Radiative Electroweak Symmetry Breaking in Standard Model Extensions

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I. Gogoladze, S. Khan, KB (2016)
Virtues of SUSY as a BSM Framework

I. Solves gauge hierarchy problem based on a symmetry principle

II. Successful gauge coupling unification

III. Natural dark matter candidate

IV. Radiative electroweak symmetry breaking
Non-SUSY extensions maintaining these virtues

Gauge hierarchy is accepted as a fact (analogous to cosmological constant)

Gauge coupling unification realized with an intermediate Pati-Salam symmetry in $SO(10)$

New physics for neutrino masses, or dark matter, can lead to radiative electroweak symmetry breaking

Models studied:
1. Type-II Seesaw models
2. Radiative neutrino mass models
3. Singlet scalar dark matter model
4. Inert Higgs doublet model
5. Model for 750 GeV scalar

S. Khan, KB (2015)
Evolution of Higgs mass parameter in SM

\[ V(\phi) = m_\phi^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \]

\[ 16\pi^2 \frac{dm_\phi^2}{dt} = m_\phi^2 \left( 6\lambda + 2 \text{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e) - \frac{9}{2} g_2^2 - \frac{9}{10} g_1^2 \right) \]

If \( m_\phi^2 \) is positive at a high energy, it will remain so at low energy.

If usual type-I seesaw mechanism is implemented, \( m_\phi^2 \) becomes more positive at low energy.

\[ 16\pi^2 \frac{dm_\phi^2}{dt} = 16\pi^2 \left( \frac{dm_\phi^2}{dt} \right)_{\text{SM}} - 4 \text{Tr}(Y_\nu Y_\nu^\dagger M_R^\dagger M_R) \]

\( M_R \) cannot exceed \( 10^7 \) GeV, or else the Higgs mass is fine-tuned.

Vissani (1997)

New TeV scale scalars can turn Higgs (mass)\(^2\) negative.
Motivations for turning positive to negative

In many extensions of the SM, Higgs boson is part of a larger multiplet that breaks a higher symmetry.

Consistency of the high scale symmetry breaking would require all physical boson \((\text{mass})^2\) to be positive.

This set includes the SM Higgs boson, which must then turn negative for EW symmetry breaking by some mechanism.

Examples:

I. \(SO(10)\) symmetry broken by a single \(144\) Babu, Gogoladze, Nath, Syed (2005)

\(144\) contains a SM singlet, as well as a Higgs doublet.

II. Trinification model based on \(SU(3)_c \times SU(3)_L \times SU(3)_R\)


Two fields in \((1,3,3^*)\) break symmetry down to SM. These fields contain SM Higgs boson.
Radiative EW symmetry breaking in type-II seesaw models

A scalar triplet $\Delta(1, 3, 1)$ acquires a small VEV and induces tiny Majorana neutrino masses directly.

$$\mathcal{L}_{Yuk} = -Y_{ij}^\Delta \ell^T_i C i\tau_2 \Delta \ell_j + h.c.$$ 

Higgs potential:

$$V = m_{\phi}^2 \phi^T \phi + m_\Delta^2 \text{Tr}(\Delta^\dagger \Delta) + \frac{\lambda}{2} (\phi^T \phi)^2 + \frac{\Lambda_1}{2} (\text{Tr}(\Delta^\dagger \Delta))^2$$

$$+ \frac{\Lambda_2}{2} \left[ (\text{Tr}(\Delta^\dagger \Delta))^2 - \text{Tr}(\Delta^\dagger \Delta \Delta^\dagger \Delta) \right] + \Lambda_4 \phi^T \phi \text{Tr}(\Delta^\dagger \Delta)$$

$$+ \Lambda_5 \phi^T [\Delta^\dagger, \Delta] \phi + \left\{ \frac{\mu}{\sqrt{2}} \phi^T i\tau_2 \Delta^\dagger \phi + h.c. \right\}$$

Boundedness conditions:

$$\lambda > 0, \quad \Lambda_1 > 0, \quad \Lambda_1 + \frac{\Lambda_2}{2} > 0, \quad \Lambda_4 \pm \Lambda_5 + 2 \sqrt{\lambda \Lambda_1} > 0$$

$$\Lambda_4 \pm \Lambda_5 + 2 \sqrt{\lambda \left( \Lambda_1 + \frac{\Lambda_2}{2} \right)} > 0$$

Arhib et al (2001)
Evolution of mass parameters in type-II seesaw models

\[
16\pi^2 \frac{dm^2_\phi}{dt} = m^2_\phi \left( -\frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 + 2 \text{Tr}(3Y^\dagger_u Y_u + 3Y^\dagger_d Y_d + Y^\dagger_e Y_e) \right) + m^2_\Delta (6\lambda_4) + 6\mu^2
\]

\[
16\pi^2 \frac{dm^2_\Delta}{dt} = m^2_\Delta \left( -\frac{18}{5} g_1^2 - 12 g_2^2 + 2 \text{Tr}(Y^\dagger_\Delta Y_\Delta) + 8\Lambda_1 + 2\Lambda_2 \right) + m^2_\phi (4\Lambda_4) + 2\mu^2
\]

\[
16\pi^2 \dot{\lambda} = 6\lambda^2 - 3\lambda \left( 3g_2^2 + \frac{3}{5} g_1^2 \right) + 3g_2^4 + \frac{3}{2} \left( \frac{3}{5} g_1^2 + g_2^2 \right)^2 + 4\lambda T - 8H + 12\Lambda_4^2 + 8\Lambda_5^2
\]

\[
16\pi^2 \dot{\Lambda}_1 = -\frac{36}{5} g_1^2 \Lambda_1 - 24g_2^2 \Lambda_1 + \frac{108}{25} g_1^4 + 18g_2^4 + \frac{72}{5} g_1^2 g_2^2 + 14\Lambda_1^2 + 4\Lambda_1 \Lambda_2 + 2\Lambda_2^2 + 4\Lambda_4^2 + 4\Lambda_5^2
\]

\[
+ 4 \text{tr} \left( Y^\dagger_\Delta Y_\Delta \right) \Lambda_1 - 8 \text{tr} \left( Y^\dagger_\Delta Y_\Delta Y^\dagger_\Delta Y_\Delta \right)
\]

\[
16\pi^2 \dot{\Lambda}_2 = -\frac{36}{5} g_1^2 \Lambda_2 - 24g_2^2 \Lambda_2 + 12g_2^4 - \frac{144}{5} g_1^2 g_2^2 + 3\Lambda_2^2 + 12\Lambda_1 \Lambda_2 - 8\Lambda_5^2 + 4 \text{tr} \left( Y^\dagger_\Delta Y_\Delta \right) \Lambda_2
\]

\[
+ 8 \text{tr} \left( Y^\dagger_\Delta Y_\Delta Y^\dagger_\Delta Y_\Delta \right)
\]

\[
16\pi^2 \dot{\Lambda}_4 = -\frac{9}{2} g_1^2 \Lambda_4 - \frac{33}{2} g_2^2 \Lambda_4 + \frac{27}{25} g_1^4 + 6g_2^4 + 8\Lambda_1 + 2\Lambda_2 + 3\lambda + 4\Lambda_4 + 2T + 2 \text{tr} \left( Y^\dagger_\Delta Y_\Delta \right) \Lambda_4 + 8\Lambda_5^2
\]

\[
16\pi^2 \dot{\Lambda}_5 = -\frac{9}{2} g_1^2 \Lambda_5 - \frac{33}{2} g_2^2 \Lambda_5 - \frac{18}{5} g_1^2 g_2^2 + 2\Lambda_1 - 2\Lambda_2 + \lambda + 8\Lambda_4 + 2T + 2 \text{tr} \left( Y^\dagger_\Delta Y_\Delta \right) \Lambda_5
\]

M. Schmidt (2007)
Bounded quartic coupling evolution in type-II seesaw models

\[ M_\Delta = 500 \text{ GeV}, \quad \Lambda_1(M_\Delta) = 0.15, \quad \Lambda_2(M_\Delta) = 0.45, \quad \Lambda_4(M_\Delta) = 0.19, \]

\[ \Lambda_5(M_\Delta) = 0.1, \quad \mu(M_\Delta) = 10^{-5} \text{ GeV}, \quad \lambda(M_Z) = 0.258, \quad \lambda(M_\Delta) = 0.1887 \]
Turning of Higgs mass-squared in type-II seesaw models

Masses of $\Delta$ fields should be below a TeV
Radiative EW symmetry breaking in loop-induced neutrino mass models

\[ \mathcal{L} = f_{ij} L^a_i L^b_j h^+ \epsilon_{ab} + g_{ij} e^c_i e^c_j k^{--} + \mu h^+ h^+ k^{--} + h.c. \]

\[ \Downarrow \]

\[ \mathcal{O}_9 = L^i L^j L^k e^c L^l e^c \epsilon_{ij} \epsilon_{kl} \]

Consistent with all neutrino oscillation data
Predicts doubly charged Higgs boson with TeV mass
One neutrino is nearly massless
CP violation in neutrino oscillation is expected
Evolution of masses in loop-induced neutrino mass models

Higgs potential:

\[
V = \frac{1}{2} \lambda_1 (H^+ H)^2 + \frac{1}{2} \lambda_2 \left(h^+ h^-(k^{++} k^{--})\right)^2 + \frac{1}{2} \lambda_3 \left(k^{++} k^{--}\right)^2
\]

\[+ \lambda_4 (H^+ H) (h^+ h^-) + \lambda_5 (H^+ H) (k^{++} k^{--}) + \lambda_6 (h^+ h^-) (k^{++} k^{--})\]

\[+ \mu^2_H H^+ H + \mu^2_h h^+ h^- + \mu^2_k k^{++} k^{--} - (\mu h^+ h^- k^{--} + h.c.)\].

Boundedness conditions:

\[\lambda_{ii} \geq 0,\]

\[\lambda_4 \geq -\sqrt{\lambda_1 \lambda_2}; \quad \lambda_5 \geq -\sqrt{\lambda_1 \lambda_3}; \quad \lambda_6 \geq -\sqrt{\lambda_2 \lambda_3},\]

\[\lambda_4 \sqrt{\lambda_3} + \lambda_6 \sqrt{\lambda_1} + \lambda_5 \sqrt{\lambda_2} + \sqrt{\lambda_1 \lambda_2 \lambda_3} \geq 0 \quad \text{or} \quad \det \lambda \geq 0.\]

\[
16\pi^2 \frac{dm^2_\phi}{dt} = m^2_\phi \left(-\frac{9}{10} g_1^2 - \frac{9}{2} g_2^2 + 2 \text{Tr} (3 Y^\dagger_u Y_u + 3 Y^\dagger_d Y_d + Y^\dagger_e Y_e) + 6 \lambda_1\right)
\]

\[+ 2 \lambda_4 m^2_h + 2 \lambda_5 m^2_k\]

\[
16\pi^2 \frac{dm^2_h}{dt} = m^2_h \left(-\frac{18}{5} g_1^2 + 8 \text{Tr} (f^\dagger f) + 4 \lambda_2\right) + 4 \lambda_4 m^2_\phi + 2 \lambda_6 m^2_k + 8 \mu^2
\]

\[
16\pi^2 \frac{dm^2_k}{dt} = m^2_k \left(-\frac{72}{5} g_1^2 + 4 \text{Tr} (h^\dagger h) + 4 \lambda_3\right) + 4 \lambda_5 m^2_\phi + 2 \lambda_6 m^2_h + 4 \mu^2
\]
Bounded quartic coupling evolution in loop induced neutrino mass models

\[ m_{h^+} = 800 \text{ GeV}, \quad m_{k^{++}} = 450 \text{ GeV}. \quad \mu(M_Z) = 500 \text{ GeV} \]

\[ \lambda_1(M_Z) = 0.26, \quad \lambda_2(M_Z) = 0.2, \quad \lambda_3(M_Z) = 0.5, \]

\[ \lambda_4(M_Z) = 0.37, \quad \lambda_5(M_Z) = 0.3, \quad \lambda_6(M_Z) = -0.25, \]

\[ f_{e\mu} = 4.97 \times 10^{-3}, \quad f_{e\tau} = 7.55 \times 10^{-3}, \quad f_{\mu\tau} = 1.3 \times 10^{-2}, \quad |h_{\mu\mu}| = 0.44 \]
Turning of Higgs mass-squared in loop neutrino models

Masses of $h^+$ and $K^{++}$ should be below a TeV
Singlet extension of SM for dark matter

A real scalar singlet $S$ is introduced with a discrete $Z_2$ symmetry that remains unbroken. $S$ is the dark matter, which annihilates through the Higgs portal.


Higgs potential:

$$V = m^2_\phi \phi^\dagger \phi + \frac{m^2_S}{2} S^2 + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{8} S^2 + \frac{\lambda_3}{2} \phi^\dagger \phi S^2$$

Boundedness conditions:

$$\lambda_1 > 0, \quad \lambda_2 > 0, \quad \lambda_3 > -\sqrt{\lambda_1 \lambda_2}$$

Evolution of masses:

$$16\pi^2 \frac{dm^2_\phi}{dt} = m^2_\phi \left( 6\lambda + 2 \text{Tr}(3Y^\dagger_u Y_u + 3Y^\dagger_d Y_d + Y^\dagger_e Y_e) - \frac{9}{2} g_2^2 - \frac{9}{10} g_1^2 \right) + \lambda_3 m^2_S$$

$$16\pi^2 \frac{dm^2_S}{dt} = 3\lambda_2 m^2_S + 4\lambda_3 m^2_\phi$$

Bounded quartic coupling evolution in singlet dark matter model

\[ m_S = 500 \text{ GeV}, \quad \lambda_2(m_S) = 0.85, \quad \lambda_3(m_S) = 0.5 \]
Turning of Higgs mass-squared in singlet dark matter models

\[ m_S^2 \] remains positive, while \( m_\phi^2 \) turns negative
Radiative EW symmetry breaking in Inert doublet models

A scalar doublet $\eta$ which is $Z_2$ odd serves as dark matter

Higgs potential:

$$V = m_\phi^2 \phi^\dagger \phi + m_\eta^2 \eta^\dagger \eta + \frac{\lambda_1}{2} (\phi^\dagger \phi)^2 + \frac{\lambda_2}{2} (\eta^\dagger \eta)^2$$

$$+ \lambda_3 (\phi^\dagger \phi) (\eta^\dagger \eta) + \lambda_4 (\phi^\dagger \eta) (\eta^\dagger \phi) + \{ \frac{\lambda_5}{2} (\phi^\dagger \eta)^2 + \text{h.c.} \}$$

Mass evolution:

$$16\pi^2 \frac{dm_\phi^2}{dt} = m_\phi^2 \left( 6\lambda_1 + 2 \text{Tr}(3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e) - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 \right) + m_\eta^2 (4\lambda_3 + 2\lambda_4)$$

$$16\pi^2 \frac{dm_\eta^2}{dt} = m_\eta^2 \left( 6\lambda_2 - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 \right) + m_\phi^2 (4\lambda_3 + 2\lambda_4)$$

Ma (2006)
Barbieri, Hall, Rychkov (2006)
Bounded quartic coupling evolution in inert doublet model

\[ m_\eta = 800 \text{ GeV}, \quad \lambda_2(m_\eta) = 0.35, \quad \lambda_3(m_\eta) = 0.2, \]
\[ \lambda_4(m_\eta) = 0.11, \quad \lambda_5(m_\eta) = 0.1 \]
Turning of Higgs mass-squared in inert doublet models

$m^2_\eta$ remains positive, while $m^2_\phi$ turns negative
Turning of Higgs mass-squared in a model for 750 GeV scalar

$s_1$ and $s_2$ are SM singlet scalars whose masses remain positive

Dev, Mohapatra, Zhang (2016)
Boundedness of Higgs potential in a model for 750 GeV scalar

\[(G^{ru})_{ii} = (G^{id})_{ii} = (G^{le})_{ii} = 0.45,\]
\[\lambda_2 = 0.15, \quad \lambda_3 = 0.3, \quad \lambda_4 = 0.17, \quad \lambda_5 = 0.2, \quad \lambda_6 = 0.05\]
Yukawa coupling evolution in a model for 750 GeV scalar

$F^{u,d,e}$, $G^{u,d,e}$ are Yukawa matrices involving vector-like fermions
Summary and Conclusions

- Several SM extensions require turning of positive \((\text{mass})^2\) to negative value by RGE running

- Additional scalars near the TeV scale is the best hope

- Type-II seesaw model, radiative neutrino mass models, scalar dark matter models all have this built-in feature

- Discovery of new scalars at the LHC may support this idea

- In models of a 750 GeV scalar, radiative EW symmetry breaking can occur, but the potential for the new scalar becomes unbounded typically due to the needed large Yukawa couplings