

# Predictive SUSY flavour GUTs & the precision analysis tool SusyTC

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USM Valparaiso, Chile; September 2016

*Amo, Valparaiso, cuanto encierras  
y cuanto irradias, novia del océano,  
hasta más lejos que tu nímbo sordo....  
Pablo Neruda*



FLASY 2016: 6th Workshop on Flavour Symmetries and  
Consequences in Accelerators and Cosmology

# Motivation: Interesting question which came up at Flasy 2013 in Niigata ...



Which predictions do SUSY flavour GUT models make for the SUSY particle spectrum?

## FLASY13

Third Workshop  
on Flavor Symmetries

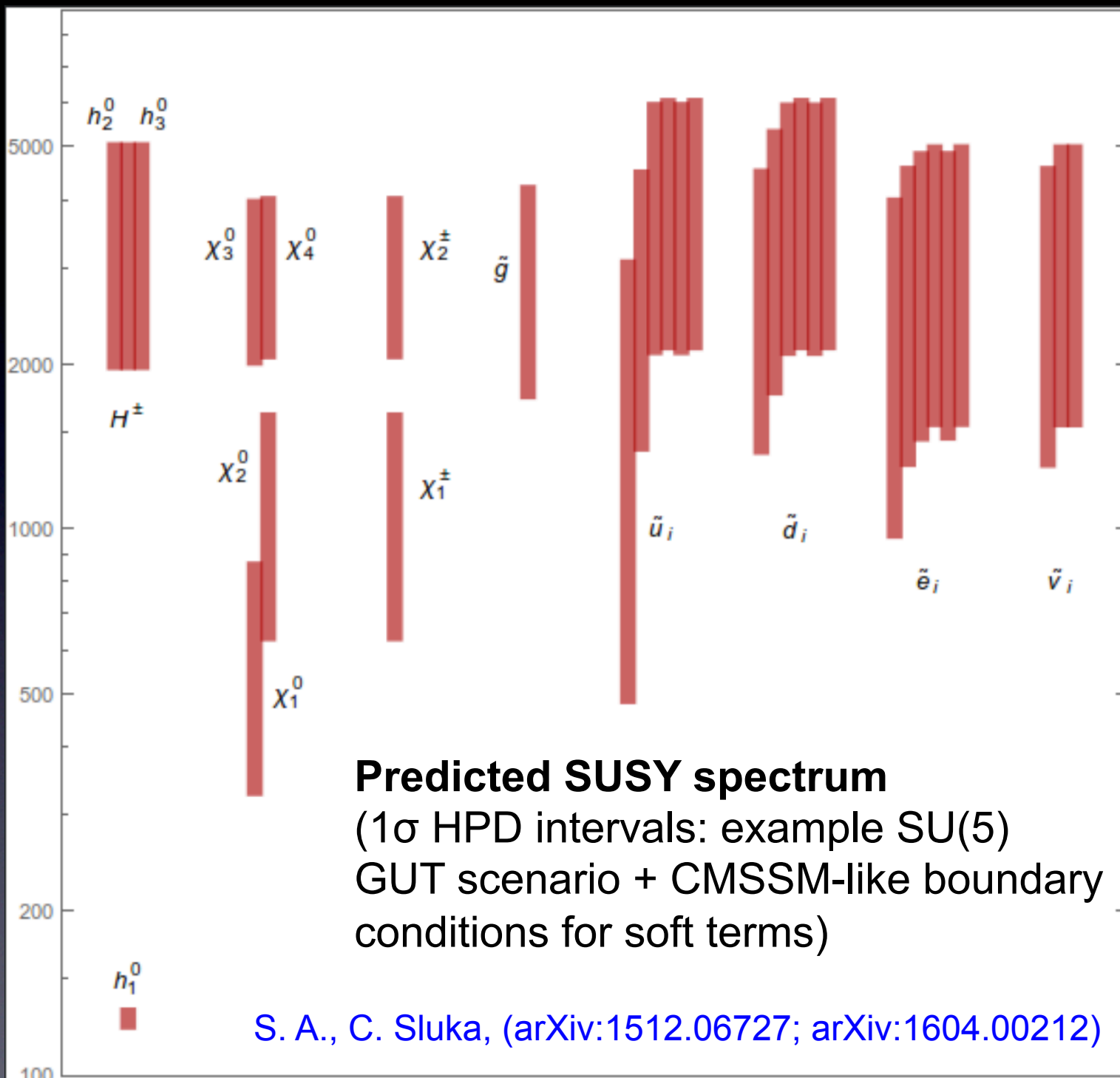
1-5 July 2013

TOKIMATE, Niigata, Japan

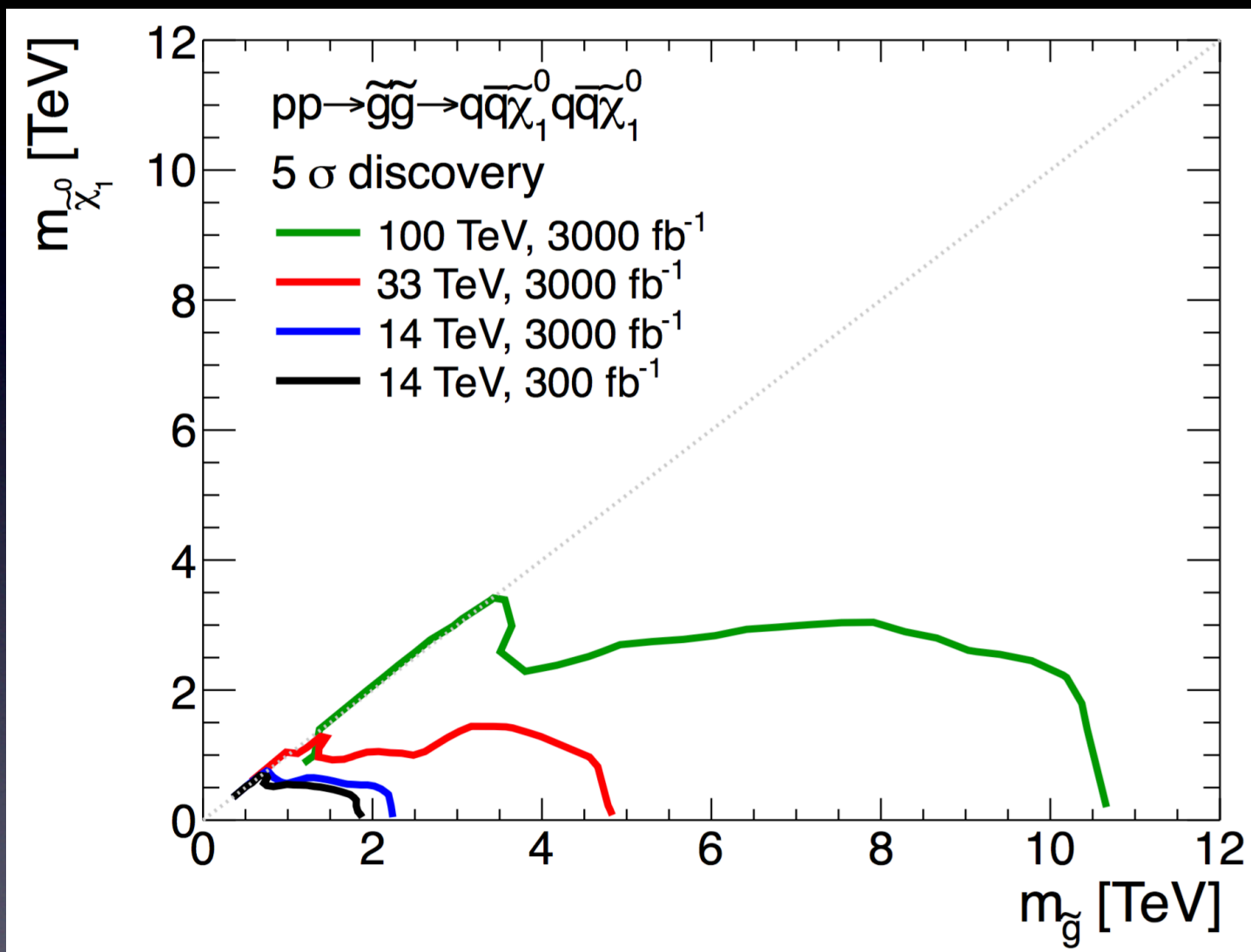


***... we finally have an answer (for the class of SU(5) GUT models we presented @ Flasy2013)***

***... and we made our software tool SusyTC public for you to check the predictions of your favourite GUT model***



# Interesting: Too heavy for LHC (run 1) but testable at a future 100 TeV pp collider (e.g. FCC-hh)



Plot from: [Cohen, Golling, Hance, Henrichs, Howe, Loyal, Padhi, Wacker \(arXiv:1311.6480\)](#)

# *Content of my talk*

## **Predictions from (supersymmetric) GUT-flavour models:**

- Interesting type of predictions: Quark-lepton mass ratios from GUTs
- SUSY loop threshold corrections: The link to the SUSY spectrum
- An example model ...

## **Model analysis & predictions for the SUSY spectrum from GUTs:**

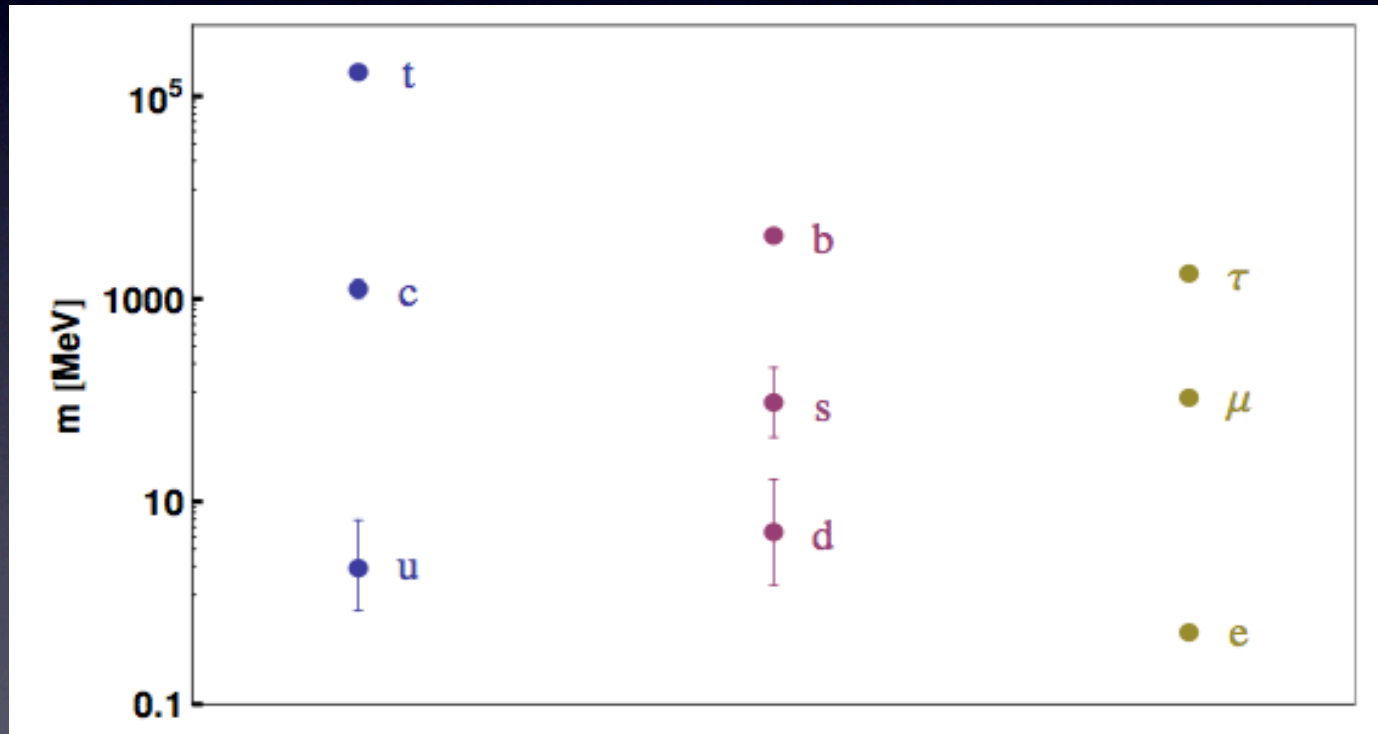
- SusyTC: a REAP extension for soft SUSY running + threshold corrections
- Predictions for the SUSY spectrum from fermion mass relations in GUTs

S. A., C. Sluka, (arXiv:1512.06727)

# *Quark-lepton mass ratios from GUTs*

# Fermion mass ratios from GUTs?

- Why are the observed masses of each family of down-type quarks and charged leptons “similar” (but not equal).



$m_b \leftrightarrow m_\tau ?$

$m_s \leftrightarrow m_\mu ?$

(running masses at the top-mass scale; errors are 3 times the  $1\sigma$  errors ...)



# GUT predictions for mass ratios

➤ Simplest example: In SU(5) GUTs

→  $m_b$  and  $m_\tau$  from one fundamental operator

MSSM Higgs  $H_d$  in GUT representation  $\bar{H}_5$

$y_{33}$   $\bar{5}_3$   $10_3$   $\langle \bar{H}_5 \rangle$

$$\langle \bar{H}_5 \rangle = (0 \quad 0 \quad 0 \quad 0 \quad v_d)$$

$$10_i = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -u_R^{cG} & u_R^{cB} & -u_L^R & -d_L^R \\ u_R^{cG} & 0 & -u_R^{cR} & -u_L^B & -d_L^B \\ -u_R^{cB} & u_R^{cR} & 0 & -u_L^G & -d_L^G \\ u_L^R & u_L^B & u_L^G & 0 & -e_R^c \\ d_L^R & d_L^B & d_L^G & e_R^c & 0 \end{pmatrix}_i$$

$$\bar{5}_i = (d_R^{cR} \quad d_R^{cB} \quad d_R^{cG} \quad e_L \quad -v_L)_i$$

# *GUT predictions for mass ratios*

➤ Simplest example: In SU(5) GUTs

→  $m_b$  and  $m_\tau$  from one fundamental operator

$$y_{33} \bar{\mathbf{5}}_3 \mathbf{10}_3 \langle \bar{H}_5 \rangle \Rightarrow \frac{m_\tau}{m_b} \Big|_{M_{GUT}} = 1$$

“b- $\tau$  unification”

Remark: In this talk “mass ratio” = “Yukawa coupling ratio”

# ***GUT predictions for mass ratios***

- GUT predictions from fundamental operators in SU(5)

→ 3rd family masses from

$$y_{33} \bar{\mathbf{5}}_3 \mathbf{10}_3 \langle \bar{H}_5 \rangle \Rightarrow \frac{m_\tau}{m_b} \Big|_{M_{GUT}} = 1 \quad \text{“b-}\tau \text{ unification”}$$

→ 2nd family masses from

MSSM Higgs  $H_d$  in representation  $\bar{H}_{45}$

$$y_{22} \bar{\mathbf{5}}_2 \mathbf{10}_2 \langle \bar{H}_{45} \rangle \Rightarrow \frac{m_\mu}{m_s} \Big|_{M_{GUT}} = 3 \quad \text{Georgi, Jarlskog ('79)}$$

# *GUT predictions for mass ratios*

- GUT predictions from fundamental operators in SU(5)

## **Remark:**

In models aiming at explaining the fermion mass hierarchies, the masses  $\ll m_t$  are typically generated by effective operators!

Froggatt, Nielsen ('79)

→ With effective operators new interesting GUT predictions for  $m_\tau/m_b$  and  $m_\mu/m_s$  can arise!

S. A., Spinrath ('09)

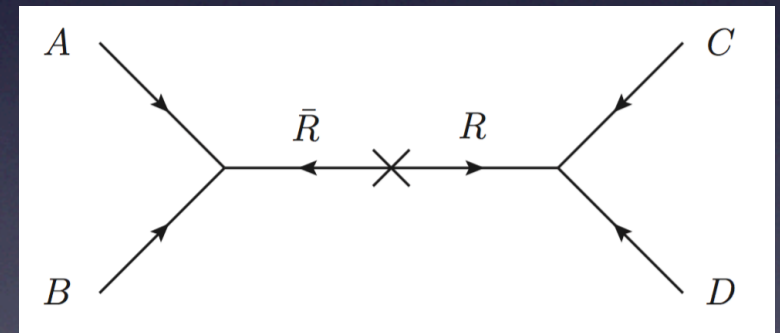
# GUT predictions for mass ratios

➤ **New GUT predictions** from effective operators, for example:

→ For the 3rd family relation  $m_\tau / m_b$  :

$$y_{33} \bar{5}_3 \frac{\langle H_{24} \rangle}{\Lambda} 10_3 \langle \bar{H}_5 \rangle$$

The vev of the GUT-Higgs  $H_{24}$  breaks  $SU(5)$  to the SM gauge symmetry by  $\langle H_{24} \rangle \sim v_{\text{GUT}} \text{diag}(2, 2, 2, -3, -3)$



| $AB$                 | $CD$                    | $R$       | $(Y_e)_{ji}/(Y_d)_{ij}$ |
|----------------------|-------------------------|-----------|-------------------------|
| $H_{24} \mathcal{F}$ | $\mathcal{T} \bar{H}_5$ | $\bar{5}$ | $-\frac{3}{2}$          |

S. A., Spinrath (arXiv:0902.4644)

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→ For the 2nd family relation  $m_\mu/m_s$ :

$$y_{22} \bar{\mathbf{5}}_2 \frac{\langle H_{24} \rangle}{\Lambda} \mathbf{10}_2 \langle \bar{H}_{45} \rangle \Rightarrow \frac{m_\mu}{m_s} \Big|_{M_{GUT}} = \frac{9}{2}$$

$$y_{22} \bar{\mathbf{5}}_2 \langle \bar{H}_5 \rangle \mathbf{10}_2 \frac{\langle H_{24} \rangle}{\Lambda} \Rightarrow \frac{m_\mu}{m_s} \Big|_{M_{GUT}} = 6$$

S. A., Spinrath (arXiv:0902.4644)

# *Which GUT scale predictions are compatible with the experimental data?*

- Procedure: RG running from high to low energies



Running masses at  $\mu = M_Z$ :  
S. A., Maurer, (arXiv:1306.6879)

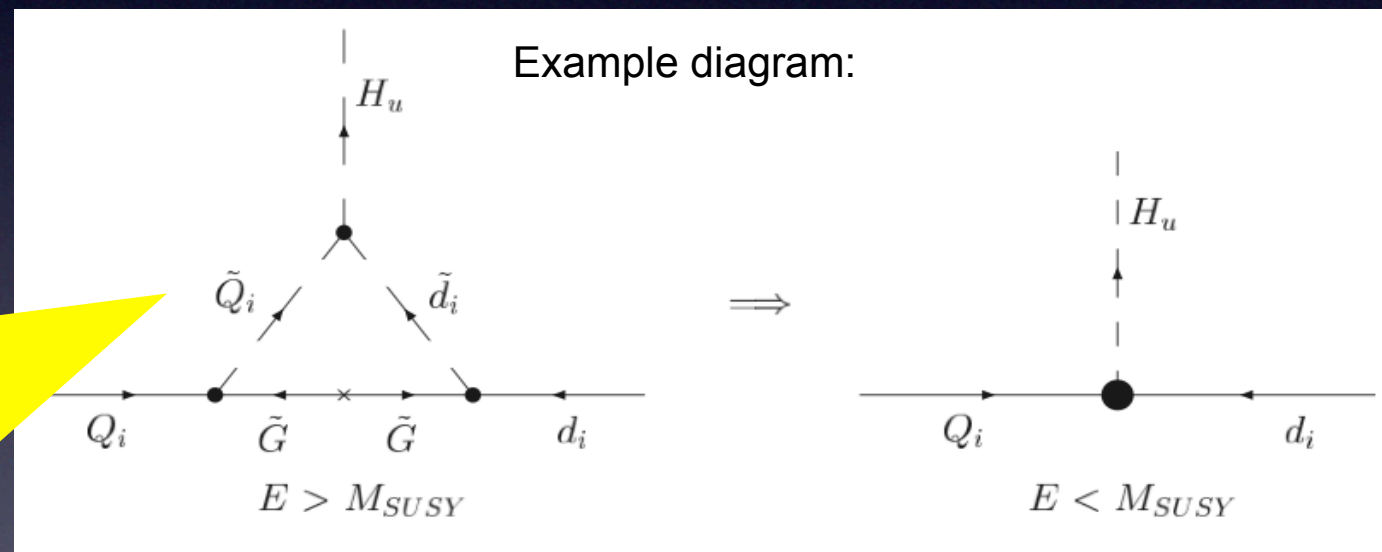


# *SUSY loop threshold corrections: Link to the SUSY spectrum*

At the SUSY scale: Matching of the MSSM to the SM @ loop level

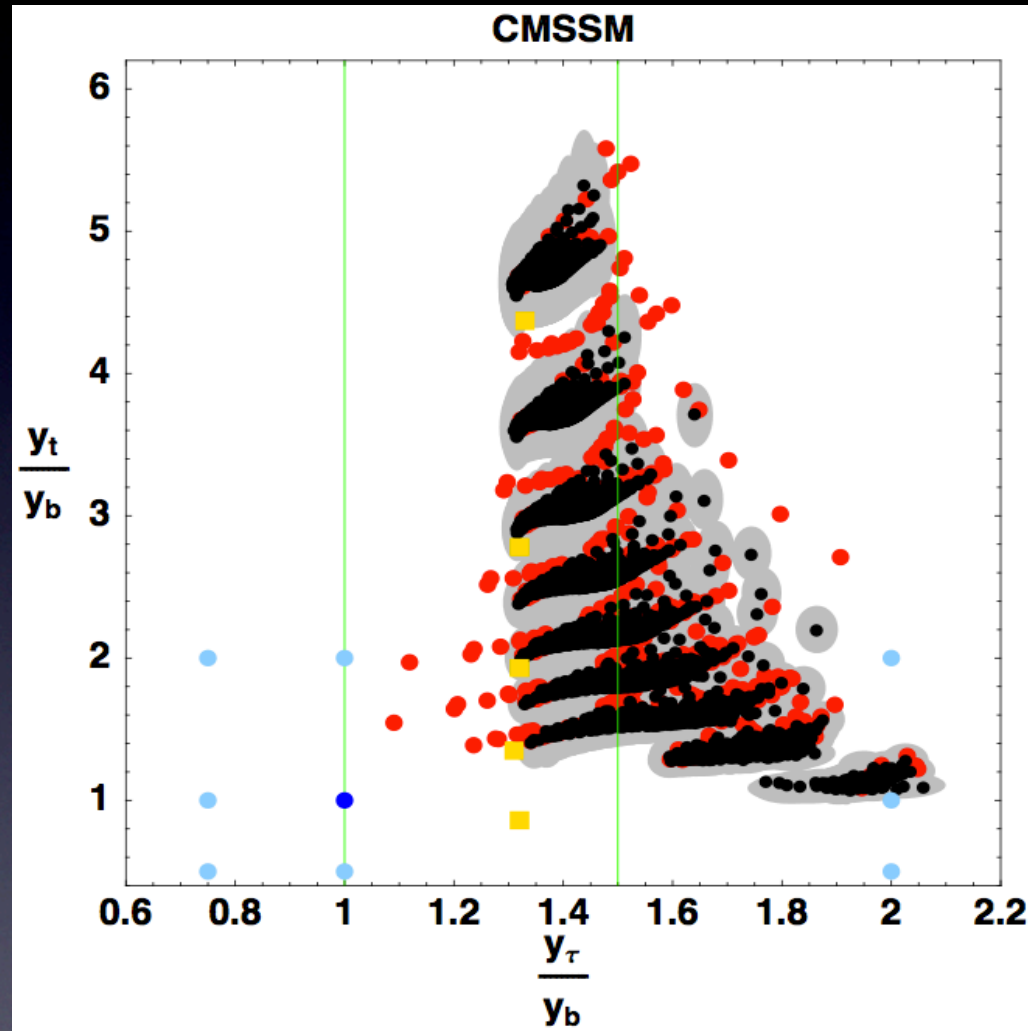
Hall, Rattazzi, Sarid ('93),  
Carena et al ('94), Blazek et al ('95),  
S.A., Spinrath ('08); S.A., Sluka, ('15)

**SUSY threshold effects at  $M_{\text{SUSY}}$  depend on the SUSY parameters, i.e. on the spectrum,  $\tan \beta$ , .... and can strongly affect the low scale results for the quark and lepton masses!**



- Example: Yukawa coupling ratios at the GUT scale for the 3rd family, CMSSM (with  $\mu > 0$ )

S. A., Spinrath (arXiv:0902.4644)

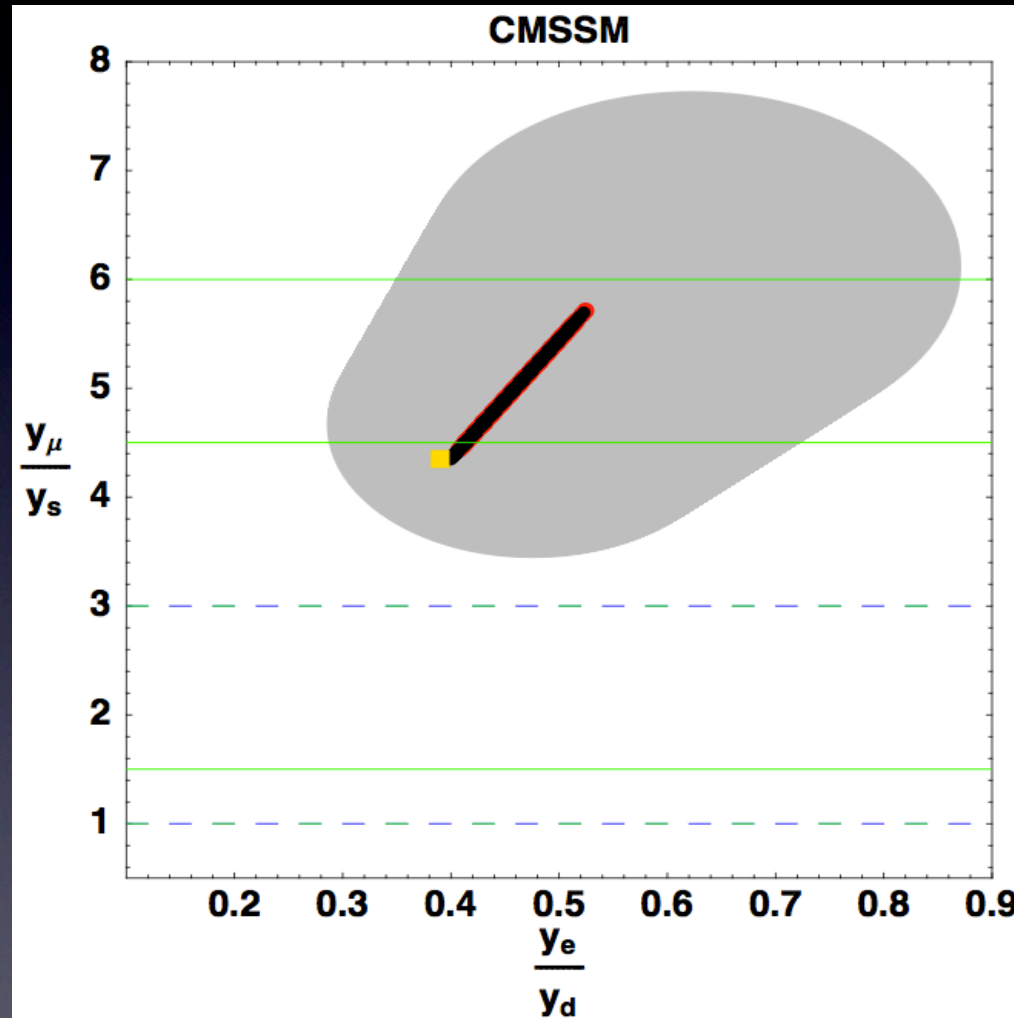


Colours:

- Black: exp. allowed
- Red: exp. disfavoured
- Yellow: no threshold effects
- Grey: exp. uncertainty

- Example: Yukawa coupling ratios at the GUT scale for the 2nd family, CMSSM (with  $\mu > 0$ )

S. A., Spinrath (arXiv:0902.4644)



Colours:

- Black: exp. allowed
- Red: exp. disfavoured
- Yellow: no threshold effects
- Grey: exp. uncertainty

# Brief: An example GUT flavour model ...

# Example for a supersymmetric $SU(5)$ GUT flavour model

S. A., C. Gross, V. Maurer, C. Sluka (arXiv:1305.6612)

Symmetries:  $SU(5) \times A_4$  x “shaping symmetries”

Charged lepton sector: GUT relations for masses & mixings

$$Y_d = \begin{pmatrix} 0 & \tilde{\epsilon}_2 & 0 \\ \tilde{\epsilon}_{ab}c_{ab} & i\tilde{\epsilon}_{ab}s_{ab} & 0 \\ 0 & \omega^2\hat{\epsilon}_\chi & \tilde{\epsilon}_3 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & 6\tilde{\epsilon}_{ab}c_{ab} & 0 \\ -\frac{1}{2}\tilde{\epsilon}_2 & i6\tilde{\epsilon}_{ab}s_{ab} & 6\omega^2\hat{\epsilon}_\chi \\ 0 & 0 & -\frac{3}{2}\tilde{\epsilon}_3 \end{pmatrix}, \quad Y_u = \begin{pmatrix} \epsilon_2^4 & \epsilon_{12}^5 & 0 \\ \epsilon_{12}^5 & \epsilon_{ab}^2 & \epsilon_{23} \\ 0 & \epsilon_{23} & y_t \end{pmatrix}$$

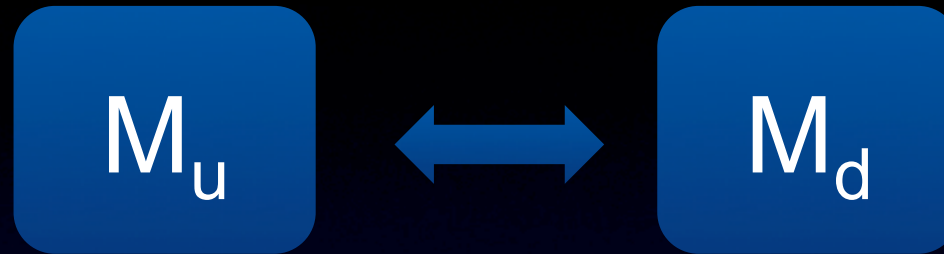
Neutrino sector: TB neutrino mixing (+ charged lepton mixing contr.)

$$Y_\nu = \begin{pmatrix} 0 & \epsilon_{N_2} \\ \epsilon_{N_1} & \epsilon_{N_2} \\ -\epsilon_{N_1} & \epsilon_{N_2} \end{pmatrix}, \quad M_R = \begin{pmatrix} M_{R_1} & 0 \\ 0 & M_{R_2} \end{pmatrix}$$

→ Normal neutrino mass hierarchy

→ Overview of predictions:

S. A., C. Gross, V. Maurer, C. Sluka (arXiv:1305.6612)



$$U_{\text{CKM}} = U_u U_d^\dagger$$

GUT  
relation

$$\theta_{13}^\nu \approx 0$$



$$U_{\text{MNS}} = U_e U_\nu^\dagger$$

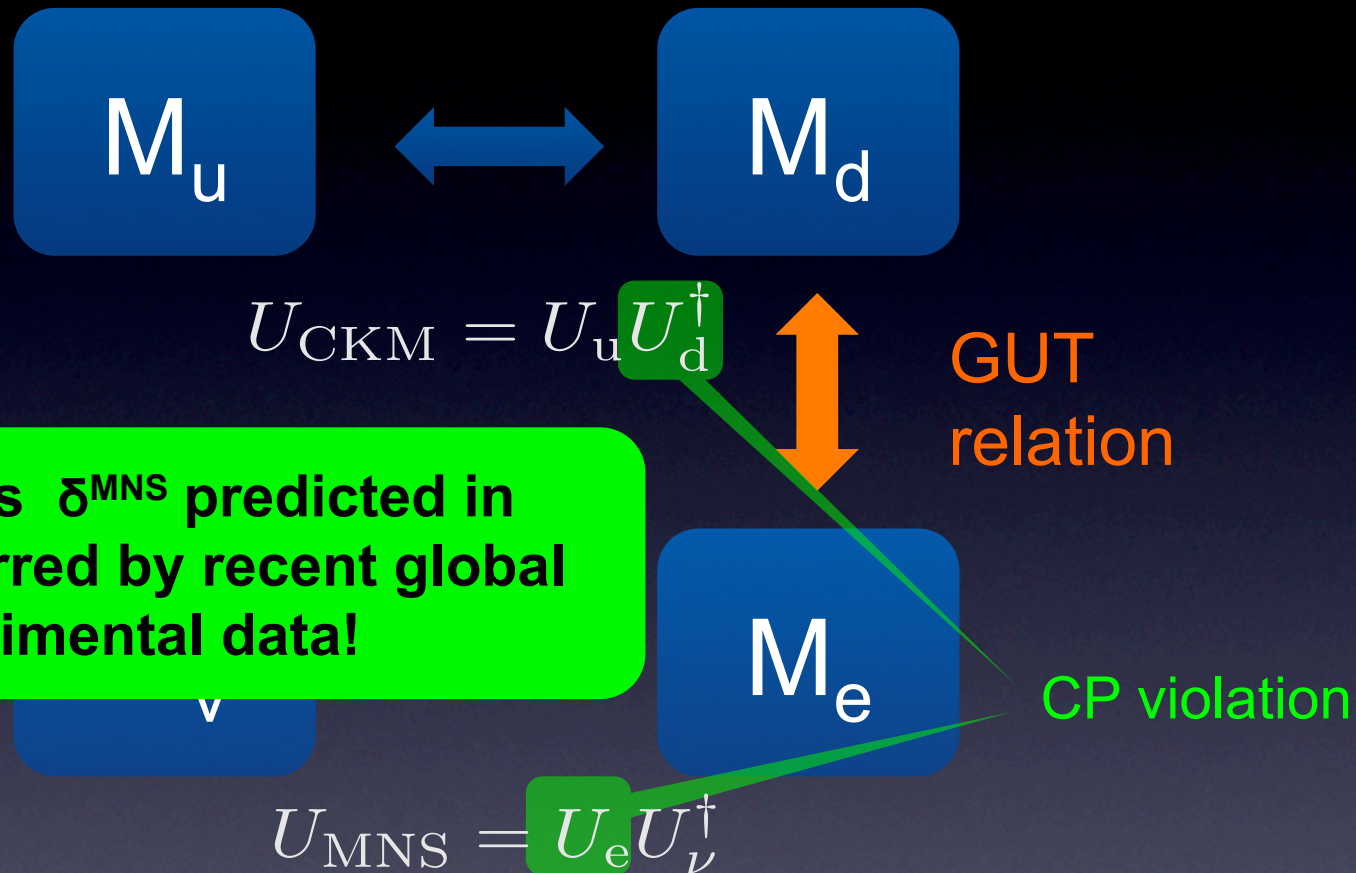
The models predict  $\theta_{23}^{\text{PMNS}} \approx 40^\circ$  (in any case  $< 45^\circ$ )

Best fit point:  
 $\chi^2/\text{d.o.f.} = 2.0$

✓ Excellent fit to the 2013 exp. data: 6 predictions, e.g.:  $\theta_{13}^{\text{PMNS}} \approx s_{23}^{\text{PMNS}} \theta_C$   
 $\delta^{\text{PMNS}} \sim 270^\circ$ , ... , (plus: constraints on SUSY spectrum)

→ Overview of predictions:

S. A., C. Gross, V. Maurer, C. Sluka (arXiv:1305.6612)

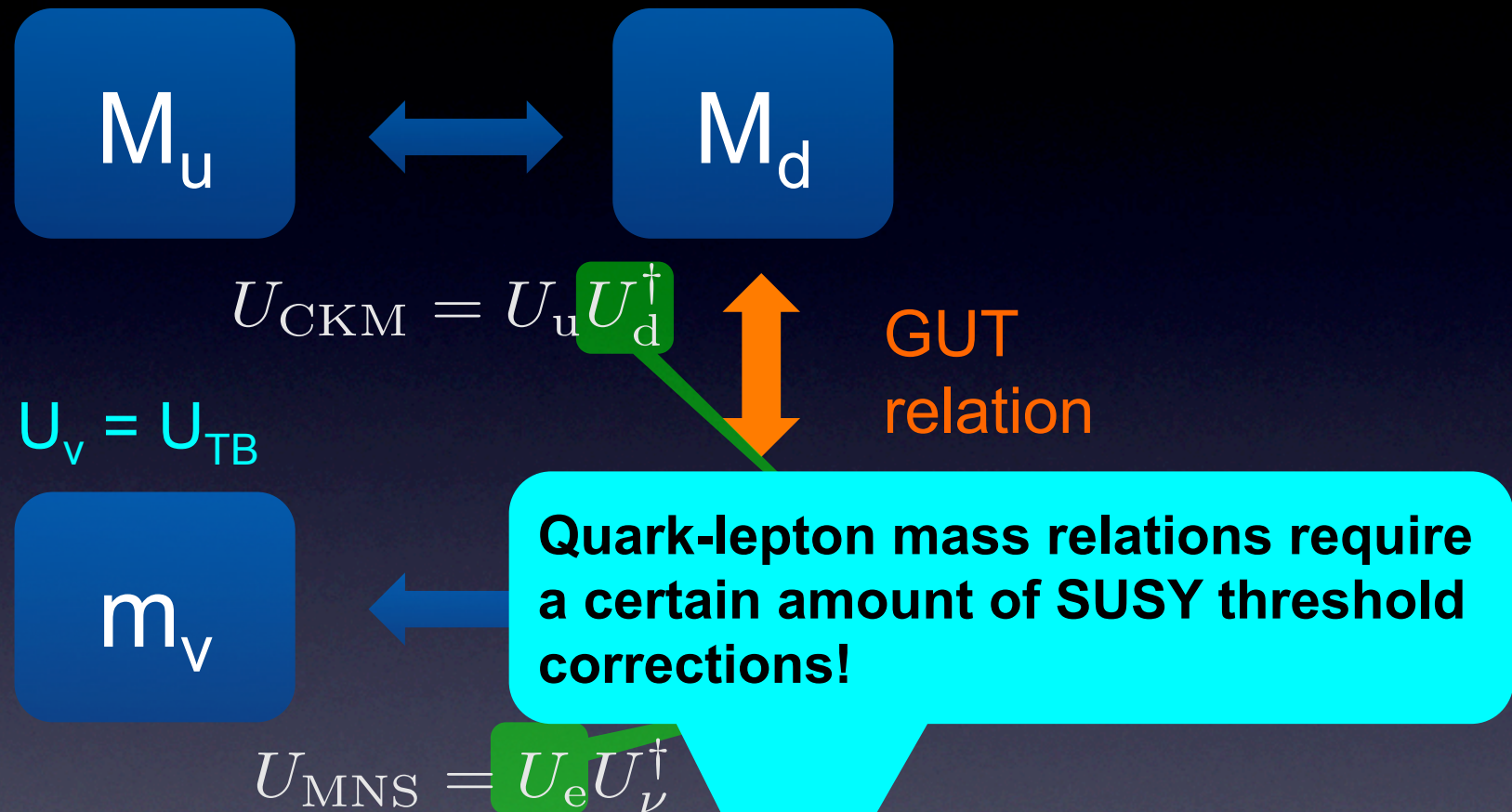


Dirac CP phases  $\delta^{MNS}$  predicted in the range preferred by recent global fits to the experimental data!

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## → Overview of predictions:

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➤ 14 parameters vs 18 measurements + additional predictions (e.g.  $\delta^{\text{PMNS}}$ )

Simplified treatment of SUSY (similar to S. A., V. Maurer; threshold corrections arXiv:1306.6879)

Observables:

Parameters:

| Parameter                            | Best fit value                           | Uncertainty  |
|--------------------------------------|--|--|
| $\tilde{\epsilon}_2$ in $10^{-4}$    | 6.83                                     | $+0.10$<br>$-0.07$   |
| $\tilde{\epsilon}_3$ in $10^{-1}$    | 2.16                                     | $\pm 0.04$   |
| $\tilde{\epsilon}_{ab}$ in $10^{-3}$ | -3.09                                    | $+0.03$<br>$-0.04$   |
| $\theta_{ab}$                        | 1.319                                    | $+0.005$<br>$-0.003$   |
| $\hat{\epsilon}_\chi$ in $10^{-2}$   | -1.27                                    | $+0.21$<br>$-0.26$   |
| $\eta_{Q_{12}}$ in $10^{-1}$         | 3.31                                     | $+1.45$<br>$-3.50$   |
| $\eta_{Q_3}$ in $10^{-1}$            | 1.93                                     | $+0.49$<br>$-0.38$   |
| $\epsilon_2$ in $10^{-2}$            | $\begin{cases} 5.27 \\ 6.07 \end{cases}$ | $\begin{cases} +0.34 \\ -0.36 \\ +0.17 \\ -0.26 \end{cases}$ |
| $\epsilon_{ab}$ in $10^{-2}$         | 4.46                                     | $+0.75$<br>$-0.19$   |
| $\epsilon_{12}$ in $10^{-1}$         | -1.65                                    | $\pm 0.05$   |
| $\epsilon_{23}$ in $10^{-2}$         | 1.78                                     | $+0.73$<br>$-0.22$   |
| $y_t$ in $10^{-1}$                   | 5.29                                     | $+0.29$<br>$-0.25$   |
| $\epsilon_{N_1}$ in $10^{-3}$        | 2.91                                     | $\pm 0.05$   |
| $\epsilon_{N_2}$ in $10^{-3}$        | 3.12                                     | $\pm 0.06$   |



| Observable  | Value at $m_t$                | Best fit result | Uncertainty            |
|---|-------------------------------|-----------------|------------------------|
| $m_u$ in MeV  | 1.22 $+0.48$<br>$-0.40$       | 1.22            | $+0.49$<br>$-0.40$     |
| $m_c$ in GeV  | 0.59 $\pm 0.08$               | 0.59            | $\pm 0.08$             |
| $m_t$ in GeV  | 162.9 $\pm 2.8$               | 162.89          | $+2.62$<br>$-2.36$     |
| $m_d$ in MeV  | 2.76 $+1.19$<br>$-1.14$       | 2.73            | $+0.30$<br>$-0.70$     |
| $m_s$ in MeV  | 52 $\pm 15$                   | 51.66           | $+5.60$<br>$-13.68$    |
| $m_b$ in GeV  | 2.75 $\pm 0.09$               | 2.75            | $\pm 0.09$             |
| $m_e$ in MeV  | 0.485 $\pm 1\%$               | 0.483           | $\pm 0.005$            |
| $m_\mu$ in MeV                                      | 102.46 $\pm 1\%$              | 102.83          | $+1.01$<br>$-0.98$     |
| $m_\tau$ in MeV                                     | 1742 $\pm 1\%$                | 1741.75         | $+17.38$<br>$-17.10$   |
| $\sin \theta_C$                                     | 0.2254 $\pm 0.0007$           | 0.2255          | $\pm 0.0007$           |
| $\sin \theta_{23}^{\text{CKM}}$                     | 0.0421 $\pm 0.0006$           | 0.0422          | $\pm 0.0006$           |
| $\sin \theta_{13}^{\text{CKM}}$                     | 0.0036 $\pm 0.0001$           | 0.0036          | $\pm 0.0001$           |
| $\delta^{\text{CKM}}$ in $^\circ$                   | 69.2 $\pm 3.1$                | 65.65           | $+1.78$<br>$-0.53$     |
| $\sin^2 \theta_{12}^{\text{PMNS}}$                  | 0.306 $\pm 0.012$             | 0.317           | $\pm 0.006$            |
| $\sin^2 \theta_{23}^{\text{PMNS}}$                  | 0.437 $+0.061$<br>$-0.031$    | 0.387           | $+0.017$<br>$-0.023$   |
| $\sin^2 \theta_{13}^{\text{PMNS}}$                  | 0.0231 $+0.0023$<br>$-0.0022$ | 0.0269          | $+0.0011$<br>$-0.0015$ |
| $\delta^{\text{PMNS}}$ in $^\circ$                  | -                             | 268.79          | $+1.32$<br>$-1.72$     |
| $\varphi_2^{\text{PMNS}}$ in $^\circ$               | -                             | 297.34          | $+8.66$<br>$-10.01$    |
| $\Delta m_{\text{sol}}^2$ in $10^{-5} \text{ eV}^2$ | 7.45 $+0.19$<br>$-0.16$       | 7.45            | $+0.18$<br>$-0.17$     |
| $\Delta m_{\text{atm}}^2$ in $10^{-3} \text{ eV}^2$ | 2.421 $+0.022$<br>$-0.023$    | 2.421           | $+0.022$<br>$-0.023$   |

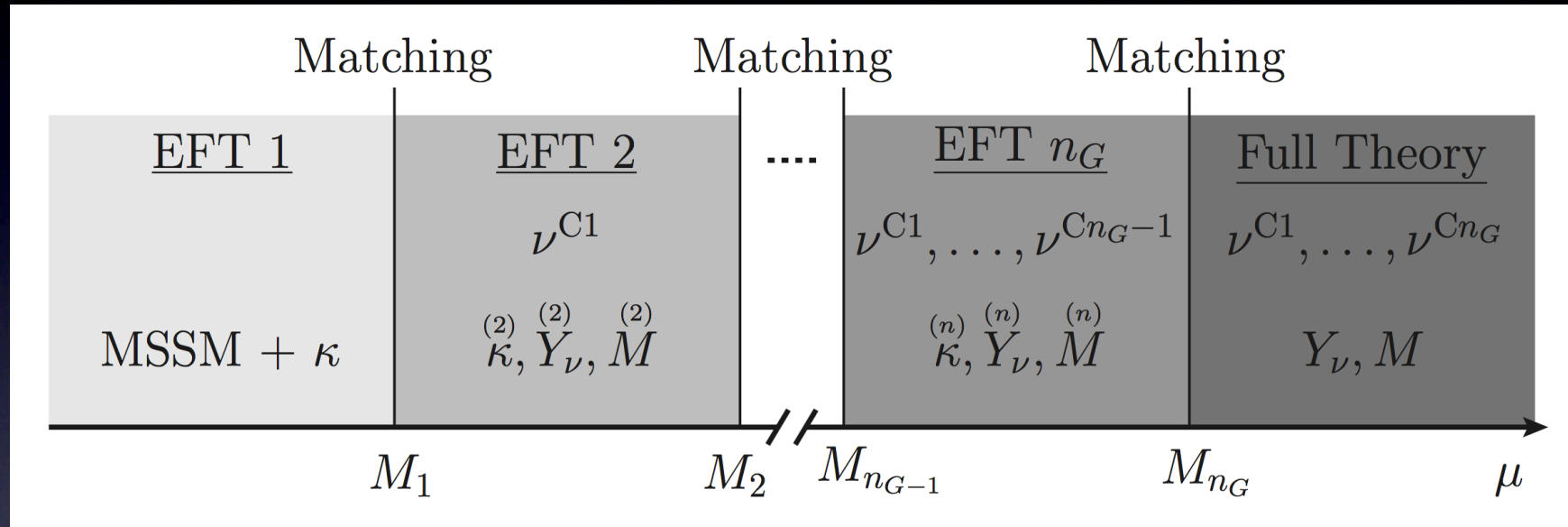
NH: S. A., C. Gross, V. Maurer, C. Sluka (arXiv:1305.6612)

# ***SusyTC: A new tool (REAP extension) for including the SUSY sector in the analysis***

S. A., C. Sluka (arXiv:1512.06727)

# REAP: A Mathematica software package for RG running in seesaw scenarios

REAP: S. A., J. Kersten, M. Lindner, M. Ratz, M. A. Schmidt (hep-ph/0501272)



- ▶ RG running of all SM + seesaw parameters (Options: SUSY (2-loop) & Non-SUSY)
- ▶ Integrates out the RH neutrinos successively at their mass thresholds (@ tree-level)
- ▶ 1-loop RN  $\nu$  thres. corr. (cf. S.A., E. Cazzato; arXiv:1509.05604) can be implemented
- ▶ Option for simplified treatment of SUSY threshold corrections at  $M_{\text{SUSY}}$

# New REAP extension: *SusyTC*

S. A., C. Sluka, arXiv:1512.06727

## Features:

- ▶ Full 2-loop running of MSSM soft SUSY breaking parameters
- ▶ Automatic calculation of  $\mu$  (@ 1-loop) and  $m_{\text{SUSY}}$
- ▶ Calculation of sparticle spectrum at  $m_{\text{SUSY}}$  (@ tree-level)
- ▶ Calculation of all SUSY threshold corrections for quarks and charged leptons
- ▶ Automatic matching to the SM
- ▶ Allows for real and complex soft breaking parameters
- ▶ Calculates  $m_A$  (CP-odd Higgs mass) for real MSSM or  $m_{H^\pm}$  for complex MSSM @ 1-loop (necessary input for e.g. FeynHiggs to calculate  $m_h$ )
- ▶ Checks (in simplified way) for charge and color-breaking minima and field directions which are 'unbounded from below'
- ▶ Option for input/output in SLHA conventions (useful for linking to external codes)

# Example application: GUT model constraints on CMSSM parameters

S. A., C. Sluka, arXiv:1512.06727

- We consider predictions for the quark lepton mass ratios as in the example flavour GUT model (after diagonalization), i.e.  $m_t/m_b = 3/2$ ,  $m_\mu/m_s \approx 6$ ,  $m_e/m_d \approx 1/2$

Approximate, in the mass basis of  $Y_d$  and  $Y_e$ : (CKM mixing now from  $Y_u$ )

$$Y_d = \begin{pmatrix} y_d & 0 & 0 \\ 0 & y_s & 0 \\ 0 & 0 & y_b \end{pmatrix}, \quad Y_e = \begin{pmatrix} -\frac{1}{2}y_d & 0 & 0 \\ 0 & 6y_s & 0 \\ 0 & 0 & -\frac{3}{2}y_b \end{pmatrix}, \quad Y_u = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{12} & y_{22} & y_{23} \\ y_{13} & y_{23} & y_{33} \end{pmatrix}$$

- We consider CMSSM boundary conditions from the soft terms at the GUT scale
- Using REAP with SusyTC 1.0, we fit the parameters to the experimental data on quark and lepton masses as well as on the mass  $m_h$  of the SM-like Higgs boson (using FeynHiggs 2.11.2)

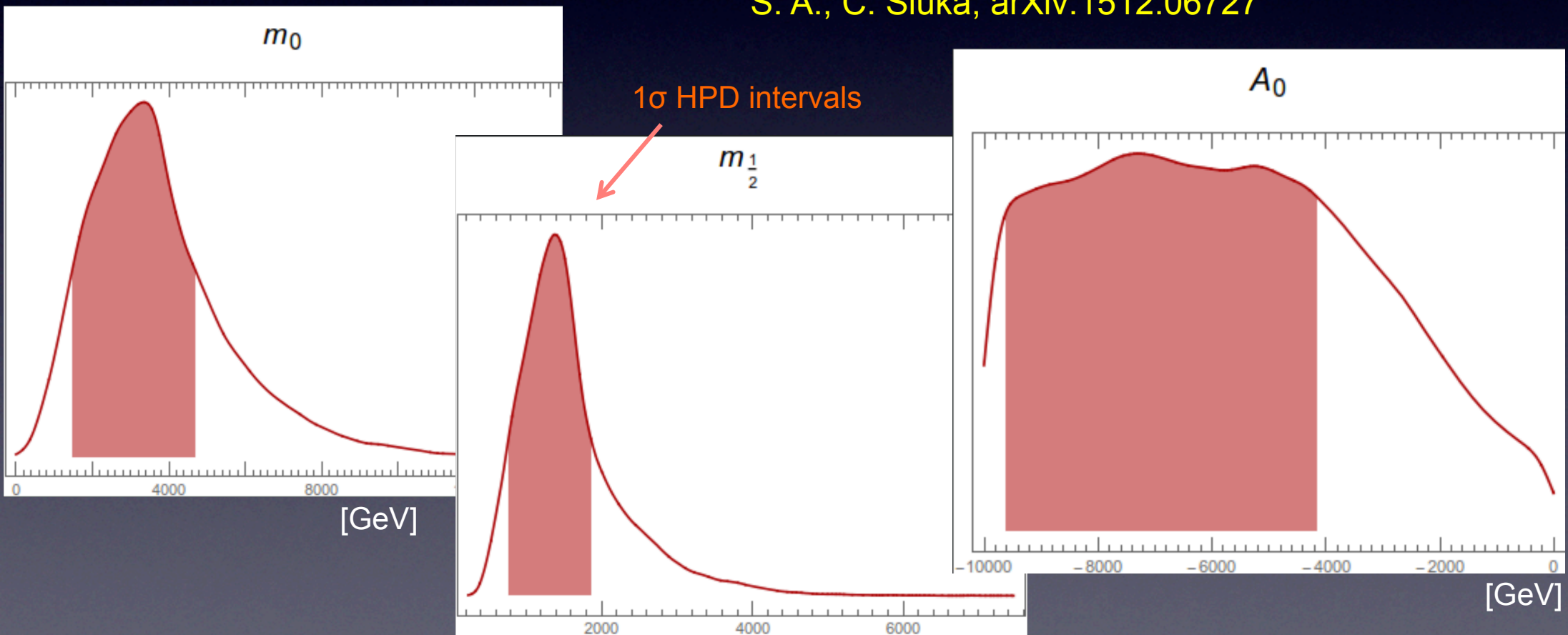
FeynHiggs: Heinemeyer, Hahn, Rzehak, Weiglein, Hollik

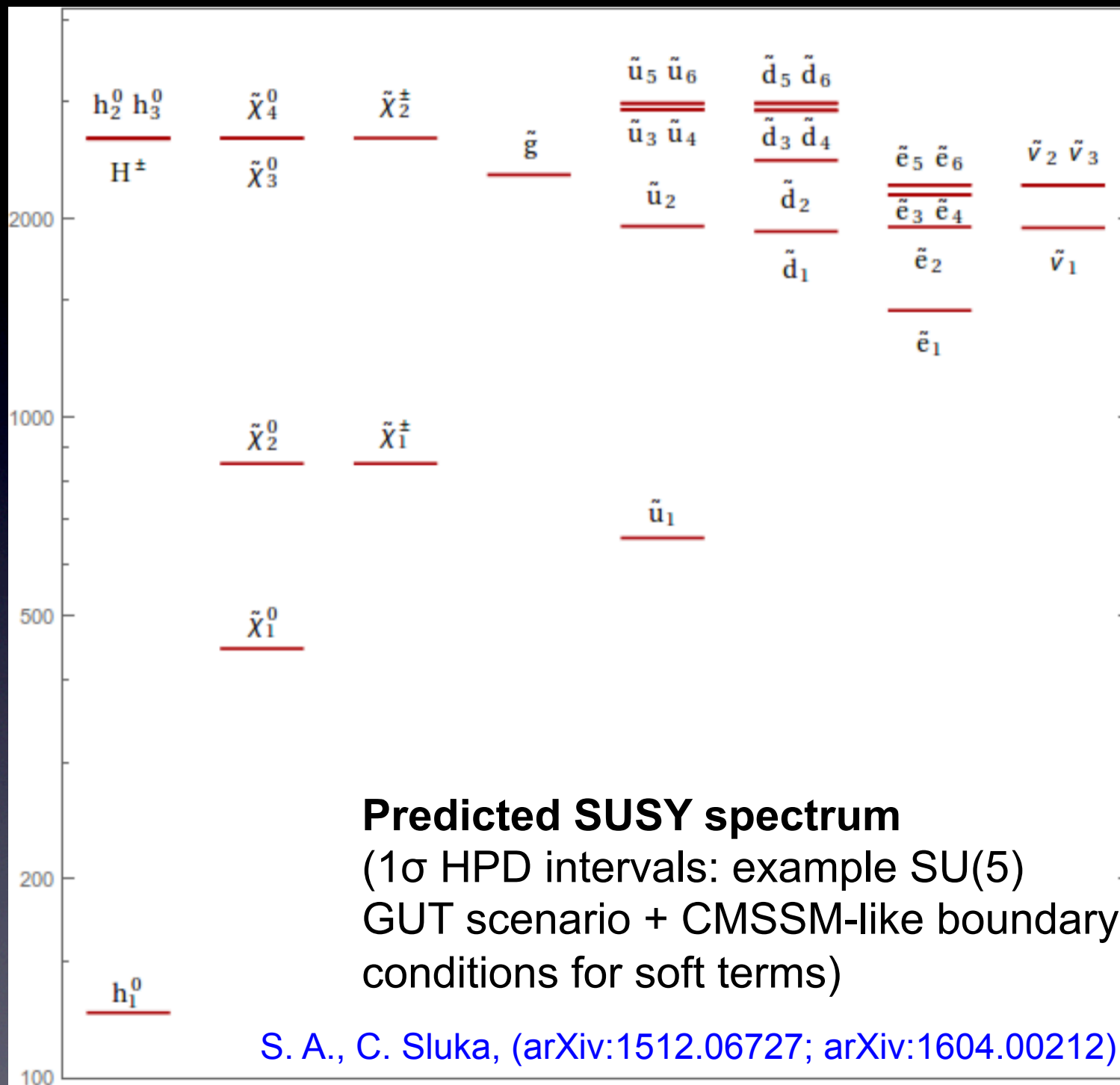
# GUT model constraints on CMSSM parameters using SusyTC: MC Monte Carlo Fit

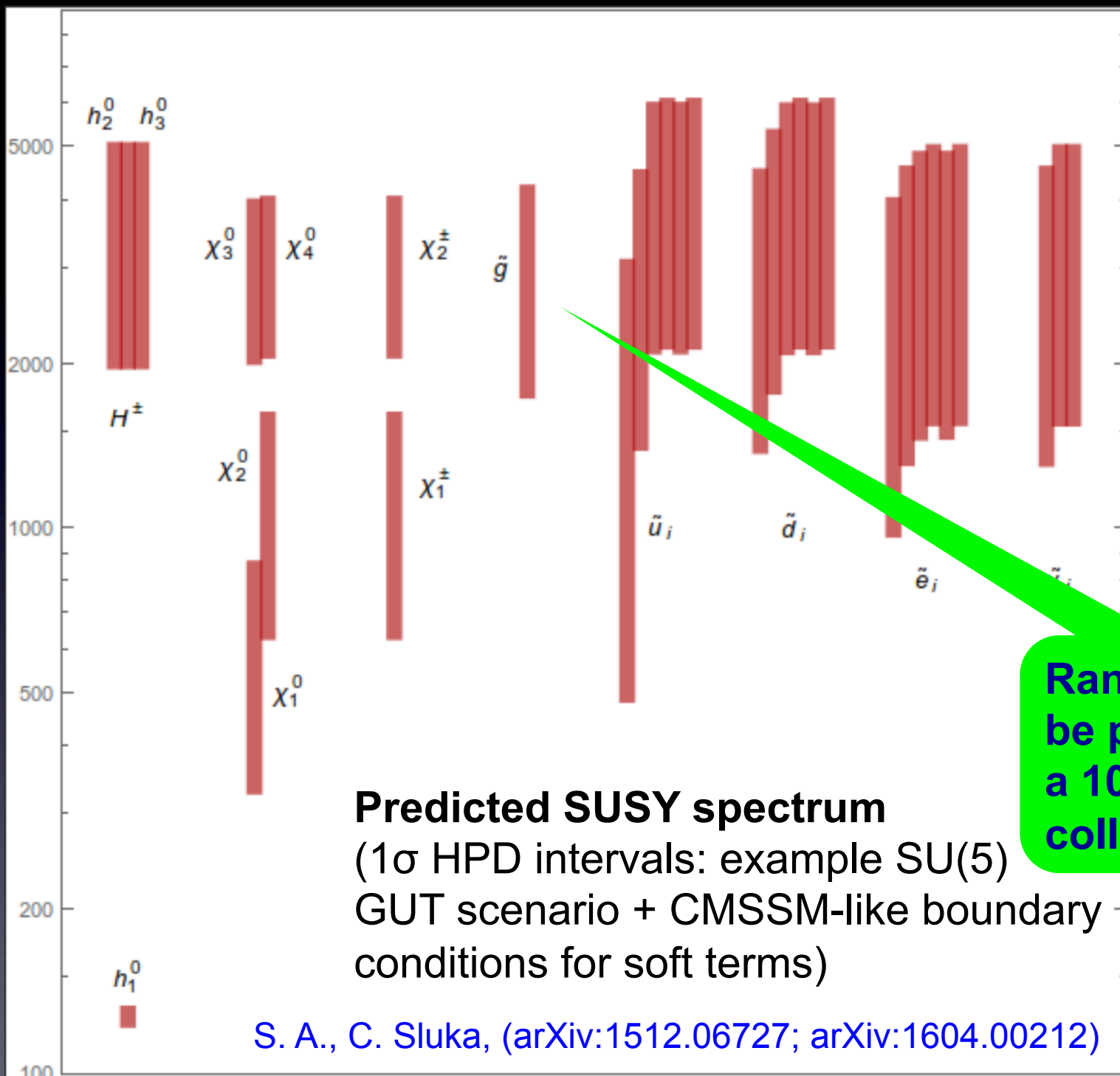
| Parameter      | $m_0$    | $A_0$     | $m_{1/2}$ |
|----------------|----------|-----------|-----------|
| Best fit value | 2119 GeV | -5822 GeV | 1008 GeV  |

Fixed  $\tan \beta = 30$   
(optimal value),  $\mu > 0$

S. A., C. Sluka, arXiv:1512.06727







Range can  
 be probed at  
 a 100 TeV pp  
 collider



# General argument: SUSY spectra from predictive GUTs

- GUT predictions for quark-lepton mass ratios require some amount of **SUSY loop threshold corrections for each generation**.

This implies that SUSY spectrum cannot be “too split”. More specifically, **the ratios of trilinear couplings, gaugino masses,  $\mu$  and sfermion masses get constrained**. Also  $\tan \beta$  cannot be too small.

In a CMSSM-like scenario  $\rightarrow$  ratios between  $m_0$ ,  $m_{1/2}$  and  $A_0$  are constrained

- With the above-described constraints, obtaining the measured value of the mass  **$m_h$  of the SM-like Higgs fixes the overall SUSY scale!**

The combination of the two effects can result in a predicted sparticle spectrum from GUT models!

S. A., C. Sluka, (arXiv:1512.06727; 1604.00212)



*From GUT-flavour perspective: No surprise SUSY was not found yet - “climb up” some more in energy to see SUSY ...*



**Thanks for your  
attention!**