Two viable models of SM fermion mass generation

Antonio Enrique Cárcamo Hernández

Universidad Técnica Federico Santa María.

FLASY 2016 Conference, Valparaíso, Chile, 29th September of 2016.
Based on: A. E. Cárcamo Hernández, arxiv:hep-ph/1512.09092v5, Eur.Phys.J. C76 (2016) no.9, 503.
Carolina Arbelaéz, A. E. Cárcamo Hernández, S. Kovalenko, I. Schmidt, arxiv:hep-ph/1602.03607

The origin of fermion masses and mixings cannot be understood within the Standard Model.

FERMIONS

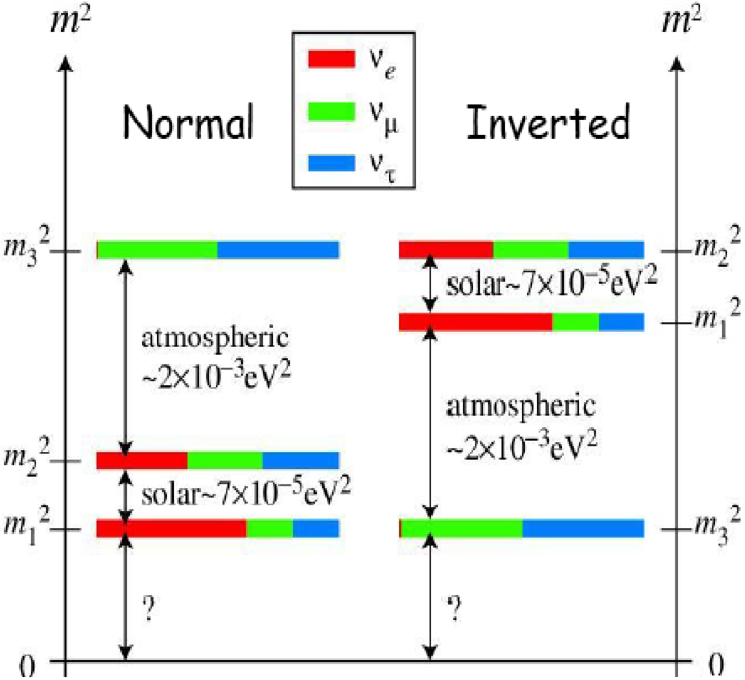
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor		Mass GeV/c^2	Electric charge
ν_L	lightest neutrino*	$(0-0.13)\times10^{-9}$	0
e	electron	0.000511	−1
ν_M	middle neutrino*	$(0.009-0.13)\times10^{-9}$	0
μ	muon	0.106	−1
ν_H	heaviest neutrino*	$(0.04-0.14)\times10^{-9}$	0
τ	tau	1.777	−1

Quarks spin =1/2

Flavor		Approx. Mass GeV/c^2	Electric charge
u	up	0.002	2/3
d	down	0.005	−1/3
c	charm	1.3	2/3
s	strange	0.1	−1/3
t	top	173	2/3
b	bottom	4.2	−1/3



m^2
Normal
ν_e
ν_μ
ν_τ
Inverted
m^2
$m_3{}^2$
$m_2{}^2$
solar~7×10^{-5}eV^2
$m_1{}^2$
atmospheric
~2×10^{-3}eV^2
atmospheric
~2×10^{-3}eV^2
$m_2{}^2$
solar~7×10^{-5}eV^2
$m_1{}^2$
$m_3{}^2$
?
?
0
0

The S_3 group has three irreducible representations: $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{2}$. Denoting the basis vectors for two S_3 doublets as $(x_1, x_2)^T$ and $(y_1, y_2)^T$ and y' a non trivial S_3 singlet, the S_3 multiplication rules are (Ishimori, et al, Prog. Theor. Phys. Suppl 2010)::

$$\begin{aligned}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}_{\mathbf{2}} &= (x_1 y_1 + x_2 y_2)_{\mathbf{1}} + (x_1 y_2 - x_2 y_1)_{\mathbf{1}'} \\ &+ \begin{pmatrix} x_2 y_2 - x_1 y_1 \\ x_1 y_2 + x_2 y_1 \end{pmatrix}_{\mathbf{2}}, \end{aligned} \quad (1)$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}_{\mathbf{2}} \otimes (y')_{\mathbf{1}'} = \begin{pmatrix} -x_2 y' \\ x_1 y' \end{pmatrix}_{\mathbf{2}}, \qquad (x')_{\mathbf{1}'} \otimes (y')_{\mathbf{1}'} = (x' y')_{\mathbf{1}}. \quad (2)$$

A $S_3 \times Z_8$ flavor model

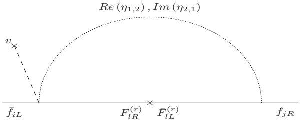
The S_3 symmetry is assumed to be preserved whereas the Z_8 discrete group is broken at the scale v_χ.

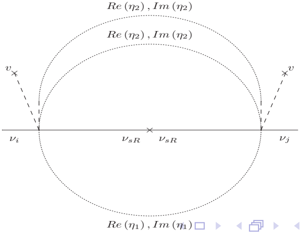
$$\phi \sim (\mathbf{1}, 1), \quad \eta = (\eta_1, \eta_2) \sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad \chi \sim (\mathbf{1}, -i),$$

$$\begin{aligned}
q_{jL} &\sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}}\right), \quad u_{kR} \sim \left(\mathbf{1}', e^{\frac{\pi i(3-k)}{2}}\right), \quad u_{3R} \sim (\mathbf{1}, 1), \\
d_{jR} &\sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}}\right), \quad l_{jL} \sim \left(\mathbf{1}, e^{-\frac{\pi i(3-j)}{2}}\right), \quad l_{jR} \sim \left(\mathbf{1}', e^{\frac{\pi i(3-j)}{2}}\right), \\
T_L^{(k)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad T_R^{(k)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \quad k = 1, 2, \\
B_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad B_R^{(j)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \quad j = 1, 2, 3, \\
E_L^{(j)} &\sim \left(\mathbf{2}, e^{-\frac{\pi i}{4}}\right), \quad E_R^{(j)} \sim \left(\mathbf{2}, e^{\frac{\pi i}{4}}\right), \\
\nu_{kR} &\sim \left(\mathbf{1}', e^{-\frac{\pi i}{4}}\right), \quad k = 1, 2. \qquad (3)
\end{aligned}$$

$$
\begin{aligned}
-\mathcal{L}_{Y}^{(U)} &= \sum_{j=1}^{3}\sum_{r=1}^{2} y_{jr}^{(u)}\overline{q}_{jL}\widetilde{\phi}\left(T_R^{(r)}\eta\right)_{\mathbf{1}}\frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{r=1}^{2}\sum_{s=1}^{2} x_{rs}^{(u)}\left(\overline{T}_L^{(r)}\eta\right)_{\mathbf{1}'} u_{sR}\frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{j=1}^{3} y_{j3}^{(u)}\overline{q}_{jL}\widetilde{\phi}u_{3R}\frac{\chi^{3-j}}{\Lambda^{3-j}} + \sum_{r=1}^{2} y_r^{(T)}\left(\overline{T}_L^{(r)}T_R^{(r)}\right)_{\mathbf{1}}\chi + h.c
\end{aligned}
$$

$$
-\mathcal{L}_{Y}^{(\nu)} = \sum_{j=1}^{3}\sum_{s=1}^{2} y_{js}^{(\nu)}\overline{l}_{jL}\widetilde{\phi}\nu_{sR}\frac{\left[\eta^{*}\left(\eta\eta^{*}\right)_{\mathbf{2}}\right]_{\mathbf{1}'}\chi^{3-j}}{\Lambda^{6-j}} + \sum_{s=1}^{2} y_s\overline{\nu}_{sR}\nu_{sR}^{C}\chi + h.c.
$$





$$
\begin{aligned}
-\mathcal{L}_{Y}^{(D)} &= \sum_{j=1}^{3}\sum_{k=1}^{3} y_{jk}^{(d)}\overline{q}_{jL}\phi\left(B_R^{(k)}\eta\right)_{\mathbf{1}}\frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{j=1}^{3}\sum_{k=1}^{3} x_{jk}^{(d)}\left(\overline{B}_L^{(j)}\eta\right)_{\mathbf{1}'} d_{kR}\frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{k=1}^{3} y_k^{(B)}\left(\overline{B}_L^{(k)}B_R^{(k)}\right)_{\mathbf{1}}\chi + h.c. \qquad (4)\\
-\mathcal{L}_{Y}^{(l)} &= \sum_{j=1}^{3}\sum_{k=1}^{3} y_{jk}^{(l)}\overline{l}_{jL}\phi\left(E_R^{(k)}\eta\right)_{\mathbf{1}}\frac{\chi^{3-j}}{\Lambda^{4-j}} \\
&+ \sum_{j=1}^{3}\sum_{k=1}^{3} x_{jk}^{(l)}\left(\overline{E}_L^{(j)}\eta\right)_{\mathbf{1}'} l_{kR}\frac{\chi^{3-k}}{\Lambda^{3-k}} \\
&+ \sum_{k=1}^{3} y_k^{(E)}\left(\overline{E}_L^{(k)}E_R^{(k)}\right)_{\mathbf{1}}\chi \qquad (5)
\end{aligned}
$$

where I set

$$
v_\chi = \lambda\Lambda, \qquad \lambda = 0.225. \qquad (6)
$$

Fermion masses and mixing.

The charged fermion mass matrices are:

$$M_U = \begin{pmatrix} \varepsilon_{11}^{(u)}\lambda^3 & \varepsilon_{12}^{(u)}\lambda^2 & y_{13}^{(u)}\lambda^2 \\ \varepsilon_{21}^{(u)}\lambda^2 & \varepsilon_{22}^{(u)}\lambda & y_{23}^{(u)}\lambda \\ \varepsilon_{31}^{(u)}\lambda & \varepsilon_{32}^{(u)} & y_{33}^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \qquad (7)$$

$$M_{D,l} = \begin{pmatrix} \varepsilon_{11}^{(d,l)}\lambda^4 & \varepsilon_{12}^{(d,l)}\lambda^3 & \varepsilon_{13}^{(d,l)}\lambda^2 \\ \varepsilon_{21}^{(d,l)}\lambda^3 & \varepsilon_{22}^{(d,l)}\lambda^2 & \varepsilon_{23}^{(d,l)}\lambda \\ \varepsilon_{31}^{(d,l)}\lambda^2 & \varepsilon_{32}^{(d,l)}\lambda & \varepsilon_{33}^{(d,l)} \end{pmatrix} \frac{v}{\sqrt{2}},$$

where the dimensionless parameters $\varepsilon_{jk}^{(f)}$ $(j,k = 1,2,3)$ with $f = u,d,l$, are generated at one loop level. The invariance of charged exotic fermion Yukawa interactions under the cyclic symmetry requires to consider the Z_8 instead of the Z_4 discrete symmetry.

The light active neutrino mass matrix is:

$$M_\nu = \begin{pmatrix} W_1^2 & W_1 W_2 \cos\varphi & W_1 W_3 \cos(\varphi - \varrho) \\ W_1 W_2 \cos\varphi & W_2^2 & W_2 W_3 \cos\varrho \\ W_1 W_3 \cos(\varphi - \varrho) & W_2 W_3 \cos\varrho & W_3^2 \end{pmatrix},$$

$$\overrightarrow{W_j} = \left(\frac{A_{j1}\sqrt{y_1 v_\chi f_1^{(\nu)}}}{64\pi^3\Lambda}, \frac{A_{j2}\sqrt{y_2 v_\chi f_2^{(\nu)}}}{64\pi^3\Lambda} \right), A_{js} = \lambda^{3-j} y_{js}^{(\nu)} \frac{v}{\sqrt{2}}$$

$$\cos\varphi = \frac{\overrightarrow{W_1} \cdot \overrightarrow{W_2}}{\left|\overrightarrow{W_1}\right| \left|\overrightarrow{W_2}\right|}, \quad \cos(\varphi - \varrho) = \frac{\overrightarrow{W}_1 \cdot \overrightarrow{W_3}}{\left|\overrightarrow{W}_1\right| \left|\overrightarrow{W_3}\right|}, \quad \cos\varrho = \frac{\overrightarrow{W_2} \cdot \overrightarrow{W_3}}{\left|\overrightarrow{W_2}\right| \left|\overrightarrow{W_3}\right|}$$

where the dimensionless parameters $f_s^{(\nu)}$ $(s = 1, 2)$ are generated at three loop level.

Since the dimensionless parameters $\varepsilon_{jk}^{(f)}$ $(j,k=1,2,3)$ with $f=u,d,l$, are generated at one loop level, I set $\varepsilon_{jk}^{(f)}=a_{jk}^{(f)}\lambda^3$. In addition, I adopt the benchmark:

$$\begin{aligned} a_{12}^{(u)} &= a_{21}^{(u)}, \qquad a_{31}^{(u)}=y_{13}^{(u)}, \qquad a_{32}^{(u)}=y_{23}^{(u)} \\ a_{12}^{(d)} &= \left|a_{12}^{(d)}\right| e^{-i\tau_1}, \qquad a_{21}^{(d)}=\left|a_{12}^{(d)}\right| e^{i\tau_1}, \\ a_{13}^{(d)} &= \left|a_{13}^{(d)}\right| e^{-i\tau_2}, \qquad a_{31}^{(d)}=\left|a_{13}^{(d)}\right| e^{i\tau_2}, \qquad a_{23}^{(d)}=a_{32}^{(d)}. \end{aligned} \tag{8}$$

The best fit values are:

$$\begin{aligned} a_{11}^{(u)} &\simeq 0.58, \qquad a_{22}^{(u)}\simeq 2.19, \qquad a_{12}^{(u)}\simeq 0.67, \\ a_{13}^{(u)} &\simeq 0.80, \qquad a_{23}^{(u)}\simeq 0.83, \qquad a_{11}^{(d)}\simeq 1.96, \\ a_{12}^{(d)} &\simeq 0.53, \qquad a_{13}^{(d)}\simeq 1.07, \qquad a_{22}^{(d)}\simeq 1.93, \\ a_{23}^{(d)} &\simeq 1.36, \qquad a_{33}^{(d)}\simeq 1.35, \qquad \tau_1\simeq 9.56^\circ, \qquad \tau_2\simeq 4.64^\circ. \end{aligned} \tag{9}$$

Observable	Model value	Experimental value
$m_u(MeV)$	1.44	$1.45^{+0.56}_{-0.45}$
$m_c(MeV)$	656	635 ± 86
$m_t(GeV)$	177.1	$172.1 \pm 0.6 \pm 0.9$
$m_d(MeV)$	2.9	$2.9^{+0.5}_{-0.4}$
$m_s(MeV)$	57.7	$57.7^{+16.8}_{-15.7}$
$m_b(GeV)$	2.82	$2.82^{+0.09}_{-0.04}$
$\sin\theta_{12}$	0.225	0.225
$\sin\theta_{23}$	0.0412	0.0412
$\sin\theta_{13}$	0.00351	0.00351
δ	64°	68°

Table: Model and experimental values of the quark masses and CKM parameters.

In the concerning to the charged lepton sector, I adopt the benchmark $a_{jk}^{(l)} = a_k^{(l)}\delta_{jk}$, so that the charged lepton masses take the form:

$$m_e = a_1^{(l)}\lambda^7\frac{v}{\sqrt{2}}, \qquad m_\mu = a_2^{(l)}\lambda^5\frac{v}{\sqrt{2}}, \qquad m_\tau = a_3^{(l)}\lambda^3\frac{v}{\sqrt{2}}.$$

The best-fit values are:

$$\begin{aligned} \varrho = \varphi/2 &\simeq 38.73^\circ, \quad W_1 \simeq -0.063 eV^{\frac{1}{2}}, \quad W_2 \simeq 0.18 eV^{\frac{1}{2}}, \\ W_3 &\simeq 0.15 eV^{\frac{1}{2}}, \quad \text{for NH} \\ a_1^{(l)} &\simeq 0.1, \quad a_2^{(l)} \simeq 1.02, \quad a_3^{(l)} \simeq 0.88, \end{aligned} \qquad (10)$$

$$\begin{aligned} \varrho &\simeq 162.26^\circ, \quad \varphi \simeq 79.44^\circ, \quad W_1 \simeq 0.22 eV^{\frac{1}{2}}, \\ W_2 &\simeq 0.15 eV^{\frac{1}{2}}, \quad W_3 \simeq 0.17 eV^{\frac{1}{2}}, \quad \text{for IH} \\ a_1^{(l)} &\simeq 0.1, \quad a_2^{(l)} \simeq 1.02, \quad a_3^{(l)} \simeq 0.88, \end{aligned} \qquad (11)$$

Observable	Model value	Experimental value
$m_e(MeV)$	0.487	0.487
$m_\mu(MeV)$	102.8	102.8 ± 0.0003
$m_\tau(GeV)$	1.75	1.75 ± 0.0003
$\Delta m^2_{21}(10^{-5}\text{eV}^2)$ (NH)	7.22	$7.60^{+0.19}_{-0.18}$
$\Delta m^2_{31}(10^{-3}\text{eV}^2)$ (NH)	2.50	$2.48^{+0.05}_{-0.07}$
$\sin^2\theta_{12}$ (NH)	0.334	0.323 ± 0.016
$\sin^2\theta_{23}$ (NH)	0.567	$0.567^{+0.032}_{-0.128}$
$\sin^2\theta_{13}$ (NH)	0.0228	0.0234 ± 0.0020
$\Delta m^2_{21}(10^{-5}\text{eV}^2)$ (IH)	7.60	$7.60^{+0.19}_{-0.18}$
$\Delta m^2_{13}(10^{-3}\text{eV}^2)$ (IH)	2.48	$2.48^{+0.05}_{-0.06}$
$\sin^2\theta_{12}$ (IH)	0.323	0.323 ± 0.016
$\sin^2\theta_{23}$ (IH)	0.573	$0.573^{+0.025}_{-0.043}$
$\sin^2\theta_{13}$ (IH)	0.0240	0.0240 ± 0.0019

Table: Model and experimental values of the charged lepton masses, neutrino mass squared splittings and leptonic mixing parameters for the normal (NH) and inverted (IH) mass hierarchies.

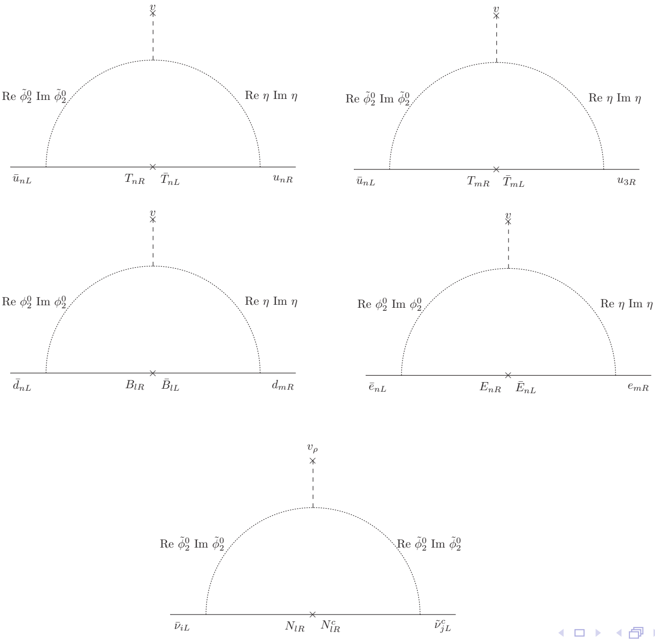
$$
\begin{aligned}
\phi_1 &\sim (\mathbf{1},1,1)\,, \qquad \phi_2 \sim (\mathbf{1},-1,1)\,,\\
\xi &\sim (\mathbf{2},1,1)\,, \qquad \eta \sim (\mathbf{1},-1,1)\,,\\
\tau &\sim \left(\mathbf{1}',1,e^{-\frac{i\pi}{6}}\right).
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
q_{1L} &\sim (\mathbf{1}',1,1)\,, \qquad q_{2L} \sim (\mathbf{1},1,1)\,,\\
q_{3L} &\sim (\mathbf{1},1,1)\,, \qquad u_{3R} \sim (\mathbf{1},1,1)\,,\\
u_{1R} &\sim (\mathbf{1}',-1,-1)\,, \qquad u_{2R} \sim \left(\mathbf{1},-1,e^{\frac{i\pi}{3}}\right)\\
D_R &= (d_{1R},d_{2R}) \sim (\mathbf{2},-1,1)\,, \quad d_{3R} \sim (\mathbf{1}',1,i)\,,\\
l_{1L} &\sim (\mathbf{1},1,1)\,, \qquad l_{2L} \sim (\mathbf{1},1,1)\,,\\
l_{3L} &\sim (\mathbf{1},1,1)\,, \qquad l_{3R} \sim (\mathbf{1}',1,i)\,,\\
l_{1R} &\sim (\mathbf{1},-1,-1)\,, \qquad l_{2R} \sim (\mathbf{1}',-1,i)\,,
\end{aligned}
$$

$$
\begin{array}{lcl}
T_{1L} & \sim & (\mathbf{1},1,1)\,, \qquad T_{1R}\sim\left(\mathbf{1}',-1,1\right), \\
T_{2L} & \sim & (\mathbf{1},1,1)\,, \qquad T_{2R}\sim(\mathbf{1},-1,1)\,, \\
B_{1L} & \sim & (\mathbf{1},1,1)\,, \qquad B_{1R}\sim\left(\mathbf{1}',-1,1\right), \\
B_{2L} & \sim & \left(\mathbf{1}',1,1\right), \qquad B_{2R}\sim(\mathbf{1},-1,1)\,, \\
E_{1L} & \sim & (\mathbf{1},1,1)\,, \qquad E_{1R}\sim(\mathbf{1},-1,1)\,, \\
E_{2L} & \sim & (\mathbf{1},1,1)\,, \qquad E_{2R}\sim(\mathbf{1},-1,1)\,, \\
N_{1R} & \sim & (\mathbf{1},-1,1)\,, \qquad N_{2R}\sim(\mathbf{1},-1,1)\,.
\end{array}
$$

$$
\begin{aligned}
\mathcal{L}_Y^U &= m_{T_1}\overline{T}_{1L}T_{1R} + m_{T_2}\overline{T}_{2L}T_{2R} + y_3^{(u)}\overline{q}_{3L}\widetilde{\phi}_1 u_{3R} \\
&+ y_1^{(u)}\overline{q}_{1L}\widetilde{\phi}_2 T_{1R} + x_1^{(u)}\overline{T}_{1L}\eta u_{3R} \\
&+ y_2^{(u)}\overline{q}_{2L}\widetilde{\phi}_2 T_{2R} + x_2^{(u)}\overline{T}_{2L}\eta u_{3R} \\
&+ x_3^{(u)}\overline{T}_{1L}\eta u_{1R}\frac{\tau^6}{\Lambda^6} + x_4^{(u)}\overline{T}_{2L}\eta u_{2R}\frac{\tau^2}{\Lambda^2}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_Y^D &= m_{B_1}\overline{B}_{1L}B_{1R} + m_{B_2}\overline{B}_{2L}B_{2R} + y_3^{(d)}\overline{q}_{3L}\phi_1 d_{3R}\frac{\tau^3}{\Lambda^3} \\
&+ x_2^{(d)}\overline{B}_{1L}\eta D_R\frac{\xi}{\Lambda} + x_4^{(d)}\overline{B}_{2L}\eta D_R\frac{\xi}{\Lambda} \\
&+ y_1^{(d)}\overline{q}_{1L}\phi_2 B_{1R} + x_1^{(d)}\overline{B}_{1L}\eta D_R\frac{\xi\xi}{\Lambda^2} \\
&+ y_2^{(d)}\overline{q}_{2L}\phi_2 B_{2R} + x_3^{(d)}\overline{B}_{2L}\eta D_R\frac{\xi\xi}{\Lambda^2} + h.c
\end{aligned}
$$


v
Re $\tilde{\phi}_2^0$ Im $\tilde{\phi}_2^0$
Re η Im η
$\bar{u}_{nL}$ T_{nR} $\bar{T}_{nL}$ u_{nR}
v
Re $\tilde{\phi}_2^0$ Im $\tilde{\phi}_2^0$
Re η Im η
$\bar{u}_{nL}$ T_{mR} $\bar{T}_{mL}$ u_{3R}
v
Re ϕ_2^0 Im ϕ_2^0
Re η Im η
$\bar{d}_{nL}$ B_{lR} $\bar{B}_{lL}$ d_{mR}
v
Re ϕ_2^0 Im ϕ_2^0
Re η Im η
$\bar{e}_{nL}$ E_{nR} $\bar{E}_{nL}$ e_{mR}
v_ρ
Re $\tilde{\phi}_2^0$ Im $\tilde{\phi}_2^0$
Re $\tilde{\phi}_2^0$ Im $\tilde{\phi}_2^0$
$\bar{\nu}_{iL}$ N_{lR} N_{lR}^c $\tilde{\nu}_{jL}^c$

$$\begin{aligned}
\mathcal{L}_Y^l &= m_{E_1}\overline{E}_{1L}E_{1R} + m_{E_2}\overline{E}_{2L}E_{2R} + y_3^{(l)}\bar{l}_{3L}\phi_1 l_{3R}\frac{\tau^3}{\Lambda^3} \\
&+ x_1^{(l)}\bar{l}_{1L}\phi_2 E_{1R} + y_1^{(l)}\overline{E}_{1L}\eta l_{1R}\frac{\tau^6}{\Lambda^6} \\
&+ x_2^{(l)}\bar{l}_{2L}\phi_2 E_{1R} + y_2^{(l)}\overline{E}_{1L}\eta l_{2R}\frac{\tau^3}{\Lambda^3} \\
&+ x_3^{(l)}\bar{l}_{1L}\phi_2 E_{2R} + y_3^{(l)}\overline{E}_{2L}\eta l_{1R}\frac{\tau^6}{\Lambda^6} \\
&+ x_4^{(l)}\bar{l}_{2L}\phi_2 E_{2R} + y_4^{(l)}\overline{E}_{2L}\eta l_{2R}\frac{\tau^3}{\Lambda^3}
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_Y^\nu &= y_{11}^{(\nu)}\bar{l}_{1L}\widetilde{\phi}_2 N_{1R} + y_{21}^{(\nu)}\bar{l}_{2L}\widetilde{\phi}_2 N_{1R} \\
&+ y_{31}^{(\nu)}\bar{l}_{3L}\widetilde{\phi}_2 N_{1R} + y_{12}^{(\nu)}\bar{l}_{1L}\widetilde{\phi}_2 N_{2R} \\
&+ y_{22}^{(\nu)}\bar{l}_{2L}\widetilde{\phi}_2 N_{2R} + y_{32}^{(\nu)}\bar{l}_{3L}\widetilde{\phi}_2 N_{2R} \\
&+ m_{1N}\overline{N}_{1R}N_{1R}^c + m_{2N}\overline{N}_{2R}N_{2R}^c + h.c
\end{aligned}$$

$$v_\xi \sim v_\tau \sim \Lambda_{int} \sim \lambda\Lambda, \qquad <\xi>= v_\xi(1,0), \qquad \lambda = 0.225. \quad (13)$$

Fermion masses and mixing.

$$M_U = \begin{pmatrix} \varepsilon_{11}^{(u)}\lambda^6 & 0 & \varepsilon_{13}^{(u)} \\ 0 & \varepsilon_{22}^{(u)}\lambda^2 & \varepsilon_{23}^{(u)} \\ 0 & 0 & y_3^{(u)} \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{14}$$

$$M_D = \begin{pmatrix} \varepsilon_{11}^{(d)}\lambda^2 & \varepsilon_{12}^{(d)}\lambda & 0 \\ \varepsilon_{21}^{(d)}\lambda^2 & \varepsilon_{22}^{(d)}\lambda & 0 \\ 0 & 0 & y_3^{(d)}\lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{15}$$

$$M_l = \begin{pmatrix} \varepsilon_{11}^{(l)}\lambda^6 & \varepsilon_{12}^{(l)}\lambda^3 & 0 \\ \varepsilon_{21}^{(l)}\lambda^6 & \varepsilon_{22}^{(l)}\lambda^3 & 0 \\ 0 & 0 & y_3^{(l)}\lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{16}$$

$$M_\nu = \begin{pmatrix} W_1^2 & \kappa W_1 W_2 & W_1 W_3 \\ \kappa W_1 W_2 & W_2^2 & \kappa W_2 W_3 \\ W_1 W_3 & \kappa W_2 W_3 & W_3^2 \end{pmatrix}, \tag{17}$$

The following benchmark is considered:

$$\varepsilon_{11}^{(u)} = a_{11}^{(u)}\lambda^2, \quad \varepsilon_{22}^{(u)} = a_{22}^{(u)}\lambda^2, \quad \varepsilon_{13}^{(u)} = a_{13}^{(u)}\lambda^3, \quad \varepsilon_{23}^{(u)} = a_{23}^{(u)}\lambda^2,$$
$$\varepsilon_{11}^{(d)} = a_{11}^{(d)}\lambda^5, \quad \varepsilon_{21}^{(d)} = a_{21}^{(d)}\lambda^4, \quad \varepsilon_{12}^{(d)} = a_{12}^{(d)}\lambda^5, \quad \varepsilon_{22}^{(d)} = a_{22}^{(d)}\lambda^4.$$

$$\varepsilon_{11}^{(l)} = a_{11}^{(l)}\lambda^2, \quad \varepsilon_{21}^{(l)} = a_{21}^{(l)}\lambda^2, \quad \varepsilon_{12}^{(l)} = a_{12}^{(l)}\lambda^2, \quad \varepsilon_{22}^{(l)} = a_{22}^{(l)}\lambda^2.$$

And the following best fit point is obtained:

$$a_{23}^{(u)} \simeq 0.81, \quad a_{13}^{(u)} \simeq 0.3e^{i\delta}, \quad \delta = -113^\circ, \quad a_{22}^{(u)} \simeq 1.43, \quad a_{11}^{(u)} \simeq 1.27,$$
$$a_{11}^{(d)} \simeq 0.84, \quad a_{12}^{(d)} \simeq 0.4, \quad a_{22}^{(d)} \simeq 0.57, \quad a_{33}^{(d)} \simeq 1.42.$$

$$\kappa \simeq 0.7, \quad W_1 \simeq 0.06eV^{\frac{1}{2}}, \quad W_2 \simeq 0.17eV^{\frac{1}{2}}, \quad W_3 \simeq 0.16eV^{\frac{1}{2}},$$
$$a_{11}^{(l)} \simeq 1.66, \quad a_{12}^{(l)} \simeq 0.58, \quad a_{22}^{(l)} \simeq 1.01, \quad y_3^{(l)} \simeq 0.88, \qquad \text{for NH}$$

$$\kappa \simeq 7.79 \times 10^{-3}, \quad W_1 \simeq 0.19eV^{\frac{1}{2}}, \quad W_2 \simeq 0.22eV^{\frac{1}{2}}, \quad W_3 \simeq 0.11eV^{\frac{1}{2}},$$
$$a_{11}^{(l)} \simeq 0.32, \quad a_{12}^{(l)} \simeq -0.35, \quad a_{22}^{(l)} \simeq 1.03, \quad y_3^{(l)} \simeq 0.88, \qquad \text{for IH}$$

Observable	Model value	Experimental value
$m_u(MeV)$	1.47	$1.45^{+0.56}_{-0.45}$
$m_c(MeV)$	641	635 ± 86
$m_t(GeV)$	172.2	$172.1 \pm 0.6 \pm 0.9$
$m_d(MeV)$	3.00	$2.9^{+0.5}_{-0.4}$
$m_s(MeV)$	59.2	$57.7^{+16.8}_{-15.7}$
$m_b(GeV)$	2.82	$2.82^{+0.09}_{-0.04}$
$\sin\theta_{12}$	0.2257	0.2254
$\sin\theta_{23}$	0.0412	0.0413
$\sin\theta_{13}$	0.00352	0.00350
δ	68°	68°

Table: Model and experimental values of the quark masses and CKM parameters.

Observable	Model value	Experimental value
$m_e(MeV)$	0.487	0.487
$m_\mu(MeV)$	102.8	102.8 ± 0.0003
$m_\tau(GeV)$	1.75	1.75 ± 0.0003
$\Delta m_{21}^2(10^{-5}\text{eV}^2)$ (NH)	7.60	$7.60^{+0.19}_{-0.18}$
$\Delta m_{31}^2(10^{-3}\text{eV}^2)$ (NH)	2.48	$2.48^{+0.05}_{-0.07}$
$\sin^2\theta_{12}$ (NH)	0.323	0.323 ± 0.016
$\sin^2\theta_{23}$ (NH)	0.567	$0.567^{+0.032}_{-0.128}$
$\sin^2\theta_{13}$ (NH)	0.0234	0.0234 ± 0.0020
$\Delta m_{21}^2(10^{-5}\text{eV}^2)$ (IH)	7.60	$7.60^{+0.19}_{-0.18}$
$\Delta m_{13}^2(10^{-3}\text{eV}^2)$ (IH)	2.38	$2.48^{+0.05}_{-0.06}$
$\sin^2\theta_{12}$ (IH)	0.323	0.323 ± 0.016
$\sin^2\theta_{23}$ (IH)	0.0573	$0.573^{+0.025}_{-0.043}$
$\sin^2\theta_{13}$ (IH)	0.0240	0.0240 ± 0.0019

Table: Model and experimental values of the charged lepton masses, neutrino mass squared splittings and leptonic mixing parameters for the normal (NH) and inverted (IH) mass hierarchies.

Conclusions

For the $S_3 \times Z_8$ flavor model:

- The top quark and the exotic fermions acquire tree level masses.
- The remaining charged fermions and the light active neutrinos get one and three loop level masses, respectively.
- The breaking of Z_8 generates the non SM fermion masses as well as the observed pattern of SM fermion masses and mixings
- The unbroken S_3 allows for natural dark matter candidates.

For the $S_3 \times Z_2 \times Z_{12}$ flavor Inert Doublet model:

- At tree level only the third generation charged fermions acquire masses and there is no quark mixings.
- The remaining fermions masses and the fermion mixing angles arise at one loop level.
- The breaking of $S_3 \times Z_{12}$ generates the observed pattern of fermion masses and mixing angles.
- The unbroken Z_2 allows for natural dark matter candidates.

Acknowledgements

Thank you very much to all of you for the attention.