EXPERIMENTAL CONSIDERATIONS ON E-BY-E FLUCTUATION SIGNALS

- Criticality at crossover
- Phase boundaries
- Experimental challenges
  - Volume fluctuations
    - Centrality determination
  - Conservation laws
- Summary

Anar Rustamov, a.rustamov@cern.ch

Baku, 2015

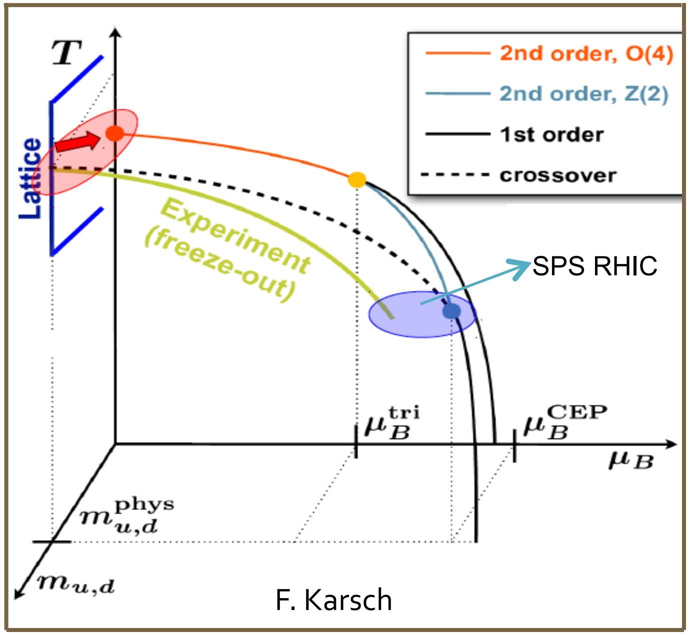
Dear Peter, Happy Birthday!

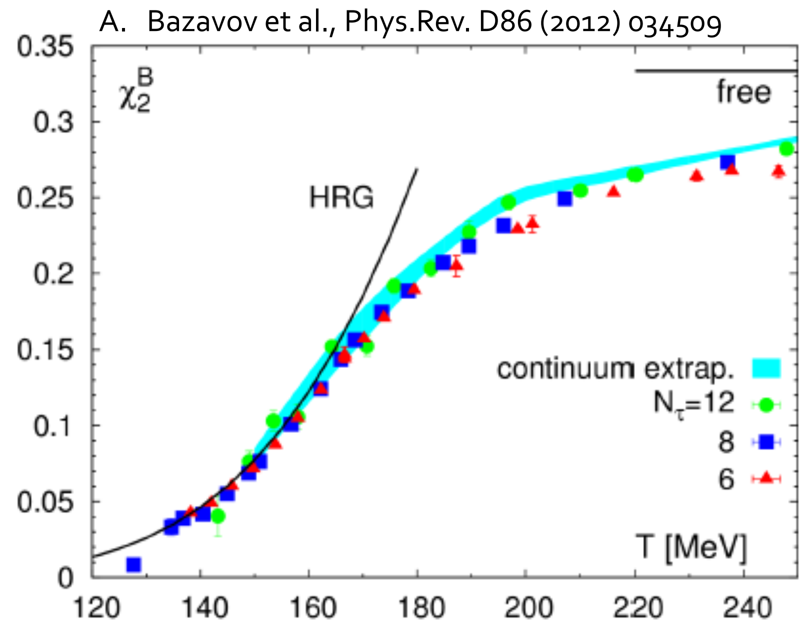
Many thanks for everything I have learnt from you and for your assistance in different activities!

Baku, 2015

CERN 2016

Criticality at crossover?



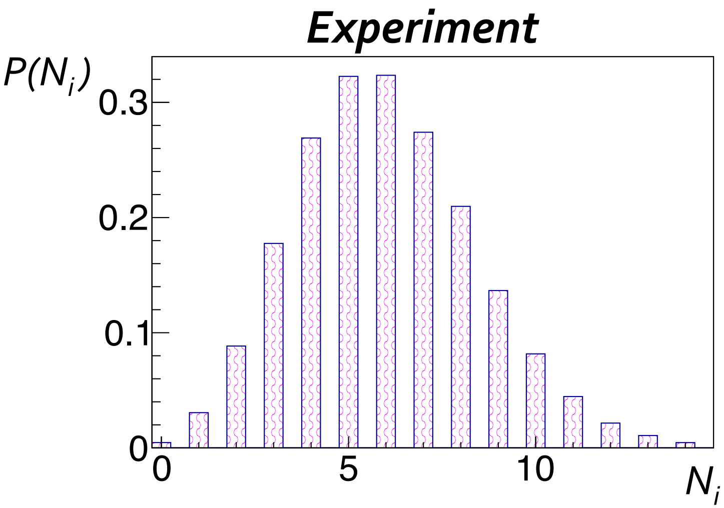


freeze-out at the phase boundary!

$$T_c^{lattice} = 154 \pm 9\, MeV, \quad T_{fo}^{ALICE} = 156 \pm 3 MeV$$

- ***E-by-E fluctuations:***
  - To study dynamics of the phase transitions
  - To locate phase boundaries

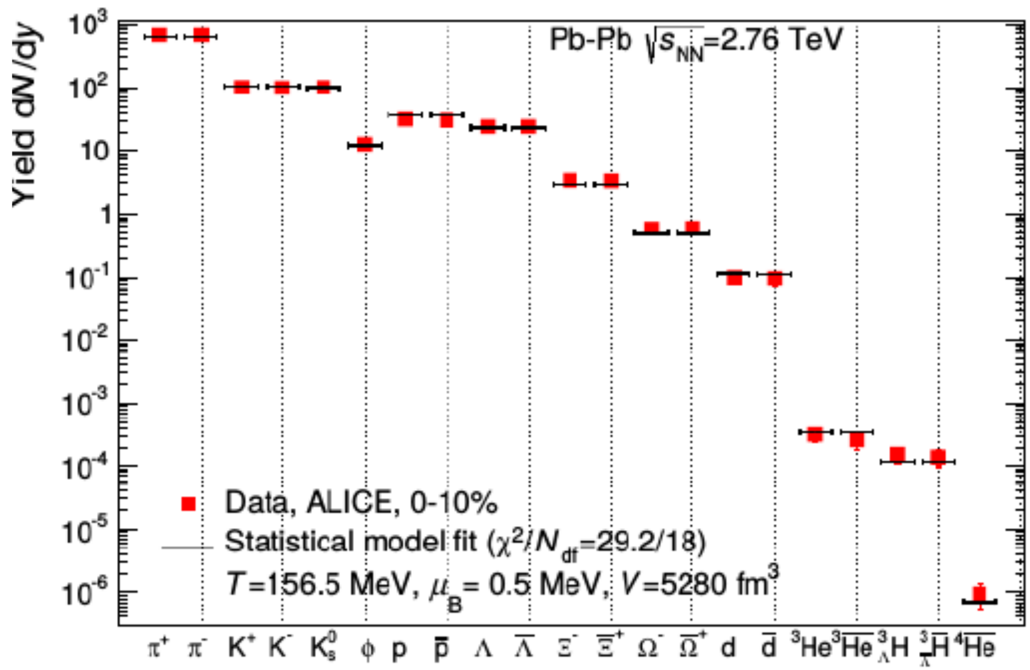
Phase boundaries from first moments



$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp\left[\left(E_i - \mu_i \right) / T \right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

$$\chi^2 = \sum_{k=1}^{n} \frac{\left(\langle N_k^{\exp} \rangle - \langle N_k^{HRG} \rangle \right)^2}{\sigma_k^2}$$



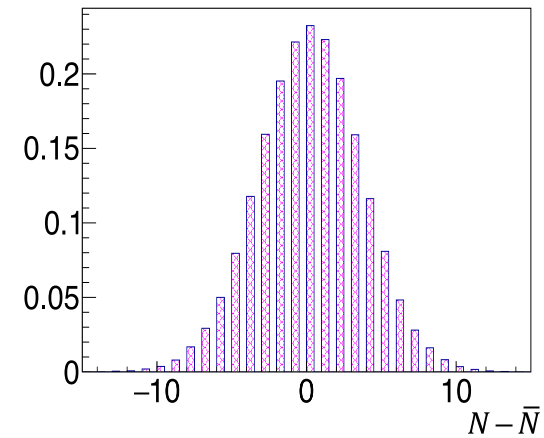
ALICE, PLB 726 (2013) 610

J. Stachel, A. Andronic, P. Braun-Munzinger and K. Redlich
J. Phys. Conf. Ser. 509 (2014) 012019

works in the energy range spanning by 3 orders of magnitude! y axis: 9 orders of magnitude!

Exploring the structure of matter

$$\hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial(\mu_N/T)^n} \qquad \frac{P}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu_{B,Q,S})$$



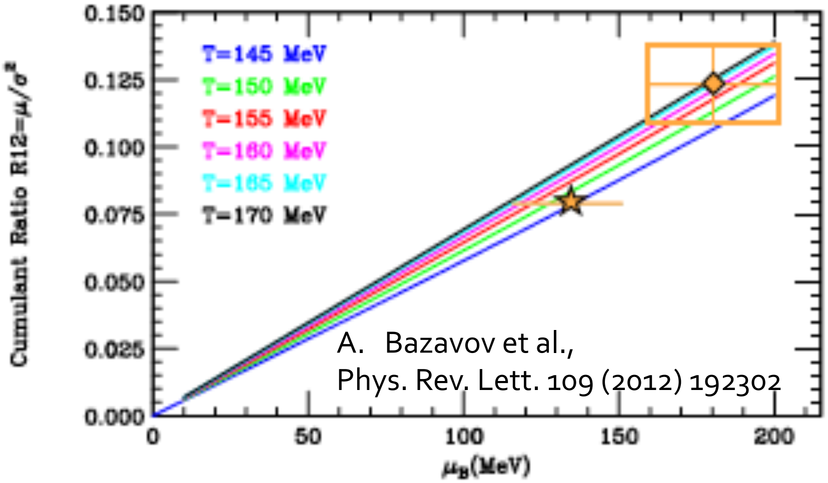
$$VT^3\,\hat{\chi}_2^N = \left\langle (N-\bar{N})^2 \right\rangle - \left\langle N-\bar{N} \right\rangle^2 \equiv c_2(N-\bar{N})$$

Do ratios eliminate volume dependencies?

$$\frac{c_4(N-\bar{N})}{c_2(N-\bar{N})} \equiv k\sigma^2 = \frac{\hat{\chi}_4^N}{\hat{\chi}_2^N} \qquad \frac{c_3(N-\bar{N})}{c_2(N-\bar{N})} \equiv S\sigma = \frac{\hat{\chi}_3^N}{\hat{\chi}_2^N}$$

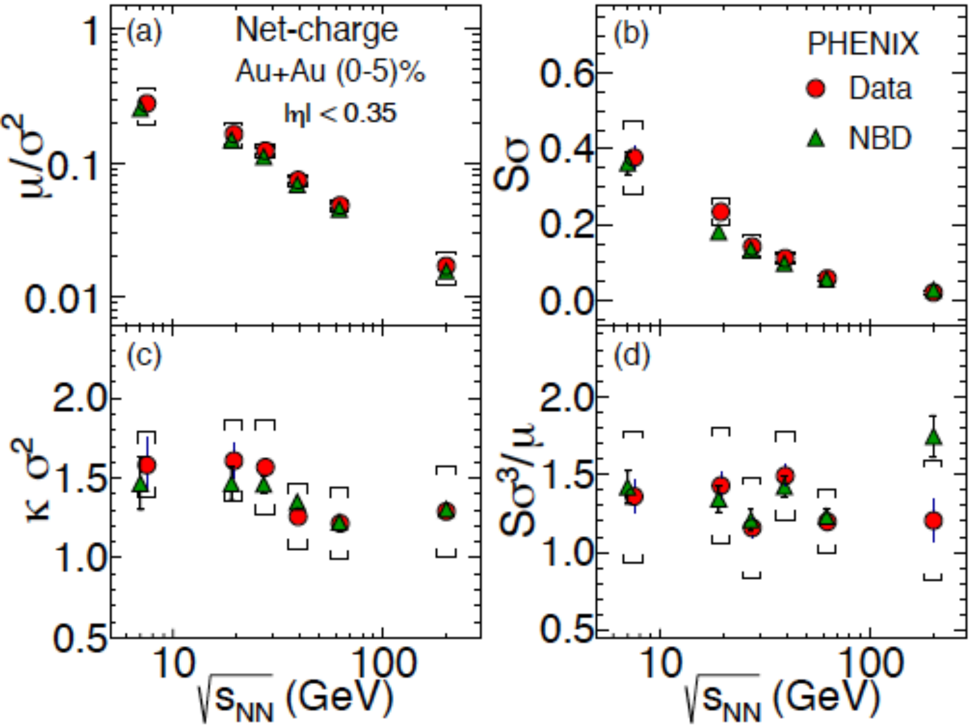
- ***Assumptions:***
  - *Volume is fixed in each event*
  - *Conservations are imposed on the averages*
- ***Reality:***
  - *Volume fluctuates from E-to-E*
  - *Conservations depend on the acceptance (p_T, y)*

locating phase boundaries



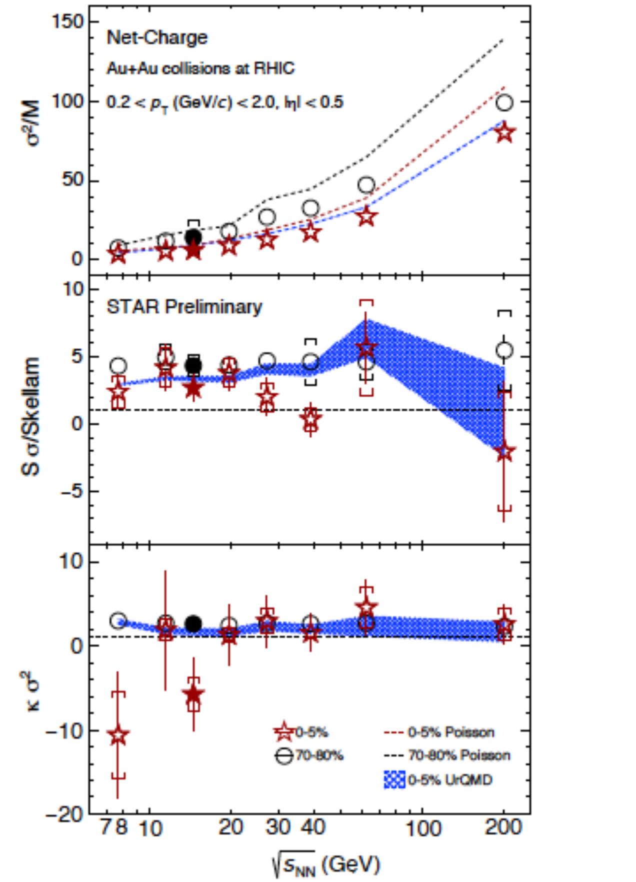
Some experimental results

PRC93 (2016) 011901(R) PHENIX



Data matches with the difference of two Negative Binomial densities fitted to the single multiplicity distributions

STAR PRL 112 32302 (2014) + QM15

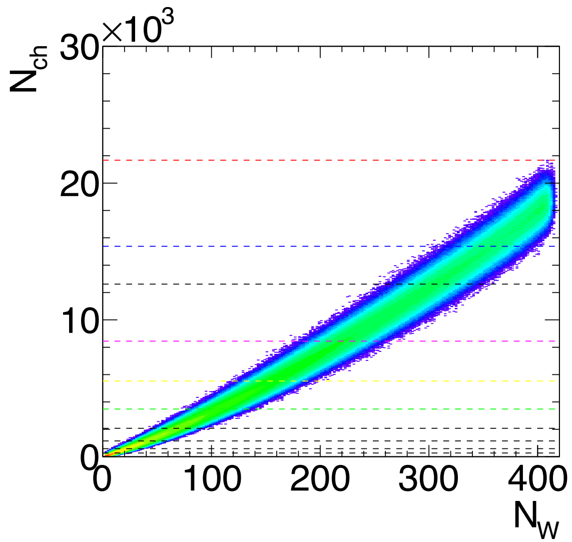


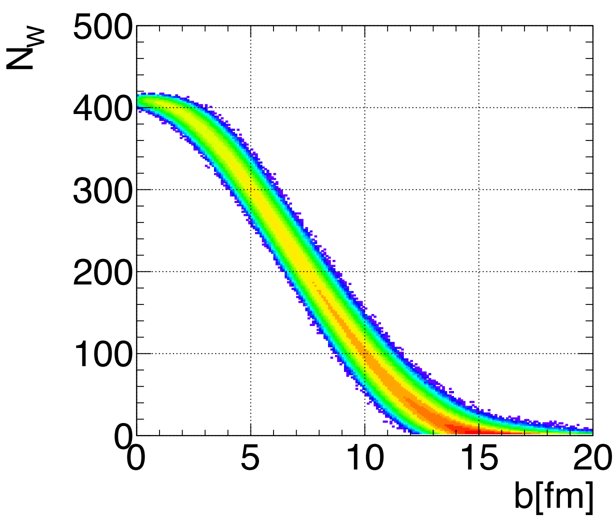
Consistent with the Skellam Baseline

Larger error bars for $k\sigma^2$

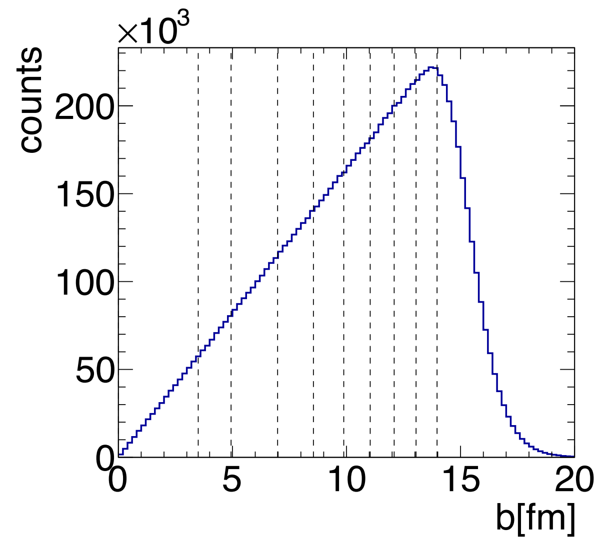
Link to the volume, different approaches

Experimental approaches:





Theoretical approach:



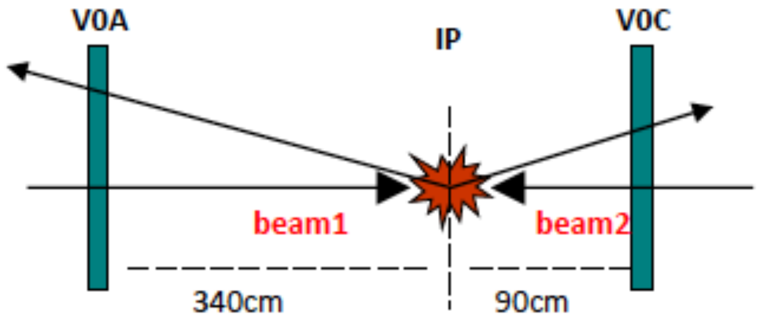
- **Each approach gives:**
  - Similar $\langle N_w \rangle$
  - Very different $\langle N_W^n \rangle$

For higher moments centrality selection is crucial!

Centrality selection in ALICE

One of the different methods

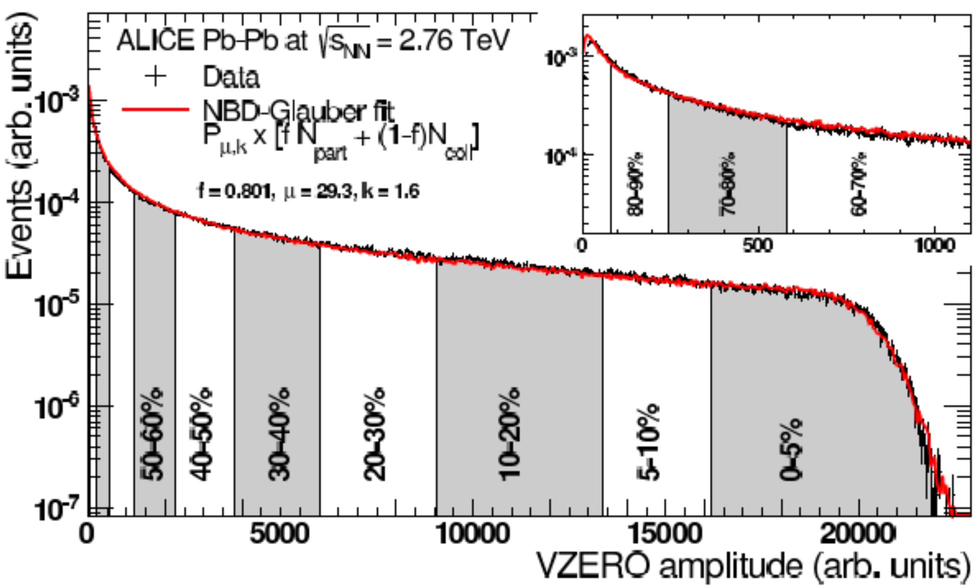
ALICE Phys.Rev. C88 (2013) no.4, 044909

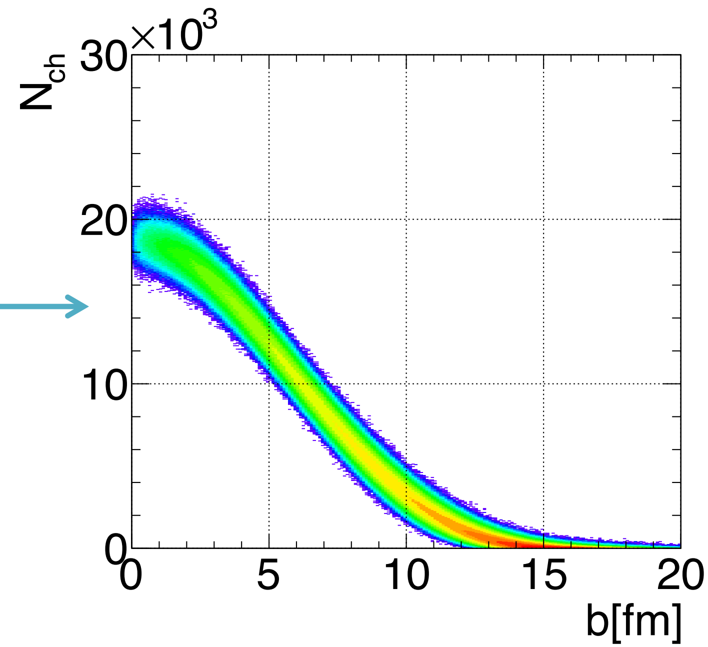


$2.8 < \eta < 5.1$ $\quad -3.7 < \eta < -1.7$

$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \frac{\left(\mu/k\right)^n}{\left(\mu/k+1\right)^{n+k}}$$

$$N = fN_W + (1-f)N_{coll}$$



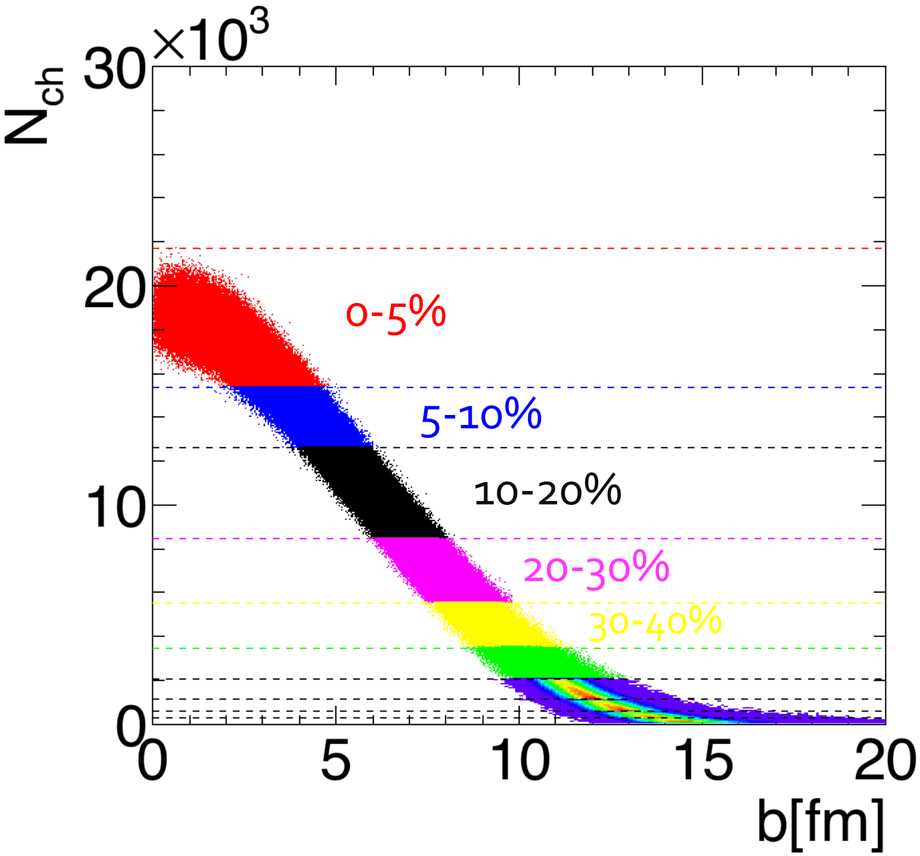


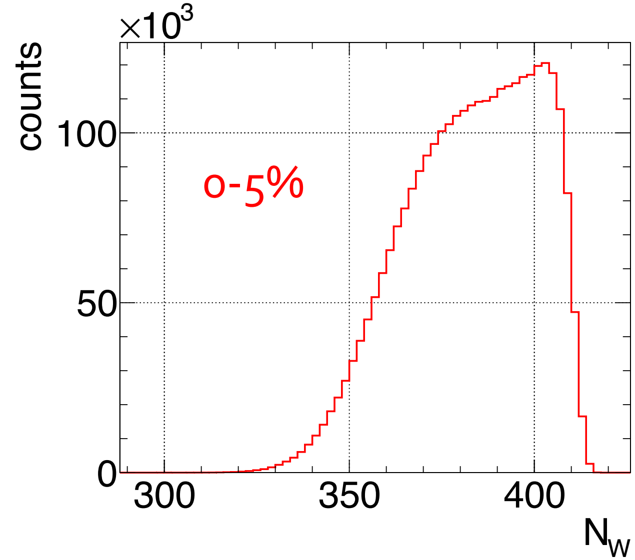
Building the model, Initial fluctuations

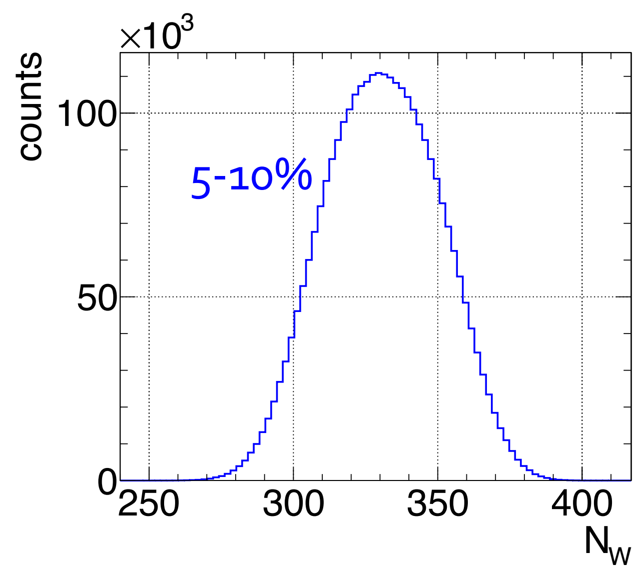
MC Glauber initial fluctuations:

input parameters from ALICE

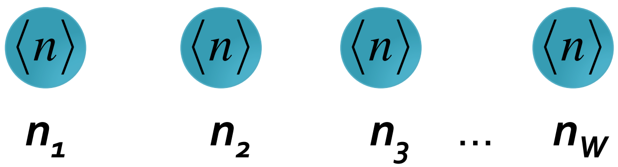
ALICE Phys.Rev. C88 (2013) no.4, 044909



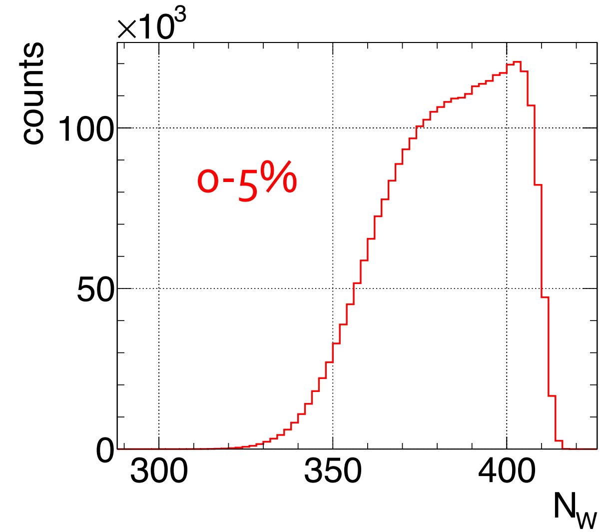


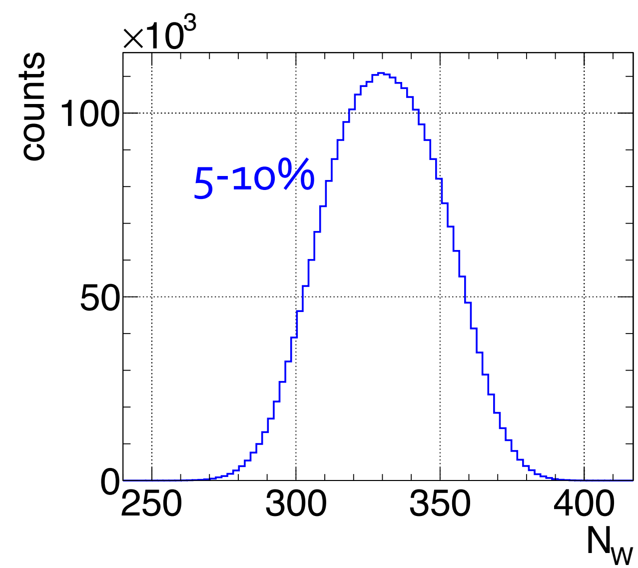


Building the model, Particle production

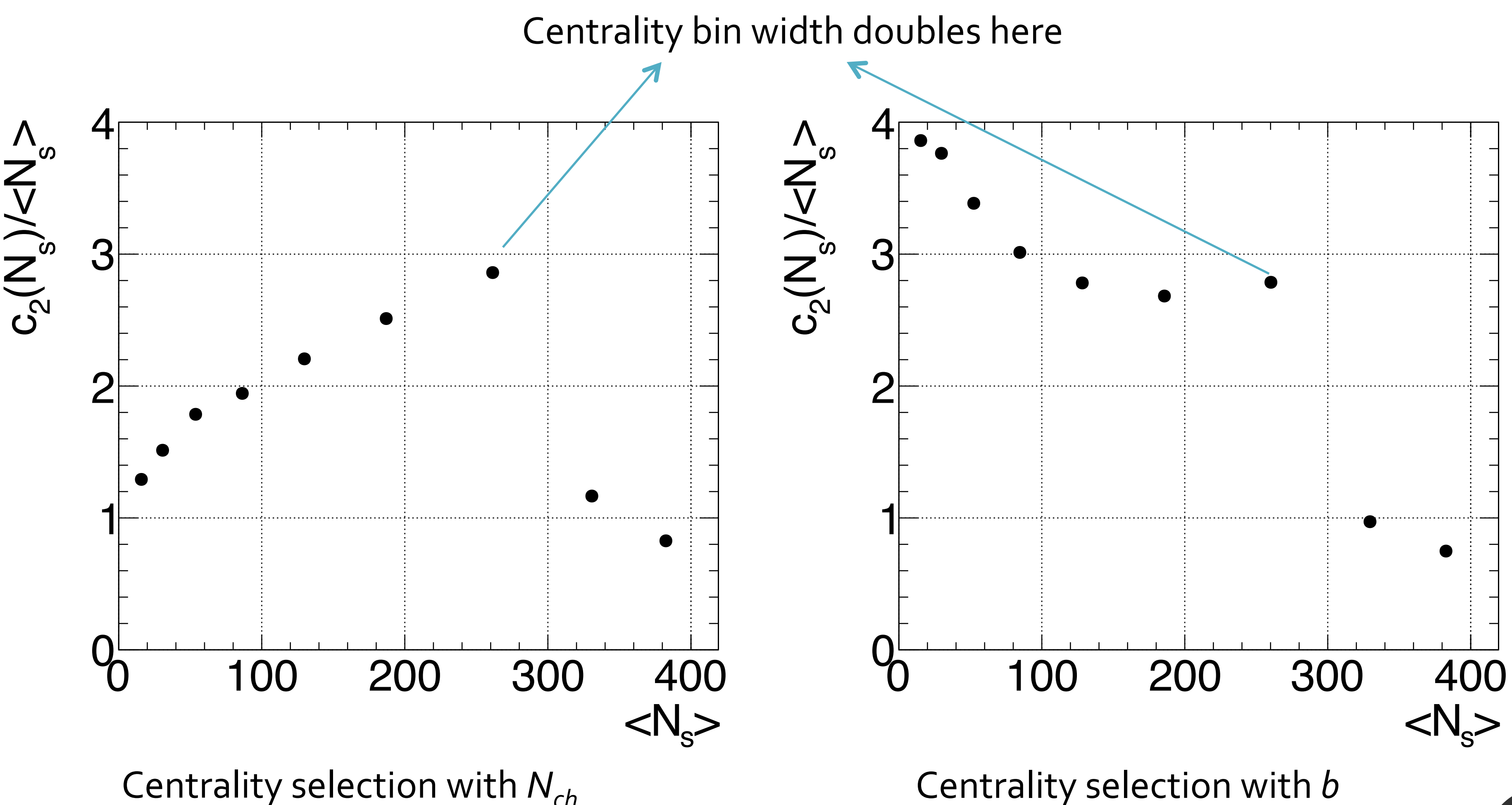


- ***N_W fluctuates with the Glauber initial conditions***
- ***Each source is treated Grand Canonically***
- ***Mean proton multiplicities $\langle p \rangle$, $\langle \bar{p} \rangle$ from ALICE Pb+Pb@2.76 TeV***
- ***Expected results without volume fluctuations:***
  - particles: $c_n = N_w \langle n \rangle = \langle p \rangle = \langle \bar{p} \rangle$
    $c_n / c_2 = 1$
  - net-particles: $c_n = \langle p \rangle + (-1)^n \langle \bar{p} \rangle$
    $c_3 / c_2 = 0, \quad c_4 / c_2 = 1$

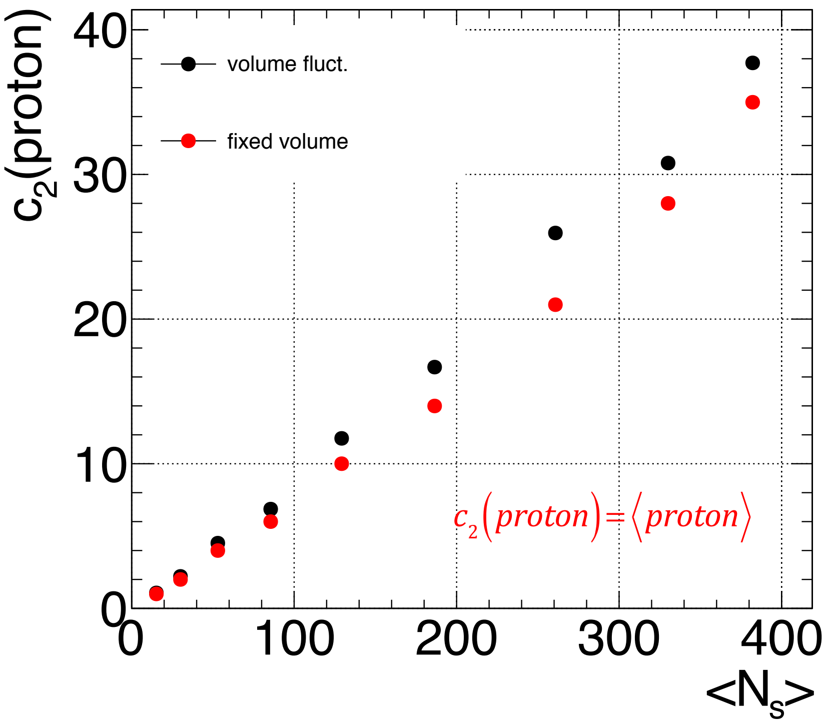


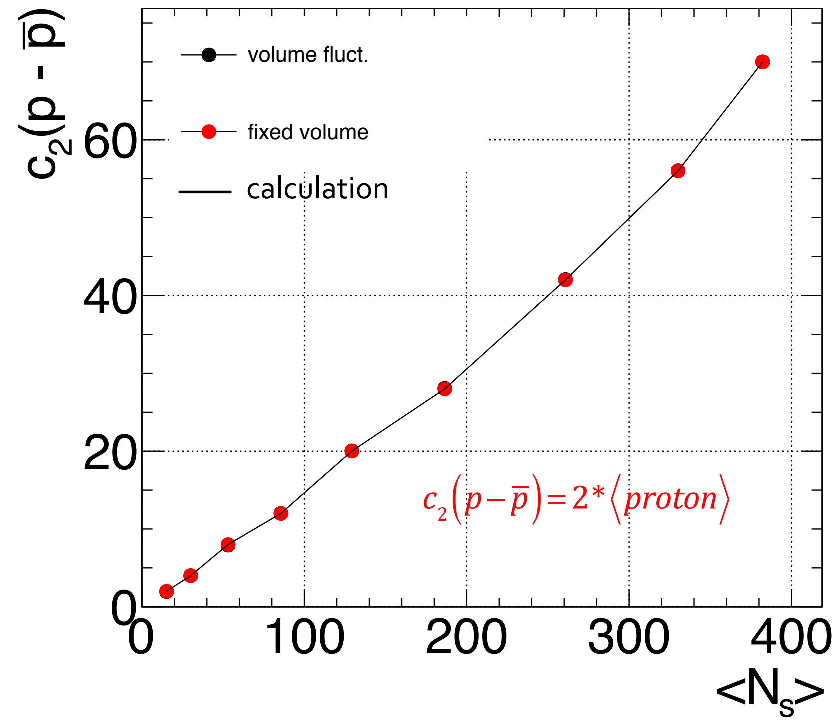


Fluctuations of wounded nucleons



Produced particles, second cumulants



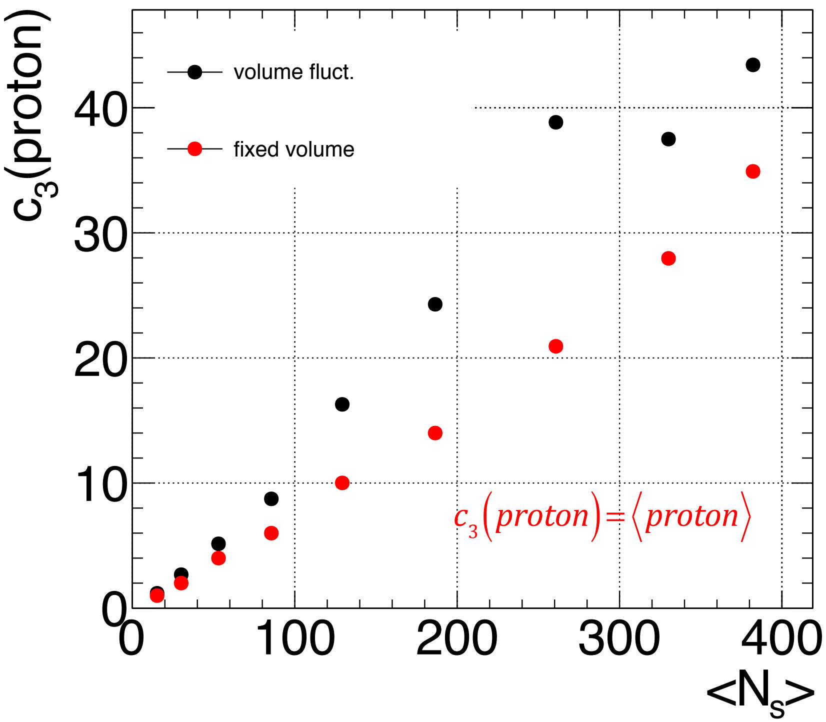


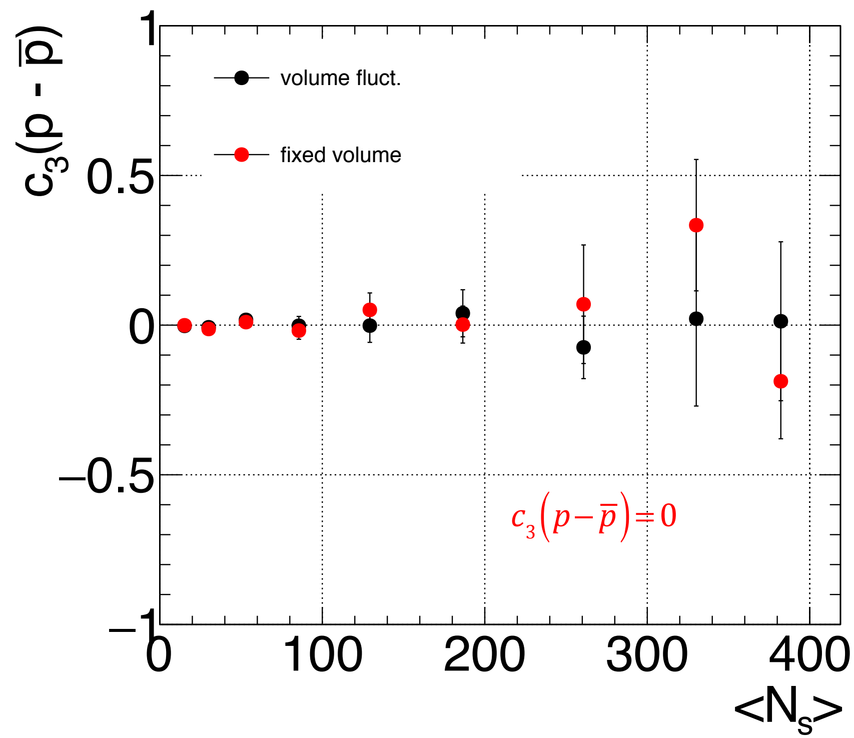
$$c_2(p-\bar{p})=\langle N_w\rangle c_2(n-\bar{n})+\langle n-\bar{n}\rangle^2 c_2(N_w)$$

$n,\bar{n}$ from single wounded nucleon

vanishes for ALICE

Third cumulants



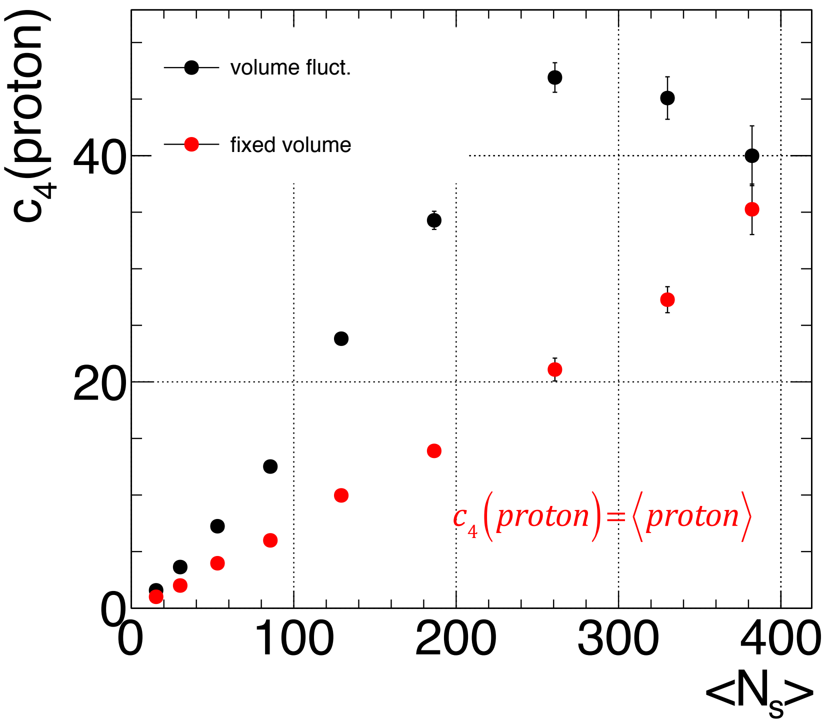


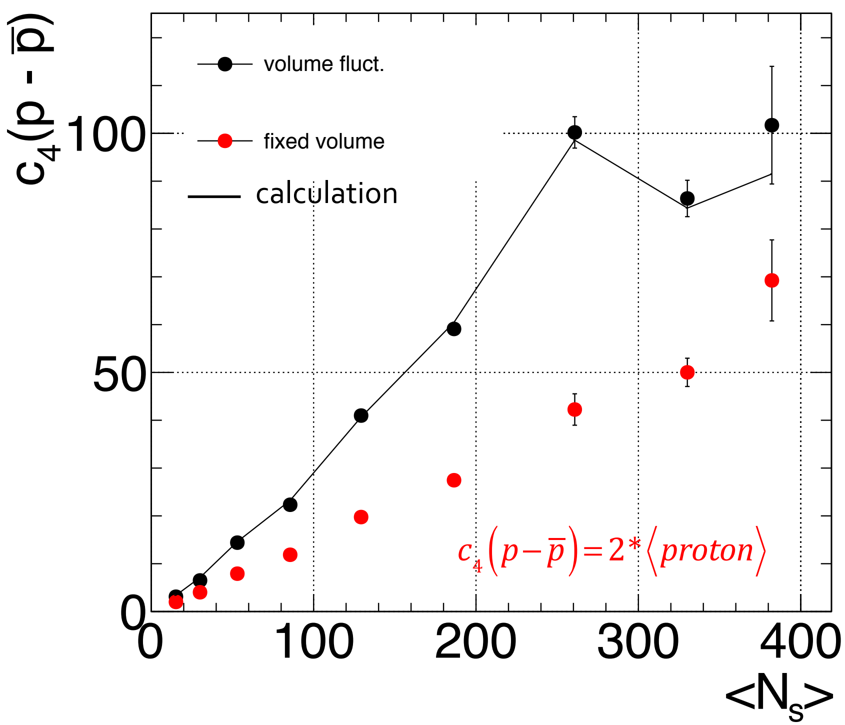
$$c_3(p-\bar{p}) = \langle N_w \rangle c_3(n-\bar{n}) + \langle n-\bar{n} \rangle (\ldots)$$

$n, \bar{n}$ from single wounded nucleon

vanishes for ALICE

Fourth cumulants





$$c_4\left(p-\bar{p}\right)=\left\langle N_w\right\rangle c_4\left(n-\bar{n}\right)+3c_2\left(n-\bar{n}\right)^2 c_2\left(N_w\right)+\left\langle n-\bar{n}\right\rangle\left(...\right)$$

vanishes for ALICE

$n,\bar{n}$ from single wounded nucleon

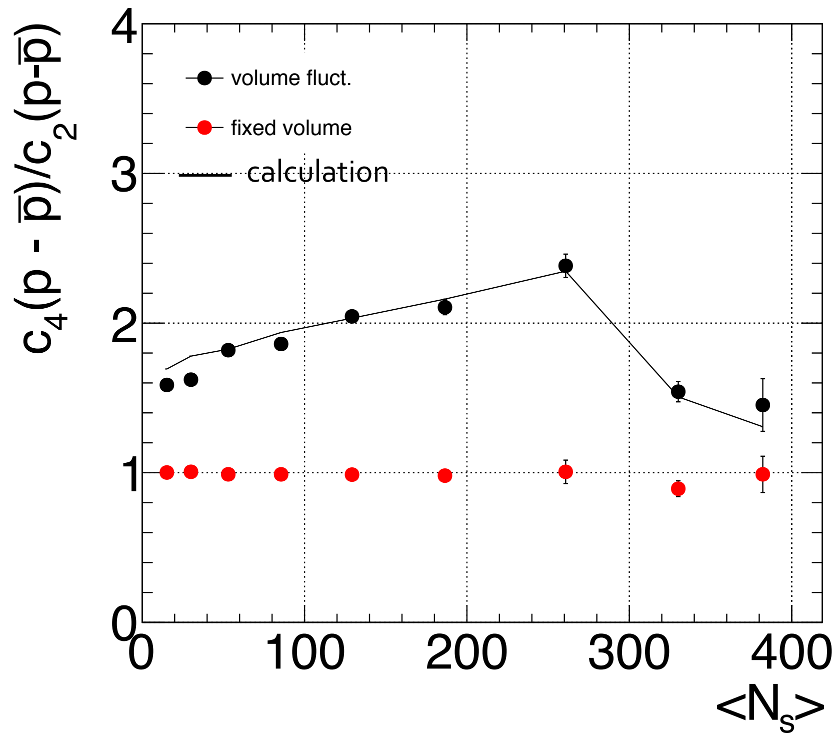
Cumulant Ratios

$$\frac{c_4(p-\bar{p})}{c_2(p-\bar{p})} = \frac{c_4(n-\bar{n})}{c_2(n-\bar{n})} + 3c_2(n-\bar{n})\frac{c_2(N_w)}{\langle N_w \rangle}$$

Additional term due to volume fluctuations

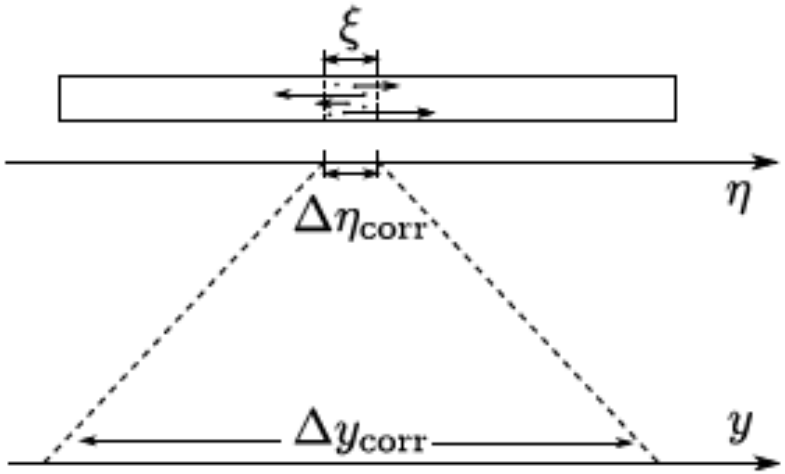
which is missing in famous:

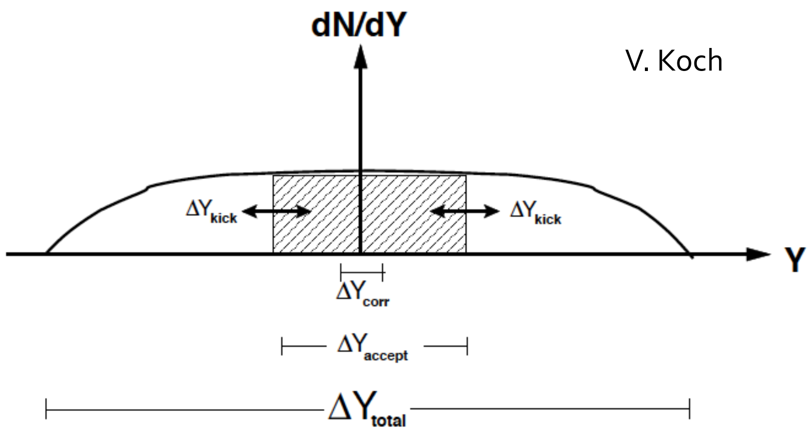
$$\frac{c_4(p-\bar{p})}{c_2(p-\bar{p})} = \frac{\chi_4^B}{\chi_2^B}$$



Acceptance selection

Bo Ling and M. Stephanov, Phys.Rev. C93 (2016) no.3, 034915





V. Koch

$$\Delta Y_{accept} \gg \Delta Y_{corr}$$

$$\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}$$

Thermal broadening of the rapidity window

Acceptance should be large and small at the same time ☺

conservation starts to dominate for larger acceptance

Finite acceptance effects

n – accepted baryons
N – 4π baryons

$$\alpha = \langle n \rangle / \langle N \rangle$$

$$B(n;N,\alpha) = \frac{N!}{n!(N-n)!} \alpha^n (1-\alpha)^{N-n}$$

$$P(n) = \sum_N B(n;N,\alpha) P(N)$$

probability distribution in 4π

$$\langle n \rangle = \alpha \langle N \rangle, \quad \langle \bar{n} \rangle = \alpha \langle \bar{N} \rangle$$

$$\langle n^2 \rangle = \alpha^2 \langle N^2 \rangle - \alpha^2 \langle N \rangle + \alpha \langle N \rangle$$

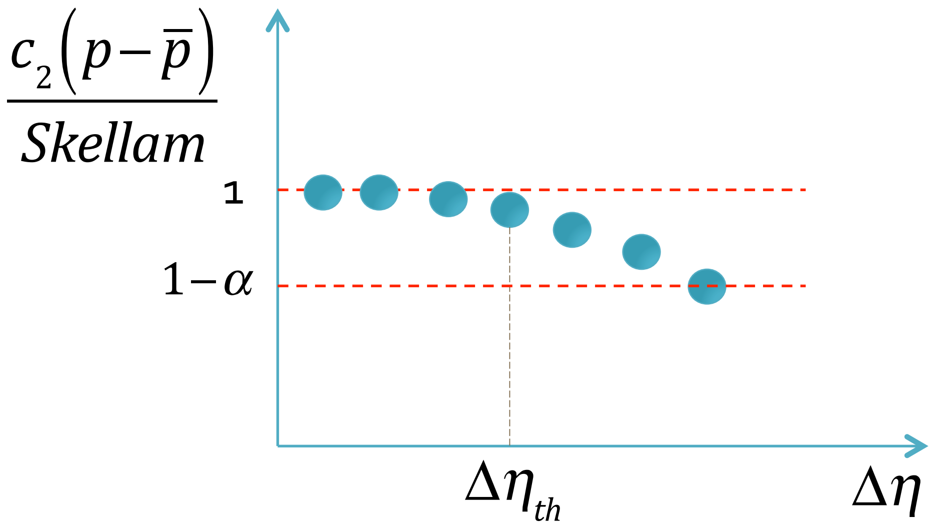
$$\langle n\bar{n} \rangle = \alpha^2 \langle N\bar{N} \rangle$$

$$\frac{c_2(n-\bar{n})}{Skellam} = \frac{c_2(n-\bar{n})}{\alpha(\langle N \rangle + \langle \bar{N} \rangle)} = \alpha \frac{c_2(N-\bar{N})}{\langle N \rangle + \langle \bar{N} \rangle} + 1 - \alpha = 1 - \alpha$$

1. $\alpha \to 1 \qquad c_2(n-\bar{n}) \to 0$
2. $\alpha \to 0 \qquad c_2(n-\bar{n}) \to 1$ (trivial fluctuations)

final result does not depend on $P(N)$

Recipe for net-particle analysis



$$\frac{c_2(p-\bar{p})}{Skellam} = 1-\alpha$$

$$\alpha = \frac{\langle n_p^{acc} \rangle}{\langle N_B^{4\pi} \rangle}$$

Proposed comparison procedure:

- Perform analysis for $\Delta\eta > \Delta\eta_{th}$
- Calculate acceptance factors based on experimental data $\alpha(\Delta\eta) = \frac{p^{acc}}{B^{4\pi}}$
- Correct the experimental data
- Compare to LQCD

Summary and Outlook

- ***E-by-E fluctuation signals are useful probes of singularities near phase boundaries***

- ***For direct comparison between experiment and theory the following have to be considered:***
  - Fluctuations of wounded nucleons have to be understood and subtracted from the data
  - The effect is more important at low energies
  - For the ALICE this does not cause a trouble up to the fourth cumulants
  - Measuring npart fluctuations leads to recipe for analysis

  - Phase space dependence of fluctuation measurements have to be studied
  - Modifications driven by the conservation lows should be corrected

Much more on these and other aspects of fluctuation measurements:
P. Braun-Munzinger, A. R, J. Stachel: ***Experimental Considerations on E-by-E fluctuations***
To be published soon