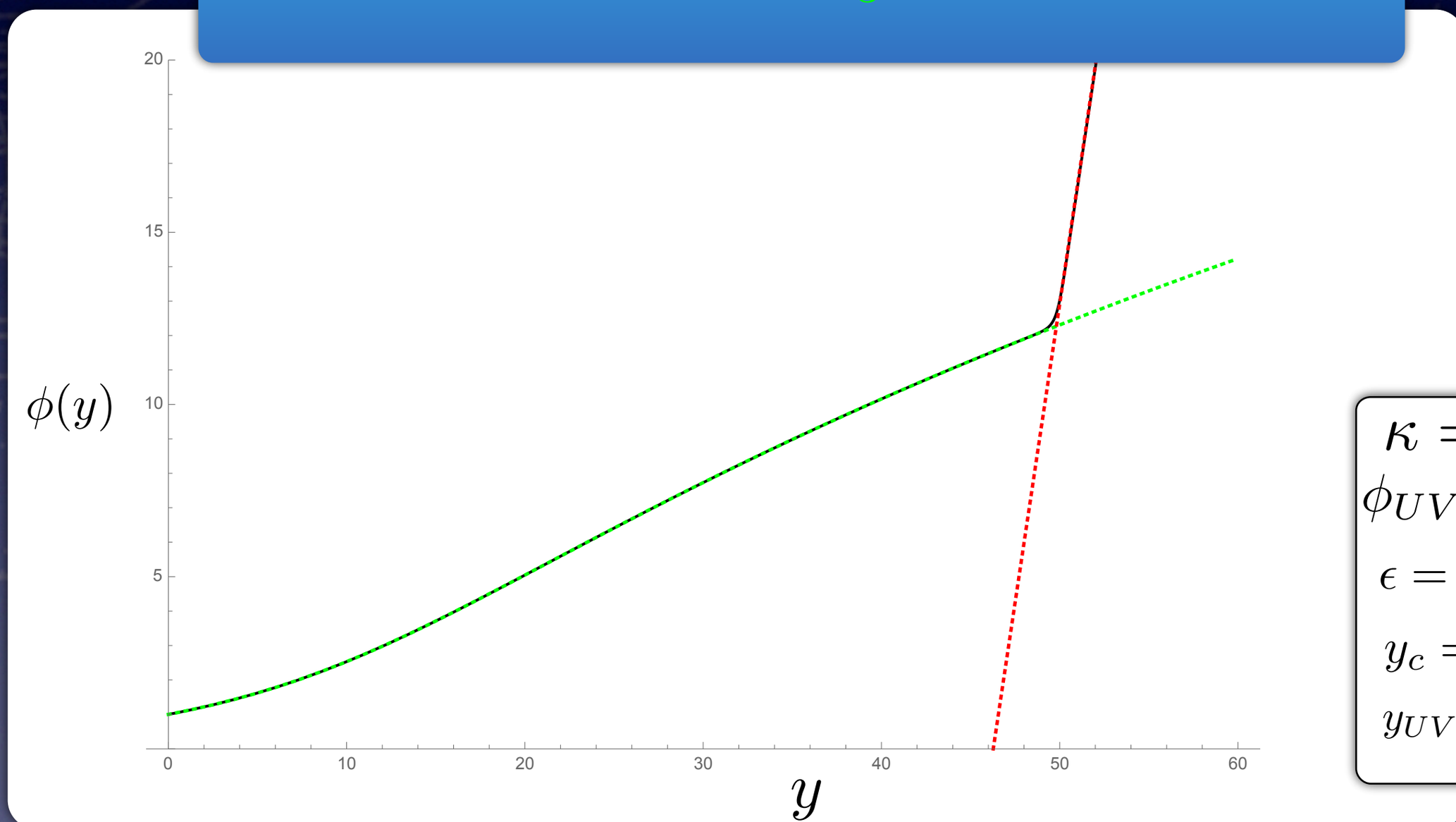


Finite Temperature



A Near Marginal Deformation

$$\ddot{\phi} = 4\left(\dot{\phi} - \frac{1}{\kappa^2}\frac{\epsilon\phi}{1+\kappa^2\frac{\epsilon}{3}\phi^2}\right)\left(1 - \frac{\kappa^2}{12}\dot{\phi}^2\right)$$



$\kappa = 1$

$\phi_{UV} = 1$

$\epsilon = 1/10$

$y_c = 50$

$y_{UV} = 0$