Perturbative Unitarity in the Dark Sector

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Pheno Conference, PITT-PACC
May 10, 2016

Based on 1412.5660, 1501.03153 and ongoing work with Matthew Cahill-Rowley, Sonia El-Hedri, and Devin Walker
The Scale of New Physics

• Historically in HEP, we’ve often known where we were going
  – Fermi theory of weak decays needed new bosons
  – Precision measurements pointed to the top quark
  – Heavy bosons needed symmetry breaking

• After the Higgs discovery, we have no map
  – The Standard Model is stubbornly good

• Where are we going, and how far away is it?
A Picture of Unitarity

\[ y = \text{Im} \tilde{T}_{ii}^J \]

Argand diagram

\[ x = \text{Re} \tilde{T}_{ii}^J \]

nth-loop

Tree Level

1/2

Schuessler and Zeppenfeld 0710.5175
Aydemir, Anber, Donoghue 1203.5153
A Picture of Unitarity

\[ a' = \frac{d_{\text{min}}}{|x_{\text{TL}}|} \]

\[ y = \text{Im} \tilde{T}_{ii}^J \]

\[ x = \text{Re} \tilde{T}_{ii}^J \]

Argand diagram

Schuessler and Zeppenfeld 0710.5175
Gauge Portal Dark Matter

• This model is characterized by the Lagrangian

\[
\mathcal{L}_{DM} \supset g' \bar{\chi} \gamma^\mu \gamma_5 Z'_\mu \chi - \lambda_\chi \bar{\chi} \Phi \chi
\]

\[
\mathcal{L}_{gauge} \supset -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} Z'_{\mu\nu} Z'_{\mu\nu} + \frac{\sin \delta}{2} Z'_{\mu\nu} B^{\mu\nu}
\]

\[
\mathcal{L}_{Higgs} \supset |D_\mu \Phi|^2 + V(H, \Phi; \lambda_1, \lambda_2, \lambda_3)
\]

• With breaking of the new symmetry by

\[
\Phi = \frac{1}{\sqrt{2}} (u + \phi^0)
\]

Babu, Kolda, March-Russell hep-ph/9710441
\[ R = \min \left\{ \frac{|2m_\chi - m_{Z'}|}{m_\chi}, \frac{|2m_\chi - m_{\phi^0}|}{m_\chi} \right\} \]
Colored Scalars and Dark Matter

• In a SUSY-inspired model, we add
  \[ \tilde{u}_R = (\tilde{u}_R, \tilde{c}_R, \tilde{t}_R) \]

• And the Lagrangian terms
  \[ \mathcal{L} \supset \frac{1}{2} M_\chi \bar{\chi} \chi + \frac{1}{2} M_{\tilde{u}}^2 \tilde{u}^* u + \lambda_{\text{dark}} \tilde{u}^* \chi \bar{P}_R u \]

• This introduces the new parameter and scales
  \[ \lambda_{\text{dark}}, M_\chi, M_{\tilde{u}} \]
Dirac Dark Matter

$m_x$ (TeV) vs. $m_{\tilde{u}_R}$ (TeV)
Direct Detection
Majorana Dark Matter

True FCC Reach
Strong Couplings and Bound States

• All of this analysis has focused on the case of very strong couplings to get high allowed mass.

• These large couplings can also lead to other effects that may be important:
  – Sommerfeld enhancements
  – Dark matter bound states
Yukawa Potential Bound States

\[ V(r) = \alpha \frac{e^{-m_{\text{med}} r}}{r} \]

\[ T_{\text{freeze}} \sim \frac{m_X}{20} \]
Cosmological Rates

\[ \Delta = \frac{m \alpha^2}{4} \]

\[ \uparrow \downarrow \text{ and } \uparrow \uparrow \text{ bound states} \]
Bethe-Salpeter Equation

- States that are strongly bound enough to matter will have momenta high enough to require relativistic treatment.
- If ladder diagrams are the dominant contribution to the binding the Bethe-Salpeter equation describes the physics.

\[ G^{(4)} + W + W + \cdots = \]

\[ = G^{(4)} + W + W + \cdots \]
Relativistic Corrections

• States with binding energies of $M/10$ or larger require relativistic corrections to the coupling of a factor of 2 or more

• This will be an important shift in the cosmological implications of strong coupling
Outlook

• Perturbativity arguments can be made fully rigorous through unitarity considerations
• These unitarity bounds provide strong constraints on dark matter dynamics
• Combined with collider searches we will be able to place strong limits on WIMPs
• Models with strong coupling like these may already be affected by new phenomena due to bound state formation
  – Investigations of cosmological impact of bound state dynamics are in progress