

# Quantum mechanics of the neutron electric dipole moment



Gordon Baym  
University of Illinois

Celebrating **Peter Braun-Munzinger** at 70

QCD thermodynamics: pressure and passion

Schloss Waldthausen

25 August 2016



## TRANSVERSE ENERGY PRODUCTION IN PROTON-NUCLEUS AND NUCLEUS-NUCLEUS COLLISIONS $\star$

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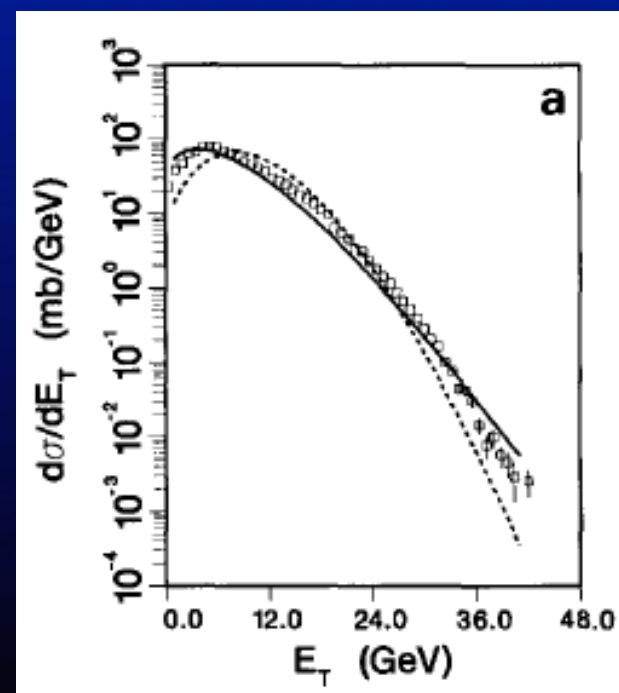


Vesa -2006

importance of rescattering in the target fragmentation region in the measured  $E_T$  spectra.

$$\frac{1}{N} \frac{dN}{dE_T} = \frac{1}{\epsilon_0} e^{-E_T/\epsilon_0} \sum_{n=1}^{\infty} \frac{\bar{n}^n}{n!((n-1)!)} e^{-\bar{n}} \left( \frac{E_T}{\epsilon_0} \right)^{n-1}$$

In conclusion, the HELIOS experiments provide strong evidence for rescattering of secondaries in the nucleus. The gross features of transverse energy production in  $^{16}\text{O-Pb}$  scattering can be understood in terms of independent nucleon scatterings on the target. However, when larger projectiles are used we expect significant deviations, in central collisions, from the independent nucleon scattering picture.

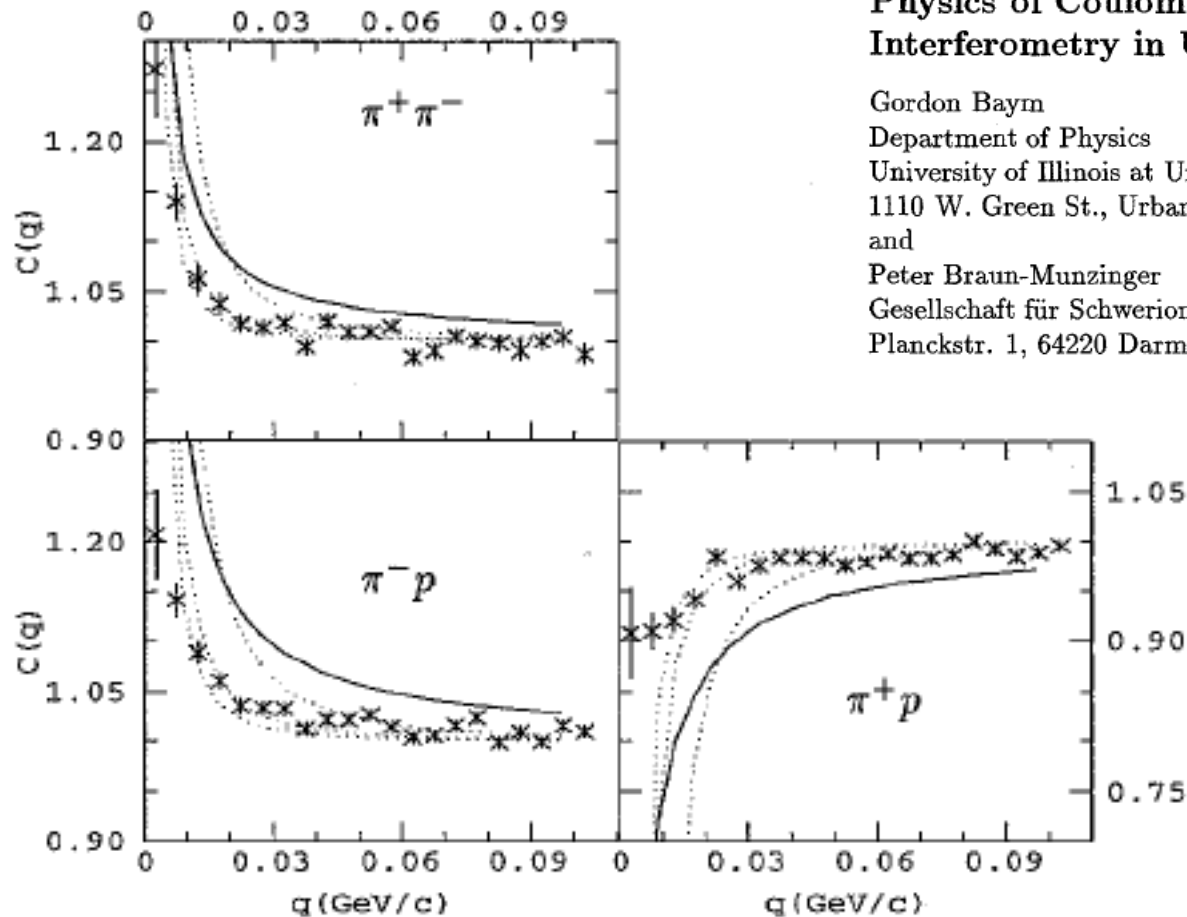




G. Baym, S. Nagamiya, and P. Braun-Munzinger,  
eds. Proc. 7th Int. Conf. on Ultrarelativistic nucleus-  
nucleus collisions (Quark Matter '88),  
Nucl. Phys. A498 (1989)

### Physics of Coulomb Corrections in Hanbury-Brown Twiss Interferometry in Ultrarelativistic Heavy Ion Collisions

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Calculate Coulomb corrections in classical toy model

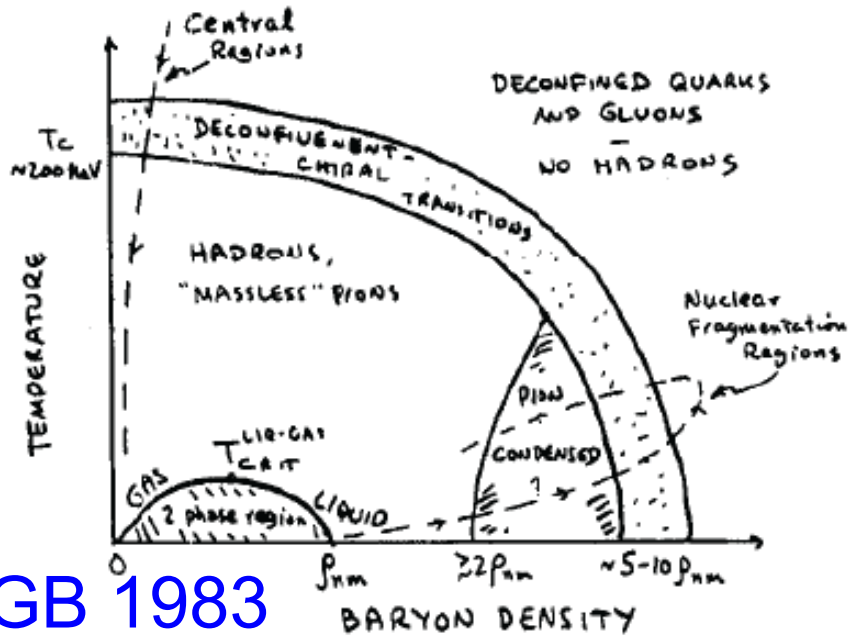
$$\frac{q^2}{2m_{red}} = \frac{q_0^2}{2m_{red}} \pm \frac{e^2}{r_0}$$

Correction of correlation fcn

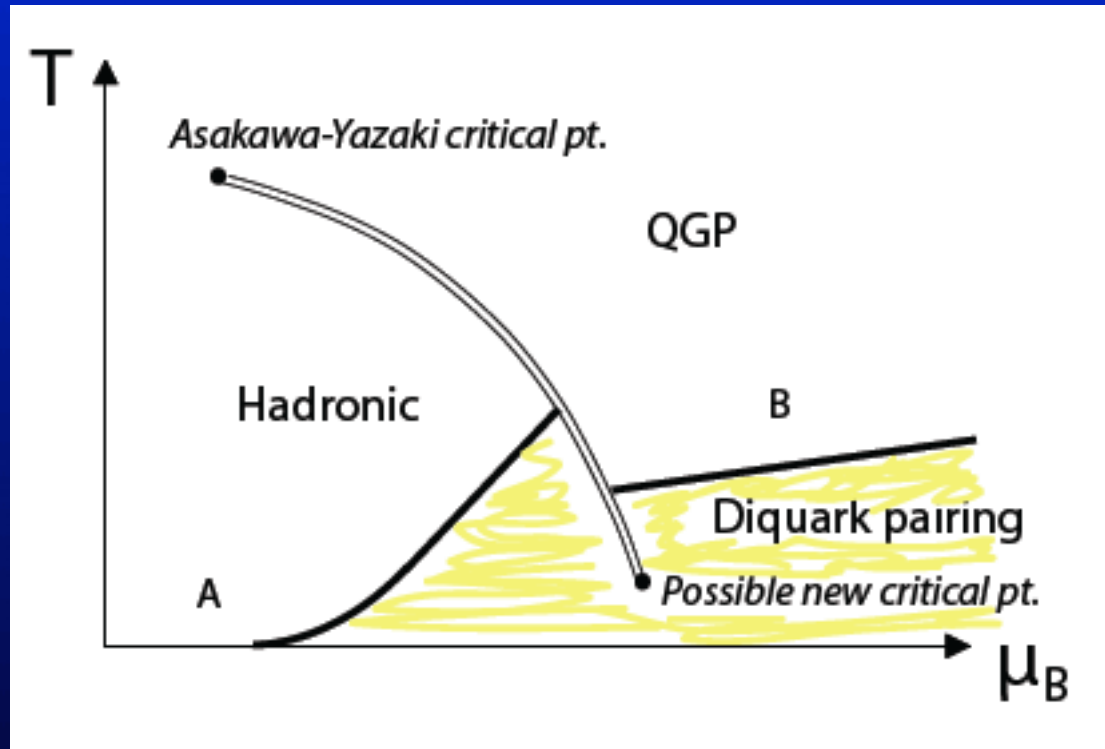
$$C(\vec{q}) = \frac{q_0}{q} C_0(\vec{q}_0) = \left( 1 \mp \frac{2m_{red}e^2}{r_0q^2} \right)^{1/2} C_0(\vec{q}_0)$$

# The QCD phase diagram

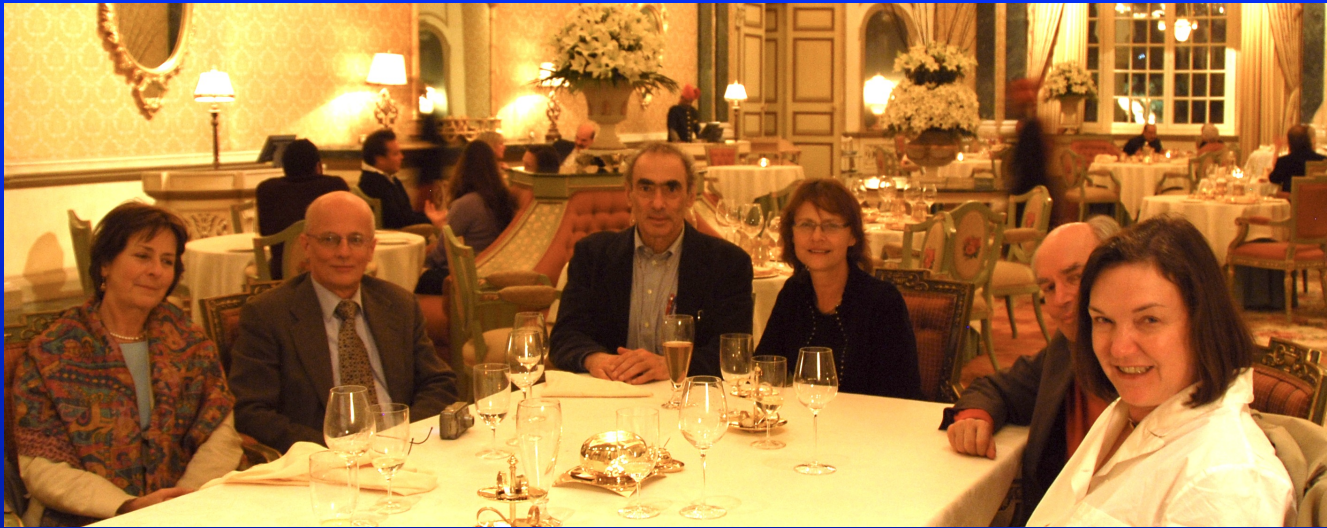
PHASE DIAGRAM OF NUCLEAR MATTER.



GB 1983



After long arguments with Peter and Johanna at the 2015 Kobe QM meeting



Jaipur 2007



Trento 2003



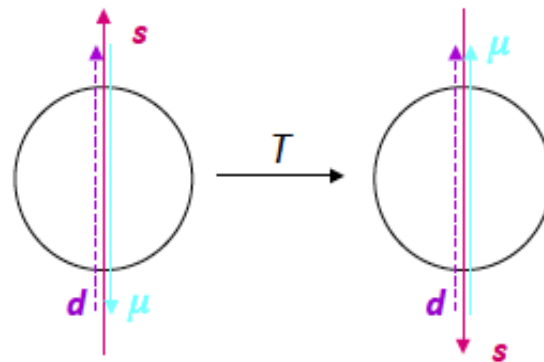
Urbana 2015

# The neutron electric dipole moment

# The neutron electric dipole moment

## T Violation and Neutron EDM

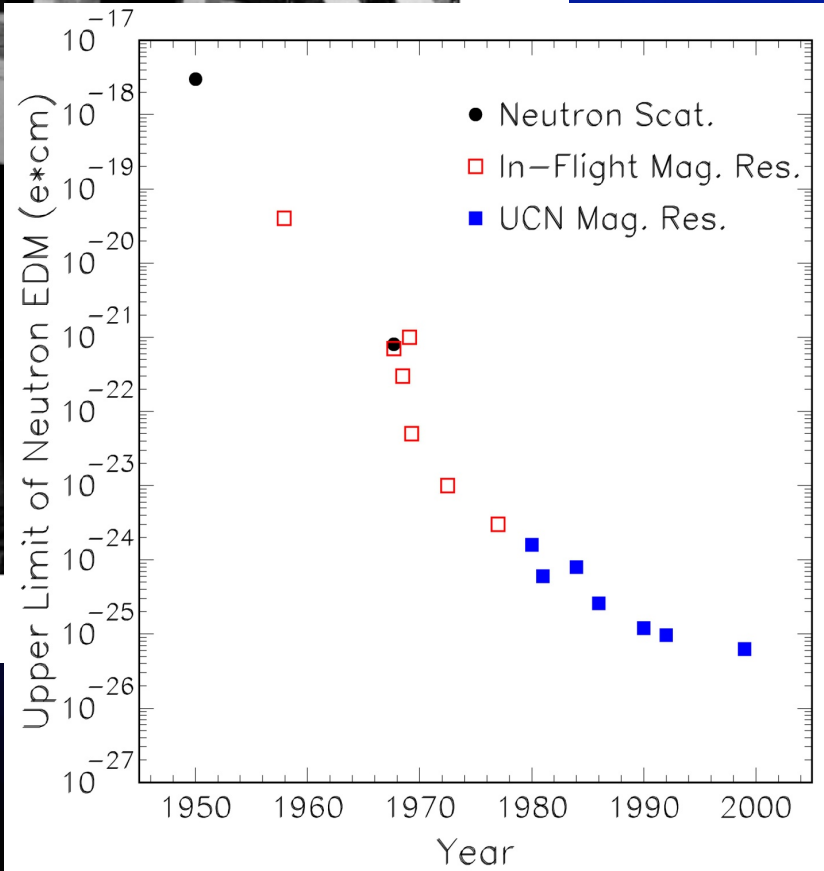
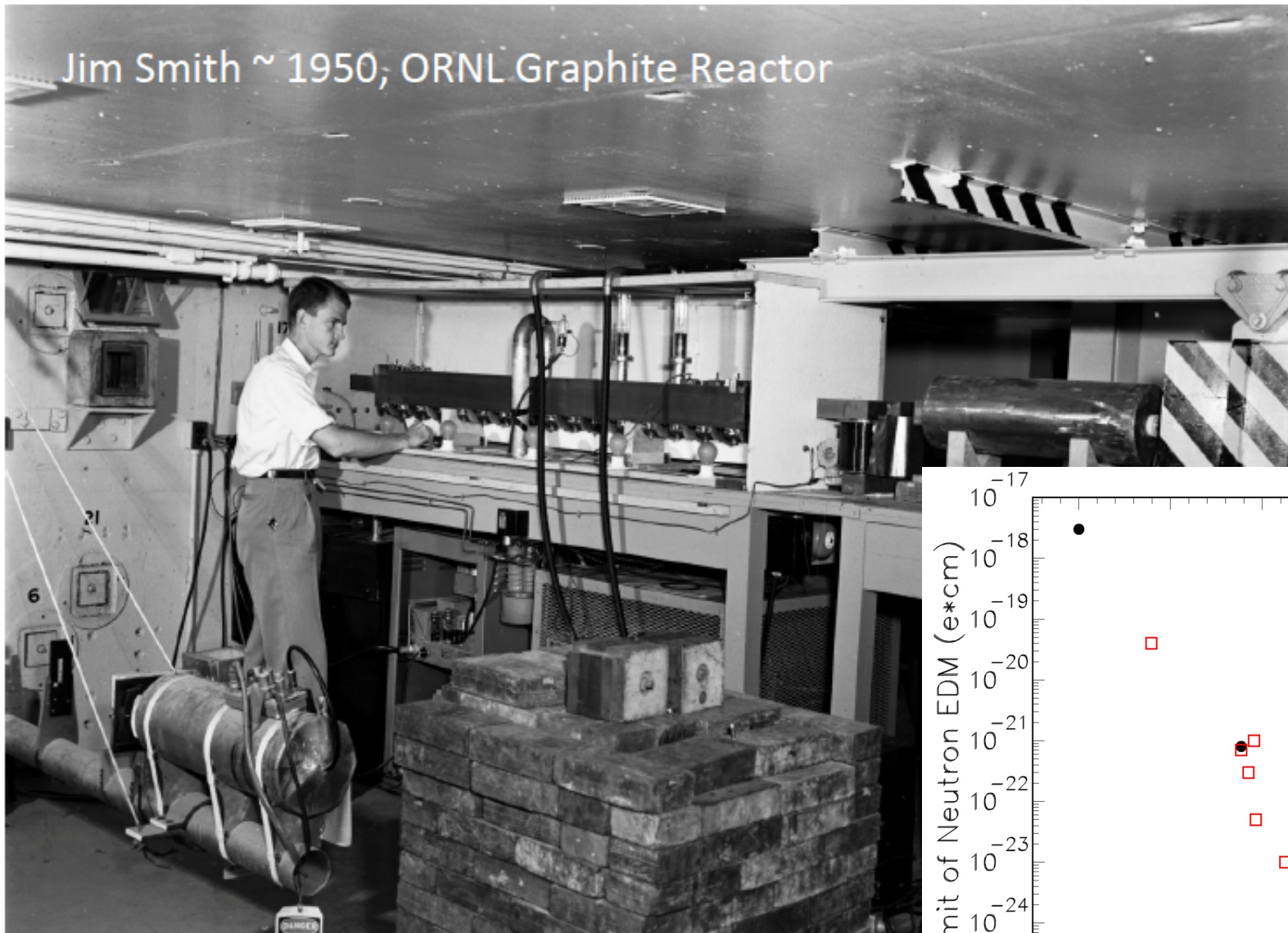
- Existence of particle EDM implies T reversal sym'y violation
  - spin is only orientation (vector) in problem:  $\vec{d} \uparrow \uparrow \vec{s}$  or  $\vec{d} \uparrow \downarrow \vec{s}$



- T reversal violation implies CP violation if CPT symmetry preserved
  - *observation*; requires only locality, Lorentz invariance and Hermitian Hamiltonian
- Not enough standard model CP violation to explain baryon asymmetry of universe
- Generic scale of neutron EDM in SUSY above current limit



# EDM Measurements



# EDM measurements

- Neutron

- Sussex, RAL, ILL

- $d_n < 2.9 \times 10^{-26} \text{ e}\cdot\text{cm}$

Baker, et al. PRL **97** (2006) 1318

- [ $d_n < 5.5 \times 10^{-26} \text{ e}\cdot\text{cm}$ ]

Serebrov, et al. arXiv:1408.6430

- Future

- Sussex, RAL, ILL

- Crystal ILL

- Gatchina ILL

- PSI

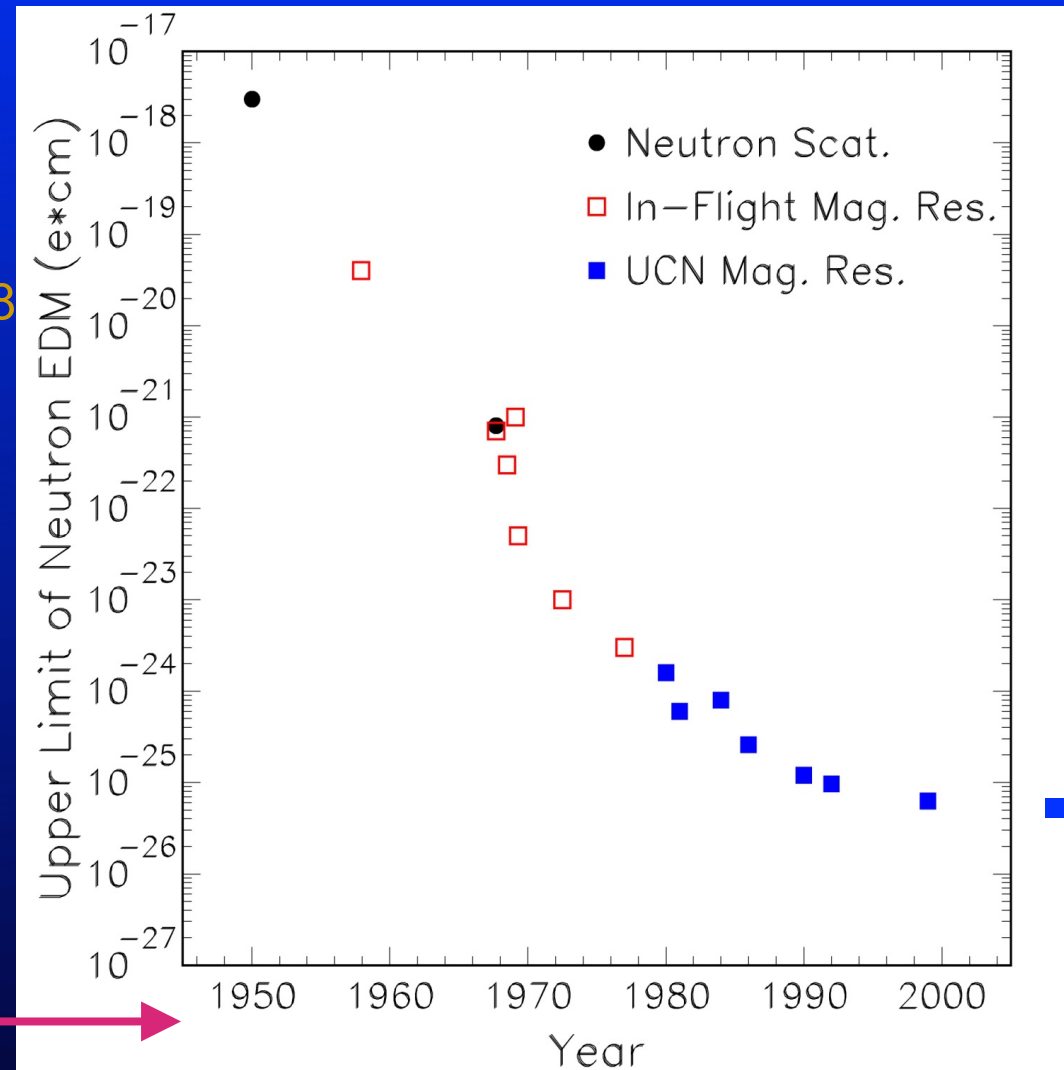
- SNS Oak Ridge

- $d_n < 5 \times 10^{-28} \text{ e}\cdot\text{cm}?$

- Japan

- TRIUMF

- TU Munich



# EDM measurements

- Electron

- paramagnetic atoms: Tl [Regan, et al. PRL \*\*88\*\* \(2002\) 071805](#)

- $d_e < 1.6 \times 10^{-27} \text{ e}\cdot\text{cm}$

- molecules:

- YbF  $d_e < 1.05 \times 10^{-27} \text{ e}\cdot\text{cm}$  [Hinds, et al. Nature \*\*473\*\* \(2011\)](#)

- ThO  $d_e < 8.7 \times 10^{-29} \text{ e}\cdot\text{cm}$  [J. Baron, et al. Science \*\*343\*\* \(2014\) 269](#)

- Nuclei

- Hg [Griffiths, et al. PRL \*\*102\*\* \(2009\) 101610](#)

- $d_{\text{Hg}} < 3.1 \times 10^{-29} \text{ e}\cdot\text{cm}$

- [Ra [trapped atom: ANL](#)]

- [p, d [storage ring proposal: BNL](#)]

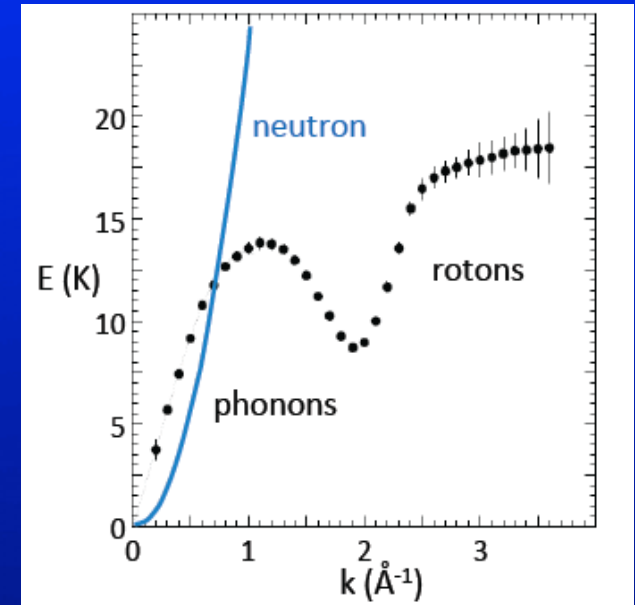
# Oak Ridge SNS experiment: $T = 0.5$ K

Concept: Golub and Lamoreaux, Phys. Rep. 237 (1994) 1

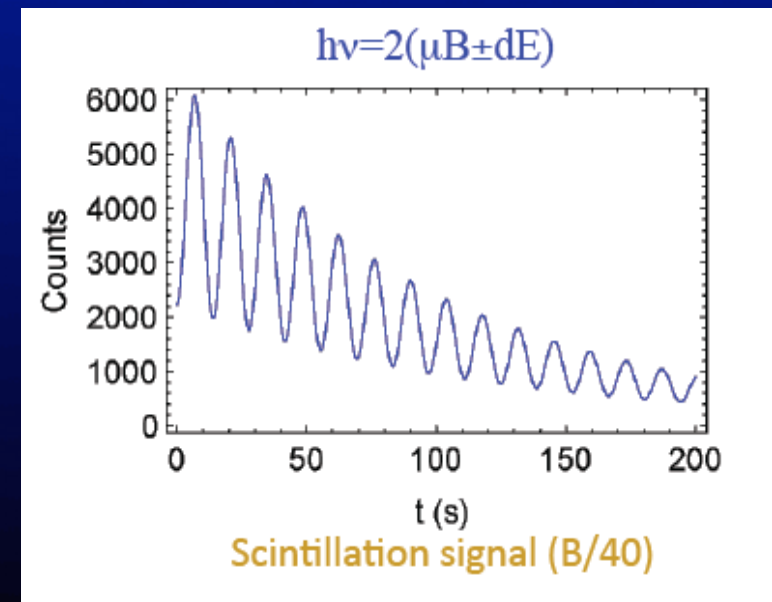
Spin-polarized ultracold neutrons on spin-polarized  $^3\text{He}$ :



$$\sigma(\vec{n} + \vec{^3\text{He}}) \sim 0, \sigma(\vec{n} + \overleftarrow{^3\text{He}}) \sim 2\text{mb}$$



- Measure neutron precession frequency in “NMR” experiment;
- look at modulation from external electric field coupled to nEDM



# Apparatus Overview

DR LHe volume  
(~300 liters)

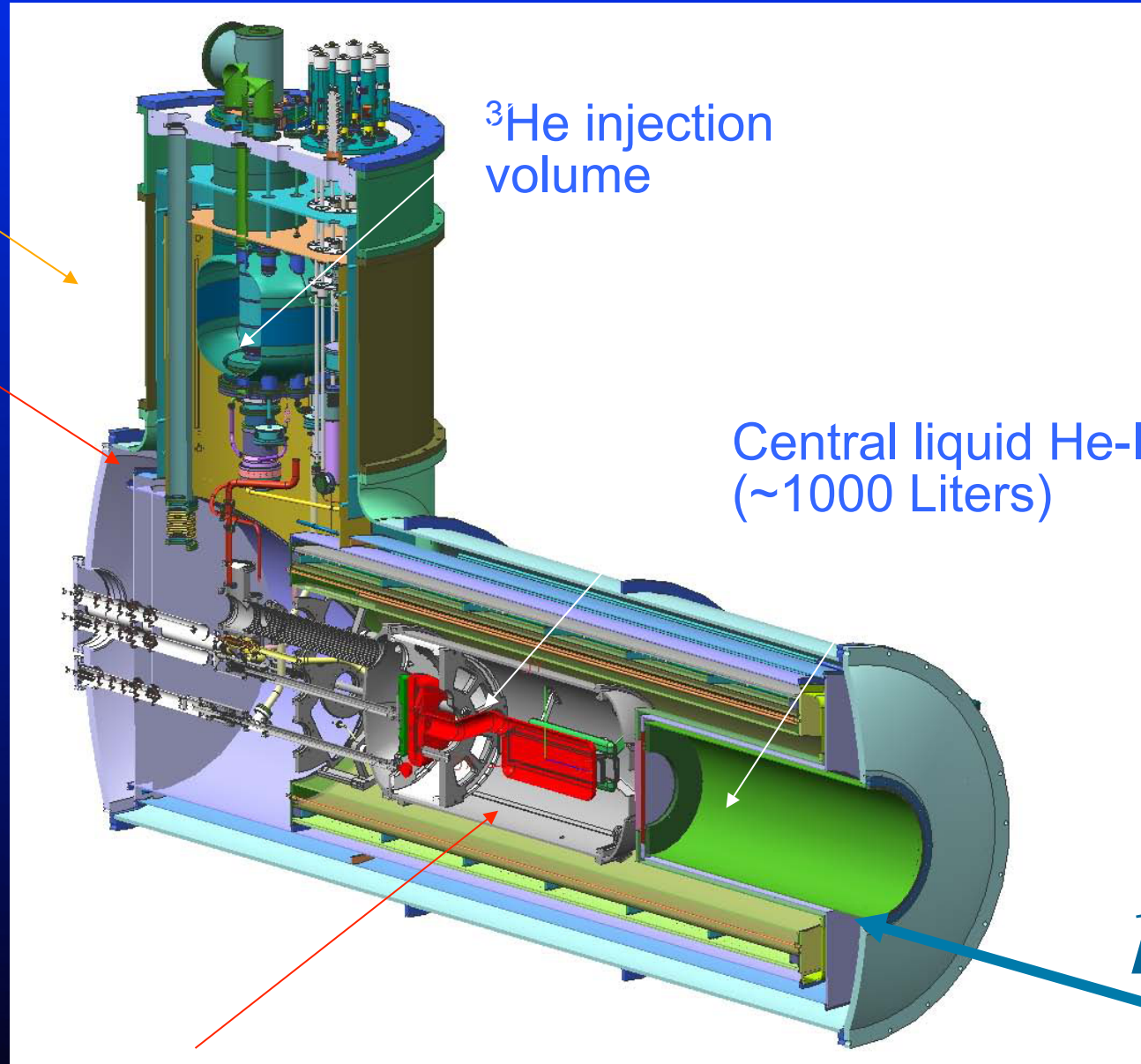
Dilution  
refrigerator  
mixing volume

$^3\text{He}$  injection  
volume

Central liquid He-II  
(~1000 Liters)

1 m

Measurement cell



# Quantum mechanics of neutron dipole moment

GB and D.H. Beck, PNAS, 113, 7438 (2016)

*“A parity-violating perturbation, corresponding physically to a permanent electric dipole moment of an electron parallel to its spin, is introduced into the Dirac equation for an electron.”*

E. Salpeter, PR 112 (1968)

$$\left[ \left( p_\mu + \frac{e}{c} A_\mu \right) \gamma_\mu - imc \right] u = -\xi \left( \frac{e\hbar}{4mc^2} \right) \gamma_5 \gamma_\mu \gamma_\nu F_{\mu\nu} u.$$

The neutron EDM must point along the spin, since the spin is the only vector in the neutron. **TRUE OR FALSE???**

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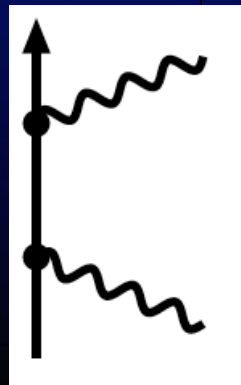
$$\left[ \left( \not{p} + \frac{e}{c} \not{A} \right) \gamma_5 - imc \right] u = -\xi \left( \frac{e\hbar}{4mc^2} \right) \gamma_5 \gamma_\mu \gamma_\nu F_{\mu\nu} u.$$

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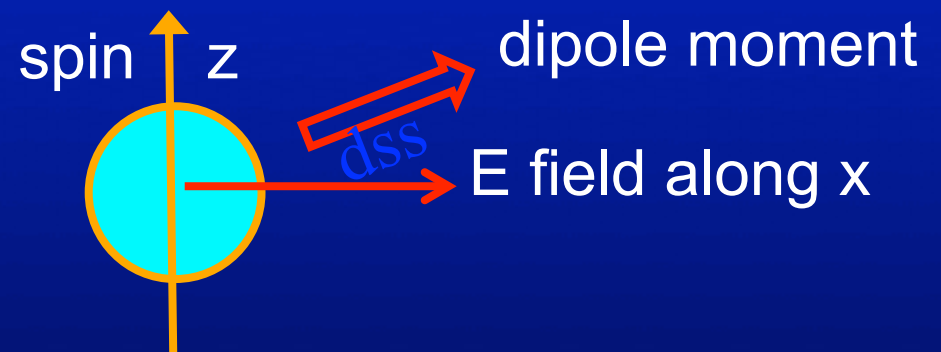
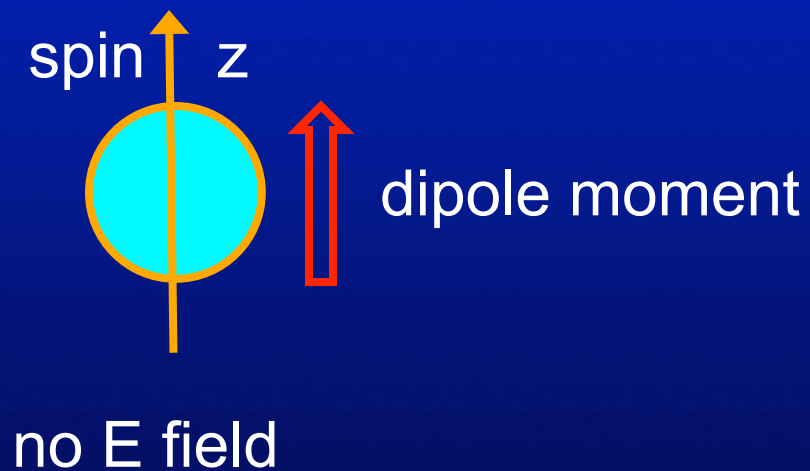
The nEDM **cannot** be locked to spin, since neutron can be polarized by external electric field

Spin operator and dipole operator are non-commuting independent operators.

Dipole moment = polarizability X electric field  
Polarizability measured in Compton scattering on n.



**Puzzle:** put neutron in very strong magnetic field along  $z$ , locking in spin. Apply electric field along  $x$ . Induce dipole moment along  $x$ , not parallel to spin!





# Ground state of the neutron in presence of CP = T violation

$$|N \uparrow\rangle = |\uparrow\rangle + \eta |(t) \uparrow\rangle$$

$|\uparrow\rangle$  = familiar ground state

$|(t) \uparrow\rangle$  = odd parity spin  $\frac{1}{2}$   
excited state of neutron

$\eta$  = very small  $< 10^{-12}$

Lowest odd-parity excited neutron states: spin  $1/2 = N(1535)$   
spin  $3/2 = N(1520)$

Electric dipole moment:

$$\vec{\mathcal{D}} \sim \int d^3r \vec{r} \sum_{\text{quarks}, i} Q_i \bar{q}_i(\vec{r}) \gamma_0 q_i(r)$$

$$\langle \vec{d}_n \rangle = e \langle N \uparrow | \vec{\mathcal{D}} | N \uparrow \rangle = e\eta \langle \uparrow | \vec{\mathcal{D}} | (t) \uparrow \rangle + c.c.$$

Spin:

$$\langle \vec{S} \rangle = \langle N \uparrow | S_z | N \uparrow \rangle = \frac{1}{2} \hat{z}$$

$\Rightarrow$

$$\langle \vec{d}_n \rangle = d_n \hat{z} = \langle \vec{S} \rangle$$

Wigner-Eckart theorem at work (in absence of applied fields).

# Response of neutron to external electric field

In weak external electric field  $\tilde{\mathbf{E}}$  state of neutron is:

$$|(E)N \uparrow\rangle = |N \uparrow\rangle + e\mathcal{V}\vec{\mathcal{D}} \cdot \vec{E} |N \uparrow\rangle$$

where  $\mathcal{V} = \sum_{n \neq \uparrow} \frac{|n\rangle\langle n|}{\omega_n}$  with  $|n\rangle =$  odd parity states.

Field mixes in spin 1/2 and 3/2

$$\mathcal{D}_z |\uparrow\rangle = \frac{1}{3} \langle \vec{\mathcal{D}}^2 \rangle^{1/2} \left( \sqrt{2} |(d)3/2\rangle - |(d)1/2\rangle \right)$$

Total dipole moment:

$$\langle \vec{d}_{tot} \rangle = d_n \hat{z} + \chi_n \vec{E}$$

Electric polarizability:

$$\chi_n = 2e^2 \langle \uparrow | \mathcal{D}_z \mathcal{V} \mathcal{D}_z | \uparrow \rangle = \frac{2e^2}{9} \langle \vec{\mathcal{D}}^2 \rangle \left( \frac{1}{\omega_{1/2}} + \frac{2}{\omega_{3/2}} \right)$$

Experimentally  $\chi_n = 1.26 \times 10^{-3} \text{ fm}^3$  *J. Schmiedmayer, et al. PRL (1993)*

*B. Holstein and A. Nathan, PRD (1994).*

so that  $\langle \vec{\mathcal{D}}^2 \rangle \simeq 0.77 \text{ fm}^2$

Neutron has big dipole moment jiggling inside, with tiny  $\langle d \rangle$ ,  
cf. hydrogen atom in ground state.

# How can the dipole moment not be parallel to the spin?

Wigner-Eckart theorem: in absence of external (e.m.) fields, in eigenstate of total angular momentum,  $J$  and  $J_z$ ,  $\langle \text{EDM} \rangle$  parallel to  $\langle \text{spin} \rangle$ .

Lorentz invariance: neutron in absence of external fields is spin  $1/2$  with only single axis.

How can an E electric field  $\Rightarrow$  dipole moment not parallel to the spin?

Admixture of spin-3/2  $\Rightarrow$  n has multiple axes.

Imagine state in the presence of E and  $\eta$  is pure spin-1/2. Transverse electric field mixes in  $S_z = -1/2$  components of the excited states, neutron no longer an eigenstate of  $S_z$ .

Wigner-Eckart theorem does not apply. Spin and induced EDM need not be parallel.

# Response of neutron to E and B fields

Coupling to external fields:  $H_{em} = -\vec{\mu}_n \cdot \vec{B} - e\vec{D} \cdot \vec{E}$

Eq. of motion of spin:  $\frac{d}{dt}\vec{S} = -i[\vec{S}, H_{em}] = 2\mu_N\vec{S} \times \vec{B} + e\vec{D} \times \vec{E}$

Thus 
$$\begin{aligned}\frac{d}{dt}\langle\vec{S}\rangle &= 2\mu_N\langle\vec{S}\rangle \times \vec{B} + \langle\vec{d}_{tot}\rangle \times \vec{E} \\ &= 2\langle\vec{S}\rangle \times (\mu_n\vec{B} + d_n\vec{E})\end{aligned}$$

For spin 1/2 induced edm does not enter because polarizability tensor is  $\sim$  unit matrix.

Not so for higher spin particles, e.g.,  $^{205}\text{Tl}$  with  $^2\text{P}_{1/2}$  ground state, where 2<sup>nd</sup> order Stark shift  $\Rightarrow$  (weak) tensor polarizability.

Potentially could have induced moment contributions to spin motion – terrible experimental complication.

# Simple non-relativistic model

Assume CP violating interaction :  $H_{CPV,N} = -a\vec{S} \cdot \vec{\mathcal{D}}$ ,  $a < 10^{-11} \text{ fm}^{-2}$

in presence of which  $|N \uparrow\rangle = \left(1 + a\mathcal{V}\vec{\mathcal{D}} \cdot \vec{S}\right) |\uparrow\rangle$   $\mathcal{V} = \sum_{n \neq \uparrow} \frac{|n\rangle\langle n|}{\omega_n}$

Realization of CP violating component of n  $\eta|(t) \uparrow\rangle = a\mathcal{V}\vec{\mathcal{D}} \cdot \vec{S} |\uparrow\rangle$ .

$$\Rightarrow a \sim 7\eta \text{ fm}^{-2}$$

Electric dipole moment:

$$\begin{aligned} \langle \vec{d}_n \rangle &= ae \langle \uparrow | \vec{\mathcal{D}} \mathcal{V} \vec{\mathcal{D}} \cdot \vec{S} | \uparrow \rangle + c.c. \\ &= \frac{ae}{3\omega_{1/2}} \langle \vec{\mathcal{D}}^2 \rangle \hat{z} \end{aligned}$$

Approximate relation of nEDM to electrical polarizability

$$\frac{d_n}{\chi_n} = \frac{3a}{2e} \frac{1}{1 + 2\omega_{1/2}/\omega_{3/2}} \simeq \frac{a}{2e}$$

# Understanding nEDM in bag models

Would like to see how CP violation of n states comes about in presence of QCD  $\Theta$  interaction:

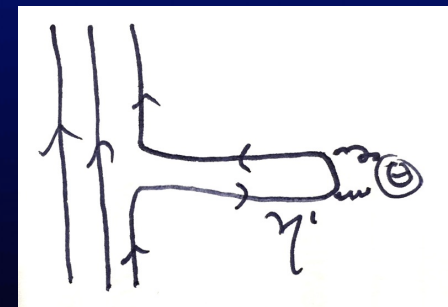
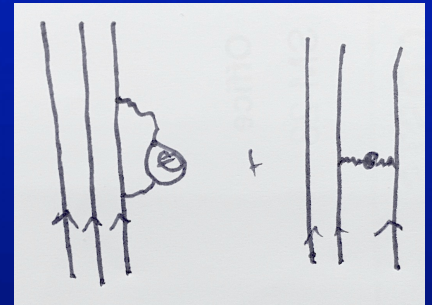
$$L_{\theta} = \theta \frac{g^2}{32\pi^2} \int d^3x F_a^{\mu\nu}(x) \tilde{F}_{\mu\nu}^a(x);$$

Direct implementation in bag models of n beset by gauge invariance non-trivialities.

Lattice QCD not yet successful.

Possible implementation in cloudy bag model:  $\eta'$  mesons in cloud. The  $\eta'$  massive through  $\theta$  term. In cloudy bag  $\theta$  term acting on  $\eta'$  in cloud can generate spontaneous n EDM.

*S. Aoki & T. Hatsuda, PR D (1992)*

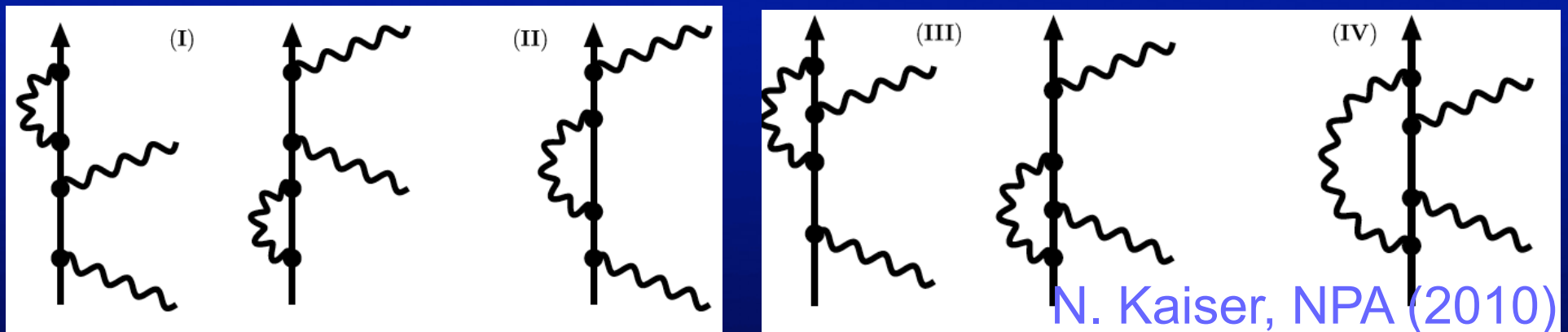


# Related problems

Extend present picture to proton and electron in atoms.

Compute electric polarizability of electron -- via  $e^+e^-$  pairs in screening cloud?

For proton, extract polarizability in Compton scattering including radiative corrections.



Leads, for forward scattering, to multiplicative factor on top of Klein-Nishina cross section

$$\left[1 + (\alpha/2\pi)(2\omega/m)^2 \left(\frac{8}{3} \ln(2\omega/m) + 11/18\right) + \dots\right]$$

# Low energy expansion of RCS cross section in terms of polarizabilities

$$\begin{aligned} \left(\frac{d\sigma}{d\Omega}\right)_{\text{Lab}} &= \left(\frac{d\sigma}{d\Omega}\right)_{\text{Lab}}^{\text{Powell}} \\ &- \frac{e^2}{4\pi M_N} \left(\frac{\nu'}{\nu}\right)^2 \nu \nu' \left\{ \frac{\alpha + \beta}{2} (1 + \cos \theta_L)^2 + \frac{\alpha - \beta}{2} (1 - \cos \theta_L)^2 \right\} \\ &+ \mathcal{O}(\nu^3) \end{aligned}$$

➔ Polarizability term : quadratic in the photon energy

➔ Angular dependence : disentangle  $\alpha$  and  $\beta$

$$\theta_L = 0^\circ \quad : \quad d\sigma \sim \alpha + \beta$$

$$\theta_L = 180^\circ \quad : \quad d\sigma \sim \alpha - \beta$$

➔ Higher terms in photon energy : can be treated in a dispersion relation formalism (see later)



**Thank you!!**



to my (not-so) old friend and colleague  
Peter

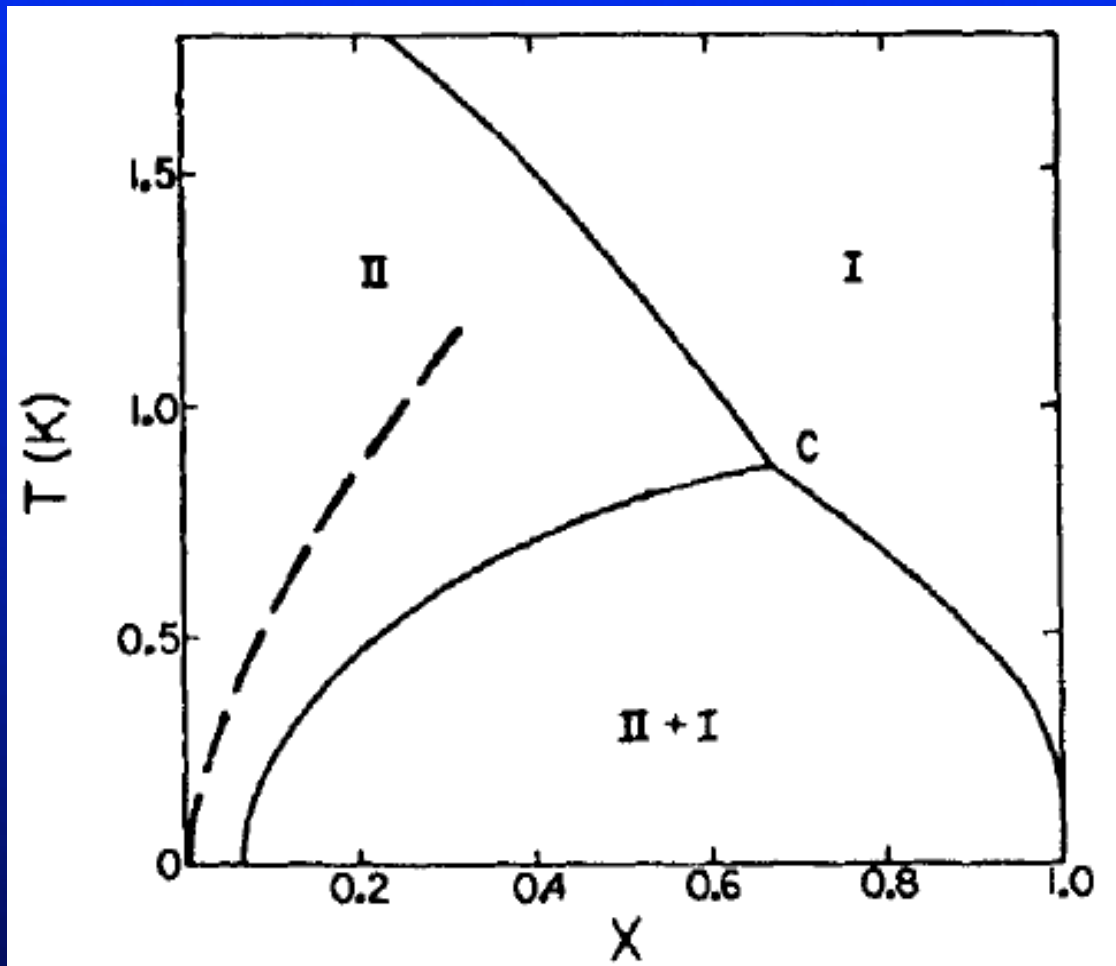
# Sweeping depolarized $^3\text{He}$ out with phonon wind

- Use phonon –  $^3\text{He}$  scattering
- Create phonons in superfluid with heater
  - phonon ‘wind’



- Basic physics worked out long ago (~1960's)
  - $^3\text{He}$  concentration,  $x_3$ , in degenerate regime
  - now need processes at relative densities  $\sim 10^{-4}$ ,  $10^{-10}$  (natural:  $10^{-7}$ )
- GB, D.H.Beck, C.Pethick, Phys. Rev. B **88** (2013) 014512, J. Low Temp. Phys. **178** (2015) 200, PR B 92, 024504 (2015)

# Dilute solutions of $^3\text{He}$ in superfluid $^4\text{He}$



Phase diagram  
 $x = n_3/n_{\text{tot}}$

Degeneracy: D. Edwards 1965

Transport expts. J. Wheatley et al. 1966+

Microscopic theory, J. Bardeen, GB, D. Pines, C. Ebner, W. Saam

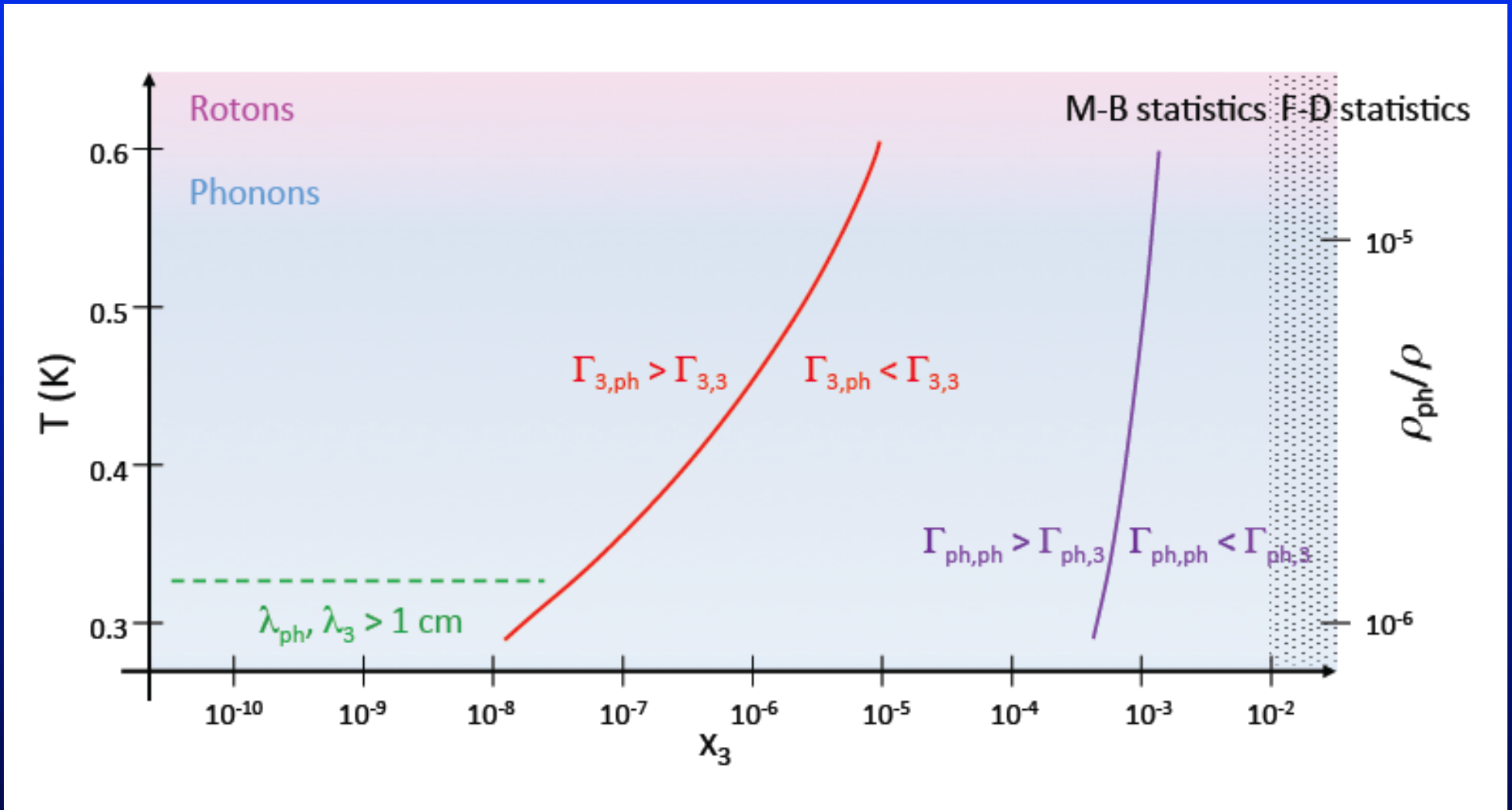
new superfluid?

# Transport in solutions of $^3\text{He}$ in superfluid $^4\text{He}$

- Goal is to determine dynamics of heat flush
  - Calculate thermal conductivities, diffusion coefficients
- Calculation based on
  - Phonon- $^3\text{He}$  scattering cross section<sup>1</sup>
  - Phonon-phonon scattering mean free path<sup>2</sup>
  - $^3\text{He}$ - $^3\text{He}$  scattering cross section<sup>3</sup>
  - Relaxation approximation for scattering from walls

1. C. Boghosian and H. Meyer, Phys. Lett. A **25** (1967) 352; B. M. Abraham, C. G. Brandt, Y. Eckstein, J. Munarin, and G. Baym, Phys. Rev. **188**, 309 (1969); G. E. Watson, J. D. Reppy and R. C. Richardson, Phys. Rev. **188** (1969) 384.  
2. D. S. Greywall, Phys. Rev. B **23**, 2152 (1981); H. J. Maris, Rev. Mod. Phys. **49** (1977) 341.  
3. A. C. Anderson, D. O. Edwards, R. Roach, R. J. Sarwinski and J. C. Wheatley, Phys. Rev. Lett. **17** (1966) 367; J. Bardeen, G. Baym and D. Pines, Phys. Rev. Lett. **17** (1966) 372, Phys. Rev. **156** (1967) 207; G. Baym and C. Ebner, Phys. Rev. **170** (1968) 346.

# Relaxation rates: physics changes with $^3\text{He}$ concentration



GB, D.H.Beck, C.Pethick, Phys. Rev. B **88** (2013) 014512, J. Low Temp. Phys. **178** (2015) 200, PR B **92**, 024504 (2015)

# Time evolution of $^3\text{He}$ driven away by phonon wind

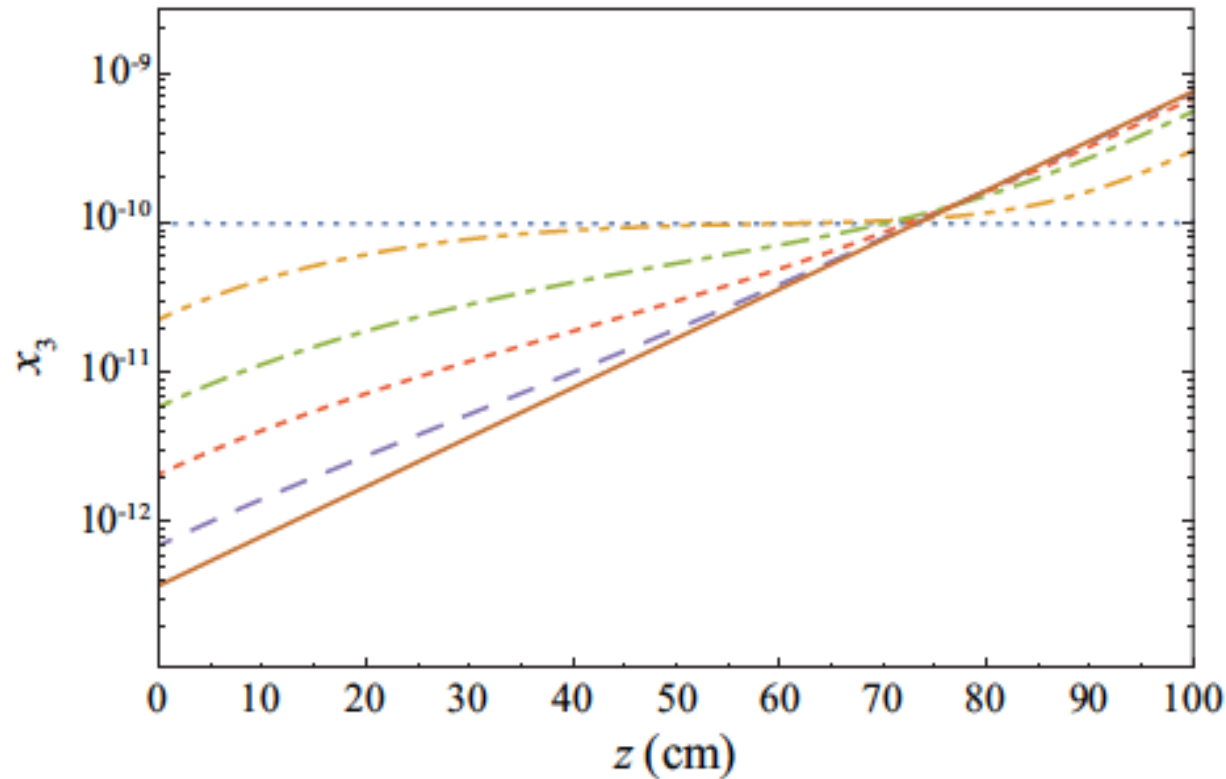


FIG. 3. (Color online) The  $^3\text{He}$  concentration,  $x_3$ , from the solution of Eq. (39) as a function of  $z$ , the distance along the pipe, for various times:  $t = 0$  (dotted), 1 (dash double dot), 3 (dash dot), 5 (short dash), 8 (long dash), and 20 s (solid). The result is shown for typical parameters in the nEDM experiment:  $x_{3,0} = 10^{-10}$  and 5 mW of heat into a 3-cm-diameter, 100-cm-long pipe at a nominal temperature of 0.45 K.

# World-wide nEDM Searches

Experiment	UCN source	cell	Measurement techniques	$\sigma_d$ Goal ( $10^{-28}$ e-cm)
ILL-PNPI	ILL turbine PNPI/Solid D <sub>2</sub>	Vac.	Ramsey technique for $\omega$ E=0 cell for magnetometer	Phase 1 < 100 < 10
ILL Crystal	Cold n Beam	solid	Crystal Diffraction Non-Centrosymmetric crystal	< 100
PSI EDM	Solid D <sub>2</sub>	Vac.	Ramsey for $\omega$ , external Cs & <sup>3</sup> He, Hg co-magnetom. Xe or Hg comagnetometer	Phase 1 ~ 50 Phase 2 < 5
Munich FRMII	Solid D <sub>2</sub>	Vac.	Room Temp. , Hg Co-mag., also external Cs mag.	< 5
RCNP/TRIUMF	Superfluid <sup>4</sup> He	Vac.	Small vol., Xe co-mag. @ RCNP Then move to TRIUMF	< 50 < 5
SNS EDM	Superfluid <sup>4</sup> He	<sup>4</sup> He	Cryo-HV, <sup>3</sup> He capture for $\omega$ , <sup>3</sup> He co-mag. with SQUIDS & dressed spins, supercond.	< 5
JPARC	Solid D <sub>2</sub>	Vac.	Under Development	< 5
JPARC	Solid D <sub>2</sub>	Solid	Crystal Diffraction Non-Centrosymmetric crystal	< 10?
LANL	Solid D <sub>2</sub>	Vac.	R & D	~ 30