

# The Meaning of Precision in Top-Quark Physics

*M. Czakon*



*CERN, 24<sup>th</sup> February 2016*

# Outline

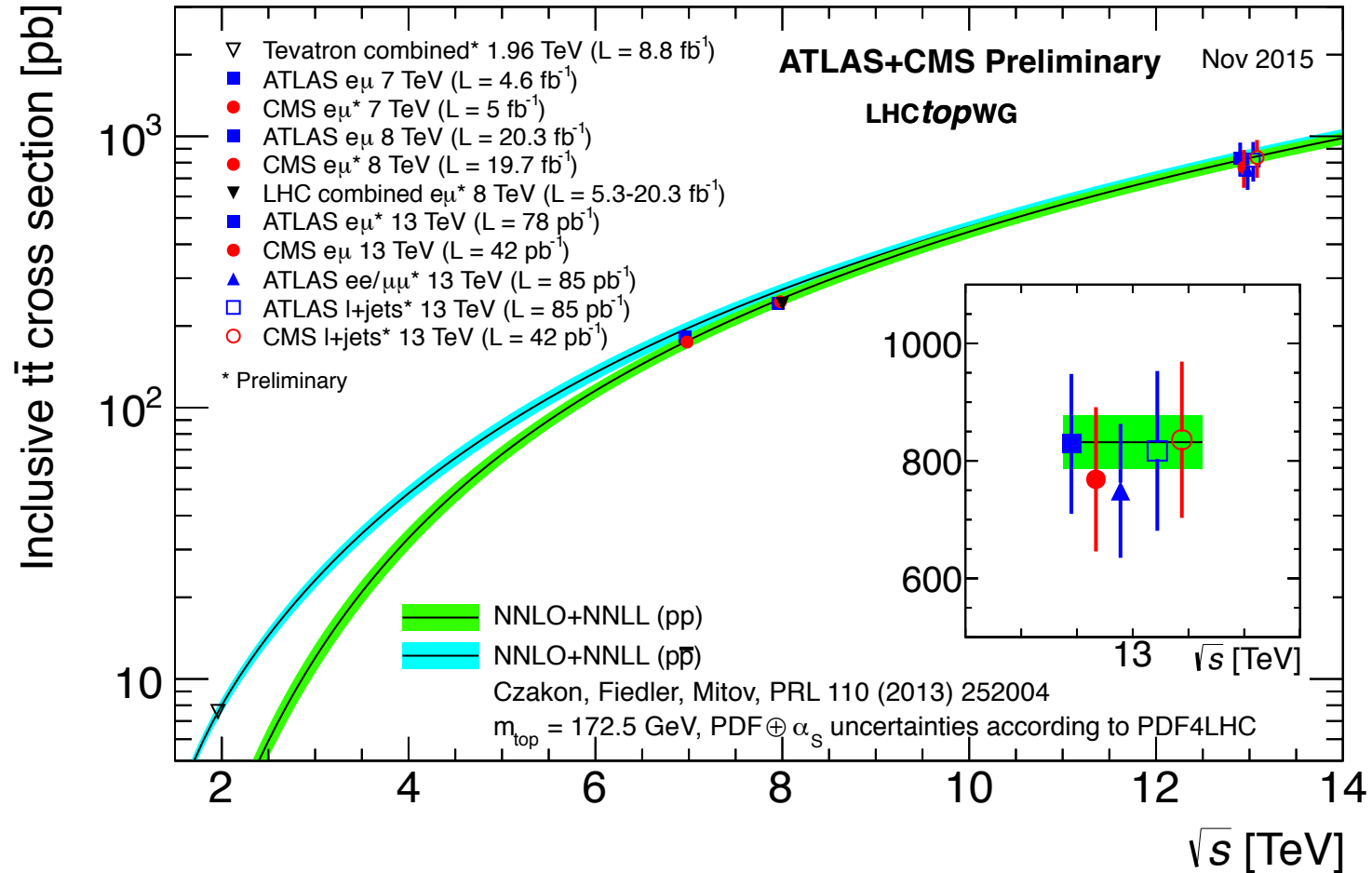
1. Precision and applications
2. Resummation
3. Fixed-order perturbation theory
4. Outlook

Collaborators:

*P. Fiedler, D. Heymes, A. Mitov*

# PRECISION & APPLICATIONS

# Total Cross Section





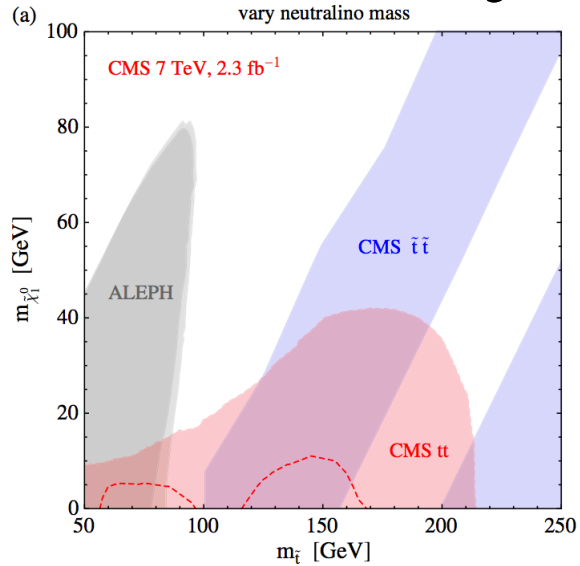
# Caveat

- Inclusive cross sections are obtained by extrapolation

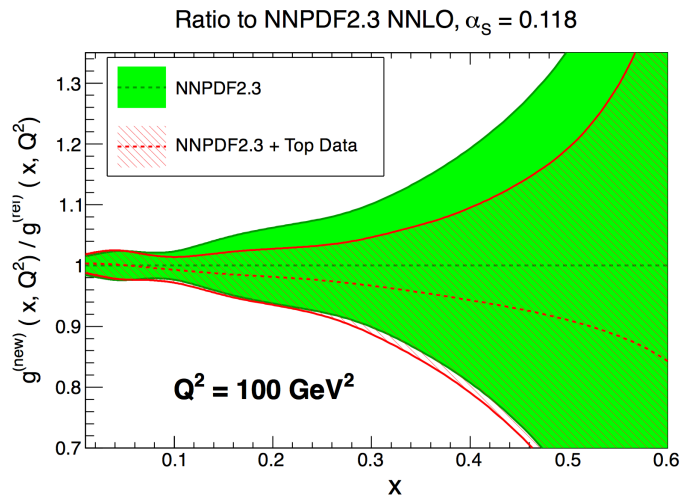
*CMS-PAS-TOP-13-004*

Source	Uncertainty [%]	
	7 TeV	8 TeV
Total (vis)	$\pm_{3.4}^{3.5}$	$\pm_{3.4}^{3.7}$
$Q^2$ scale (extrapol.)	$\pm_{0.0}^{0.4}$	$\pm_{0.1}^{0.2}$
ME/PS matching (extrapol.)	$\mp_{0.1}^{0.1}$	$\pm_{0.3}^{0.3}$
Top $p_T$ (extrapol.)	$\pm_{0.2}^{0.4}$	$\pm_{0.4}^{0.8}$
PDF (extrapol.)	$\mp_{0.1}^{0.2}$	$\mp_{0.2}^{0.1}$
Total	$\pm_{3.4}^{3.6}$	$\pm_{3.5}^{3.8}$

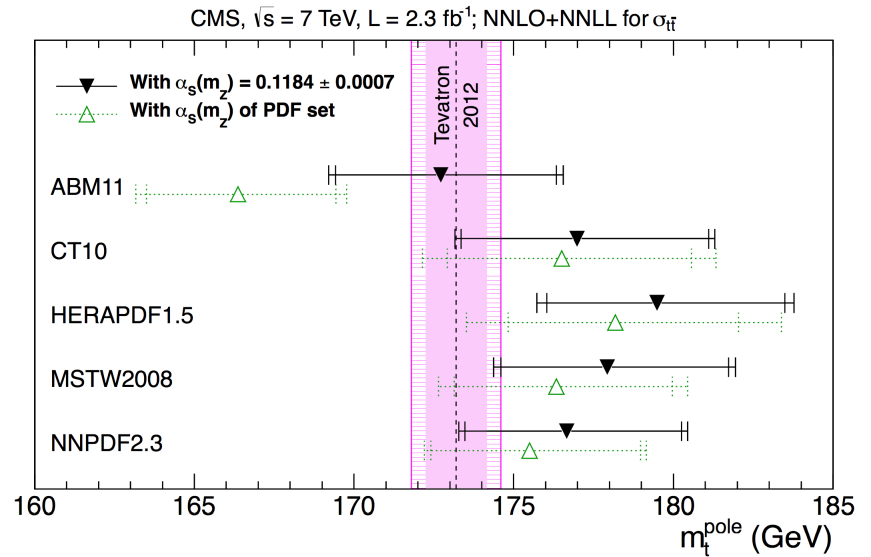
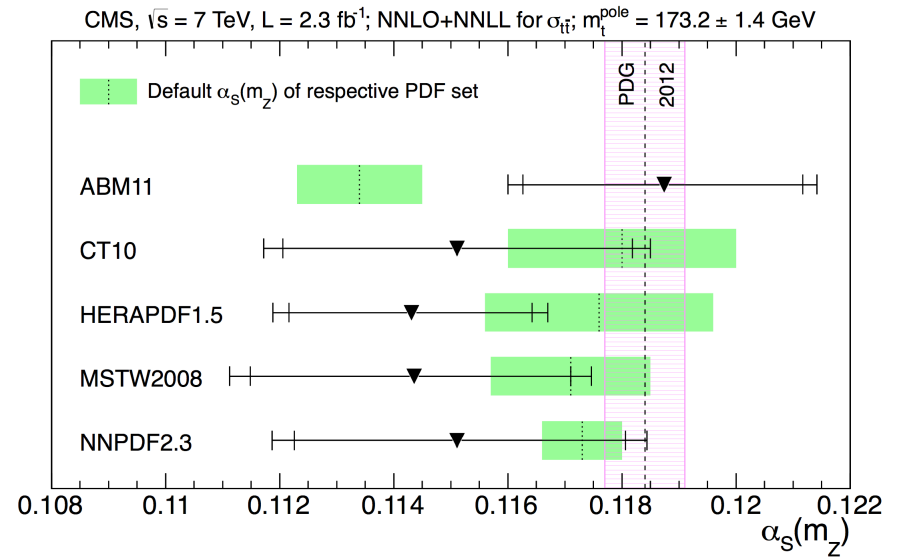
# Early Applications



*MC, Mitov, Papucci, Ruderman, Weiler, '14*



*MC, Mangano, Mitov, Rojo '13*

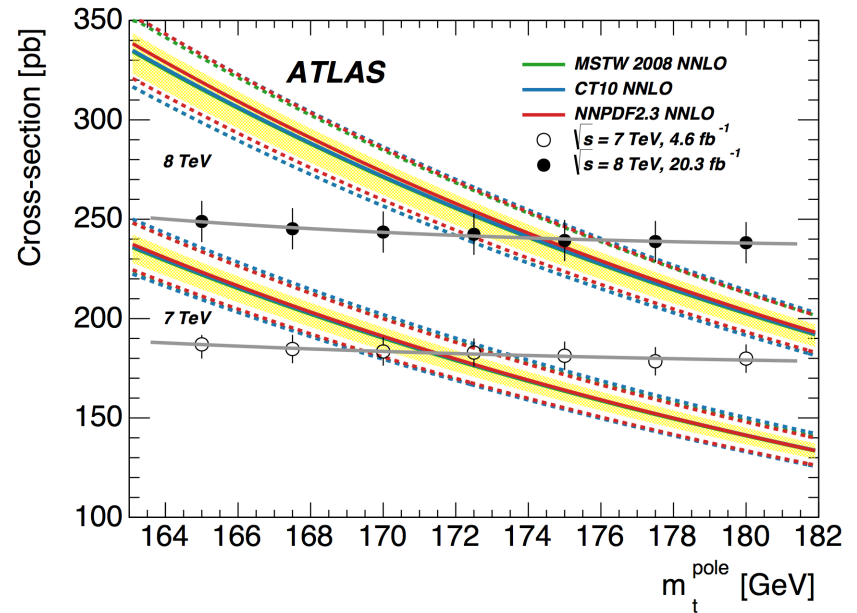


*arXiv:1307.1907 (CMS-TOP-12-022)*

# Top Quark Mass

arXiv:1406.5375

Collected by M. Vos '14



Experiment	pole mass	data	theory	comment
D0 [25]	$169.1^{+5.9}_{-5.2} \text{ GeV}$	$p\bar{p}, 1.96 \text{ TeV}, 1 \text{ fb}^{-1}$		
Langenfeld et al. [24]	$168.9^{+3.5}_{-3.4} \text{ GeV}$	idem	through $\bar{M}S$ mass	
D0 [26]	$167.5^{+5.4}_{-4.9} \text{ GeV}$	$p\bar{p}, 1.96 \text{ TeV}, 5.3 \text{ fb}^{-1}$	approx NNLO	
CMS [32]	$176.7 \pm 2.9 \text{ GeV}$	$pp, 7 \text{ TeV}, 5 \text{ fb}^{-1}$	approx NNLO	
ATLAS [28]	$172.9^{+2.5}_{-2.6} \text{ GeV}$	$pp, 7 \text{ TeV}, 5 \text{ fb}^{-1}$ + $8 \text{ TeV}, 20 \text{ fb}^{-1}$	NNLO+NNLL	full PDF4LHC [34] envelope of 3 PDF sets
CMS [29]	$173.6^{+1.7}_{-1.8} \text{ GeV}$	idem	idem	NNPDF3.0 [35], preliminary
CMS [29]	$173.9^{+1.8}_{-1.9} \text{ GeV}$	idem	idem	MMHT2014, preliminary
CMS [29]	$174.1^{+2.1}_{-2.2} \text{ GeV}$	idem	idem	CT14 [36], preliminary
ATLAS [37]	$173.7^{+2.3}_{-2.1} \text{ GeV}$	$pp, 7 \text{ TeV}, 5 \text{ fb}^{-1}$	NLO $t\bar{t} + 1 \text{ jet}$	full PDF4LHC [34]

# Strong Coupling Constant

## NNLL + NNLO with NNPDF23

Exp.	$E_{\text{CM}}$ [GeV]	$\alpha_s(M_Z)$	Exp.	scale	PDF	$m_{\text{top}}$	$E_{\text{beam}}$	total
ATLAS	7000	0.1207	$\pm 0.0017$	$\pm 0.0014$	$\pm 0.0014$	$\pm 0.0018$	$\pm 0.0009$	$\pm 0.0033$
ATLAS	8000	0.1168	$\pm 0.0018$	$\pm 0.0015$	$\pm 0.0013$	$\pm 0.0018$	$\pm 0.0008$	$\pm 0.0033$
CMS	7000	0.1184	$\pm 0.0016$	$\pm 0.0014$	$\pm 0.0014$	$\pm 0.0018$	$\pm 0.0008$	$\pm 0.0032$
CMS	8000	0.1174	$\pm 0.0017$	$\pm 0.0015$	$\pm 0.0013$	$\pm 0.0018$	$\pm 0.0008$	$\pm 0.0033$
CDF&D0	1960	0.1201	$\pm 0.0032$	$\pm 0.0013$	$\pm 0.0010$	$\pm 0.0013$	$\pm 0.0000$	$\pm 0.0038$
<b>unweighted</b>	<b>average</b>	<b>0.1187</b>						

## plain NNLO with NNPDF23

Exp.	$E_{\text{CM}}$ [GeV]	$\alpha_s(M_Z)$	Exp.	scale	PDF	$m_{\text{top}}$	$E_{\text{beam}}$	total
ATLAS	7000	0.1223	$\pm 0.0018$	$\pm 0.0025$	$\pm 0.0014$	$\pm 0.0018$	$\pm 0.0009$	$\pm 0.0040$
ATLAS	8000	0.1182	$\pm 0.0019$	$\pm 0.0026$	$\pm 0.0013$	$\pm 0.0019$	$\pm 0.0009$	$\pm 0.0041$
CMS	7000	0.1199	$\pm 0.0017$	$\pm 0.0025$	$\pm 0.0014$	$\pm 0.0018$	$\pm 0.0008$	$\pm 0.0039$
CMS	8000	0.1189	$\pm 0.0018$	$\pm 0.0026$	$\pm 0.0013$	$\pm 0.0018$	$\pm 0.0008$	$\pm 0.0040$
TEV	1960	0.1215	$\pm 0.0034$	$\pm 0.0027$	$\pm 0.0010$	$\pm 0.0014$	$\pm 0.0000$	$\pm 0.0047$
<b>unweighted</b>	<b>average</b>	<b>0.1201</b>						

Workshop on high-precision  $\alpha_s$  measurements: from LHC to FCC-ee  
CERN, 13 October 2015

**$\alpha_s$  from  $\sigma(\text{ttbar})$ :  
preliminary new results**

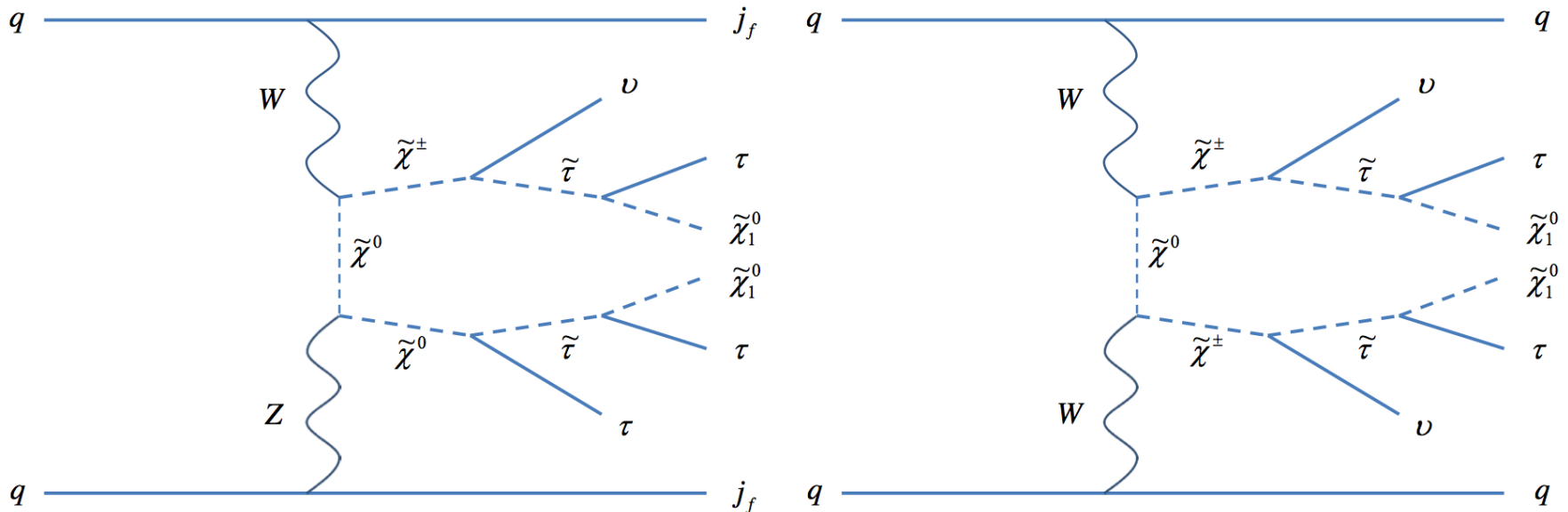
Gavin Salam (CERN), work in progress  
with Sigi Bethke, Günther Dissertori and Thomas Kljnsma

Open question of choice of theory: NNLL+NNLO v. NNLO.  
Latter increases result and uncertainty.

# Search for Supersymmetry

- Search for supersymmetry in the vector-boson fusion topology in proton-proton collisions

LHC @ 8 TeV



CERN-PH-EP/2015-213  
2015/09/01

# Search for Supersymmetry

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LHC @ 8 TeV

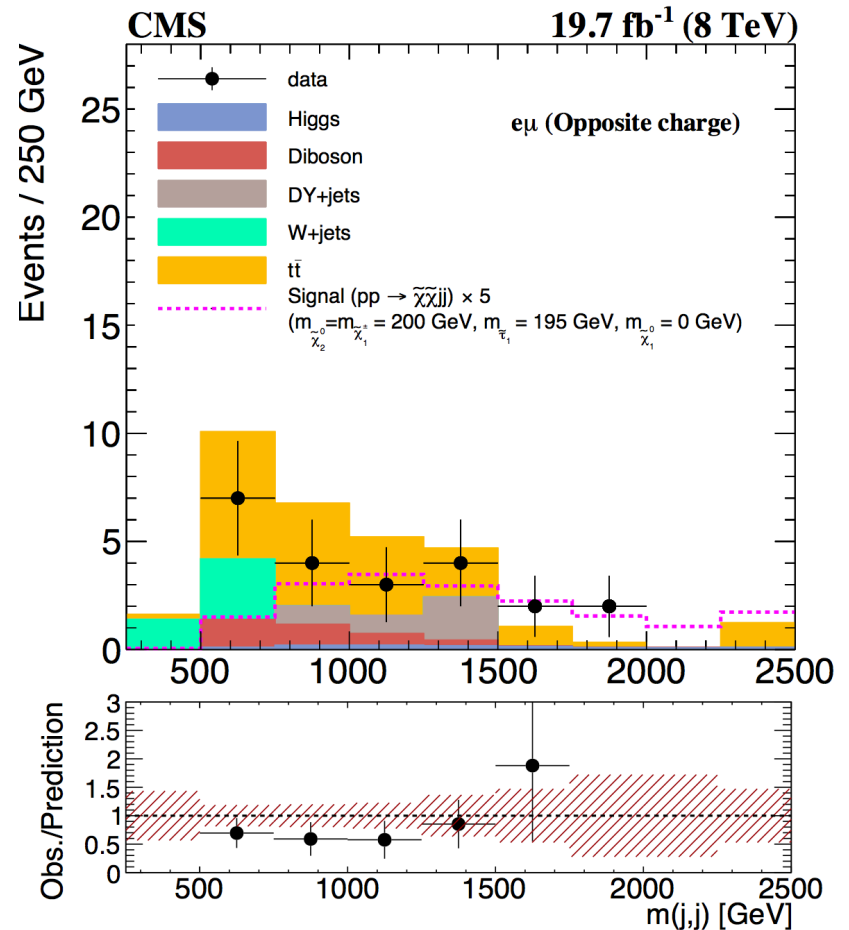
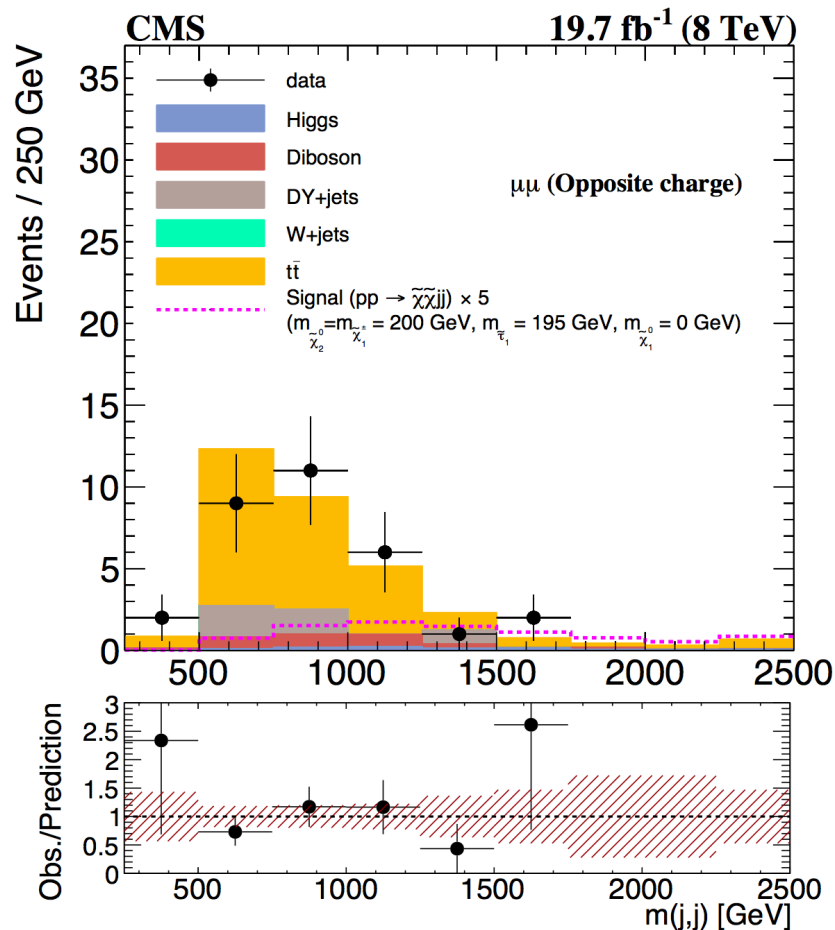
Process	$\mu^\pm \mu^\mp jj$	$e^\pm \mu^\mp jj$	$\mu^\pm \tau_h^\mp jj$	$\tau_h^\pm \tau_h^\mp jj$
Z+jets	$4.3 \pm 1.7$	$3.7^{+2.1}_{-1.9}$	$19.9 \pm 2.9$	$12.3 \pm 4.4$
W+jets	$<0.1$	$4.2^{+3.3}_{-2.5}$	$17.3 \pm 3.0$	$2.0 \pm 1.7$
VV	$2.8 \pm 0.5$	$3.1 \pm 0.7$	$2.9 \pm 0.5$	$0.5 \pm 0.2$
$t\bar{t}$	$24.0 \pm 1.7$	$19.0^{+2.3}_{-2.4}$	$11.7 \pm 2.8$	—
QCD	—	—	—	$6.3 \pm 1.8$
Higgs boson	$1.0 \pm 0.1$	$1.1 \pm 0.5$	—	$1.1 \pm 0.1$
VBF Z	—	—	—	$0.7 \pm 0.2$
Total	$32.2 \pm 2.4$	$31.1^{+4.6}_{-4.1}$	$51.8 \pm 5.1$	$22.9 \pm 5.1$
Observed	31	22	41	31



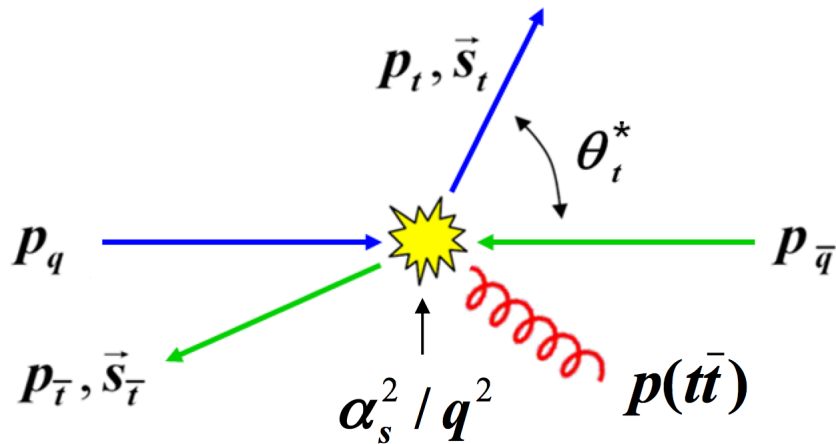
# Search for Supersymmetry

- Search for supersymmetry in the vector-boson fusion topology in proton-proton collisions

LHC @ 8 TeV



# Forward-Backward Asymmetry at the TeVatron

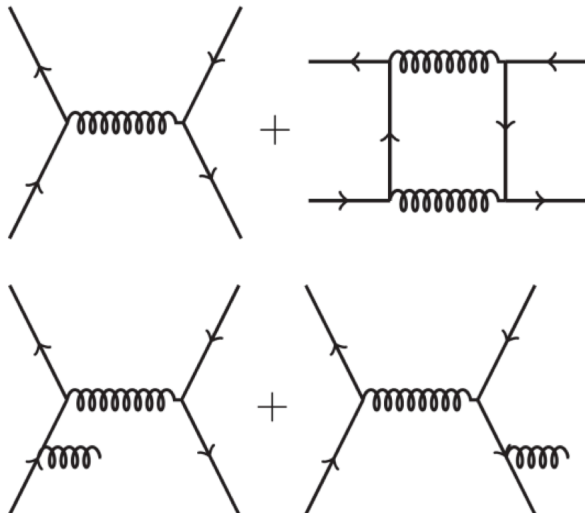


measure in  $\Delta y = y_t - y_{\bar{t}}$

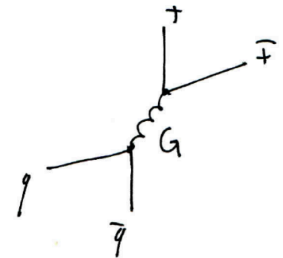
$$A^{t\bar{t}} = \frac{N(\Delta y > 0) - N(\Delta y < 0)}{N(\Delta y > 0) + N(\Delta y < 0)}$$

BSM ideas

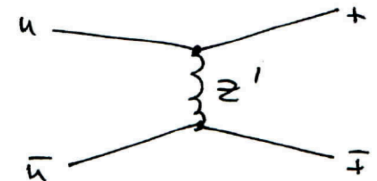
SM



- s-channel
  - massive chiral color octets
  - RS gluon



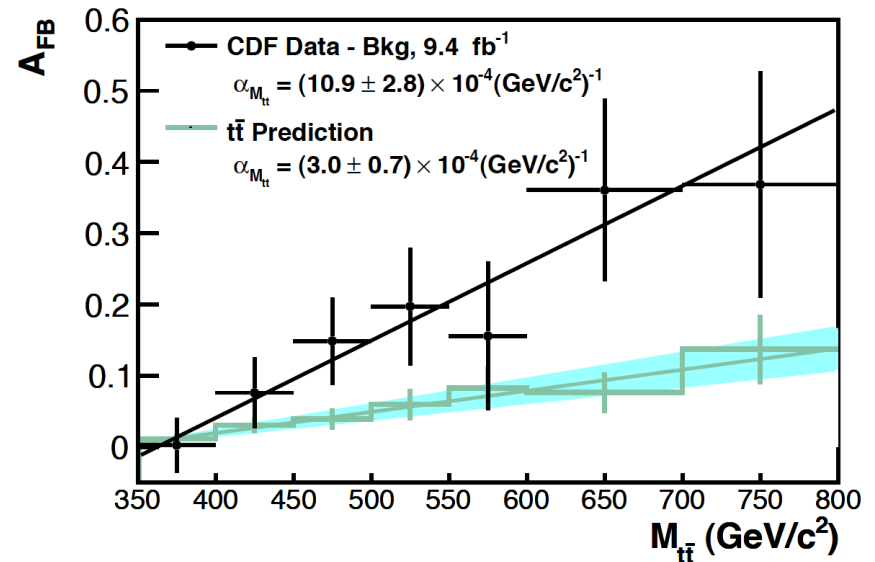
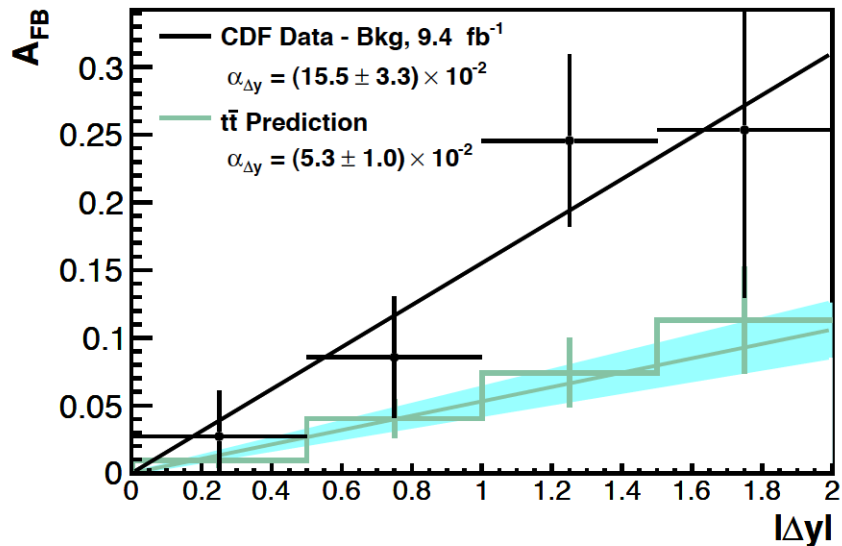
- t-channel
  - $W'Z'$
  - color triplets, sextets





# Forward-Backward Asymmetry at the Tevatron

- ... and the CDF measurement versus (previously known) SM

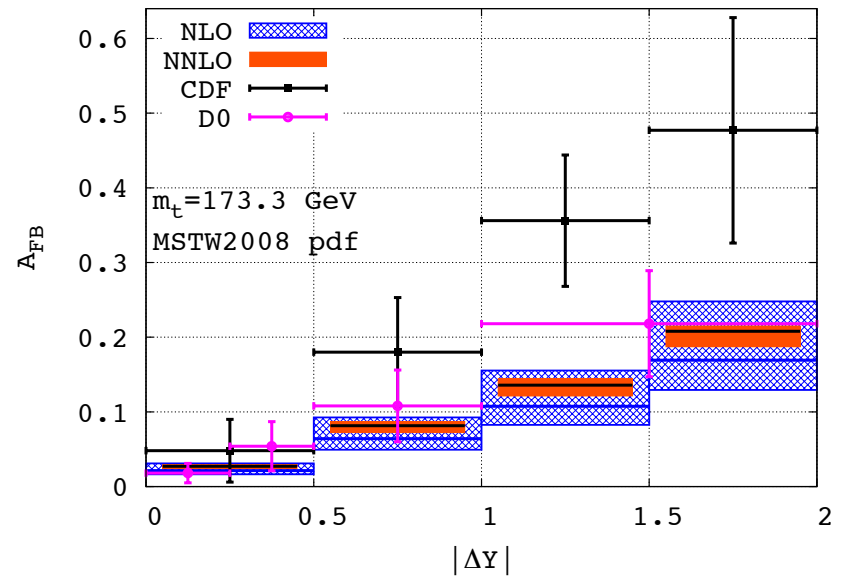
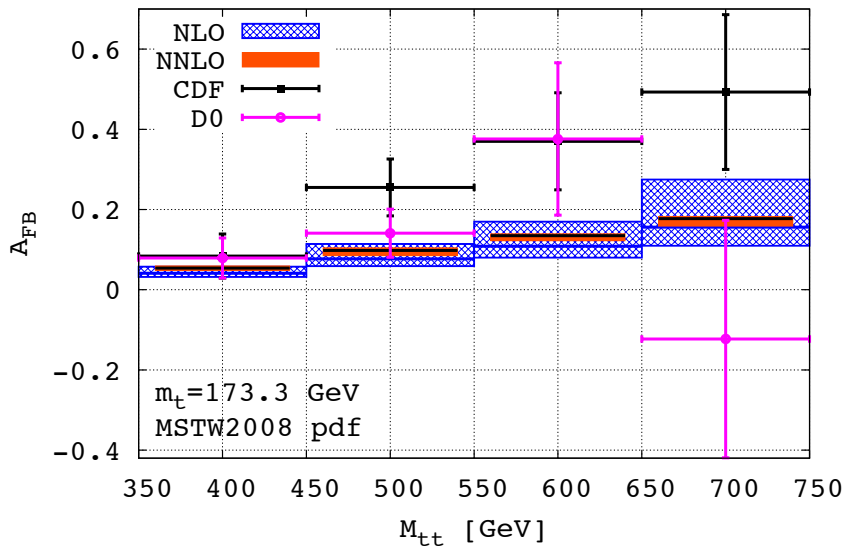
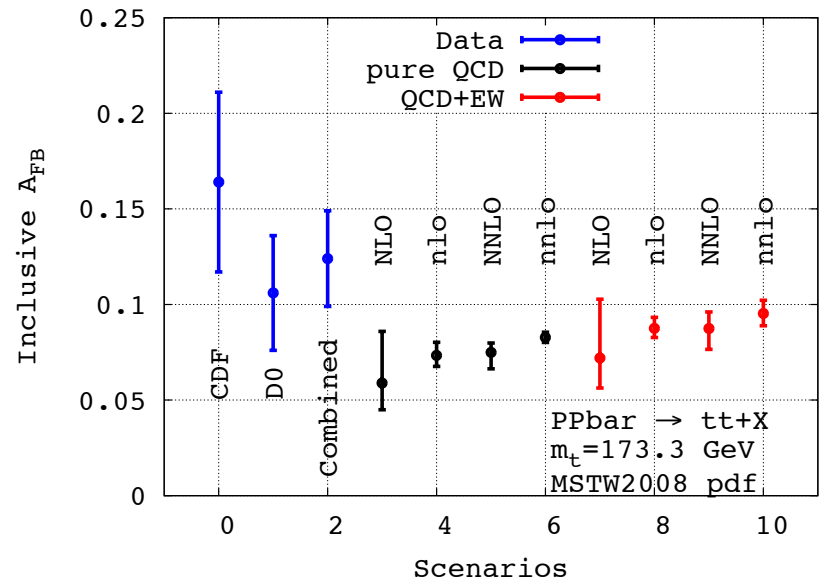


Discrepancy  $\leq 3\sigma$

New D0 measurement (2014):  
much lower than CDF and in good agreement with SM

# Data vs Precision QCD

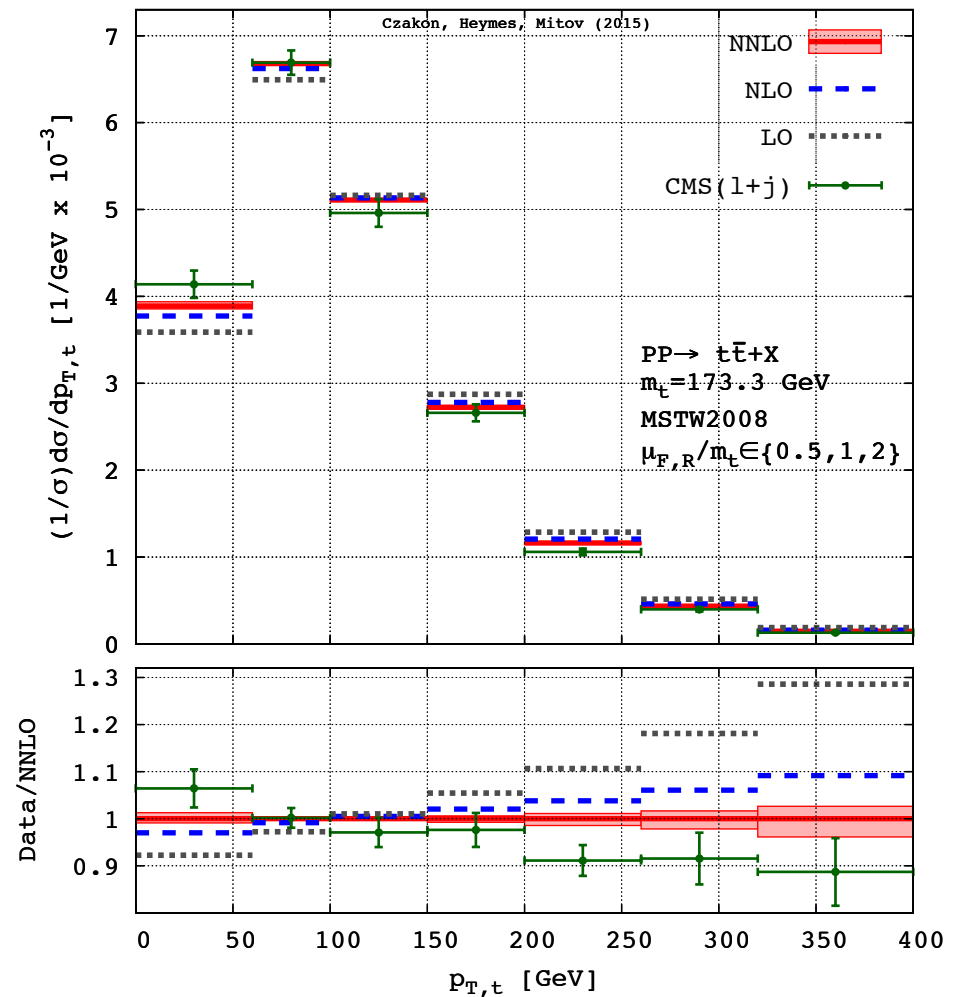
*MC, Fiedler, Mitov '14*



# Differential Distributions @ LHC

- Even with fixed scale the agreement with data quite good
- Apparently convergence poor in normalized distributions

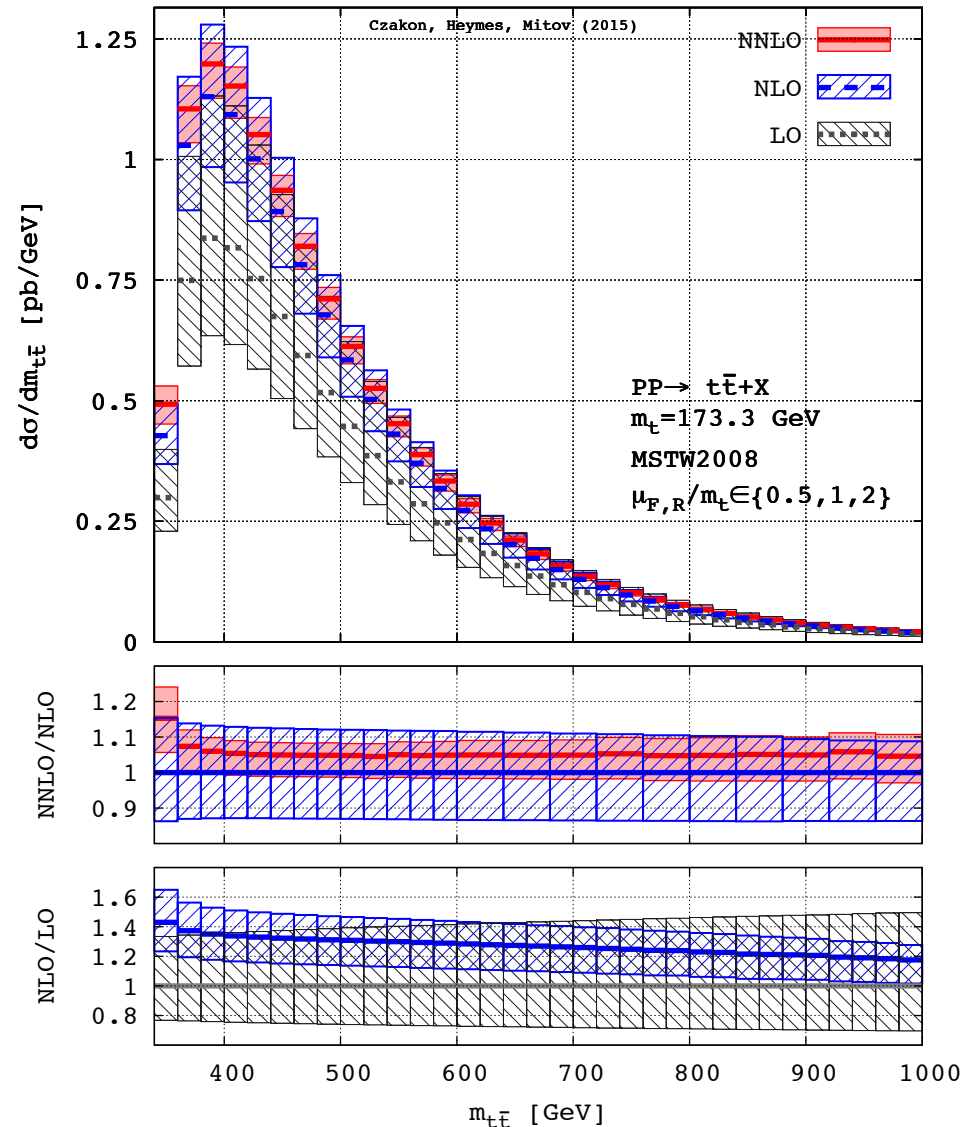
*MC, Heymes, Mitov '15*



# Differential Distributions @ LHC

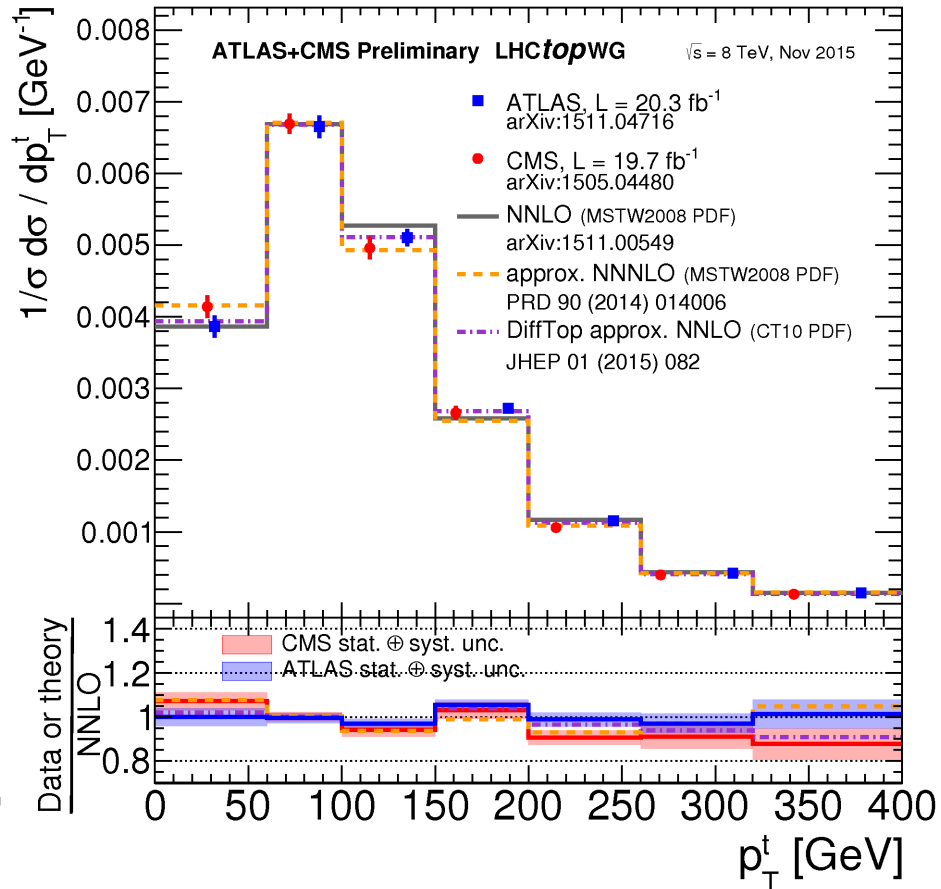
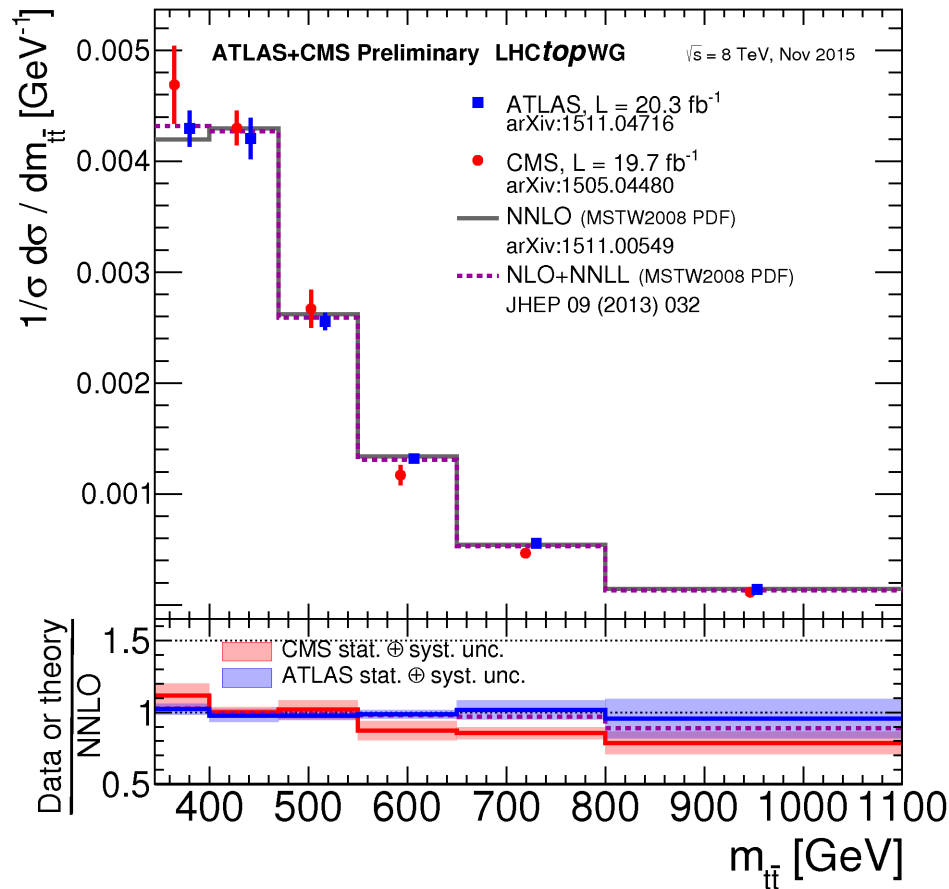
- Much better impression of convergence for absolute distributions
- Stability of invariant mass important for searches
- Limited kinematical range only

*MC, Heymes, Mitov '15*



# Differential Distributions @ LHC

- Much better agreement with ATLAS data
- Lesson for the theorist: “spot-on agreement” may be dangerous

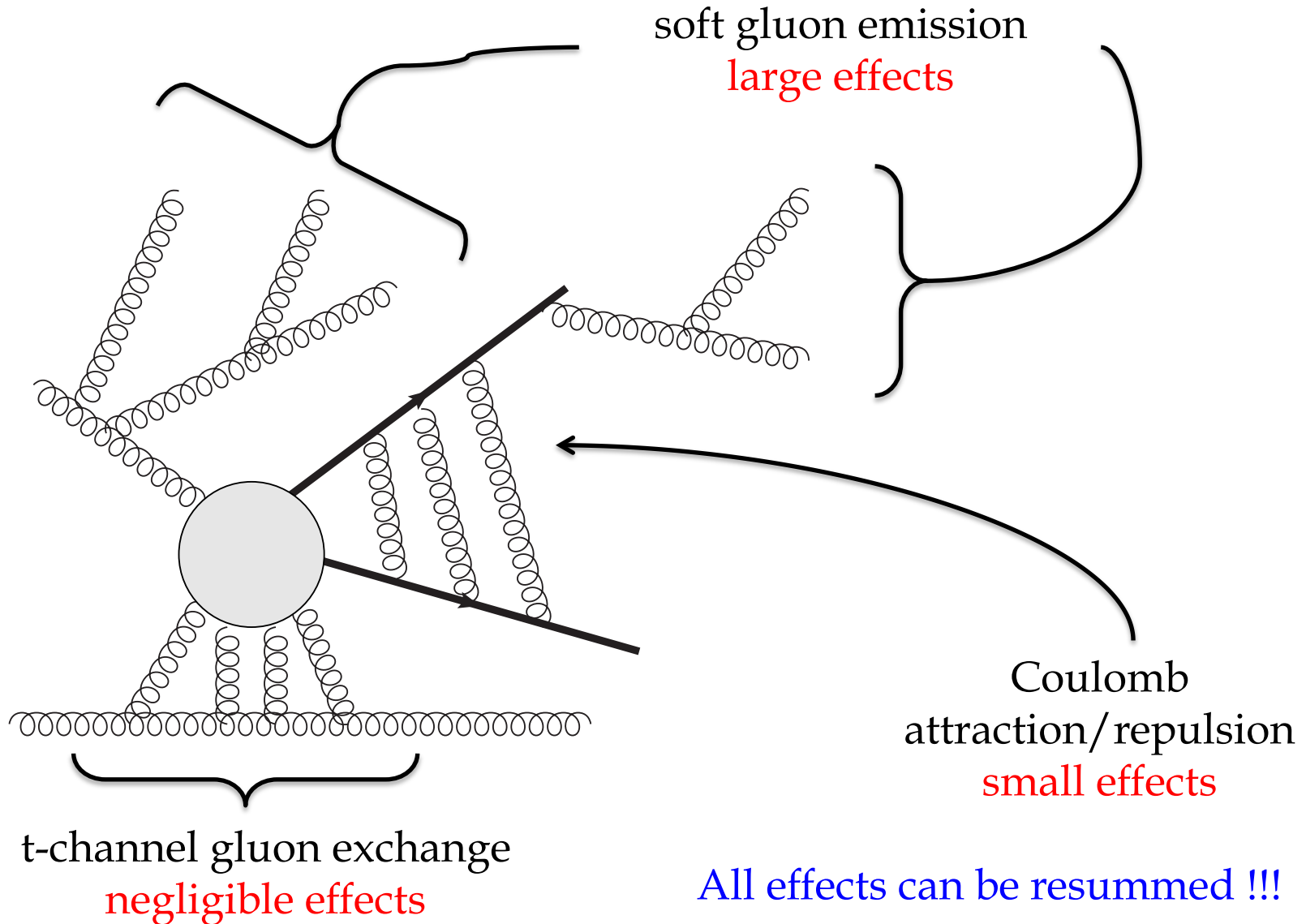


# RESUMMATION

# General Remarks

- Soft-gluon resummation up to NNLL well understood thanks to the work of many
  - *Kidonakis*
  - *Moch, Uwer*
  - *Almeida, Sterman, Vogelsang*
  - *Ahrens, Ferroglia, Neubert, Pecjak, Yang*
  - *Beneke, Falgari, Schwinn*
  - *Cacciari, MC, Mitov, Mangano, Nason*
  - *Becher, Neubert*
  - *Broggio, Papanastasiou, Signer*
  
- The “boosted” regime resummation builds on this by adding collinear singularities
  - *Ferroglia, Pecjak, Scott, Yang`13*

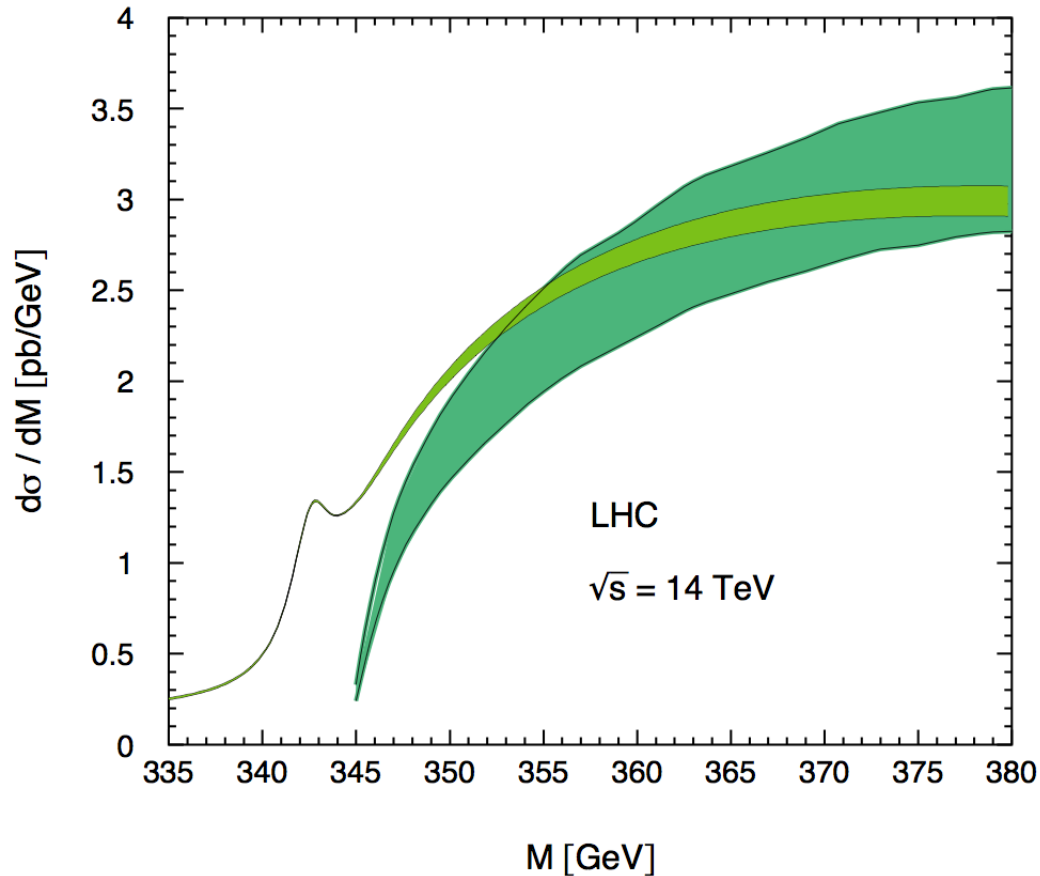
# Physical Effects for the “Bulk”





# Physical Effects for the “Bulk”

- NLO vs NLO+NLL+Coulomb



*Kiyo et al. '08*

- NNLO+NNLL for total cross section derived in *Beneke, Falgari, Schwinn '11*

# Physical Effects in the “Tails”

- Additionally to the potentially small gluon energies,  $m_t$  is small
- In this “boosted” regime there are two kinds of logs

$$\text{soft logs: } [\ln^n(1-z)/(1-z)]_+ \quad (z \equiv M_{t\bar{t}}^2/\hat{s})$$

$$\text{small-mass (collinear) logs: } \ln m_t/M_{t\bar{t}}$$

- Widely separated scales

$$\text{Soft Limit: } \hat{s}, t_1, m_t^2 \gg \hat{s}(1-z)^2$$

$$\text{Boosted Soft Limit: } \hat{s}, t_1 \gg m_t^2 \gg \hat{s}(1-z)^2 \gg m_t^2(1-z)^2$$

- Factorization possible

$$d\tilde{\sigma}_{ij}(\mu_f) = \text{Tr} \left[ \tilde{\mathbf{U}}_{ij}(\mu_f, \mu_h, \mu_s) \mathbf{H}_{ij}(M, \cos \theta, \mu_h) \tilde{\mathbf{U}}_{ij}^\dagger(\mu_f, \mu_h, \mu_s) \right. \\ \left. \times \tilde{s}_{ij} \left( \ln \frac{M^2}{\bar{N}^2 \mu_s^2}, M, \cos \theta, \mu_s \right) \right] \times \tilde{U}_D^2(\mu_f, \mu_{dh}, \mu_{ds}) C_D^2(m_t, \mu_{dh}) \tilde{s}_D^2 \left( \ln \frac{m_t}{\bar{N} \mu_{ds}}, \mu_{ds} \right) \\ + \mathcal{O} \left( \frac{1}{N} \right) + \mathcal{O} \left( \frac{m_t^2}{M^2} \right)$$

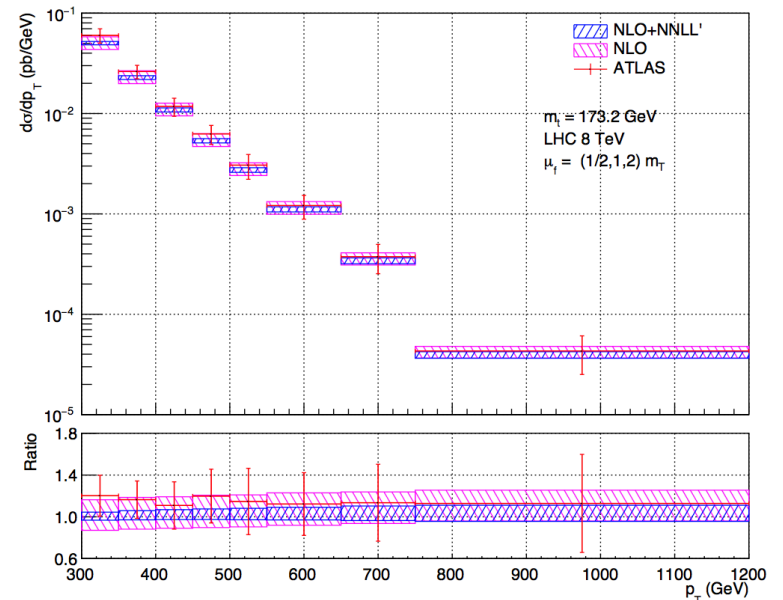
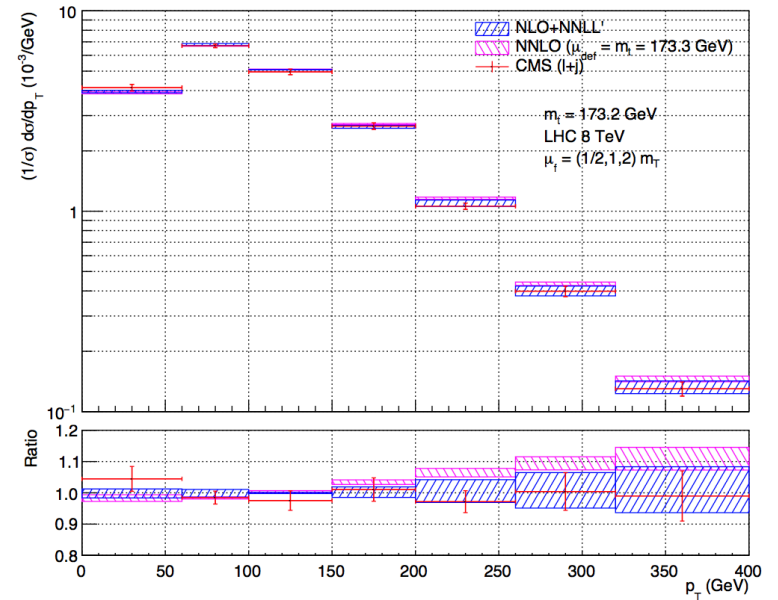
*Ferrogli, Pecjak, Scott, Yang '13*

- Notice that there are 5 (!) scales now

# Results for the LHC

- Transverse momentum distribution modified by dynamical scales and resummation
- At low  $p_T$  better description of CMS data, slightly worse for ATLAS (not shown)
- Larger scale dependence?

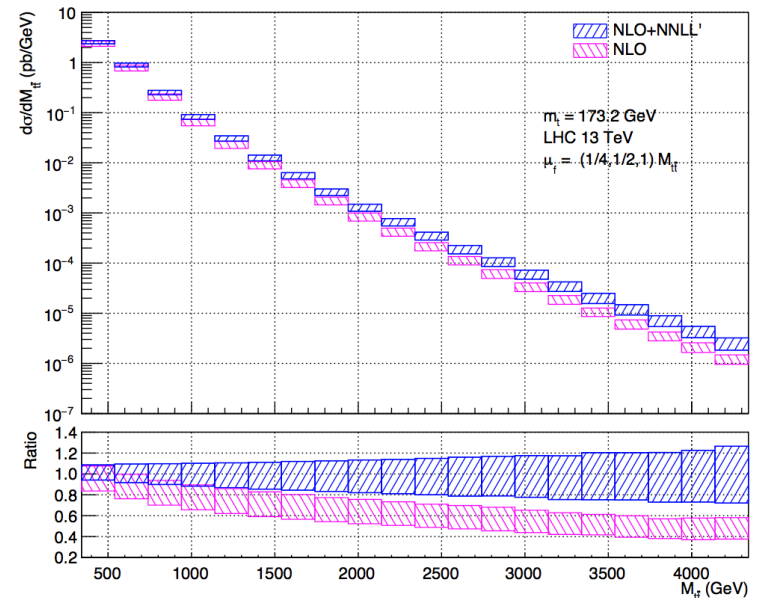
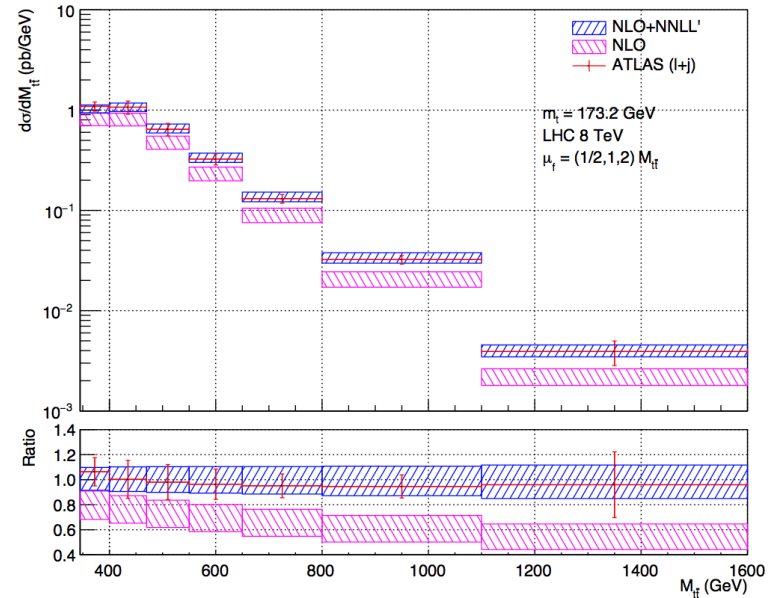
*Pecjak, Scott, Wang, Yang '15*



# Results for the LHC

- Observable dependent scale
- Results presented for 13 TeV as well
- At some point consistent matching to NNLO will become necessary
- When is true resummation needed?

*Pecjak, Scott, Wang, Yang '15*



# **FIXED-ORDER PERTURBATION THEORY**

# General Remarks

- High precision should be associated with fixed order perturbation theory:
  - Clear advantage: not many ambiguities
  - But: beware of range of applicability
  - Currently at next-to-next-to-leading order for on-shell production

*MC, Bärnreuther, Fiedler, Heymes, Mitov '12 - '15*

- Partial independent results by:

*Abelof, Gehrmann-De Ridder, Maierhofer, Pozzorini '14  
Catani, Grazzini, Torre '14 - '15*

# Contributions

- 2-loop virtual corrections (V-V)

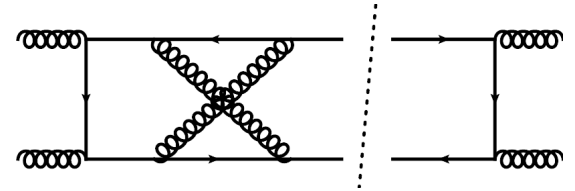
*MC '07, Bärnreuther, MC, Fiedler '13*

*complete numerical results partial analytical results:*

*Bonciani, Ferroglia, Gehrmann, Maitre, von Manteuffel, Studerus '08-'13*

*divergences of two-loop amplitudes:*

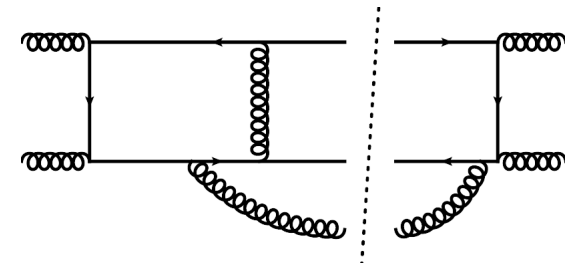
*Ferroglia, Neubert, Pecjak, Yang '09*



- 1-loop virtual with one extra parton (R-V)

*from next-to-leading order corrections to tt+jet*

*code by Stefan Dittmaier*



- 2 extra emitted partons at tree level (R-R)

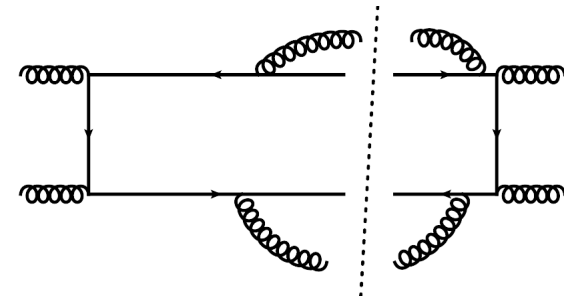
*MC '10 '11 new subtraction scheme STRIPPER*

*MC, Heymes '14 4-d formulation of STRIPPER*

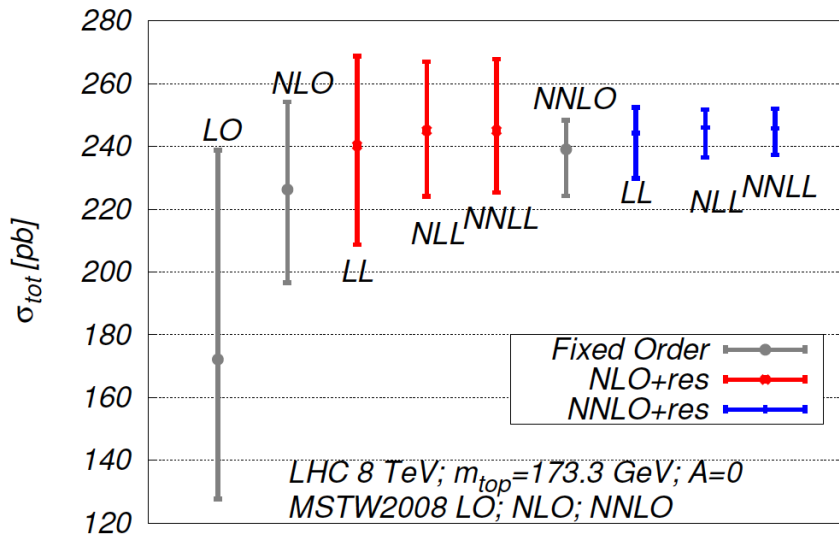
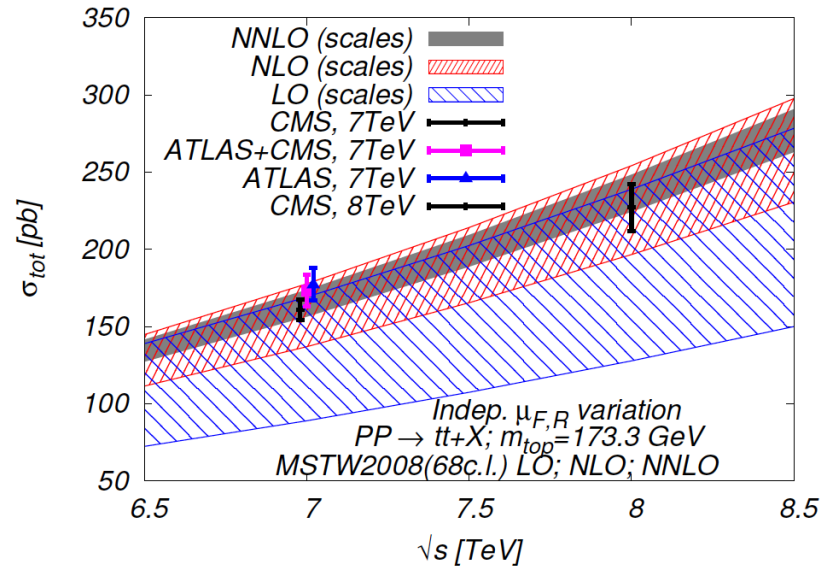
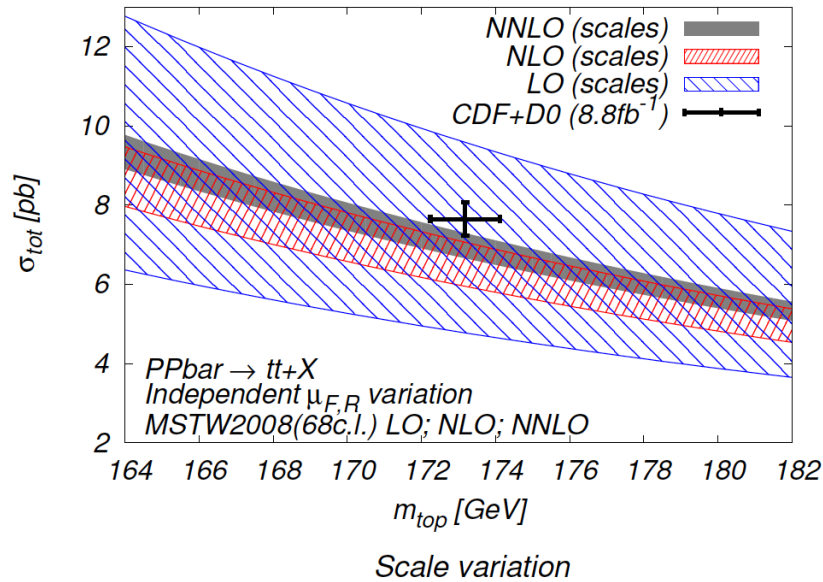
- One-loop squared amplitudes

*original results not used:*

*Körner, Merebashvili, Rogal '07, Anastasiou, Aybat '08*



# Perturbation Theory Convergence



## Concurrent uncertainties:

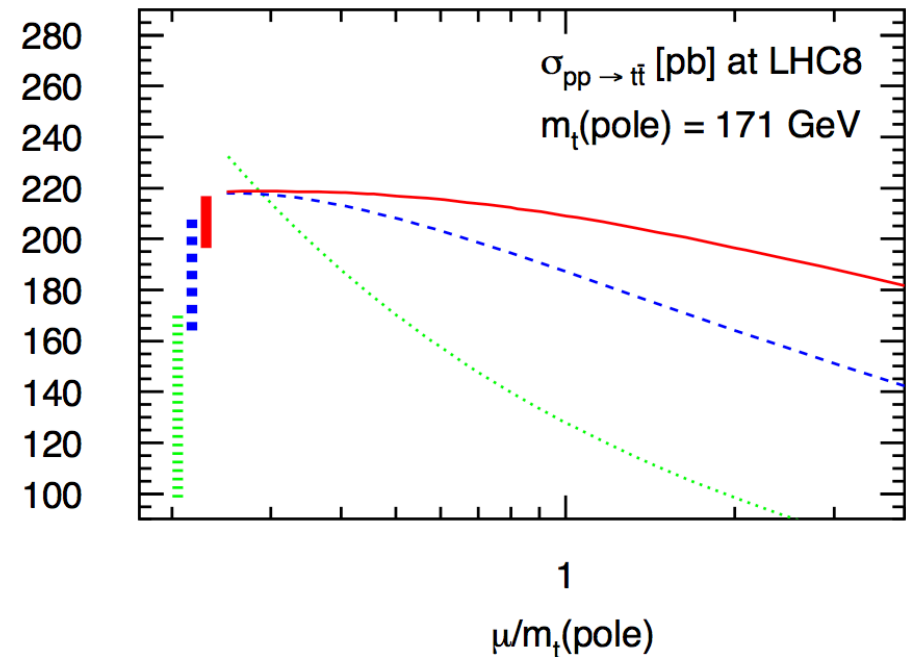
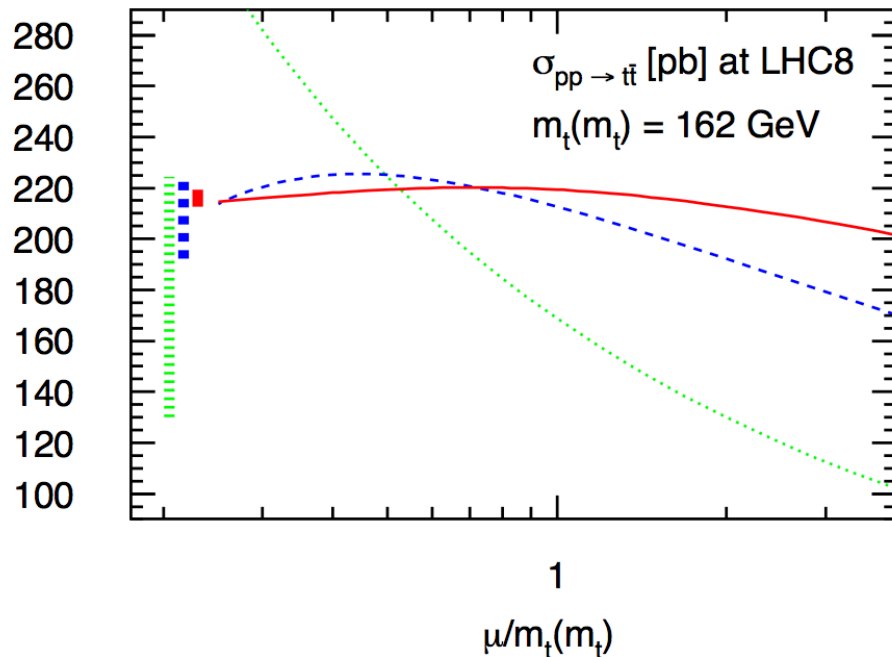
Scales	$\sim 3\%$
pdf (at 68%cl)	$\sim 2\text{-}3\%$
$\alpha_S$ (parametric)	$\sim 1.5\%$
$m_{top}$ (parametric)	$\sim 3\%$

Soft gluon resummation makes a difference:  $5\% \rightarrow 3\%$

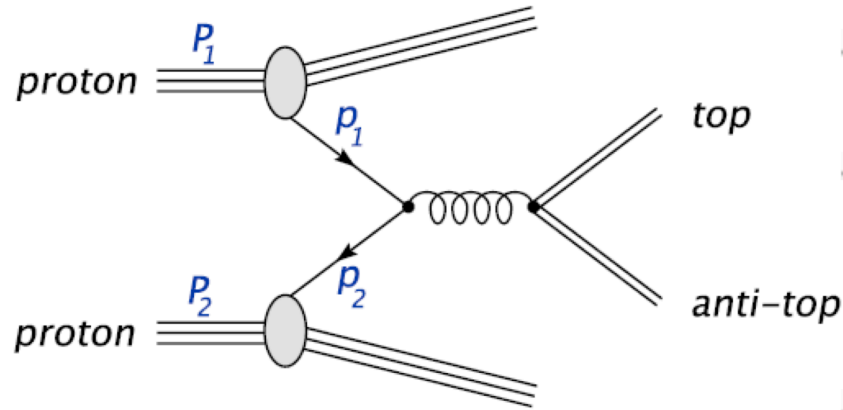


# Perturbation Theory Convergence

- It has been argued that it is better to use the  $\overline{\text{MS}}$  mass to improve convergence
- Is there a better scale in the on-shell scheme?
- Relevant for differential Monte Carlo description



# Searching for the Right Scale



- Cross section from factorization

$$\sigma_{h_1 h_2}(s, m_t) = \sum_{ij} \int dx_1 dx_2 \phi_{i/h_1}(x_1, \mu_F) \phi_{j/h_2}(x_2, \mu_F) \hat{\sigma}_{ij}(x_1 x_2 s, m_t, \alpha_s(\mu_R), \mu_R, \mu_F)$$

$\sigma_{h_1, h_2}$  hadronic cross section

$h_{1,2}$  hadrons

$s$  square of collider energy

$m_t$  top quark mass

$\phi_{i/h}$  PDF for parton  $i$  in hadron  $h$

$\hat{\sigma}_{ij}$  partonic cross section

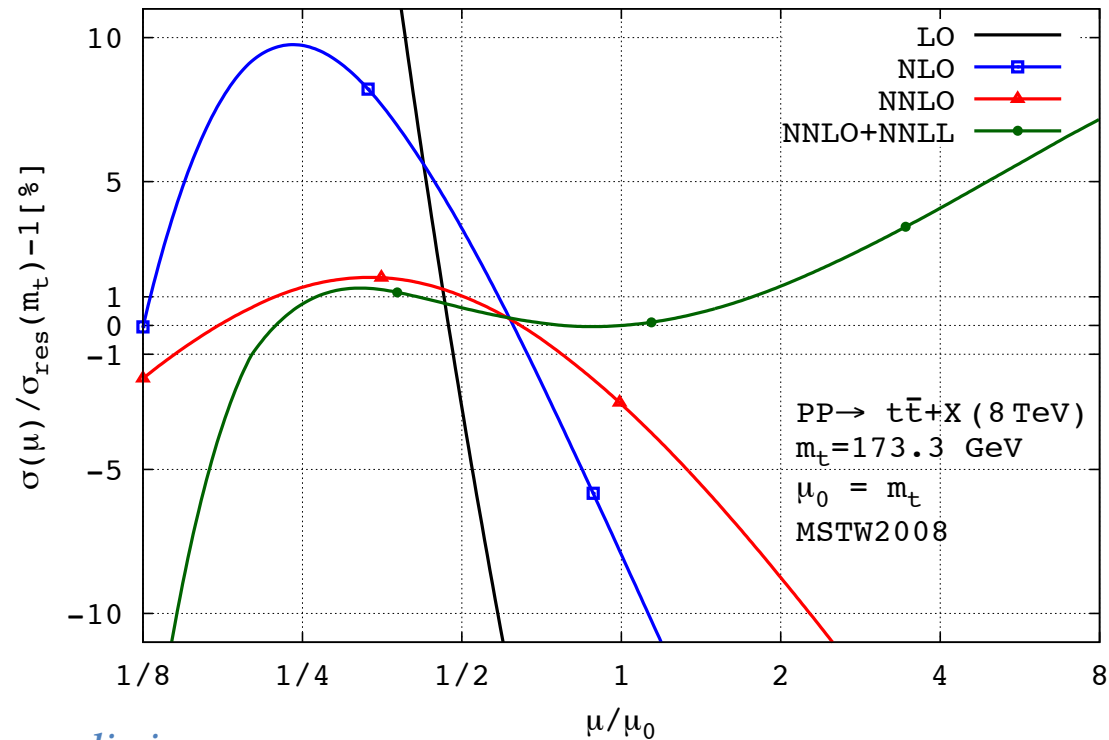
$\mu_R$  renormalization scale

$\mu_F$  factorization scale

- In fixed order perturbation theory the only ambiguity is in the two-scale choice

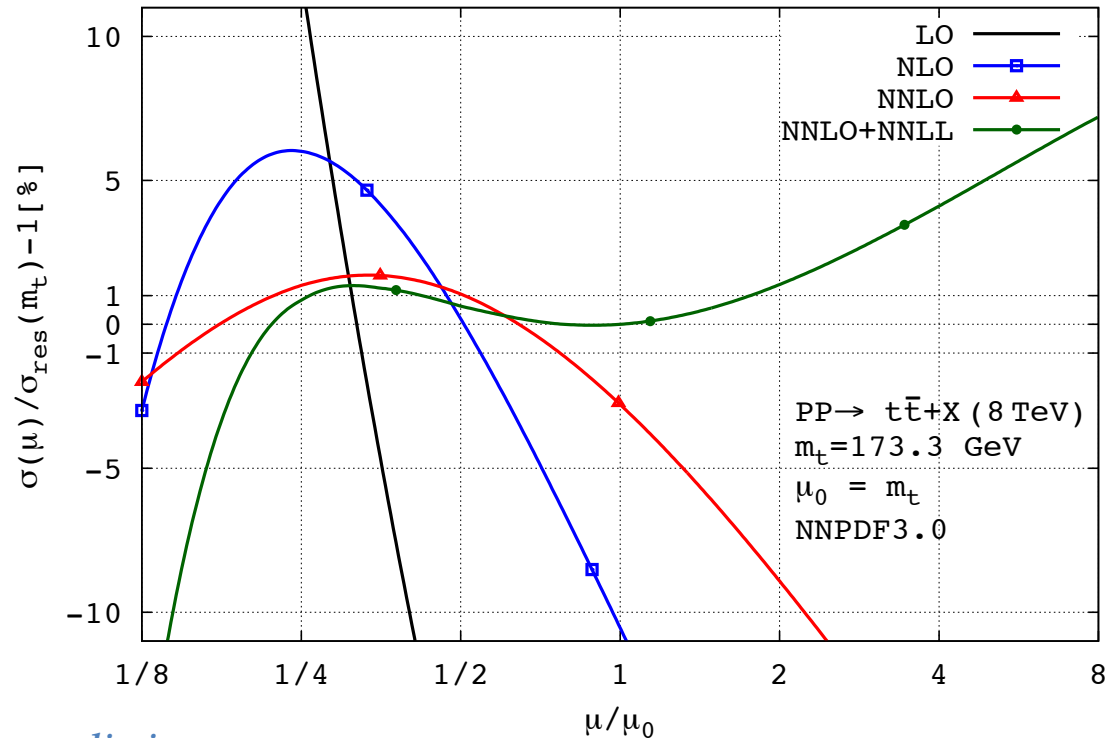
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- Total cross section depends only on the top-quark mass if the collider energy is fixed
- In principle, the scale must therefore be related to the mass
- Convergence improved at lower scales



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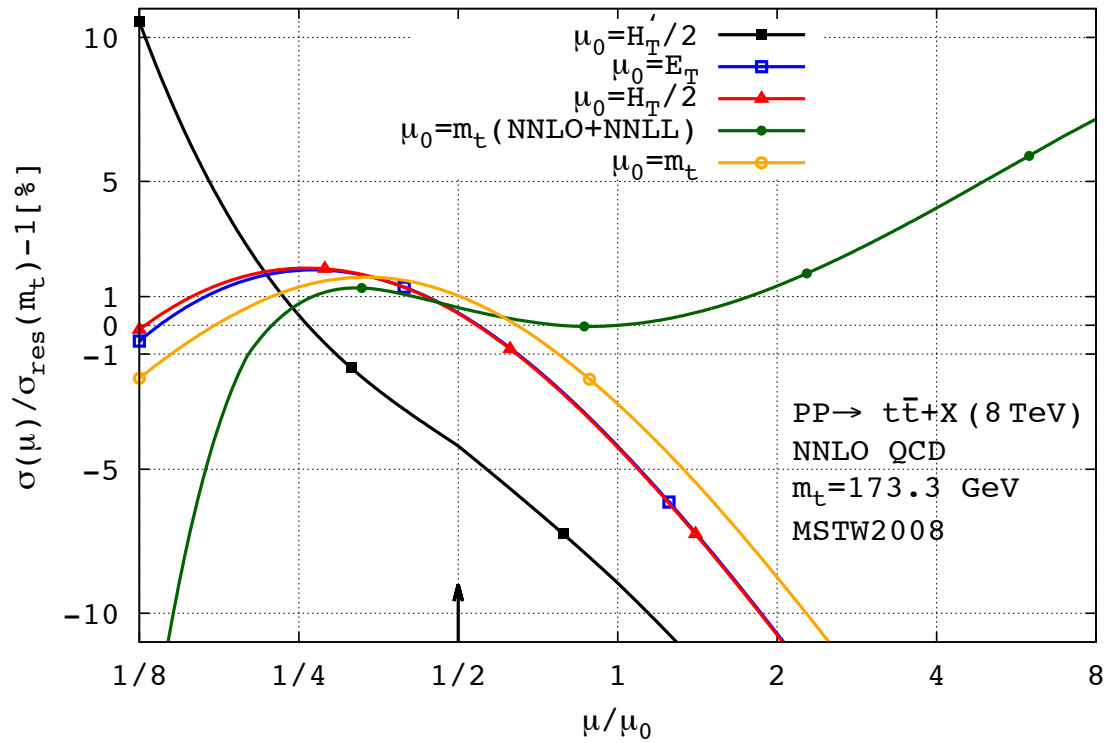
*MC, Heymes, Mitov, preliminary*

Careful with conclusions based on one PDF set only (particular attention to  $\alpha_s$ )

# Searching for the right scale

- Monte Carlo simulations use dynamical scales since they are fully differential
- Several possible choices based on

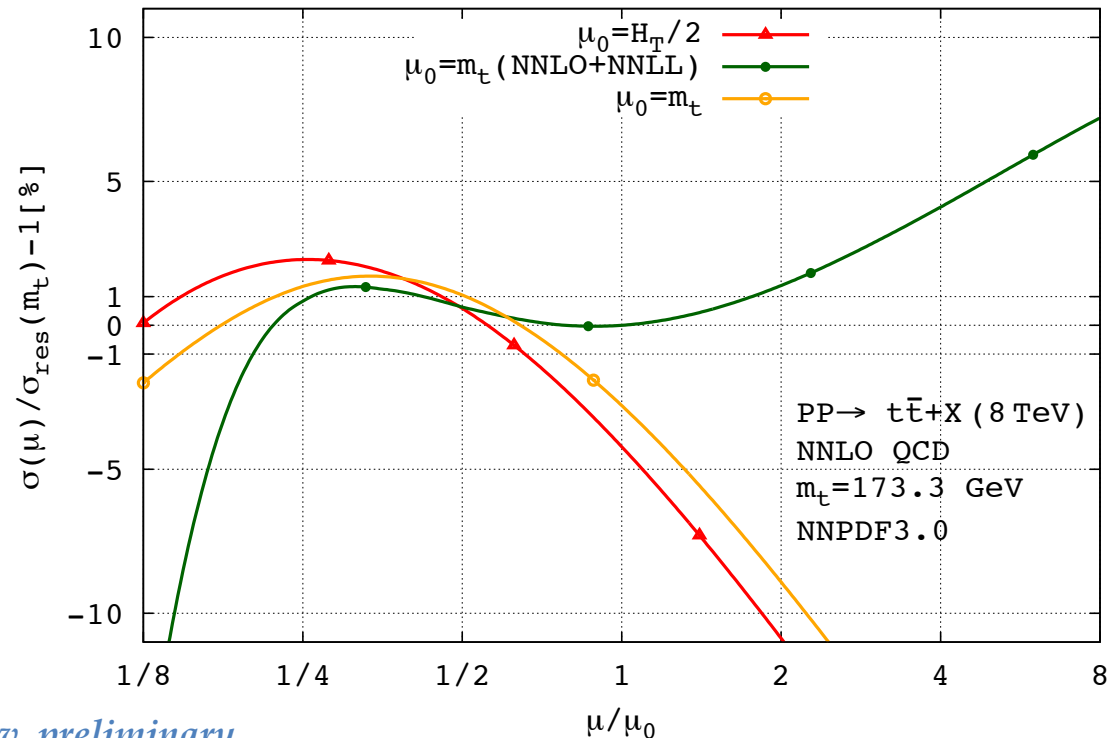
$$H_T = \sqrt{m_t^2 + p_{Tt}^2} + \sqrt{m_t^2 + p_{T\bar{t}}^2} \quad H'_T = H_T + \sum_i p_{Tj_i} \quad E_T = \sqrt{\sqrt{m_t^2 + p_{Tt}^2} \sqrt{m_t^2 + p_{T\bar{t}}^2}}$$



# Searching for the right scale

- Monte Carlo simulations use dynamical scales since they are fully differential
- Several possible choices based on

$$H_T = \sqrt{m_t^2 + p_{Tt}^2} + \sqrt{m_t^2 + p_{T\bar{t}}^2} \quad H'_T = H_T + \sum_i p_{Tj_i} \quad E_T = \sqrt{\sqrt{m_t^2 + p_{Tt}^2} \sqrt{m_t^2 + p_{T\bar{t}}^2}}$$



*MC, Heymes, Mitov, preliminary*

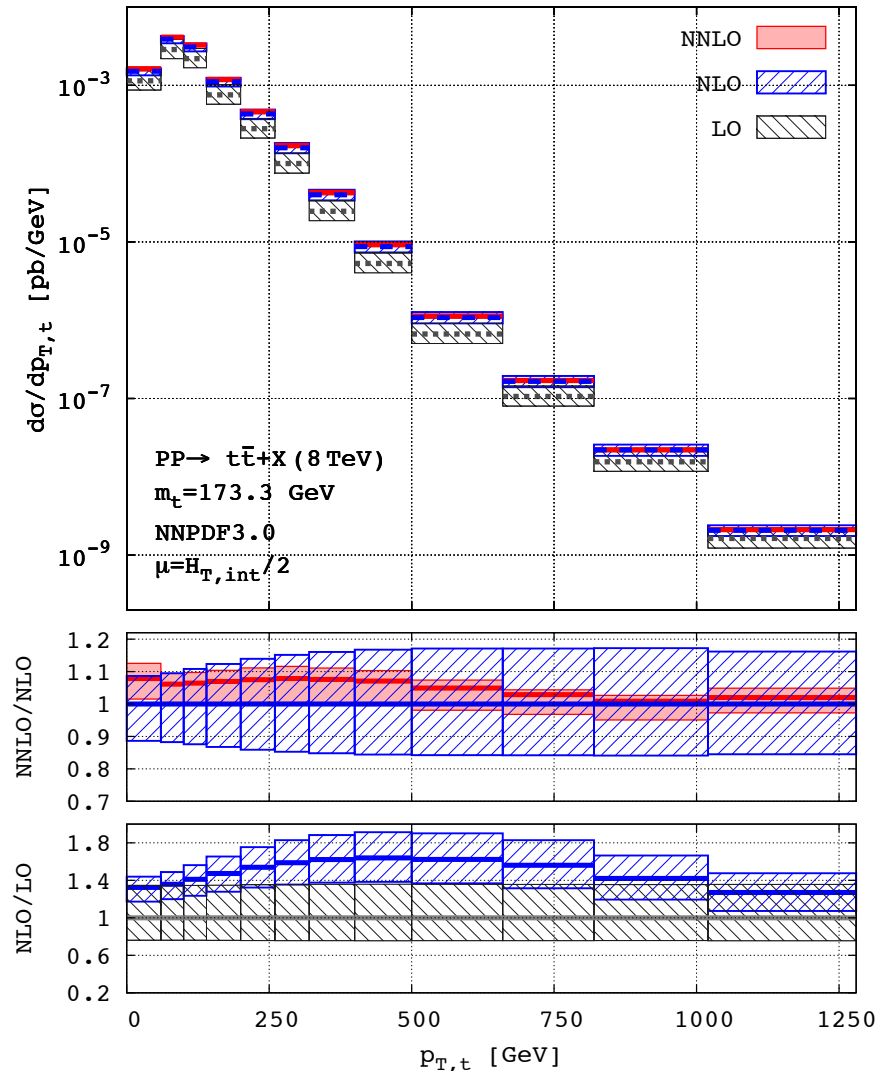
- Conclusions stable w.r.t. the PDF set

# Differential Distributions

- Single-differential distributions introduce an additional scale, e.g.  $p_{Tt}$  or  $m_{t\bar{t}}$
- It might make sense to interpolate between regimes

$$H_{T,\text{int}} = \sqrt{(m_t/2)^2 + p_{Tt}^2} + \sqrt{(m_t/2)^2 + p_{T\bar{t}}^2}$$

- Total cross section reproduced
- Excellent K-Factor at high  $p_T$

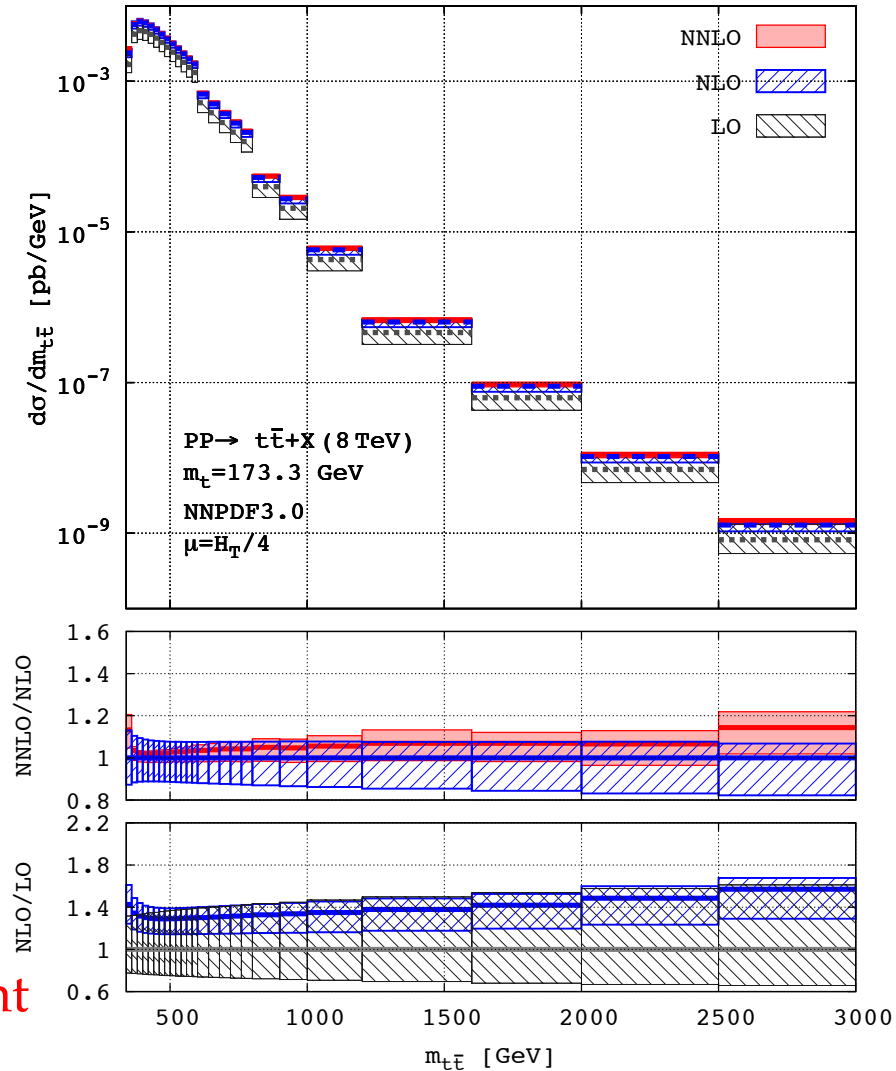


# Differential Distributions

- Single-differential distributions introduce an additional scale, e.g.  $p_{Tt}$  or  $m_{t\bar{t}}$
- A different interpolation is better for  $m_{t\bar{t}}$

$$H_T = \sqrt{m_t^2 + p_{Tt}^2} + \sqrt{m_t^2 + p_{T\bar{t}}^2}$$

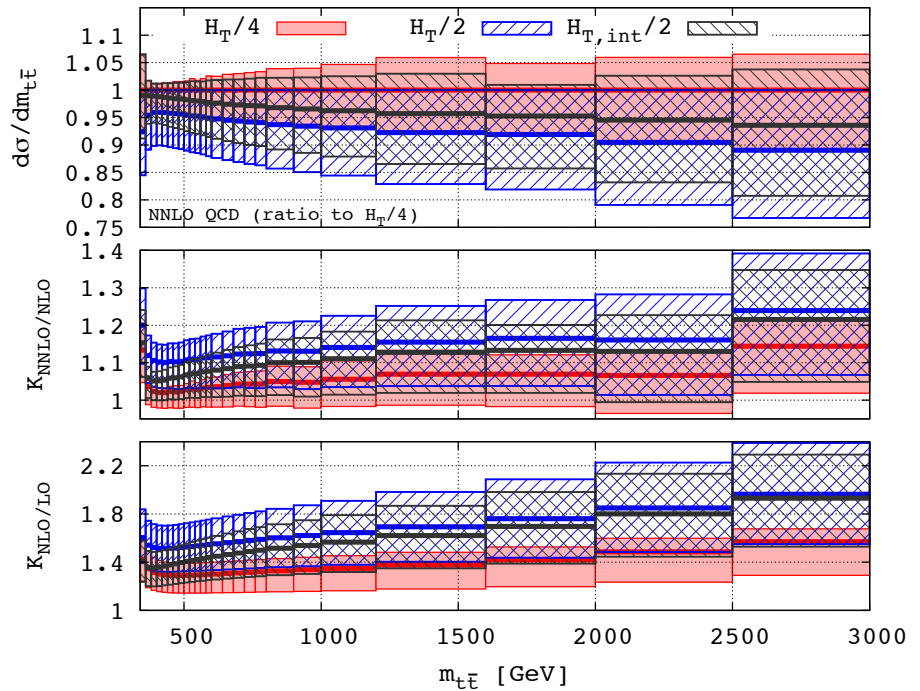
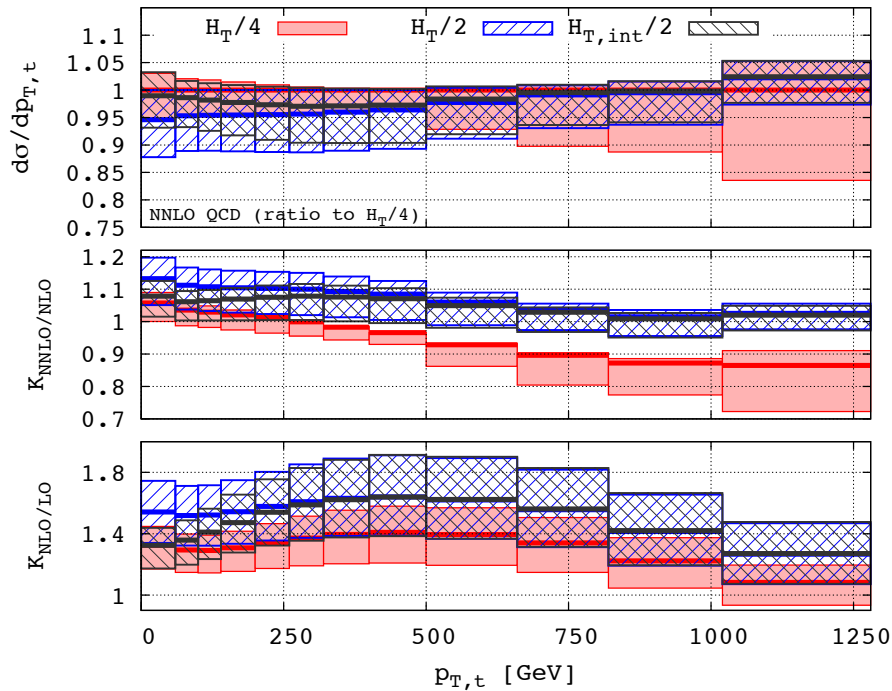
- Total cross section reproduced
- Excellent scale variation at high  $m_{t\bar{t}}$
- Introducing different scales for different observables is typical of resummation, but not usual in Monte Carlo studies





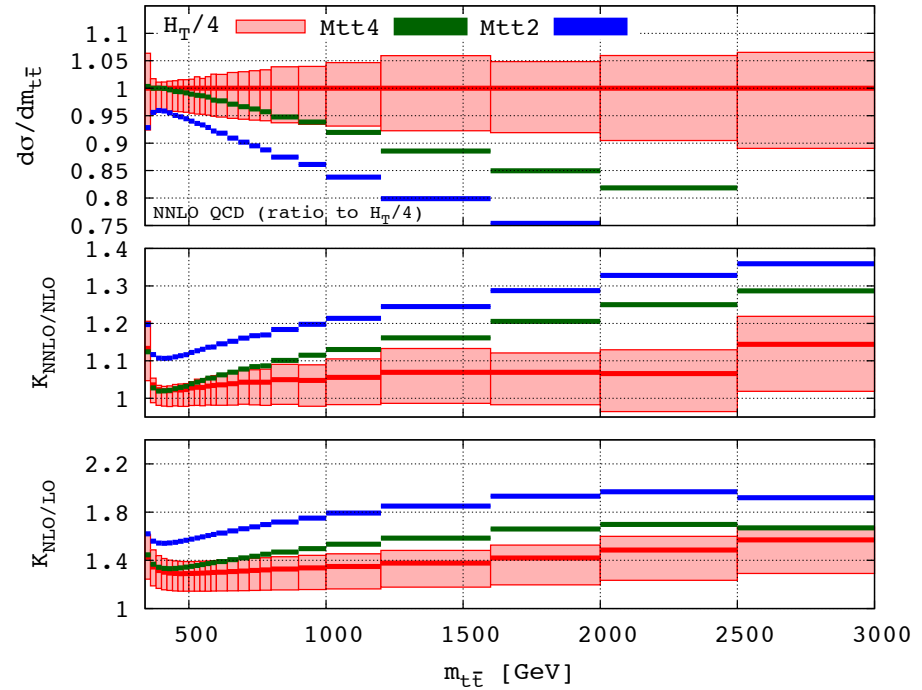
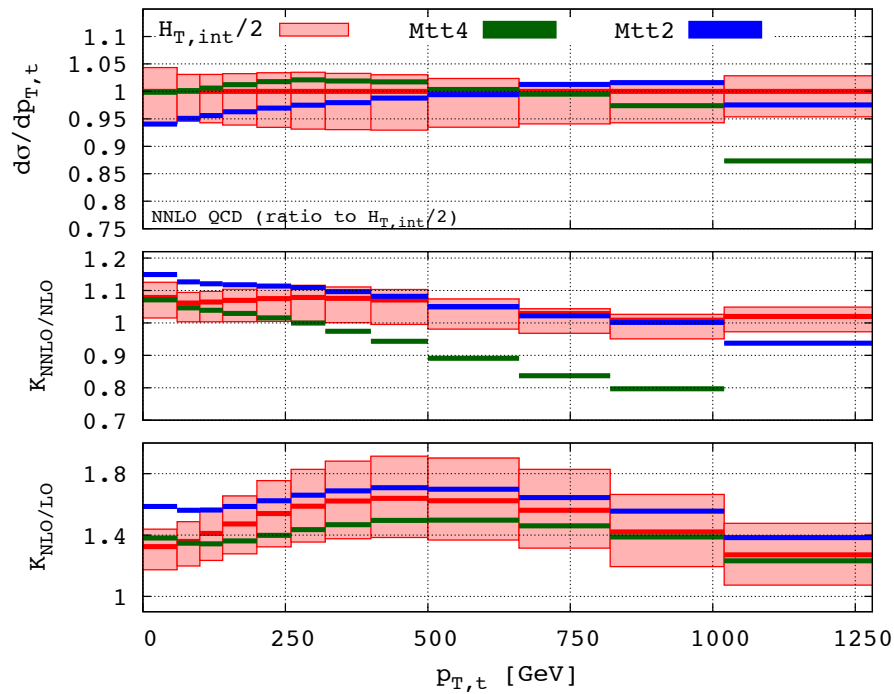
# Differential Distributions

- The issue is not that relevant once at NNLO
- It seems that the effect is largest on the scale dependence



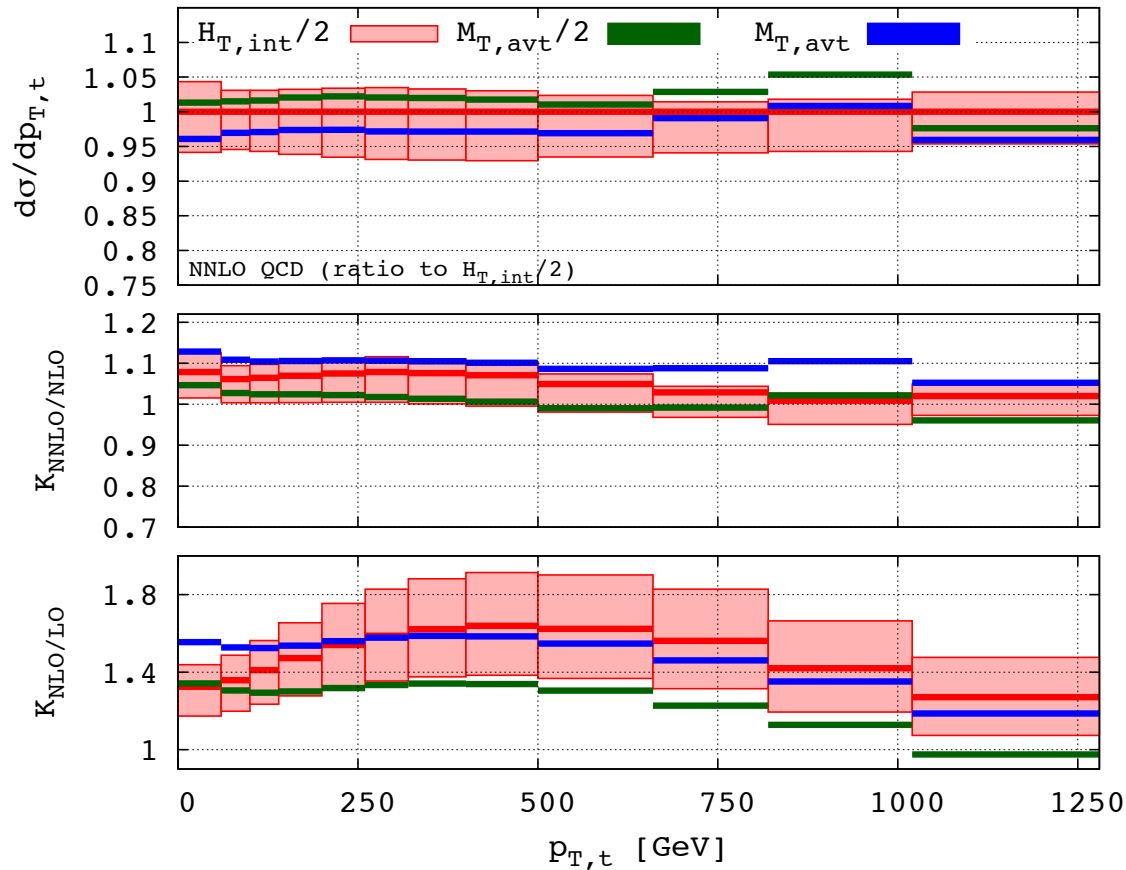
# Differential Distributions

- Alternative scales based on the invariant mass do not lead to improvement



# Differential Distributions

- Independent scales for the top-quark and anti-top quark much better



# OUTLOOK

# Decay Modeling @ NLO

- **Narrow-width approximation**

NLO corrections to both production and decay, neglecting non-factorizable corrections, including spin correlations at NLO

- Double differential angular distributions to probe spin correlations

*Bernreuther, Brandenburg, Si, Uwer '04*

- Flexible Monte Carlo implementation, fully differential level
- Spin correlations of top anti-top via decay products
- $pp \rightarrow tt + X \rightarrow WWbb + X \rightarrow lv lv bb + X$  (di-lepton)
- $pp \rightarrow tt + X \rightarrow WWbb + X \rightarrow ud lv bb + X$  (lepton + jet)

*Melnikov, Schulze '09*

- Can be implemented at NNLO
- Decay at this level is already known

*Gao, Li, Zhu '12*

*Brucherseifer, Caola, Melnikov '13*

# Decay Modeling @ NLO

- Off-shell effects through direct simulation of the final state WWbb

*Denner, Dittmaier, Kallweit, Pozzorini '11*

*Bevilacqua, MC, van Hameren, Papadopoulos, Worek '11*

*Heinrich, Maier, Nisius, Schlenk, Winter '13*

- Off-shell effects with massive b-quarks (simultaneous top-pair and single-top)

*Frederix '13*

*Cascioli, Kallweit, Maierhöfer, Pozzorini '13*



Very fancy interpolating scales

# Effects on Total Rates (Fiducial)

NWA

Off-shell

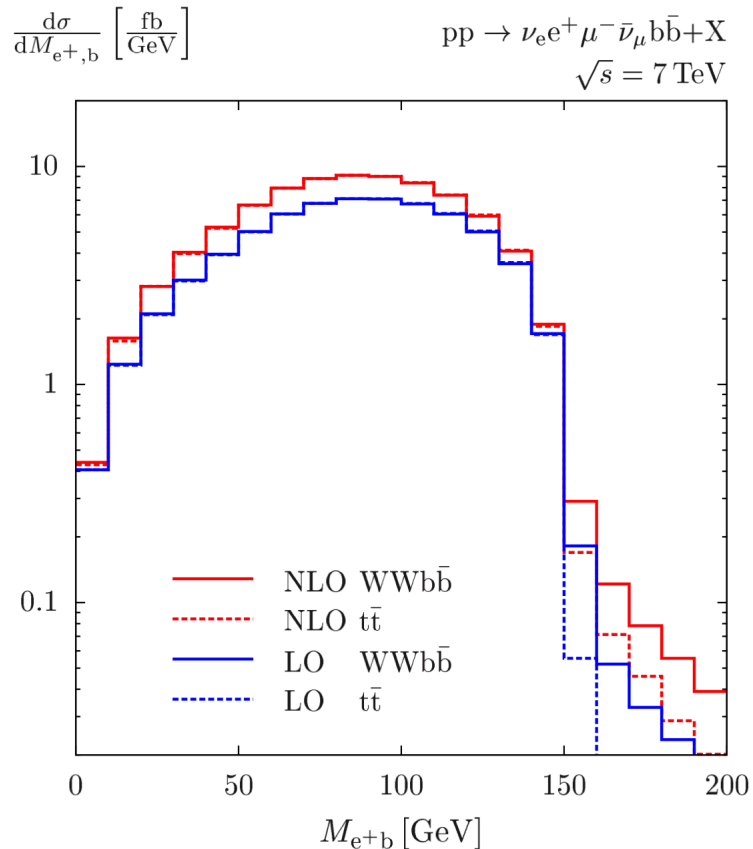
Collider	$\sqrt{s}$ [TeV]	approx.	$\sigma_{t\bar{t}}$ [fb]	$\sigma_{WWb\bar{b}}$ [fb]	$\sigma_{t\bar{t}}/\sigma_{WWb\bar{b}} - 1$	Expected
Tevatron	1.96	LO	$44.691(8)^{+19.81}_{-12.58}$	$44.310(3)^{+19.68}_{-12.49}$	+ 0.861(19)%	+ 0.8%
		NLO	$42.16(3)^{+0.00}_{-2.91}$	$41.75(5)^{+0.00}_{-2.63}$	+ 0.98(14)%	+ 0.9%
LHC	7	LO	$659.5(1)^{+261.8}_{-173.1}$	$662.35(4)^{+263.4}_{-174.1}$	- 0.431(16)%	- 0.4%
		NLO	$837(2)^{+42}_{-87}$	$840(2)^{+41}_{-87}$	- 0.41(31)%	- 0.2%
LHC	14	LO	$3306.3(1)^{+1086.8}_{-763.6}$	$3334.6(2)^{+1098.5}_{-771.2}$	- 0.849(7)%	- - -
		NLO	$4253(3)^{+282}_{-404}$	$4286(7)^{+283}_{-407}$	- 0.77(19)%	- - -

*Denner, Dittmaier, Kallweit, Pozzorini, Schulze '12*

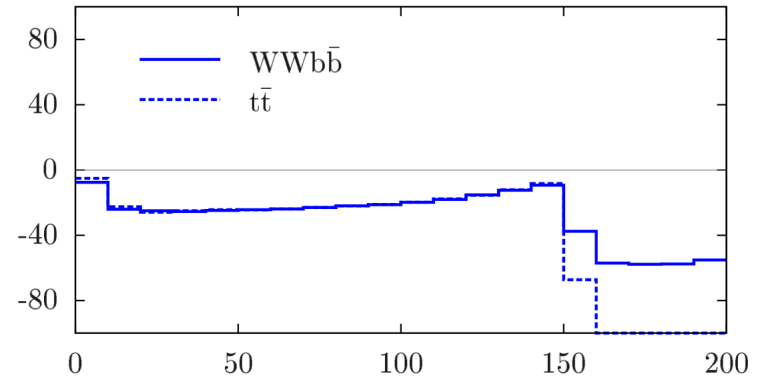
Tevatron (LHC)  $R = 0.4 (0.5)$   $p_{T,b\text{-jet}} > 20 (30) \text{ GeV}, |\eta_{b\text{-jet}}| < 2.5$

$p_{T,\text{miss}} > 25 (20) \text{ GeV}$   $p_{T,l} > 20 \text{ GeV}$  and  $|\eta_l| < 2.5$

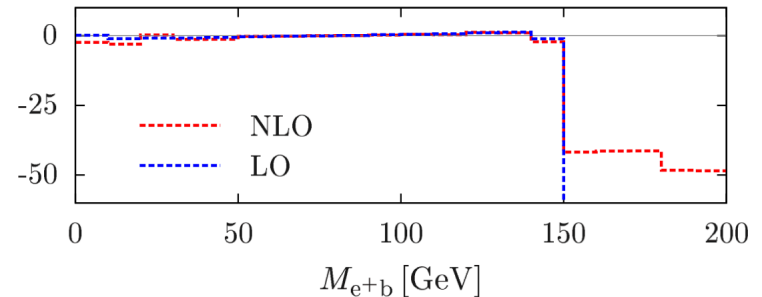
# Finite Width Sensitive Observables



LO/NLO - 1 [%]



$t\bar{t}/WWb\bar{b} - 1$  [%]



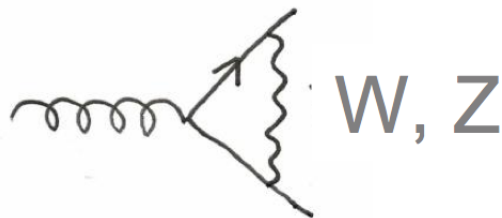
*Denner, Dittmaier, Kallweit, Pozzorini, Schulze '12*

- Large effects easily found by reaching past kinematic end-points



# Electroweak corrections

- Long history
  - *Beennakker, Denner, Hollik, Mertig, Sack, Wackerath '94*
  - *Bernreuther, Fücker, Si '05, '06*
  - *Moretti, Nolten, Ross '06*
  - *Kühn, Scharf, PU '05, '06, '14*
- Typically only virtual corrections due to W/Z
  - Large effects are negative

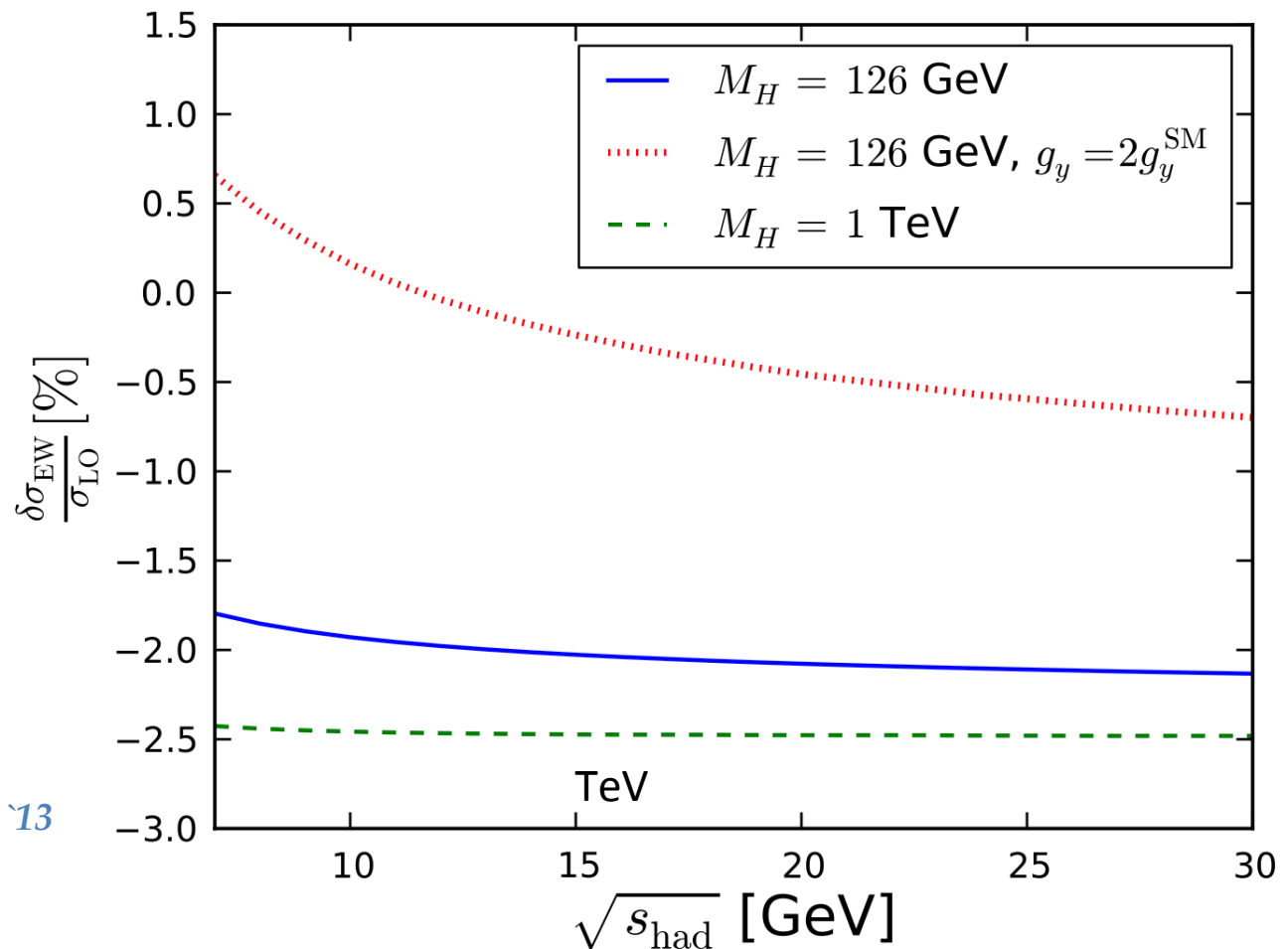


$$\log^2(p_{Tt}/M_{W,Z})$$

- When is QCD enough ?

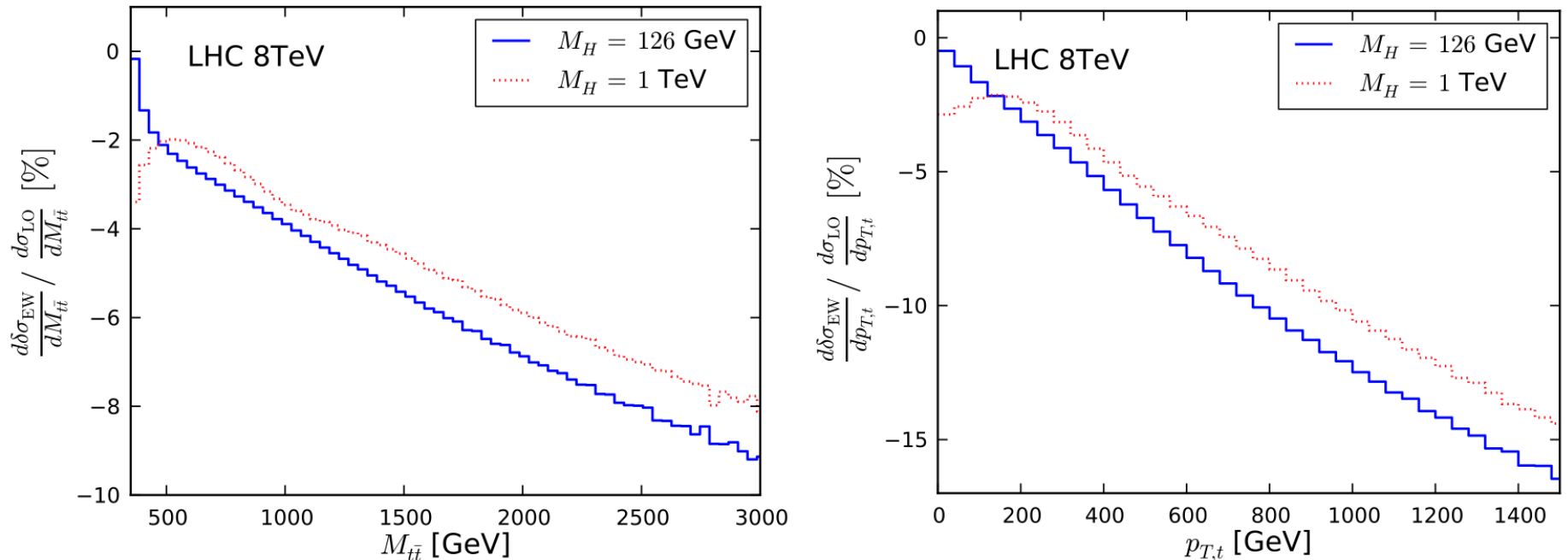
# Total cross sections

- Expectedly small corrections, which justify the use of pure QCD
- In the plot beware of the normalization to LO



# Sudakov Effects in the “Tails”

- Clearly, the “boosted” regime requires the inclusion of EW effects



*Kühn, Scharf, Uwer '13*

- These effects might be reduced by including real-radiation corrections from W/Z
  - Complete cancellation impossible due to isospin of the initial state

# Conclusions

- **Precent level precision** achieved thanks to many simplifications
- Reliable/transparent description at the level of **fiducial cross sections within grasp**
- Precision only usable when Monte Carlo systems used in data analysis: calculations **cannot replace Monte Carlo's**