LGC: A NEW REVISED VERSION

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Abstract

A new version of LGC (Logiciel Général de Compensation[1]) has been developed over the last few years. A completely different functional model and an improved stochastic model have been implemented, and the software has entirely been rewritten. New observation types have been developed to respond to new requirements such as: unlevelled[2] stations making polar measurements; more flexibility when processing offset observations (lines and planes introduced); and processing camera sensors (BCAM). For a new accelerator line monitoring system a way to define assemblies of objects has also be implemented.

The stochastic model has also been modified to allow a better breakdown and parametrization of the instrument and observation errors; and a better error propagation by means of weighted unknown parameters (coordinates and transformation parameters). Special care has been taken testing the program. Unit and functionality tests have been added to assure future development, and an in depth comparison with the previous version has been made.

Furthermore, the calculation structure has also been designed to allow new processing modules, such as a preprocessing model to calculate initial coordinate values, to be added more easily.

This paper will give an overview of the new software.

INTRODUCTION

In the last decade, new equipments and sensors were integrated at CERN so as to allow a larger field of applications and measurements, such as the unlevelled polar measurement with the laser tracker or the use of Brandeis CCD angle Monitor (BCAM) in usual measurement field. New accelerator lines request an increasing amount of monitoring system, with more material and assembly constraints. The HIE-ISOLDE project illustrates this new request. BCAMs are installed in a complex hierarchy of tables to be able to measure points inside cryo-modules. The LGC program was rewritten to include this new instrument as well as unlevelled polar measurement , define objects instead of points, and answer new network constraints [1].

[1] General processing of the observed measurements through a least squares algorithm

[2] unlevelled: not linked to the gravity

NEW OBSERVATIONS

UVEC and UVD: Measurements by camera

For the new monitoring systems, such as HIE-ISOLDE, non-typical geodetic instruments and sensors are used for the positioning. A BCAM makes observations to one or more light sources on another BCAM. Alternatively it can also measure on glass balls [2].

As shown in the Fig. 1, the initial BCAM measurement is the position of the measured target on the CCD (dx and dy), which is transformed on a unit vector, u, with $u = (i, j, k)$ representing the direction of the measurement. To express the BCAM observation,

- UVD, Unit Vector with a distance (see equation 1.) and

- UVEC, Unit VECtor (see equation 2.), as a simplification of UVD

have been defined.

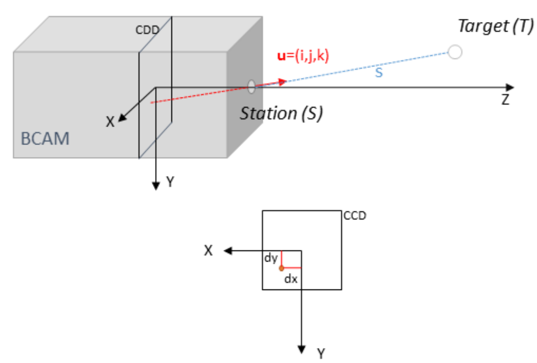

Figure 1: BCAM representation

$$s \begin{pmatrix} i \\ j \\ k \end{pmatrix} - \begin{pmatrix} X_T - X_S \\ Y_T - Y_S \\ Z_T - Z_S \end{pmatrix} = 0 \quad (1)$$

$$\begin{pmatrix} i \\ j \end{pmatrix} - \frac{k}{Z_T - Z_S} \cdot \begin{pmatrix} X_T - X_S \\ Y_T - Y_S \end{pmatrix} = 0 \quad (2)$$

Where s is the distance between the lens of the BCAM (considered as a station measuring) and the target.

Usually, cameras are not linked to the gravity and are installed on metrological plates. This installation is described by a free-rotation frame known in the superior frame (the frame will be described in the next section).

PLR3D: Polar measurements from unlevelled stations

A new way to treat laser trackers has been introduced. The instrument can now be used without gravity connection. The instrument orientation, described by the rotation matrix $RzRyRx$ in equation 3, can be redetermined with the observations. An option keyword in the input file allows to define whether the rotation matrix is known, hence to define if the instrument is levelled. Moreover, the distance and angle measurements are linked to model better the measurement interactions.

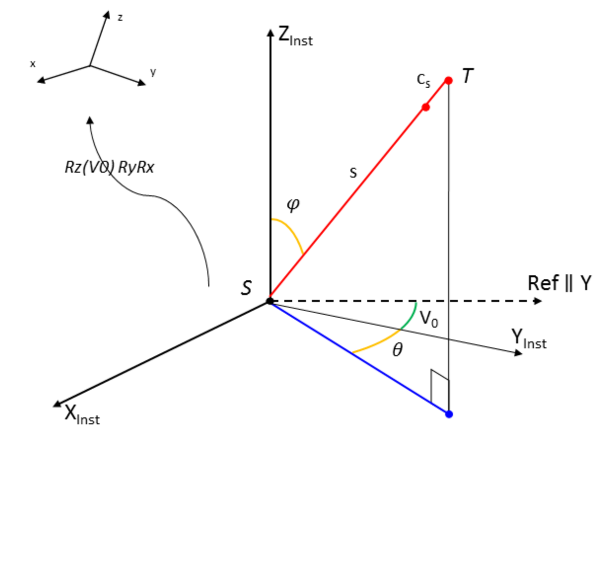

Figure 2: PLR3D instrument

$$(s+c_s)\begin{pmatrix} sin\theta\cdot sin\phi \\ cos\theta\cdot sin\phi \\ cos\phi \end{pmatrix} - R_z\cdot R_y\cdot R_x \begin{pmatrix} X_T - X_S \\ Y_T - Y_S \\ Z_T - Z_S \end{pmatrix} = 0 \quad (3)$$

Due to the measurement dependencies, the equations are not linear, so the least square process must use the combined case, also known as Gauss Markov approach [3].

Other observation improvements

Some observations have been rewritten using a reference object such as a plane or a line. Levelling measurements are calculated according to an average plane; ECHO, ECVE, ECSP or ECTH, (specific offset measurements made at CERN for the alignment campaign), are also calculated according to an average line or plane.

In the previous version [4], the plane or the line were defined with known points for the offset measurements, and the medium plane did not exist for levelling measurements. Now, every object is initially defined as the mean of every respective set of measurements. During the least square process, these objects are also re-determined. Some of the parameters are fixed, others are variable.

Taking one example to illustrate the standard, ECHO (offset measurement respect to a vertical plane, ec, see Fig. 3): the plane, p, is defined by a normal vector, n, and a reference point, REF, on the plane. The plane is vertical, so there is only its orientation in XY plan to determine (angle α in equation 4). The reference point , which is initialized as the mean of all stations coordinates, allows fixing the plan in an area around the stations and is considered as a fixed point. The stations at the extremities are used to predetermine the orientation, which will be balanced by least square.

$$ec = -cos\alpha\cdot(X_S - X_{Ref}) + sin\alpha\cdot(Y_S - Y_{Ref}) - c_{ec} \quad (4)$$

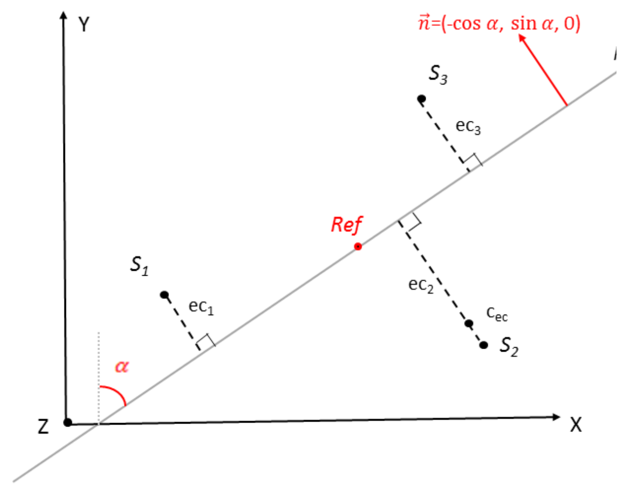

Figure 3: ECHO measurement representation shown in the XY plane

Parametrization of instrument and observation errors

In the previous version, the stochastic model use fixed parameters, except the standard deviation of the measurement, which was dynamic. Now, more parameters can be dynamically set. A single standard deviation is recalculating, for these parameters, using the law of standard deviation propagation to fill the weight matrix. The law of standard deviation propagation affords to take into account more parameters to define the instruments and the observation error with better accuracy.

$$\sigma_l^2 = \sum_{i=0}^{n} (\frac{\partial_l}{\partial_{x_i}})^2 \sigma_i^2 \quad (5)$$

Where:
x_i – Parameters with their standard deviation σ_i.
σ_l – Standard deviation of the observation.

The main used parameters are, as before, the standard deviation of the measurement, distance correction, ppm, and new parameters as the standard deviation of target and station centering.

MANAGE COMPLEX NETWORK ASSEMBLIES

Data structure

Points and transformations representation was changed to simplify the computation. Homogenous coordinates are used.

The classic point coordinates are $x = (x, y, z)$. In homogeneous coordinates, the coordinates becomes $x^h = (x, y, z, 1)$, where the last component is a scale factor, usually used for 3D representation in software to manage the perspective.

The Helmert transformation is simplified by a simple matrix multiplication (see equation 10.), instead of a matrix multiplication, $R_i(\omega, \phi, \kappa)$, a vector addition, T_i, and a scale factor, l_i.

Applying n transformations to a point, x_0, the old equation was:

$$x_n = \prod_{i=1}^{n} l_i R_i x_0 + \sum_{m=1}^{n} (\prod_{i=n}^{m} l_i R_i) T_{m-1} T_n \tag{6}$$

Now, it is simplified by:

$$x_n^h = \prod_{i=1}^{n} M_i x_0^h \tag{7}$$

Where:

$$R_i(\omega, \phi, \kappa) = \begin{pmatrix} r_{i,11} & r_{i,12} & r_{i,13} \\ r_{i,21} & r_{i,22} & r_{i,23} \\ r_{i,31} & r_{i,32} & r_{i,33} \end{pmatrix} \tag{8}$$

$$T_i = \begin{pmatrix} t_{i,1} \\ t_{i,2} \\ t_{i,3} \end{pmatrix} \tag{9}$$

And

$$M_i = \begin{pmatrix} r_{i,11} & r_{i,12} & r_{i,13} & t_{i,1}/l_i \\ r_{i,21} & r_{i,22} & r_{i,23} & t_{i,2}/l_i \\ r_{i,31} & r_{i,32} & r_{i,33} & t_{i,3}/l_i \\ 0 & 0 & 0 & 1/l_i \end{pmatrix} \tag{10}$$

Furthermore, transformations used at CERN have been rewritten using this new way of point and transformation representation: CCS (CERN Coordinate System) to CGRF (CERN Geodetic Reference Frame), CGRF to Local Geodetic (LG), Local Astronomic (LA) or Modified Local Astronomic (MLA).

Frame

A frame section is a logical block which can contain points, measurements and further frames. A frame is for example useful to group points that can only move together. Such a set of points must be declared using CALA inside the frame declaration to achieve a moving point group.

A frame is defined by three translations, three rotations and a scale factor relative to its parent frame. The transformation into the specified frame is done using the transformation matrix with homogeneous coordinates. The rotation matrix used has form $Rxyz = Rx \cdot Ry \cdot Rz$, i.e. firstly rotation around Z-axis is applied, followed by a rotation around Y-axis, and ending with a rotation around the X-axis.

Moreover, points and frame parameters can be now weighted, using standard deviation to define them.

HIE-ISOLDE, an example of complex use.

The HIE-ISOLDE monitoring project was one driving force to implement new LGC functionnalities. The monitoring system is based on BCAM measurements. These instruments are mounted on a complex assembly of metrological plates and tables. In the LGC input file, the assembly is represented by some frames, and the BCAM, by UVD measurements. Fig. 4 shows the complex structure of the assembly.

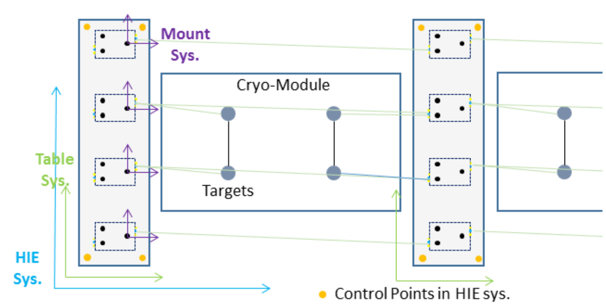

Figure 4: scheme of HIE-ISOLDE assembly

The monitoring system is composed of tables oriented according to the beam system. Each table has four metrologic elements providing the position of the BCAM on the table's reference system. Then another system is added describing the position of the CCD in the BCAM and having the measurement direction according to the Z-axis of the BCAM system. Each BCAM measures points in the cryomodule and on his front BCAM. Inside the cryo-module, there is a hierarchy of frames describing the position of the target compared to main elements.

The LGC input file, Fig. 5, is a text file describing a list of information. Some keywords are used to define the type of data to read and store in the software core. In the header

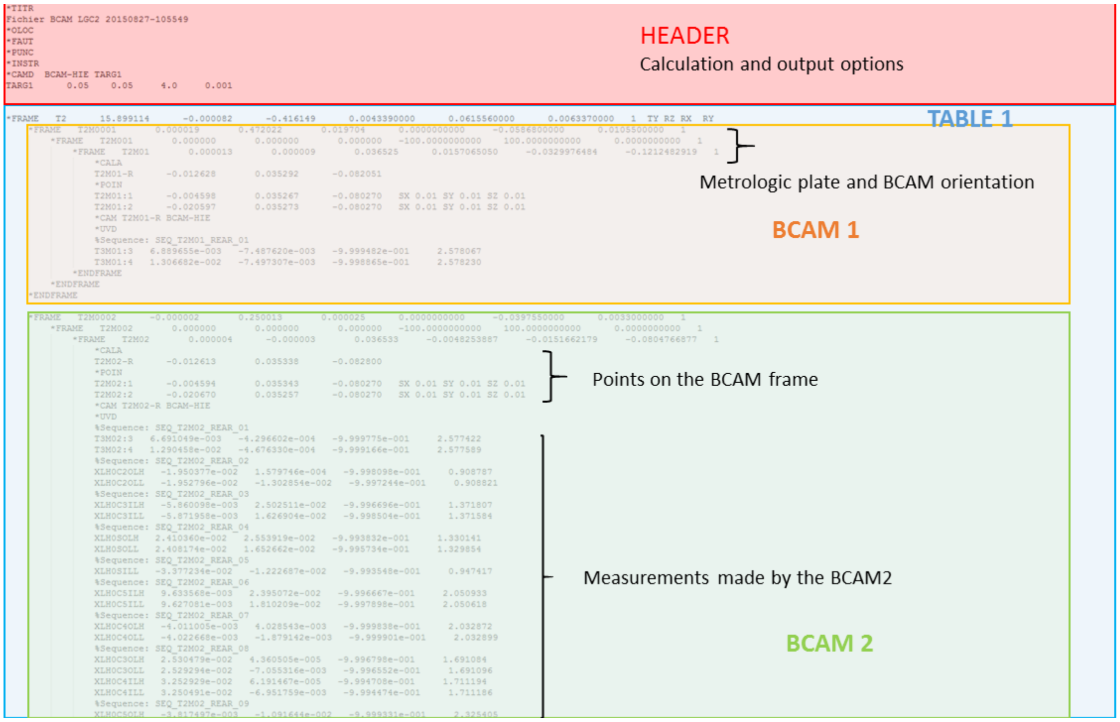

Figure 5: example of a LGC input file for BCAM

part, shown in red in Fig. 5, some keywords set the calculation and the output format. Then the network is described with the point list and the frames shown in blue, orange and green rectangles in Fig. 5. Measurements are written at their network position (in the frame if the instrument is defined inside) ending the file. The file format has not been completely modified, only the frame and the new measurements change the structure.

STOCHASTIC MODEL

In the previous version, all the observations were linear, no weight could describe the unknowns accuracy. Then only the parametric least square solution was used. In the new version, some non linear equations have been implemented, and a new functionality to constrain better the unknown accuracy is used. To solve the new observations and the new data structure, new least square solutions have been implemented.

Combined case

The combined case is used for non linear equations, here PLR3D and UVD require this case.

The equation to solve is:

$$F(\bar{X}, \bar{L}) = 0 \quad (11)$$

After linearization, the previous equation can be written:

$$W + AX + BV = 0 \quad (12)$$

where:

- X is the unknown vector,
- V is the residual vector,
- A is the first design matrix (partial derivative respect to the unknowns),
- B is the second design matrix (partial derivative respect to the observed parameters) and
- W is the misclosure vector.

Or in the hyper matrix form:

$$\begin{pmatrix} P & B^T & 0 \\ B & 0 & A \\ 0 & A^T & 0 \end{pmatrix} \begin{pmatrix} \hat{V} \\ \hat{K} \\ \hat{X} \end{pmatrix} + \begin{pmatrix} 0 \\ W \\ 0 \end{pmatrix} = 0 \quad (13)$$

Using Lagrange's method, the solution vectors are:

$$\hat{X} = -(A^T(BP^{-1}B]^T)^{-1}A)^{-1}A^T(BP^{-1}B^T)^{-1}W \quad (14)$$

$$\hat{K} = (BP^{-1}B^T)^{-1}(A\hat{X} + W) \quad (15)$$

$$\hat{V} = -P^{-1}B^T\hat{K} \quad (16)$$

Weighted unknowns

In this case, a system with an uncertainty on the unknowns can be solved. In LGC, the weighted unknowns are the coordinates and the frame parameters.

Two weight matrices should be defined. One for the observations (P_v, which correspond to the P matrix is the other least square solution) and another for the unknowns (P_x).

$$F(\bar{X}, \bar{L}) = 0 \tag{17}$$

with

$$P = \begin{pmatrix} P_v & 0 \\ 0 & P_x \end{pmatrix} \tag{18}$$

In the hyper matrix form, P_x may be singular.

$$\begin{pmatrix} P_v & B^T & 0 \\ B & 0 & A \\ 0 & A^T & P_x \end{pmatrix} \begin{pmatrix} \hat{V} \\ \hat{K} \\ \hat{X} \end{pmatrix} + \begin{pmatrix} 0 \\ W \\ 0 \end{pmatrix} = 0 \tag{19}$$

The solution vectors are:

$$\hat{X} = -(P_x + A^T(BP^{-1}B]^T)^{-1}A)^{-1}A^T(BP_v^{-1}B^T)^{-1}W \tag{20}$$

$$\hat{K} = (BP_v^{-1}B^T)^{-1}(A\hat{X} + W) \tag{21}$$

$$\hat{V} = -P_v^{-1}B^T\hat{K} \tag{22}$$

CONCLUSION

The new version of LGC is a command line software, for which unit tests and comparison with the previous version have been developed as well. The use of the software for HIE-ISOLDE monitoring confirms the promising results. In a near future, a Graphical User Interface is being considered so that the user can easily visualize results. The graphical representation becomes the next priority, simplifying the frames representation, and improving the way of browsing results with a system of filters.

The new version of LGC was developed because of new measurement technologies used at CERN, and to better answer to the future precision request in the accelerator positioning. The new functionalities of the software allow a larger field of use, as for the monitoring.

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