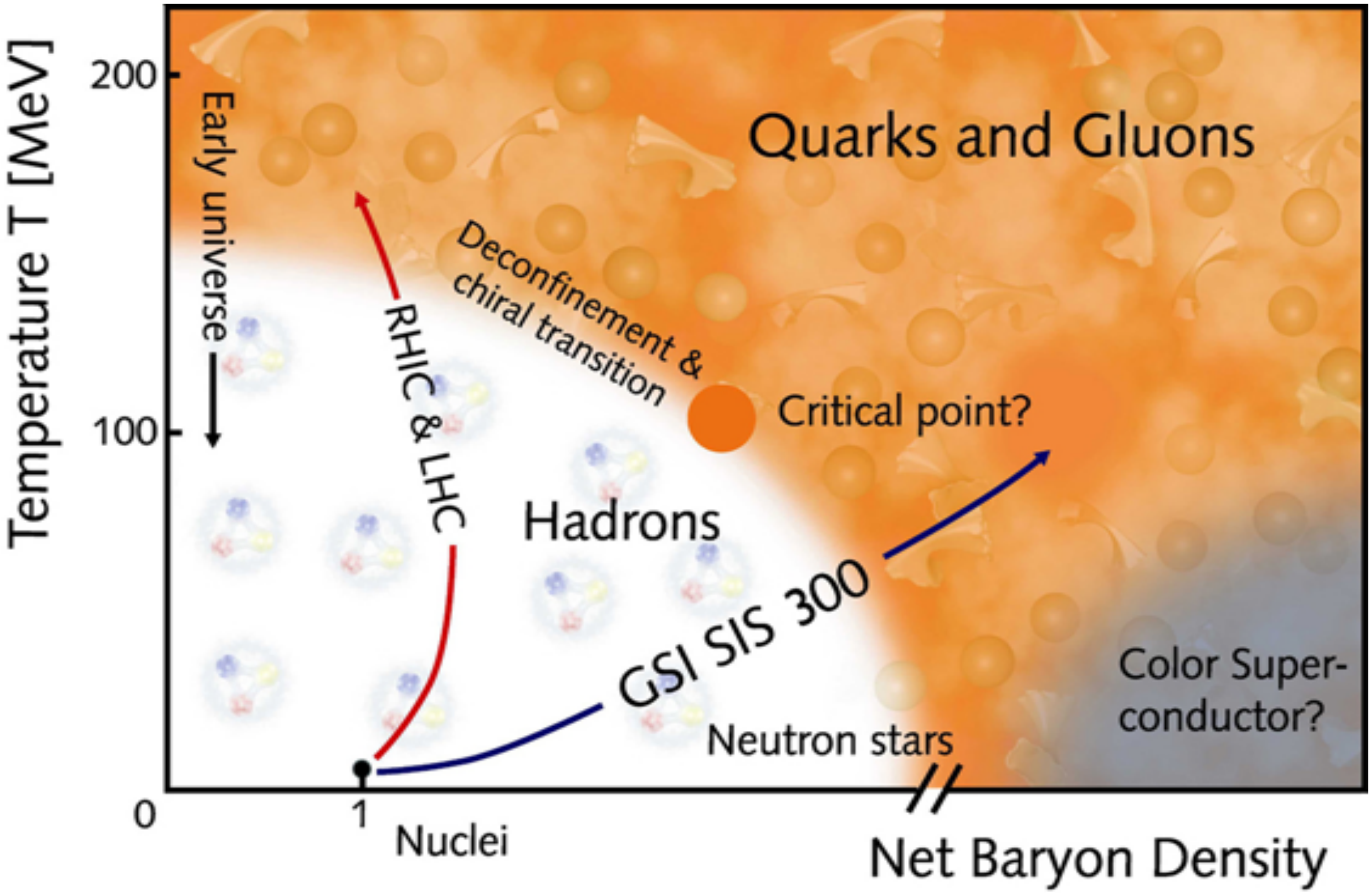
Fluctuations and Correlations:
New lessons from the RHIC beam energy scan

- Introduction
- Cumulants and Correlations
- Correlations in the preliminary STAR data

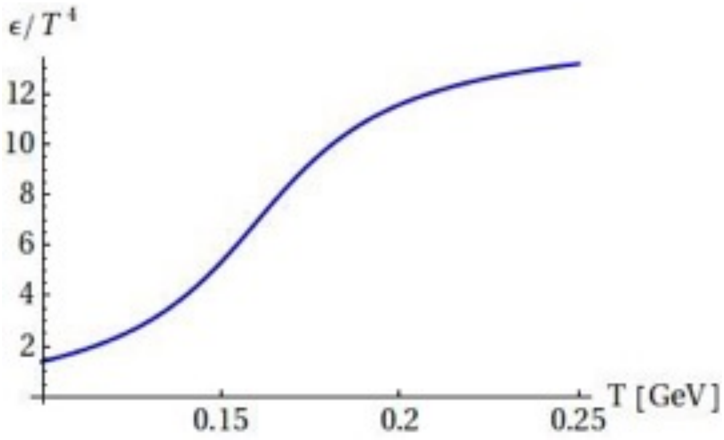
With Adam Bzdak and Nils Strodthoff: arXiv:1607.07375

Fluctuations and face transitions

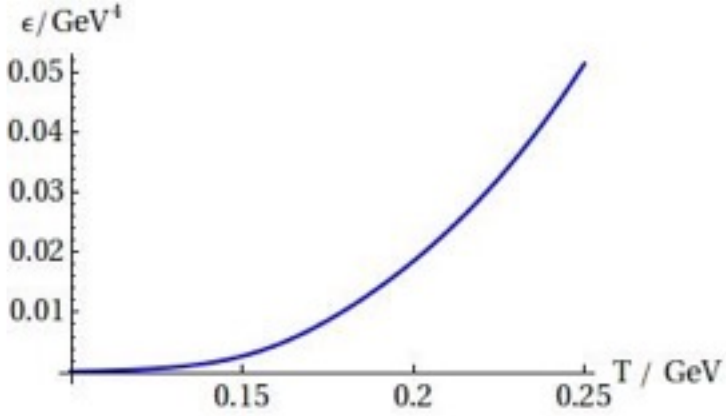
The QCD Phase diagram



Why cumulants are useful



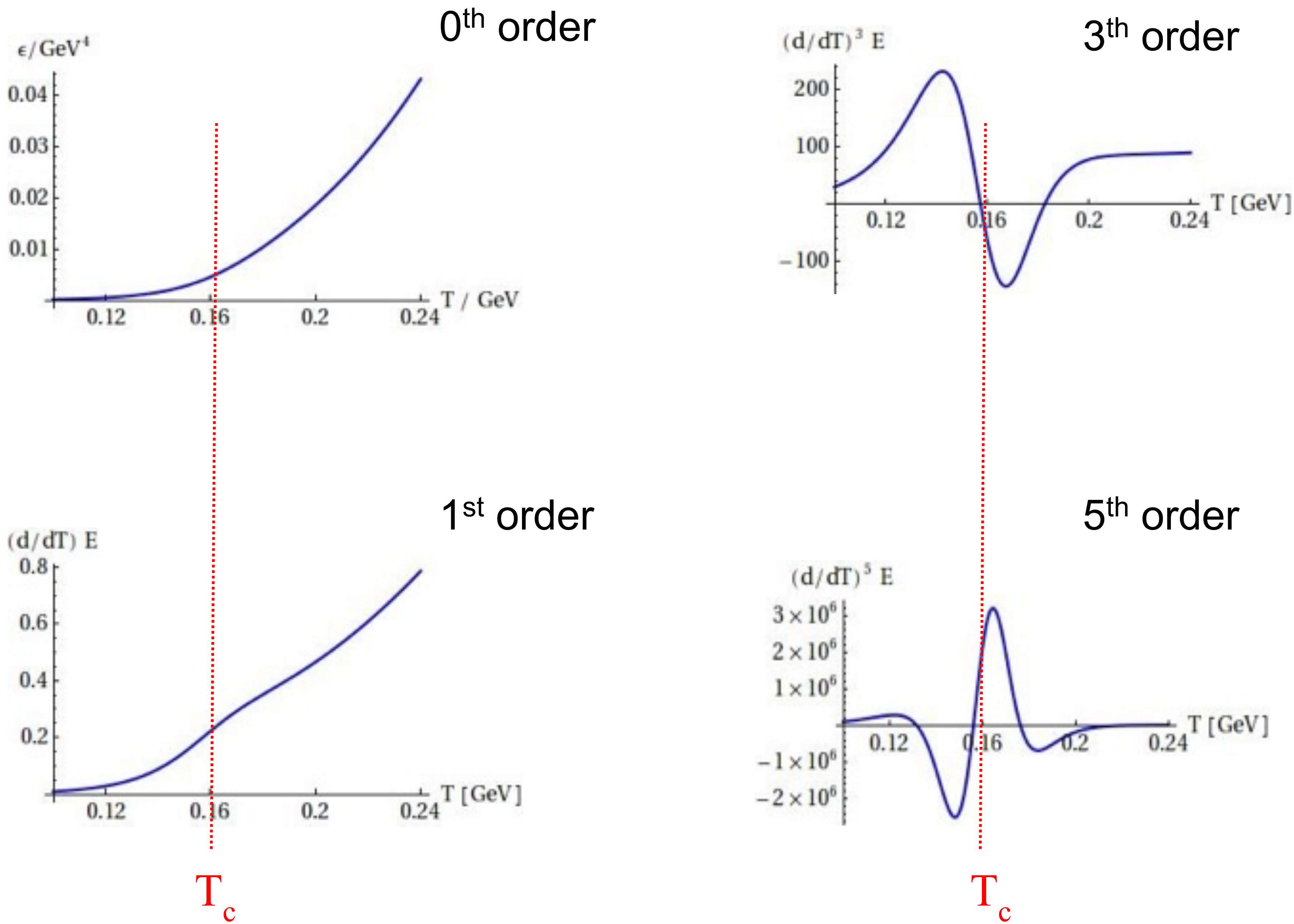
What we always see....



What it really means....

"T_c" ~ 160 MeV

Derivatives



How to measure derivatives

At μ = 0:

$$Z = tr\, e^{-\hat{E}/T + \mu/T \hat{N}_B}$$

$$\langle E \rangle = \frac{1}{Z}\, tr\, \hat{E}\, e^{-\hat{E}/T + \mu/T \hat{N}_B} = -\frac{\partial}{\partial 1/T} \ln(Z)$$

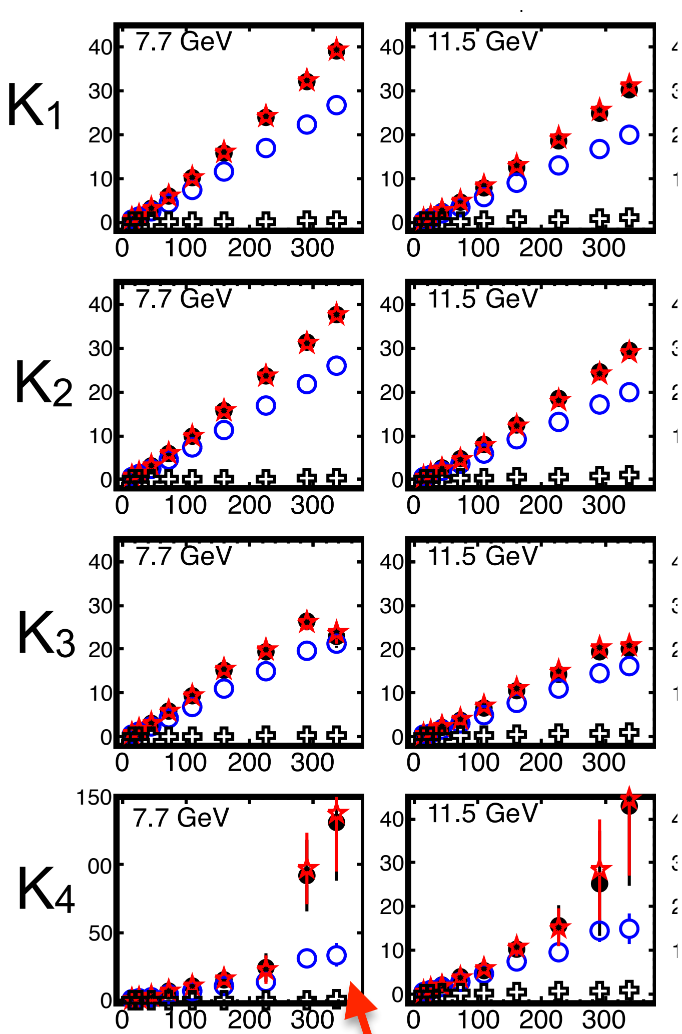
$$\langle (\delta E)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 = \left(-\frac{\partial}{\partial 1/T} \right)^2 \ln(Z) = \left(-\frac{\partial}{\partial 1/T} \right) \langle E \rangle$$

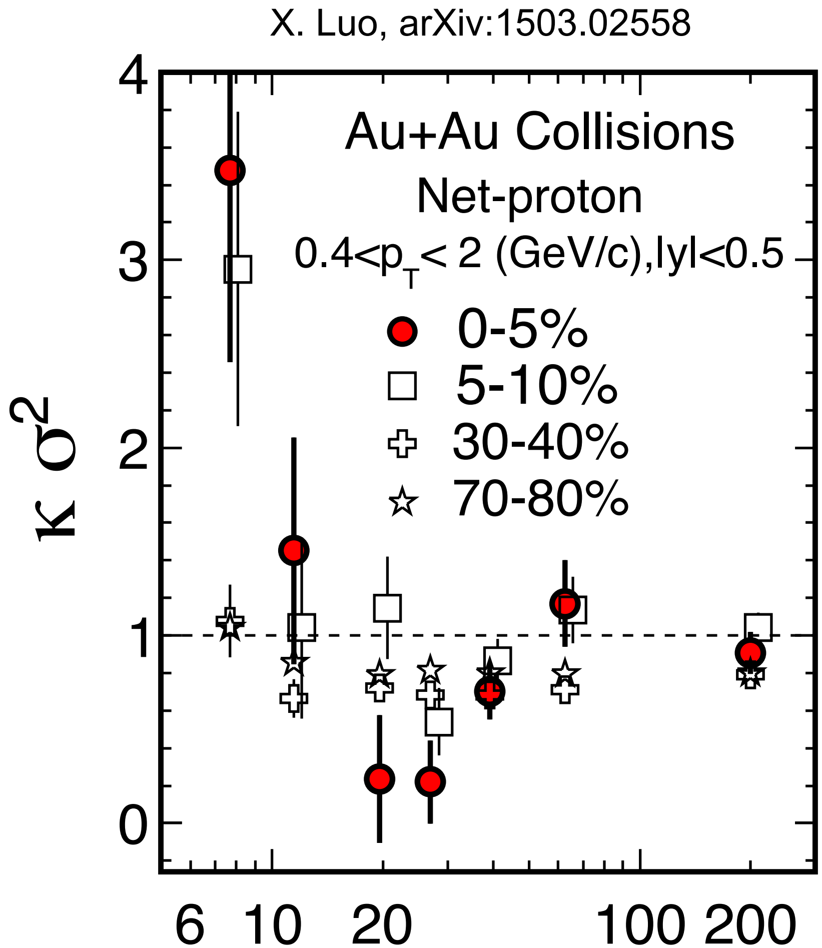
$$\langle (\delta E)^n \rangle = \left(-\frac{\partial}{\partial 1/T} \right)^{n-1} \langle E \rangle$$

Cumulants of Energy measure the temperature derivatives of the EOS

Cumulants of Baryon number measure the chem. pot. derivatives of the EOS

Latest STAR result on net-proton cumulants





Unfolding makes huge difference in new STAR data!

Things to consider

- Fluctuations of conserved charges ?!
- Volume Fluctuations
- Net-protons different from net-baryons
  - Isospin fluctuations
- "Stopping" fluctuations
- Higher cumulants probe the tails. Statistics!
- The detector "fluctuates" !
  - Efficiency effects
- …….

From Cumulants to Correlations

Cumulants $K_n = \frac{\partial^n}{\partial \hat{\mu}^n} P/T^4$

$$K_2 = \langle N - \langle N \rangle \rangle^2 = \left\langle (\delta N)^2 \right\rangle$$

$$\rho_2(p_1, p_2) = \rho_1(p_1)\rho_1(p_2) + C_2(p_1, p_2),$$

Correlation Function

$$K_3 = \left\langle (\delta N)^3 \right\rangle$$

$$\rho_3(p_1, p_2, p_3) = \rho_1(p_1)\rho_1(p_2)\rho_1(p_3) + \rho_1(p_1)\underline{C_2(p_2, p_3)} + \rho_1(p_2)\underline{C_2(p_1, p_3)} + \rho_1(p_3)\underline{C_2(p_1, p_2)} + \underline{C_3(p_1, p_2, p_3)}$$

From Cumulants to Correlations
(no anti-protons)

Factorial moments:

$$F_n = \langle N(N-1)\ldots(N-n+1)\rangle = \int dp_1 \ldots dp_2 \rho_n(p_1,\ldots,p_n)$$

$$F_1 = \int dp \rho_1(p) = \langle N\rangle$$

$$F_2 = \int dp_1 dp_2 \rho_2(p_1,p_2) = \langle N\rangle^2 + C_2$$

$$F_3 = \int dp_1 dp_2 dp_3 \rho_3(p_1,p_2,p_3) = \langle N\rangle^3 + 3\langle N\rangle C_2 + C_3$$

and so on…

Integrated correlations function

$$C_n = \int dp_1 \ldots dp_n C_n(p_1,\ldots,p_n)$$

From cumulants to correlations

$$F_1 = \int dp \rho_1(p) = \langle N \rangle$$

$$F_2 = \int dp_1 dp_2 \rho_2(p_1, p_2) = \langle N \rangle^2 + C_2$$

$$F_3 = \int dp_1 dp_2 dp_3 \rho_3(p_1, p_2, p_3) = \langle N \rangle^3 + 3 \langle N \rangle C_2 + C_3$$

$$K_1 \equiv \langle N \rangle = F_1,$$

$$K_2 \equiv \langle (\delta N)^2 \rangle = F_1 - F_1^2 + F_2,$$

$$K_3 \equiv \langle (\delta N)^3 \rangle = F_1 + 2F_1^3 + 3F_2 + F_3 - 3F_1(F_1 + F_2),$$

Can express correlations C_n in terms of cumulants K_n

$$C_2 = -K_1 + K_2,$$

$$C_3 = 2K_1 - 3K_2 + K_3,$$

$$C_4 = -6K_1 + 11K_2 - 6K_3 + K_4, .$$

Correlations near the critical point

M. Stephanov, 0809.3450, PRL 102

Scaling of Cumulants K_n with correlation length ξ

$$K_2 \sim \xi^2, \;\; K_3 \sim \xi^{4.5}, \;\; K_4 \sim \xi^7$$

Cumulants from Correlations

$$K_2 = \langle N \rangle + C_2$$
$$K_3 = \langle N \rangle + 3C_2 + C_3$$
$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

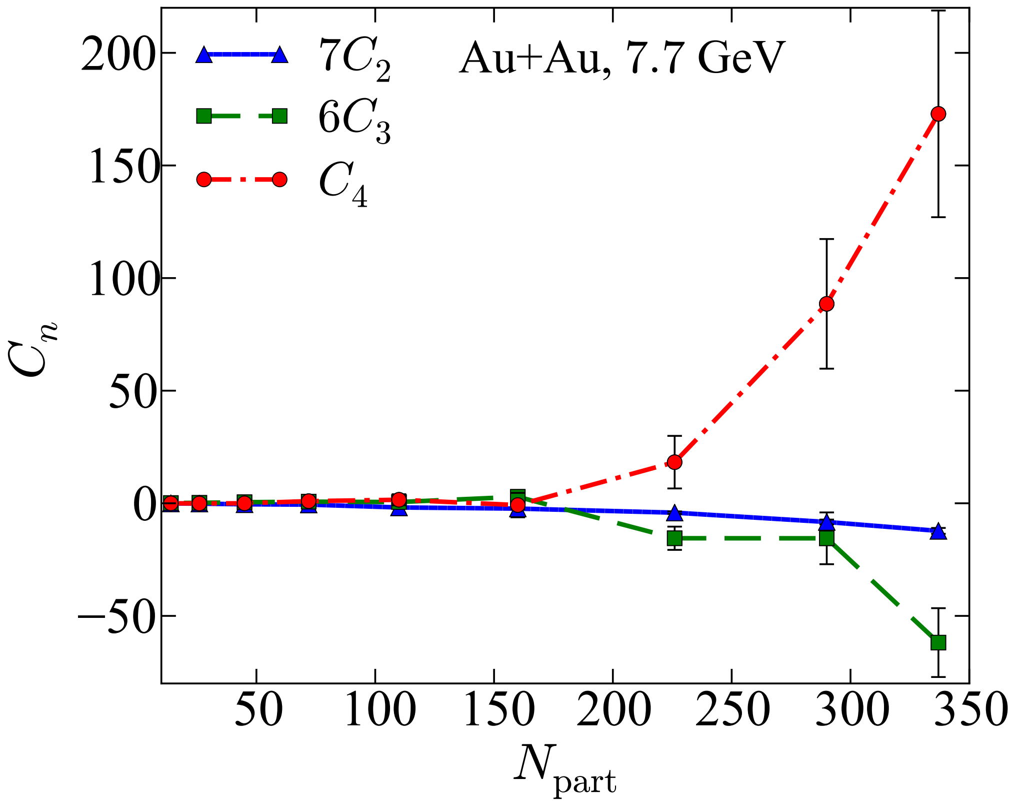
Consequently:

$$C_2 \sim \xi^2, \;\; C_3 \sim \xi^{4.5}, \;\; C_4 \sim \xi^7$$

Correlations C_n pick up the most divergent pieces of cumulants K_n!

Preliminary Star Data

(X. Luo, PoS Cpod 2014 (019))



Significant four particle correlations!

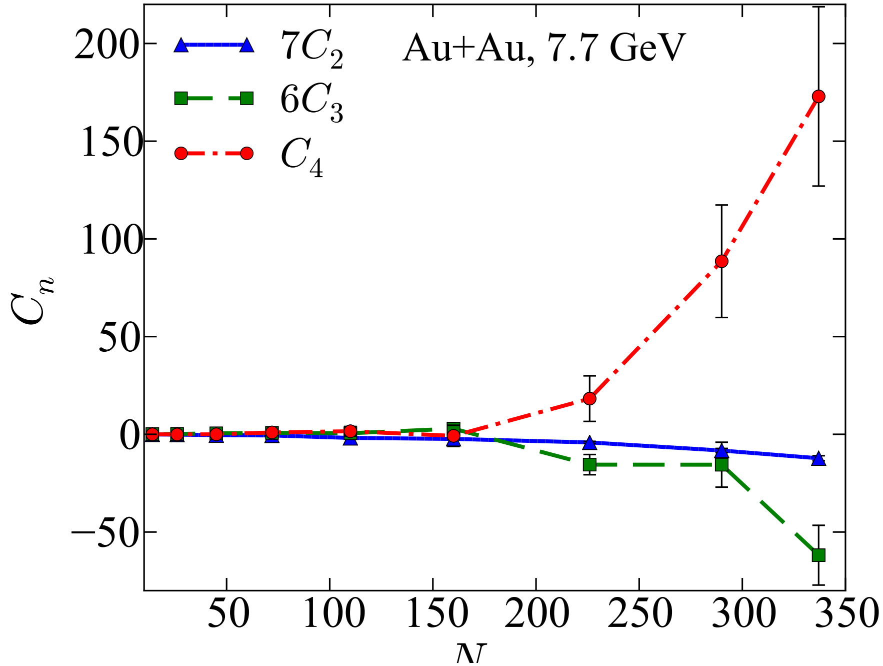
Four particle correlation dominate K_4 for central collisions at 7.7 GeV

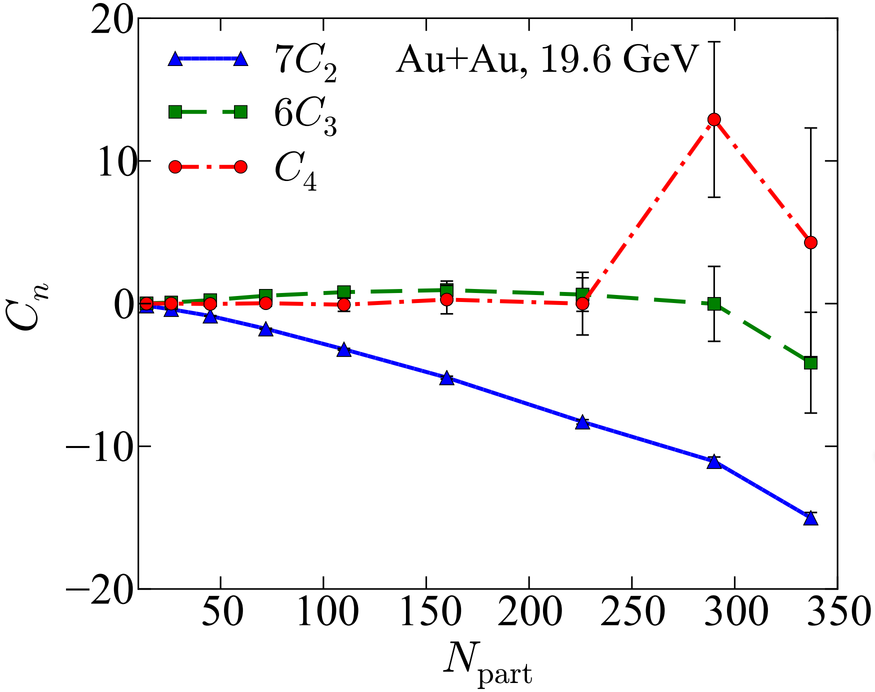
$$K_2 = \langle N \rangle + C_2$$

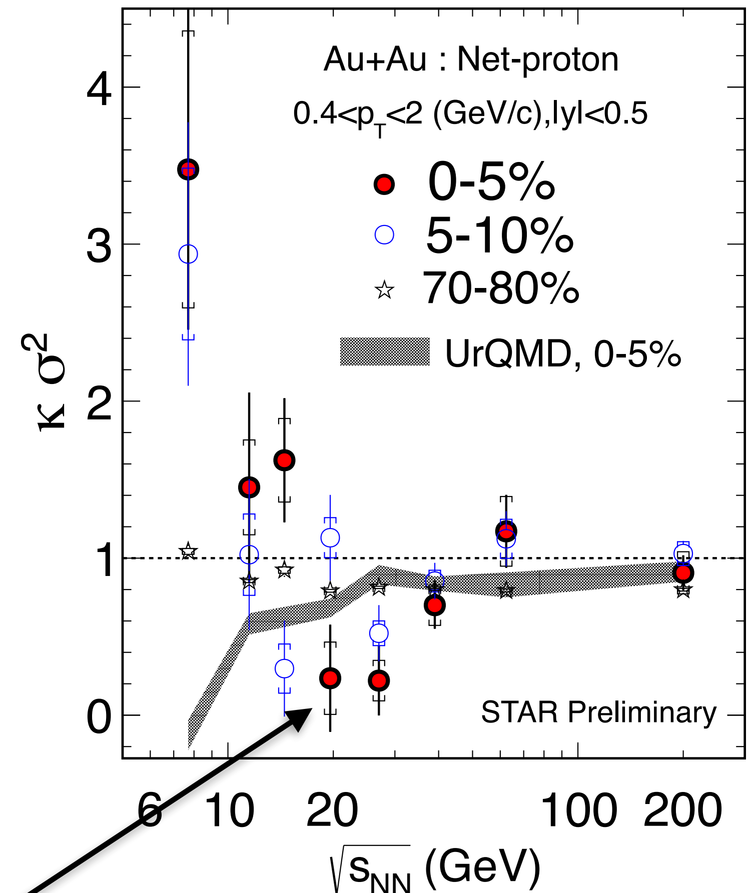
$$K_3 = \langle N \rangle + 3C_2 + C_3$$

$$K_4 = \langle N \rangle + 7C_2 + 6C_3 + C_4$$

Correlations







Dip at 19.6 GeV from NEGATIVE C_2 !

Reduced correlation function

Reduced correlation function

$$c_k = \frac{\int \rho_1(y_1)\cdots\rho_1(y_k)\, c_k(y_1,...,y_k)\, dy_1\cdots dy_k}{\int \rho_1(y_1)\cdots\rho_1(y_k)\, dy_1\cdots dy_k} \qquad C_k = \langle N\rangle^k c_k$$

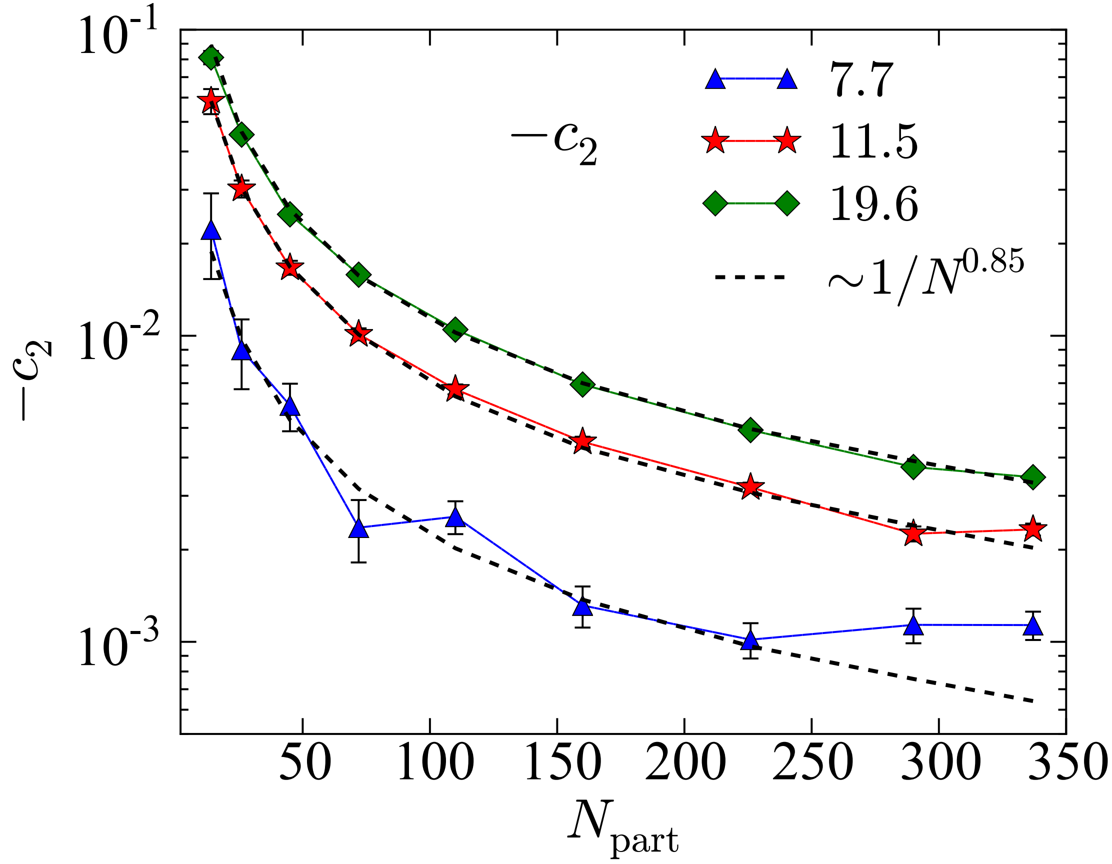
Independent sources such as resonances, cluster, p+p:

$$c_k \sim \frac{\langle N_s\rangle}{\langle N\rangle^k} \sim \frac{1}{\langle N\rangle^{k-1}}$$

For example two particle correlations:

$$c_2 \sim \frac{\text{Number of sources}}{\text{Number of all pairs}} = \frac{\text{Number of correlated pairs}}{\text{Number of all pairs}} = \frac{1}{\langle N\rangle}$$

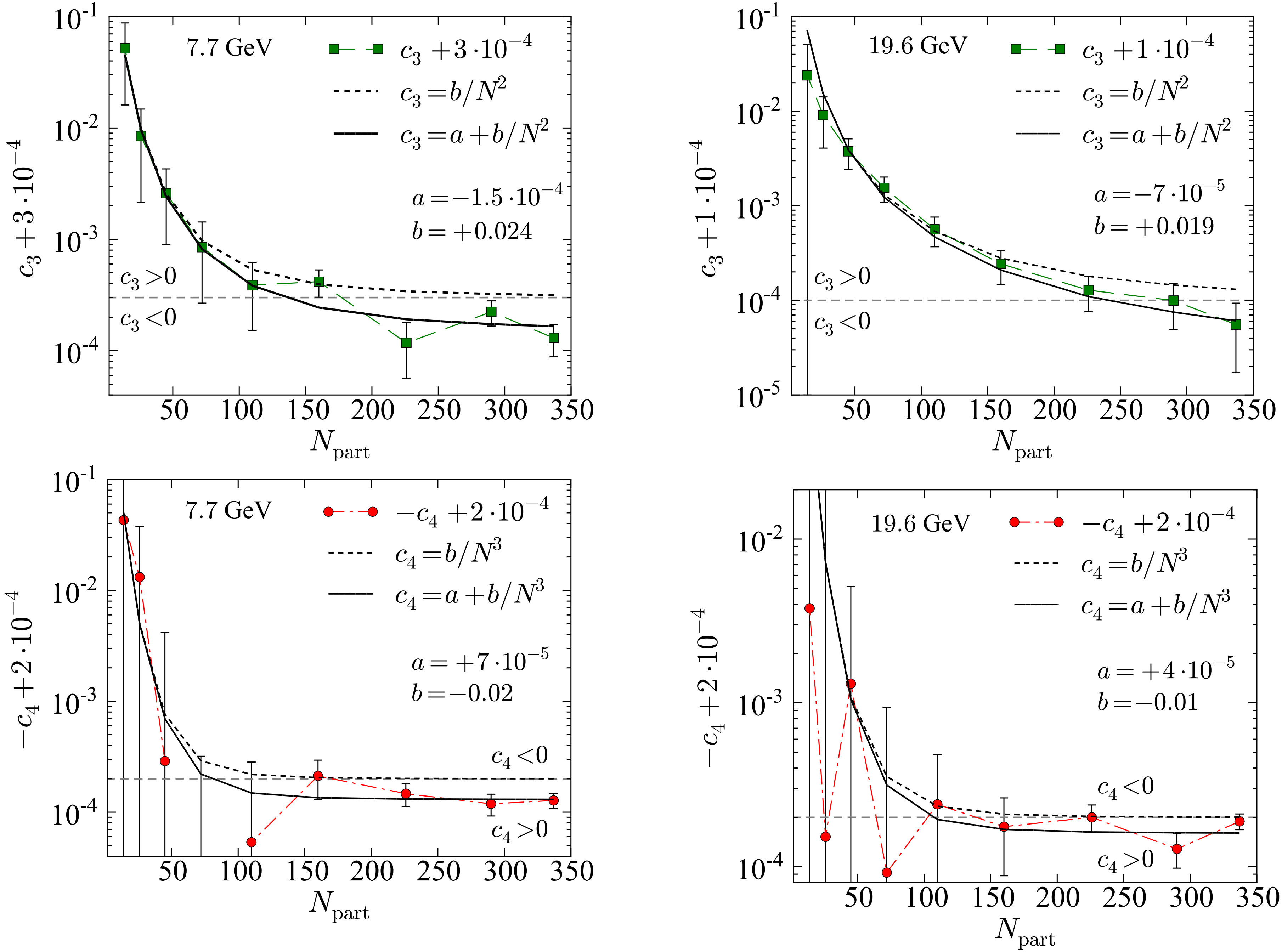
Centrality dependence



Centrality dependence

C_3

C_4



7.7 GeV

19.6 GeV

Rapidity dependence

$$C_k(\Delta Y) = \int_{\Delta Y} dy_1 \dots dy_k \rho_1(y_1) \dots \rho_1(y_k) c_k(y_1, \dots, y_k)$$

Assume: $\rho_1(y) \simeq const.$

short range correlations:

$$c_k(y_1, \dots, y_k) \sim \delta(y_1 - y_2) \dots \delta(y_{n-1} - y_k)$$

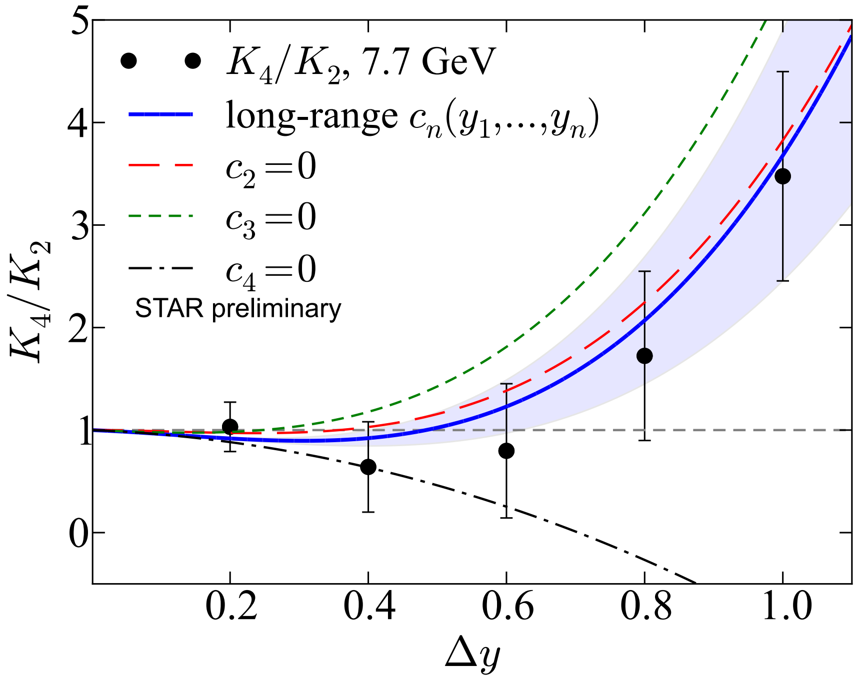
$$C_k(\Delta Y) \sim \Delta Y \rightarrow K_k \sim \Delta Y$$

Long range correlations:

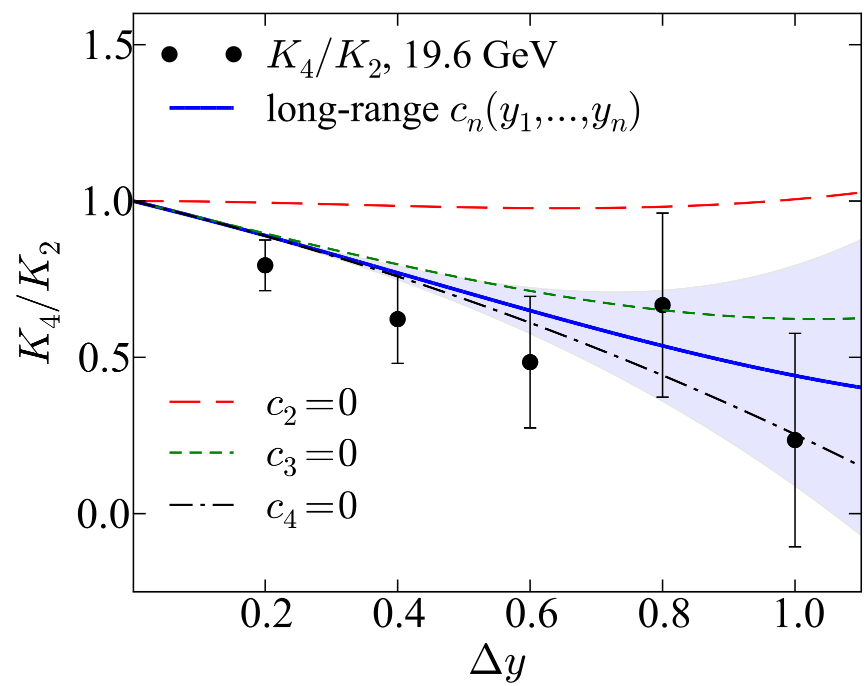
$$c_k(y_1, \dots, y_k) = const.$$

$$C_k(\Delta Y) \sim (\Delta Y)^k$$

Preliminary Star data are consistent with long range correlations

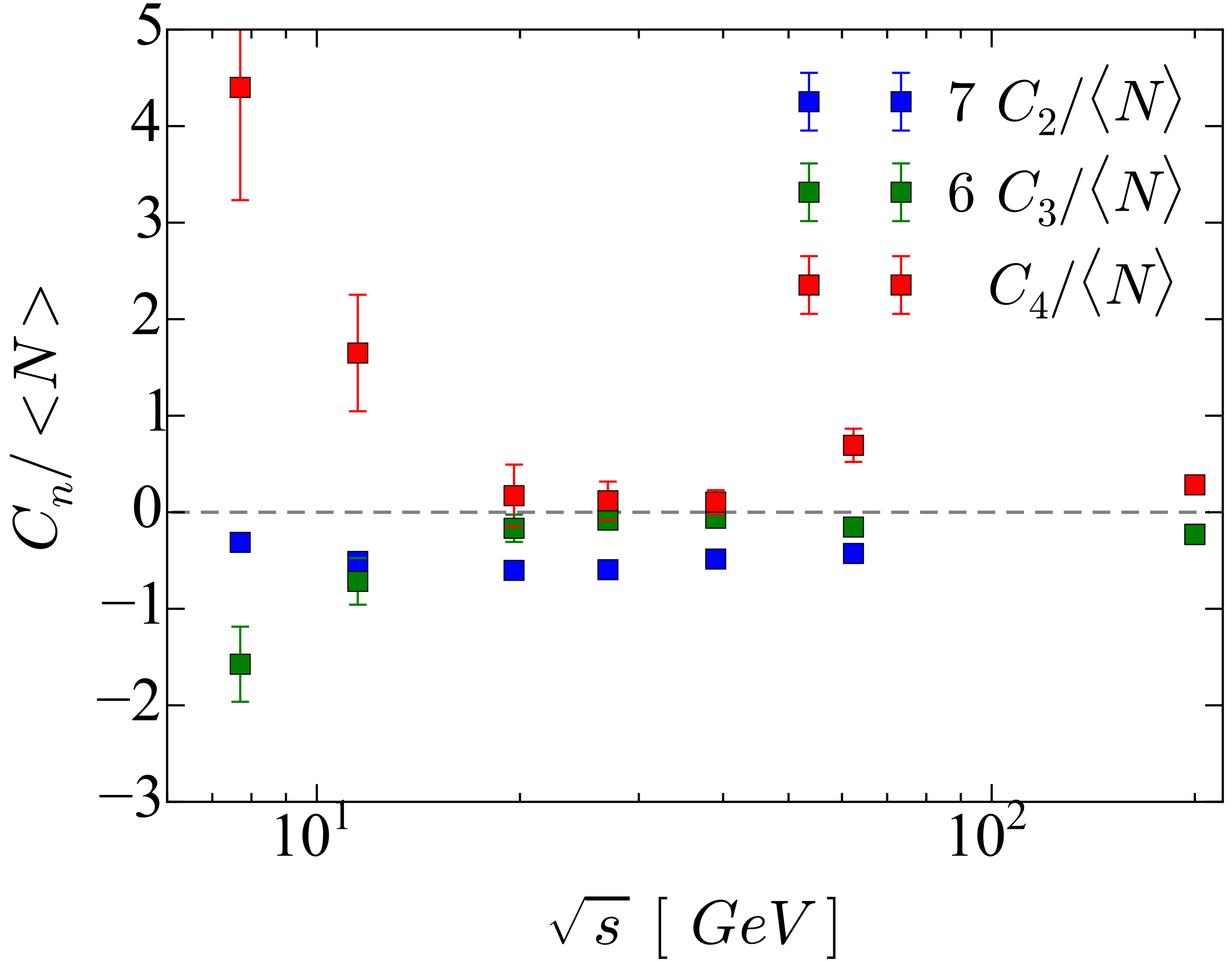


7.7 GeV
central



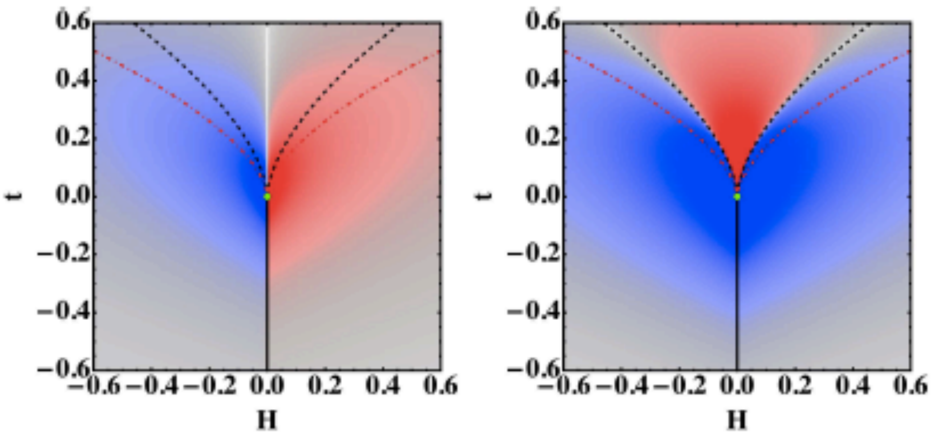
19.6 GeV
central

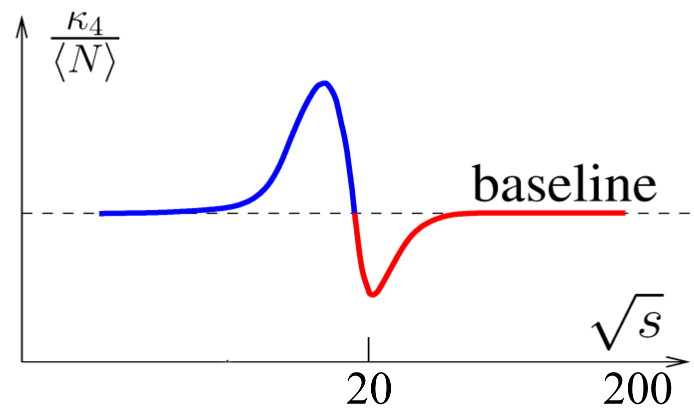
Energy dependence

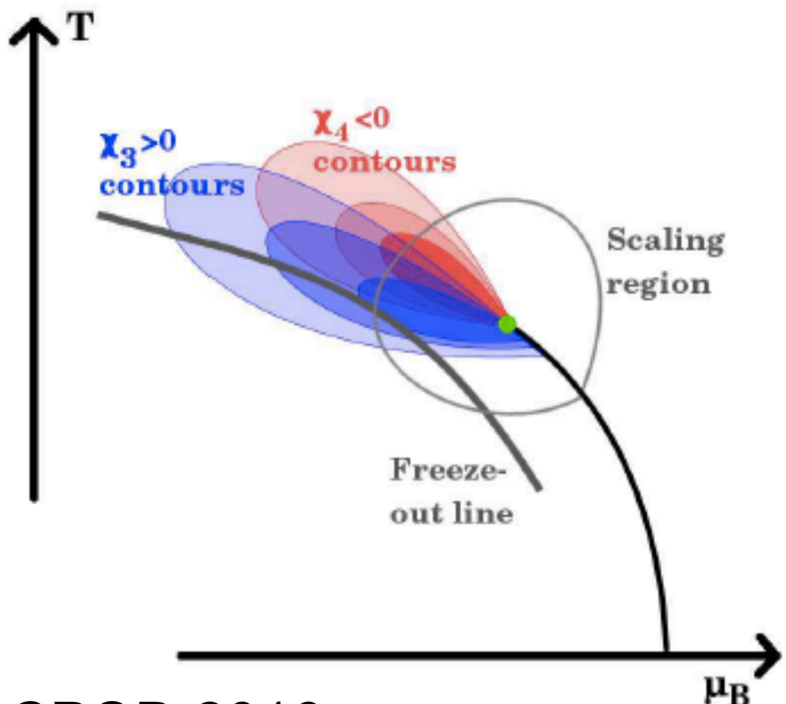


Note: anti-protons are non- negligible above 19.6 GeV

Expectation from Calculations





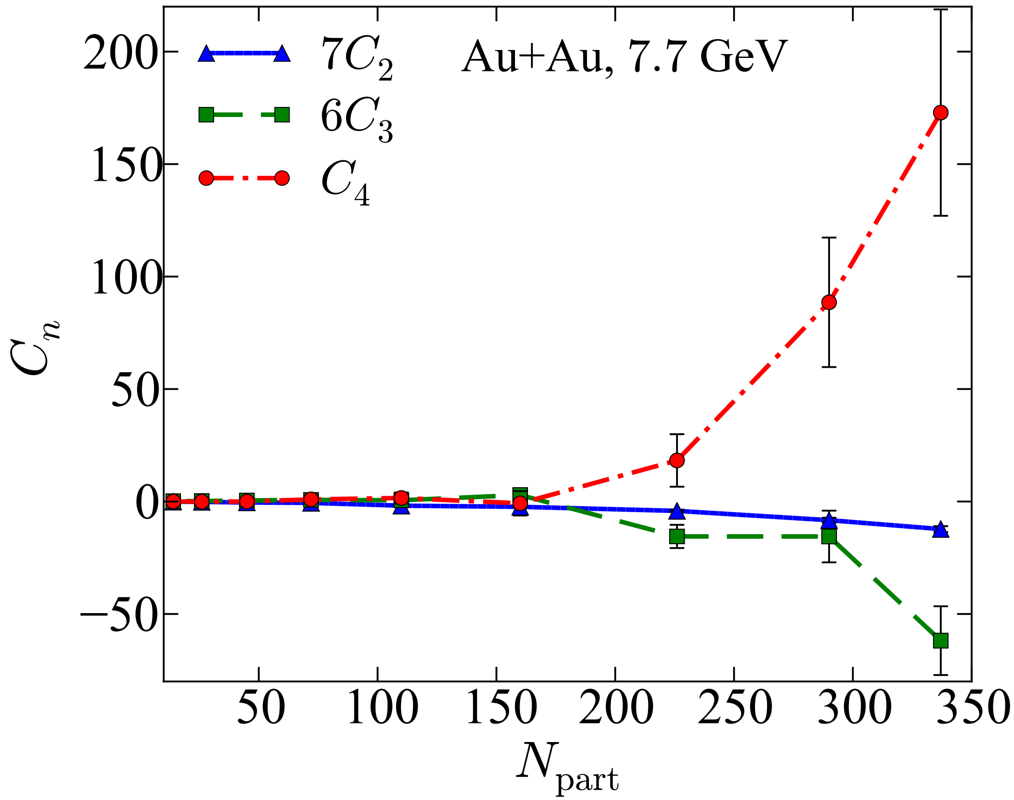


Characteristic "Oscillating pattern" is expected for the QCD critical point but *the exact shape depends on the location of freeze-out with respect to the location of CP*

- M. Stephanov, ***PRL107***, 052301(2011)
- V. Skokov, Quark Matter 2012
- J.W. Chen, J. Deng, H. Kohyyama, arXiv: 1603.05198, Phys. Rev. **D93** (2016) 034037

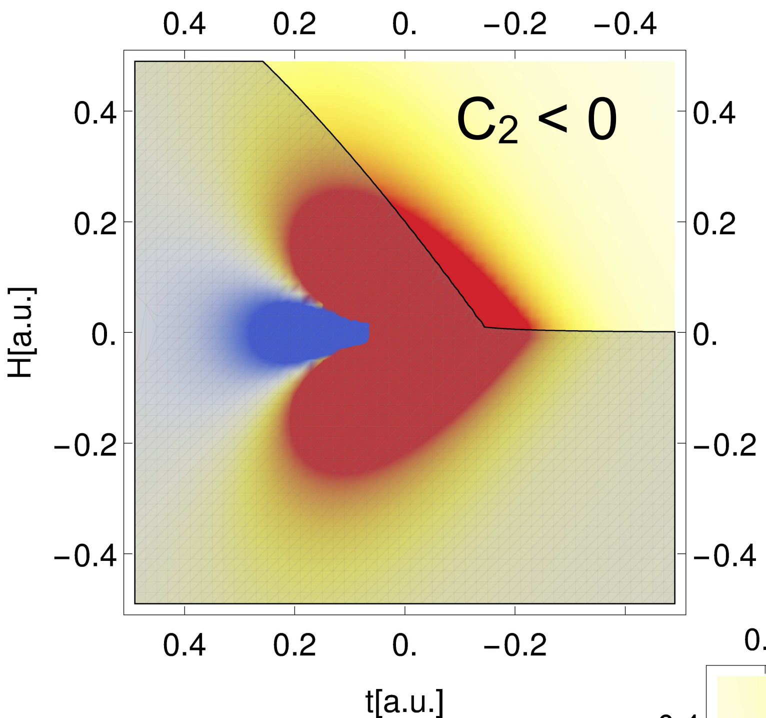
N. Xu, CPOD 2016

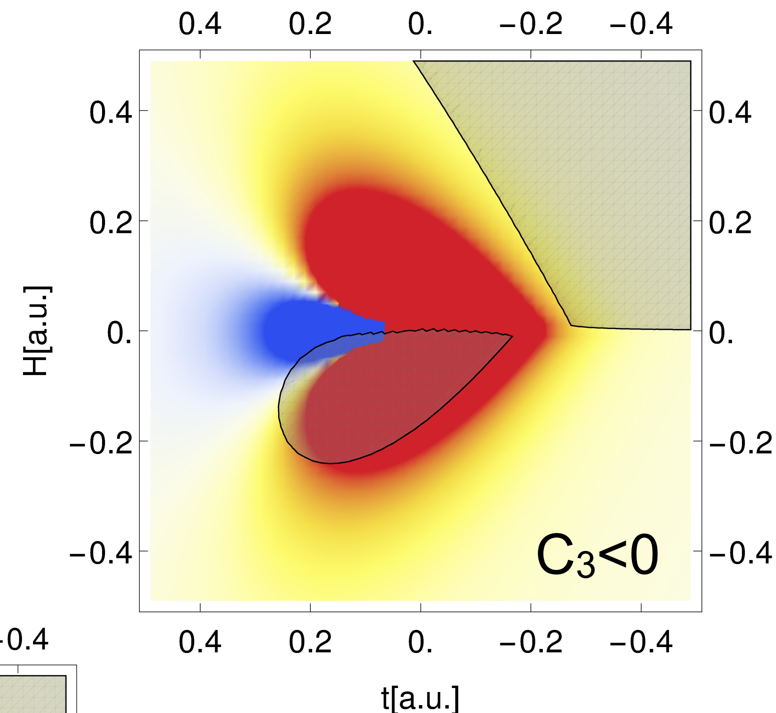
Sign of C_n

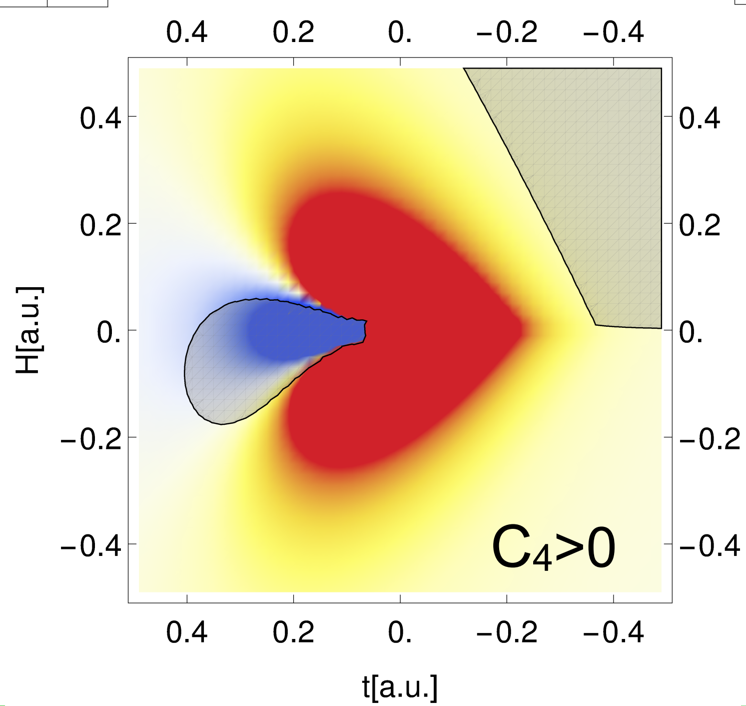


$C_2 < 0$
$C_3 < 0$
$C_4 > 0$

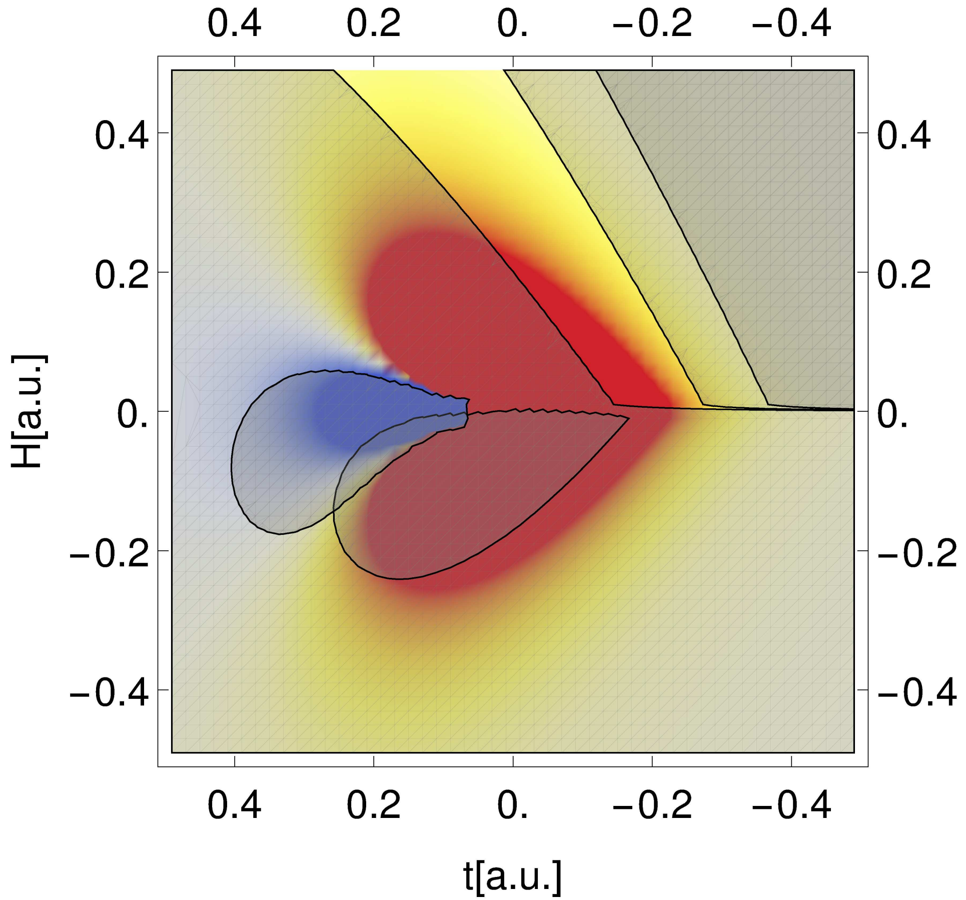
Exclusion plots





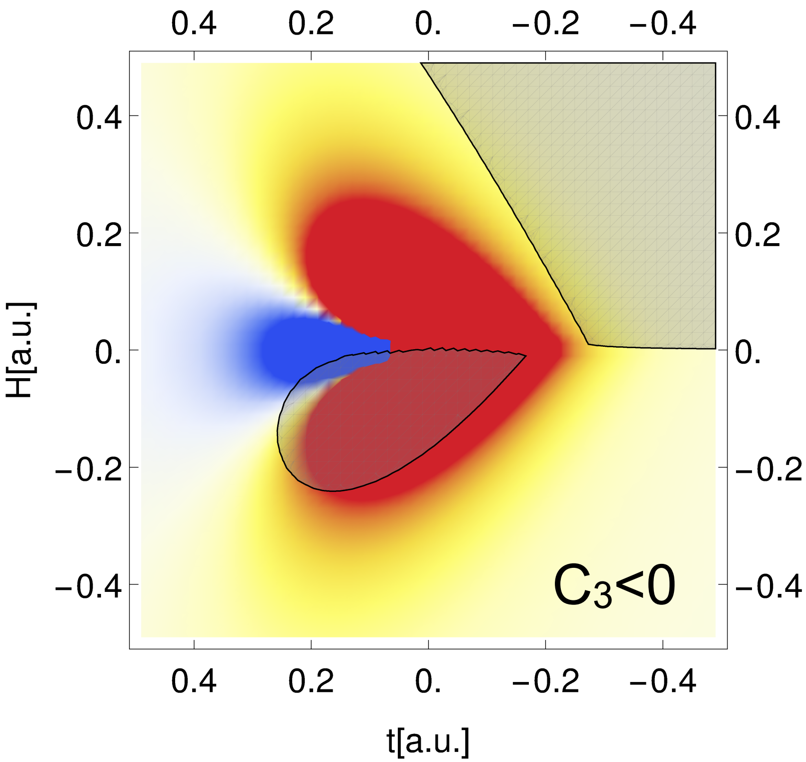


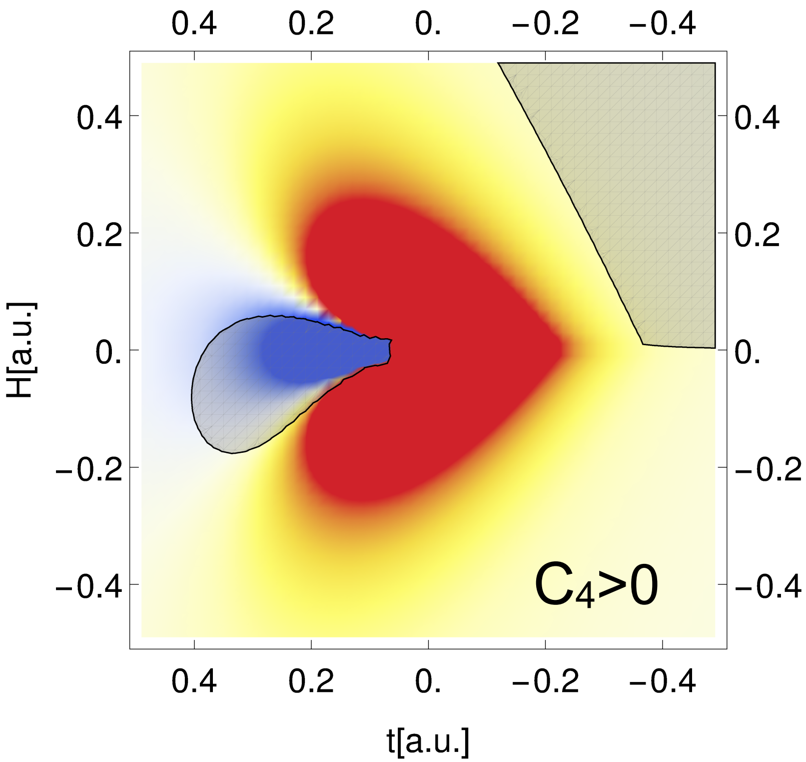
Excluding regions of the phase diagram



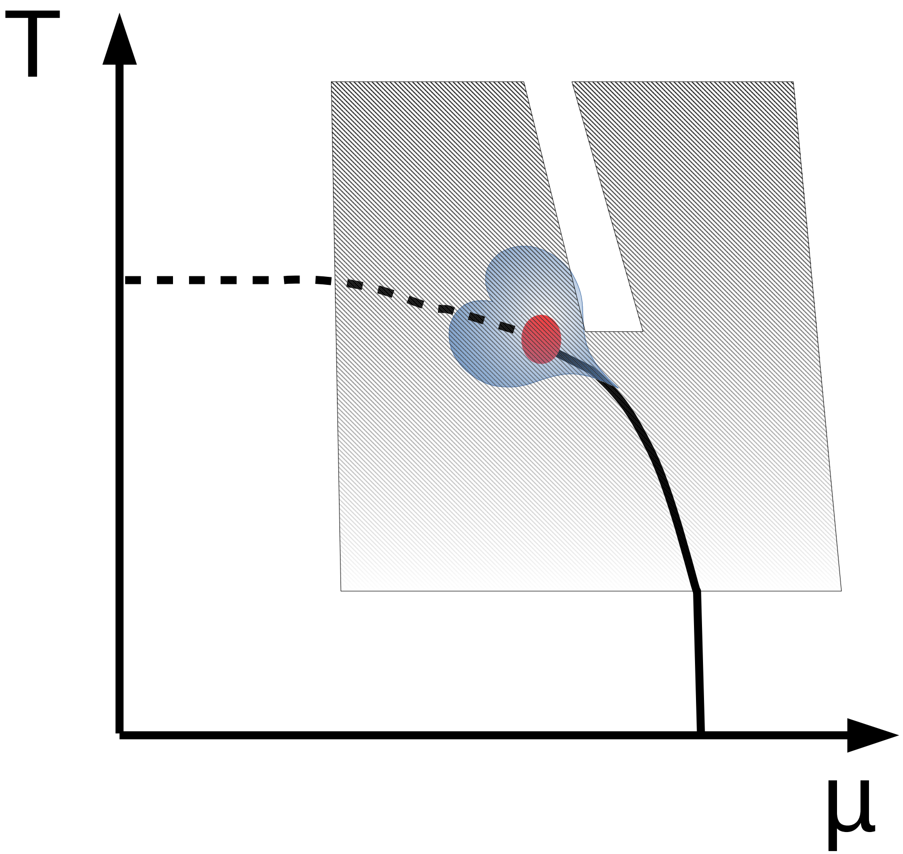
$C_2<0$, $C_3<0$, $C_4>0$

Ignore C_2

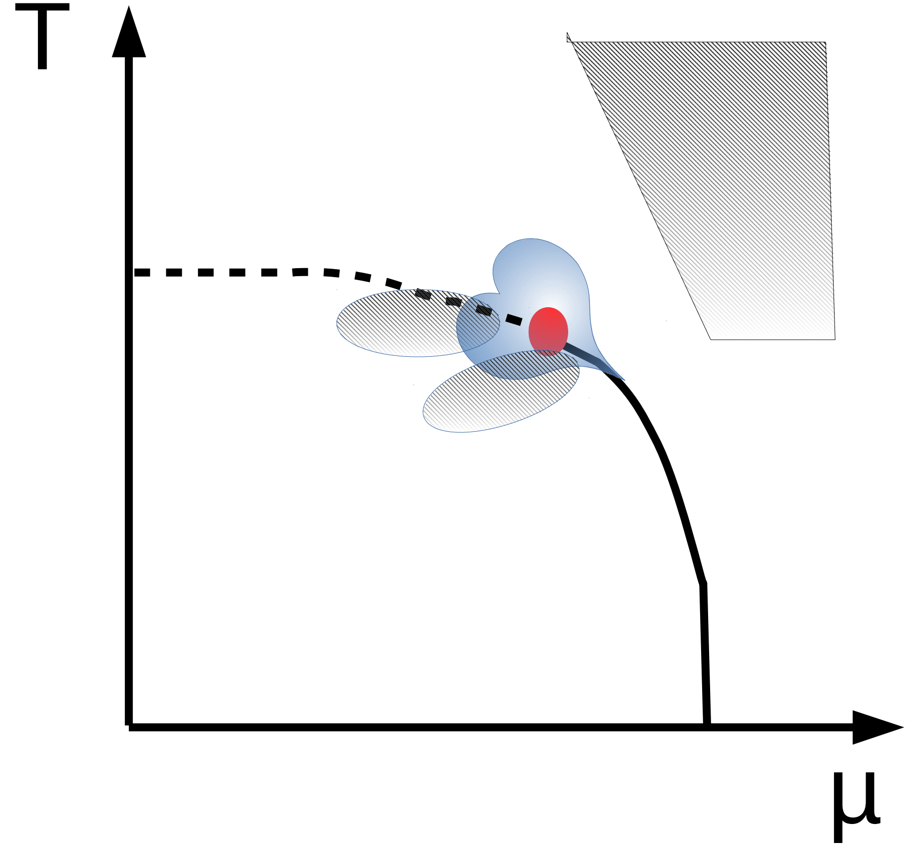




Map onto QCD phase diagram



$C_2<0, C_3<0, C_4>0$



$C_3<0, C_4>0$

Summary

- Fluctuations sensitive to phase structure:
  - measure “derivatives” of EOS
- Cumulants contain information about correlations
- Preliminary STAR data:
  - Significant four particle correlations at 7.7 and 11.5 GeV
  - Dip in K_4/K_2 at 19.6 GeV is due to negative two-particle correlations
  - Centrality dependence (at 7.7 GeV) indicates independent sources for $N_{part} < 150$ and “collective” correlations for $N_{part}>200$.
  - At about the same centrality three- and four particle correlations change sign!
    - New dynamics????? Or trivial stuff: Volume fluctuations etc.

Summary

- Preliminary STAR data continued:
  - For central 7.7 and 11.5 GeV two and three particle correlations are negative and four particle are positive.
    - This would rule out a large area around the critical point
- The STAR data are still preliminary!
- Other more mundane effects may contribute
- Correlations help chasing these effects down.

It's a long road….

Happy Birthday, Peter!