Python based particle tracking code for monitor design

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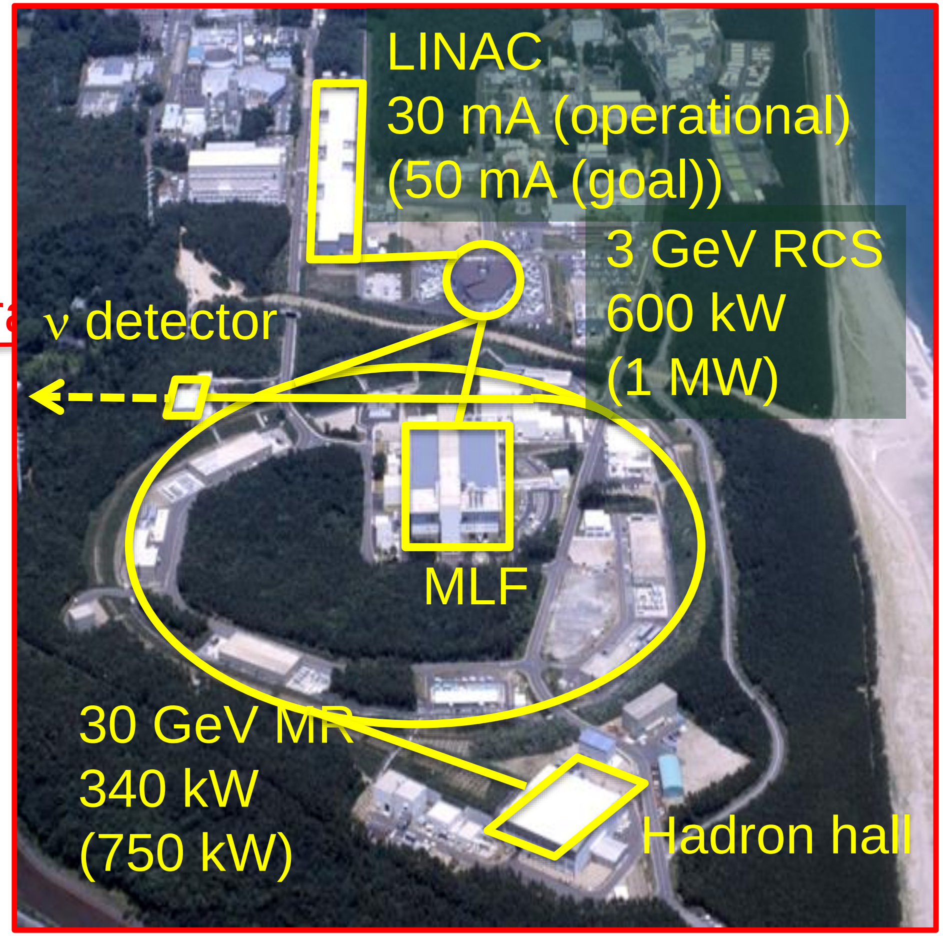
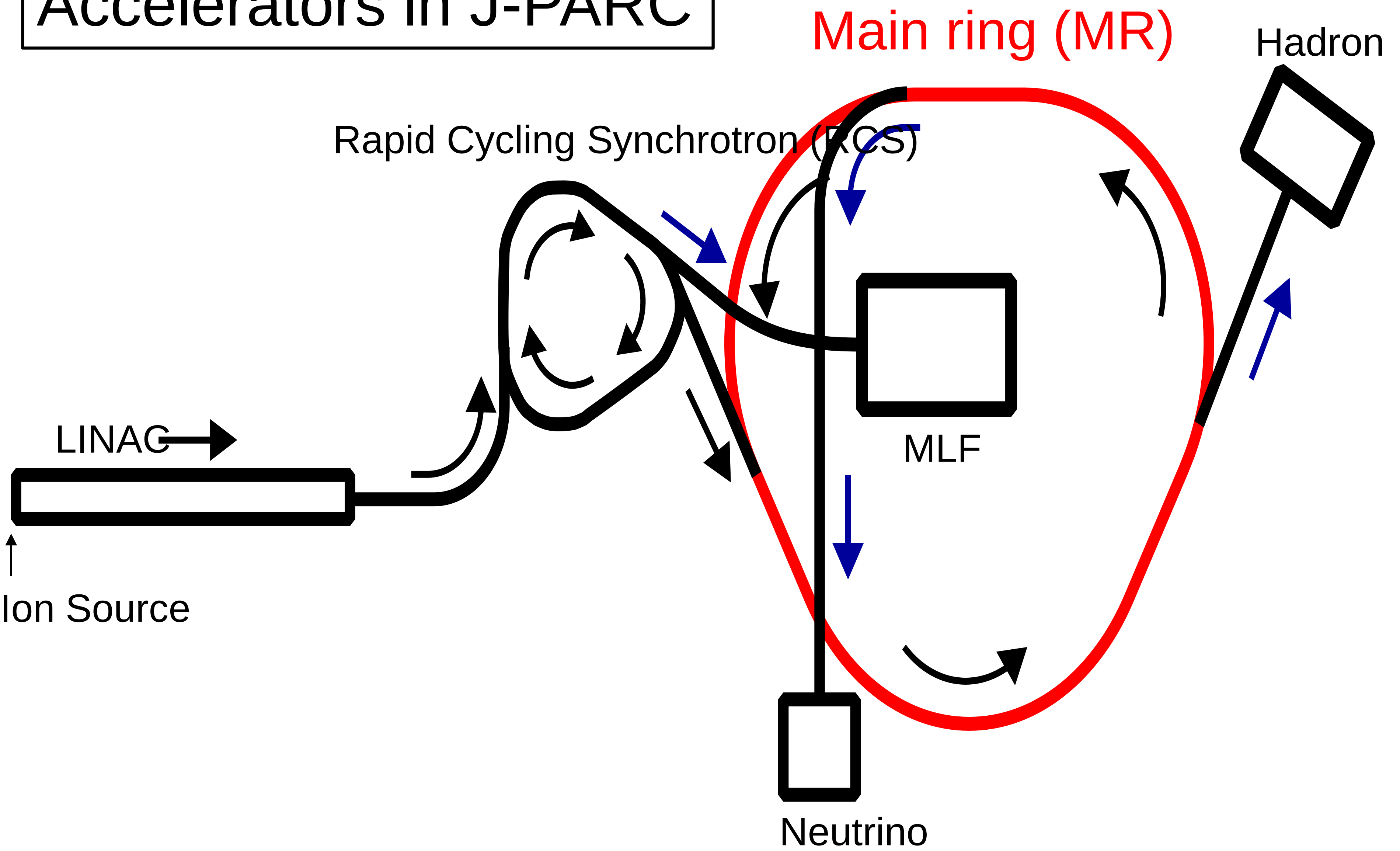
Japan **P**roton **A**ccelerator **R**esearch **C**omplex

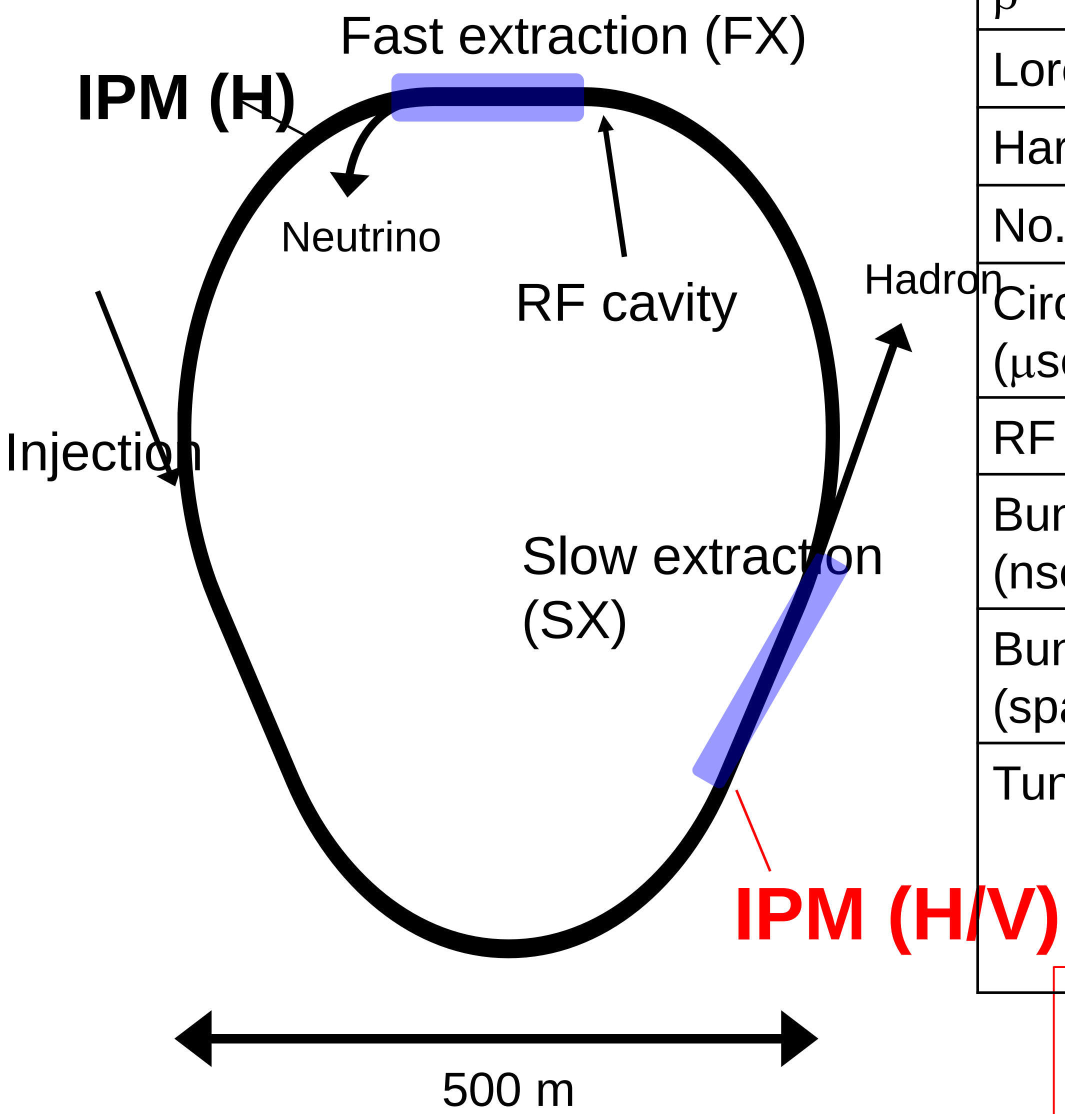


Accelerators in J-PARC



http://j-parc.jp/Acc/ja/abou

Main ring (MR)

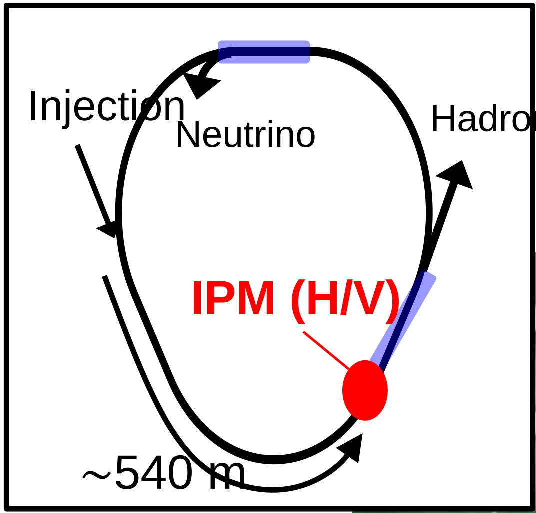


Total length (m)	1568	
Energy (GeV)	3	30
β	0.9712	0.9995
Lorentz γ	4.20	32.97
Harmonic number	9	
No. of bunches	8	
Circulating period (μsec)	5.38	5.23
RF frequency (MHz)	1.67	1.72
Bunch length (time) (nsec)	200	70
Bunch length (space) (m)	60	20
Tune	FX: ν_x=22.40, ν_y=20.75 SX: ν_x=22.30, ν_y=20.78	

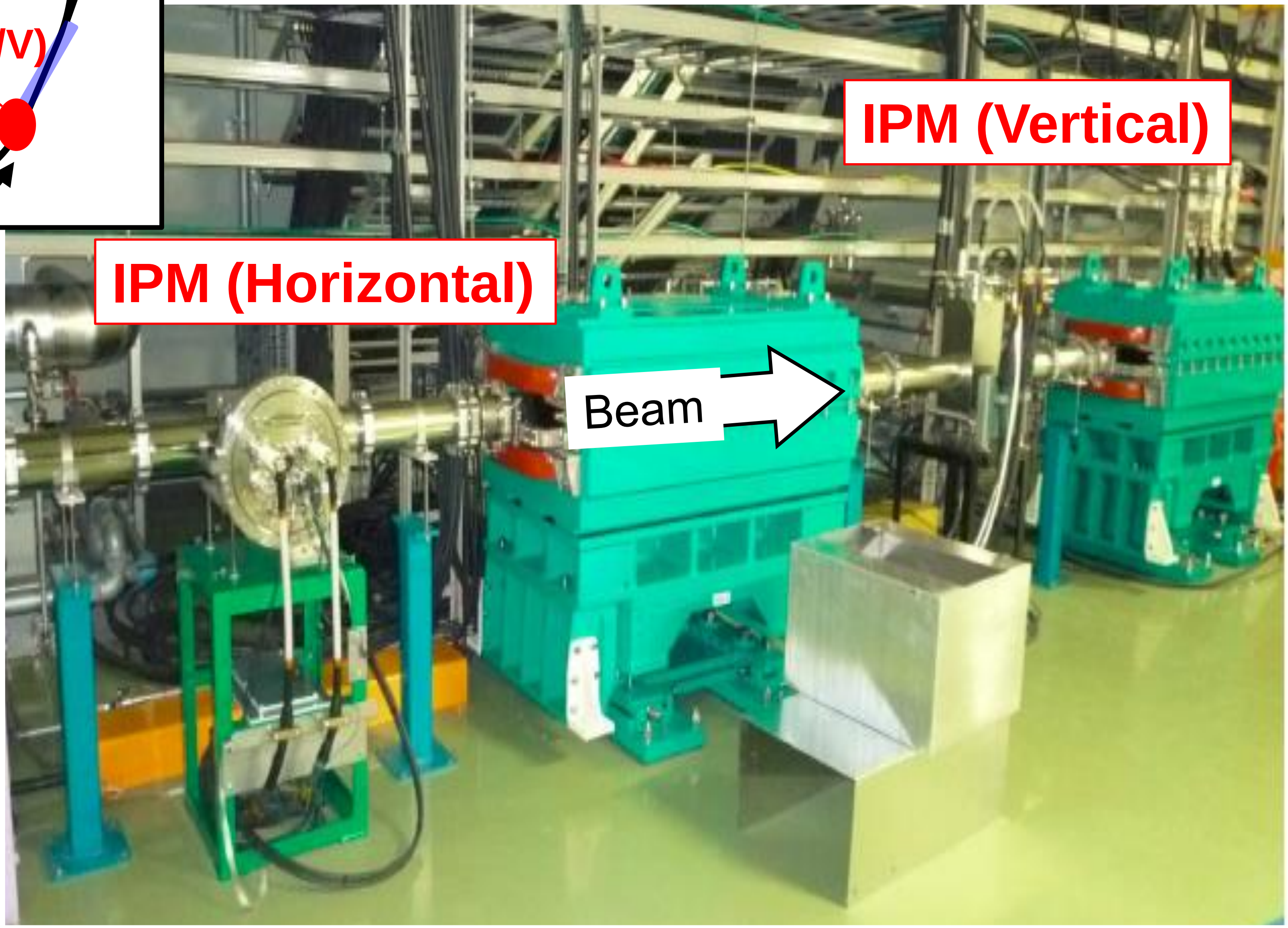
High intensity⇒reduction of bea

⇩

Beam orbit tuning with accurate beam profile infor



- 540 m downstream from injection point
- Dispersion = 0



contents

- Position displacement?
  - Main Source?
  - Equation of motion of electron in IPM
- New python based simulation code
- Demonstration of the code
- No summary

Tracking Error Sources

- The error sources which induce position x displacement are,,,,
- Error E field: Space charge of the beam and guide E field

Which one is the most important???
How much we should reduce the error source???
A simulation code will answer but it is worth while to check analytically

- Initial velocity of the detached electron (or ion)
- Electron-gas molecule collision -> Outside of this presentation

Eq. of motions (preliminary)

$$\vec{E}(x,y,z,t) = \overrightarrow{E_g}(x,y,z) + \overrightarrow{E_s}(x,y,z,t)$$

$$\vec{B}(x,y,z) = \overrightarrow{B_g}(x,y,z)$$

And using $B_x/B=\theta_x$, $B_y/B=1$, $B_z/B=\theta_z$

$$\frac{\vec{E}\times\vec{B}}{B^2} \approx \left(\frac{E_y}{B}\theta_z - \frac{E_z}{B}, \frac{E_z}{B}\theta_x - \frac{E_x}{B}\theta_z, \frac{E_x}{B} - \frac{E_y}{B}\theta_x\right)$$

$$a_x = -\frac{e}{m}\left(E_x + v_y B_z - v_z B_y\right)$$

$$a_y = -\frac{e}{m}\left(E_y + v_z B_x - v_x B_z\right)$$

$$a_z = -\frac{e}{m}\left(E_z + v_x B_y - v_y B_x\right)$$

$$a_x = -\omega\frac{\vec{E}\times\vec{B}}{B^2}(z) - \frac{e}{m}\left(\vec{v}\times\vec{B}\right)(x) - \frac{e}{m}E_y\theta_x$$

$$a_z = \omega\frac{\vec{E}\times\vec{B}}{B^2}(x) - \frac{e}{m}\left(\vec{v}\times\vec{B}\right)(z) - \frac{e}{m}E_y\theta_z$$

Using the imaginary variables and parameters,,,,

$\tilde{a} = a_x + i a_z$, $\tilde{v} = v_x + i v_z$, $\tilde{r} = x + iz$, $\tilde{\theta} = \theta_x + i\theta_z$, $\tilde{E} = E_x + iE_z$, $\widetilde{v\times B} = \left(\vec{v}\times\vec{B}\right)(x) + i\left(\vec{v}\times\vec{B}\right)(z)$, $\frac{\widetilde{E\times B}}{B^2} = \frac{\vec{E}\times\vec{B}}{B^2}(x) + i\frac{\vec{E}\times\vec{B}}{B^2}(z)$

$$\tilde{a} = -i\omega\tilde{v} - \omega\tilde{\alpha} - i\omega^2\alpha_y\tilde{\theta}t$$

$\frac{\tilde{E}}{B}$ $\frac{E_y}{B}$

Eq. of motions (preliminary)

$$\tilde{a} = -i\omega\tilde{v} - \omega\tilde{\alpha} - i\omega^2\alpha_y\tilde{\theta}t$$

If α and θ is constant value

Initial velocity

$$\tilde{v} = \tilde{v}_0 e^{-i\omega t} - \omega\alpha_y\tilde{\theta}t + i(\alpha_y\tilde{\theta} - \tilde{\alpha})(e^{-i\omega t} - 1)$$

$$\tilde{r} = \frac{i}{\omega}\widetilde{v_0}(e^{-i\omega t} - 1) - \omega\alpha_y\tilde{\theta}\cdot\frac{1}{2}t^2 + i(\tilde{\alpha} - \alpha_y\tilde{\theta})\left\{-\frac{i}{\omega}(e^{-i\omega t} - 1) + t\right\}$$

Initial position is (0,0)

On the detector plane

Flight length

$$\tilde{r}(t = TOF)$$
$$= \frac{v_0}{\omega}\left\{e^{-i(\phi+\phi')} + e^{-i\left(\frac{\pi}{2}+\phi'\right)}\right\} + \tilde{\theta}\cdot FL$$
$$- i(\tilde{\alpha} - \alpha_y\tilde{\theta})\frac{1}{\omega}\left(e^{-i\phi} + e^{-i\frac{\pi}{2}}\right) + i(\tilde{\alpha} - \alpha_y\tilde{\theta})\cdot TOF$$

Here, $ie^{-i\omega\cdot TOF} = e^{-i\phi}$ $\quad \widetilde{v_0} = v_0 e^{-i\phi'}$

Eq. of motions (preliminary)

The position displacement due to initial TKE and error fields can be expressed as,

$$\tilde{r}(t = TOF) = \frac{v_0}{\omega}\left\{e^{-i(\phi+\phi')} + e^{-i\left(\frac{\pi}{2}+\phi'\right)}\right\} + \tilde{\theta}\cdot FL - i\left(\tilde{\alpha} - \alpha_y\tilde{\theta}\right)\frac{1}{\omega}\left(e^{-i\phi} + e^{-i\frac{\pi}{2}}\right) + i\left(\tilde{\alpha} - \alpha_y\tilde{\theta}\right)\cdot TOF$$

Therefore,

$$\Delta x(t = TOF) = \frac{1}{\omega}v_0\left\{cos(\phi+\phi') + cos\left(\phi' + \frac{\pi}{2}\right)\right\} - \theta_x\cdot FL + \frac{1}{\omega}\left\{r\cdot sin(\phi+\eta_x) + \left(\alpha_x - \alpha_y\theta_x\right)\right\} - \left(\alpha_z - \alpha_y\theta_z\right)\cdot TOF$$

$$\Delta z(t = TOF) = -\frac{1}{\omega}v_0\left\{sin(\phi+\phi') + sin\left(\phi' + \frac{\pi}{2}\right)\right\} - \theta_z\cdot FL + \frac{1}{\omega}\left\{r\cdot sin(\phi+\eta_z) + \left(\alpha_z - \alpha_y\theta_z\right)\right\} + \left(\alpha_x - \alpha_y\theta_x\right)\cdot TOF$$

Where,

$$r = \sqrt{\left(\alpha_x - \alpha_y\theta_x\right)^2 + \left(\alpha_z - \alpha_y\theta_z\right)^2},$$

$$cos(\eta_x) = \frac{\alpha_x - \alpha_y\theta_x}{r}, sin(\eta_x) = \frac{\alpha_z - \alpha_y\theta_z}{r}$$

$$cos(\eta_z) = \frac{\alpha_z - \alpha_y\theta_z}{r}, sin(\eta_z) = \frac{\alpha_x - \alpha_y\theta_x}{r}$$

$v_0 = \sqrt{\frac{2\cdot TKE}{m}} = 5.9E5\sqrt{TKE}\ [\frac{m}{s}]$, where TKE is in unit of [eV]

X position displacement due to error E field (preliminary)

In case the magnetic field error is negligible small, the position x displacement can be expressed as,

$$\Delta x(t = TOF) = \frac{1}{\omega} v_0 \left\{ cos(\phi + \phi') + cos\left(\phi' + \frac{\pi}{2}\right) \right\} + \frac{1}{\omega}\left\{ \sqrt{\alpha_x{}^2 + \alpha_z{}^2} \cdot sin(\phi + \eta_x) + \alpha_x \right\} - \alpha_z \cdot TOF$$

In a real IPM chamber,

Gyro motion
Initial TKE

Gyro motion
Field error: Ex, Ez

E × B drfift (Ez × By)
Field error: Ez

$$\Delta x(t = TOF) < \frac{2}{\omega} v_0 + \frac{1}{\omega}\left\{ \sqrt{\alpha_{max,x}{}^2 + \alpha_{max,z}{}^2} + \alpha_{max,x} \right\} + \alpha_{max,z} \cdot TOF$$

$\propto \frac{1}{B}$

$\propto \frac{1}{B^2}$

$\propto \frac{1}{B}$

X position displacement due to error B field (preliminary)

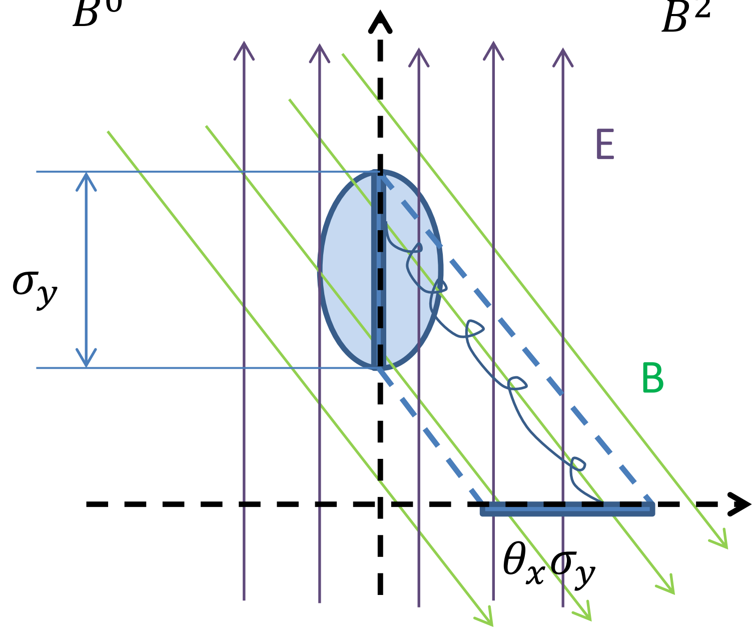
Geometrical shift
Bx/B

Gyro motion
Field error: Bx/B, Bz/B

E × B drfift (Ey × Bz)
Field error: Bz

$$\Delta x(t = TOF)$$
$$< \theta_{max,x} \cdot \sigma_y + \frac{1}{\omega}\left\{\alpha_y\sqrt{{\theta_{max,x}}^2 + {\theta_{max,z}}^2} - \alpha_y\theta_{max,x}\right\} + \alpha_y\theta_{max,z} \cdot TOF$$

$\propto \frac{1}{B^0}$ $\propto \frac{1}{B^2}$ $\propto \frac{1}{B^2}$



Python based particle tracking code

- Python based particle tracking code to design beam monitors which utilizes charged particles like IPM (detached electron and ions), Laser wire scanner (detached electron), Faraday cup based CT (secondary electron escape), etc.
- The fields in which the particles are moving,
  - Beam space charge effect
    - Only 2D electric field: Relativistic beam condition
  - 3D electric and magnetic fields from another code which will be used to control the particle motion
  - Space charge effect of the tracked particles ensemble are not included -> single particle motion
- Was used to check CPS IPM (under development) performance

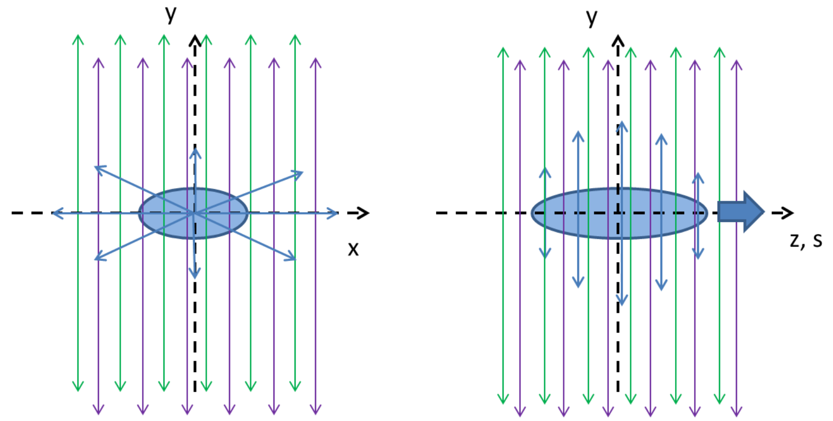
Beam profile is 3 D Gaussian profile and stable -> Frozen model

$$\rho_t(x,y)\rho_l(t) = \frac{1}{2\pi\sigma_x\sigma_y} exp\left\{-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(y-y_0)^2}{2\sigma_y^2}\right\} \frac{Q_b}{(2\pi)^{1/2}\beta_b C\sigma_t} exp\left\{-\frac{(z-\beta_b Ct + z_0)^2}{2(\beta_b C\sigma_t)^2}\right\}$$

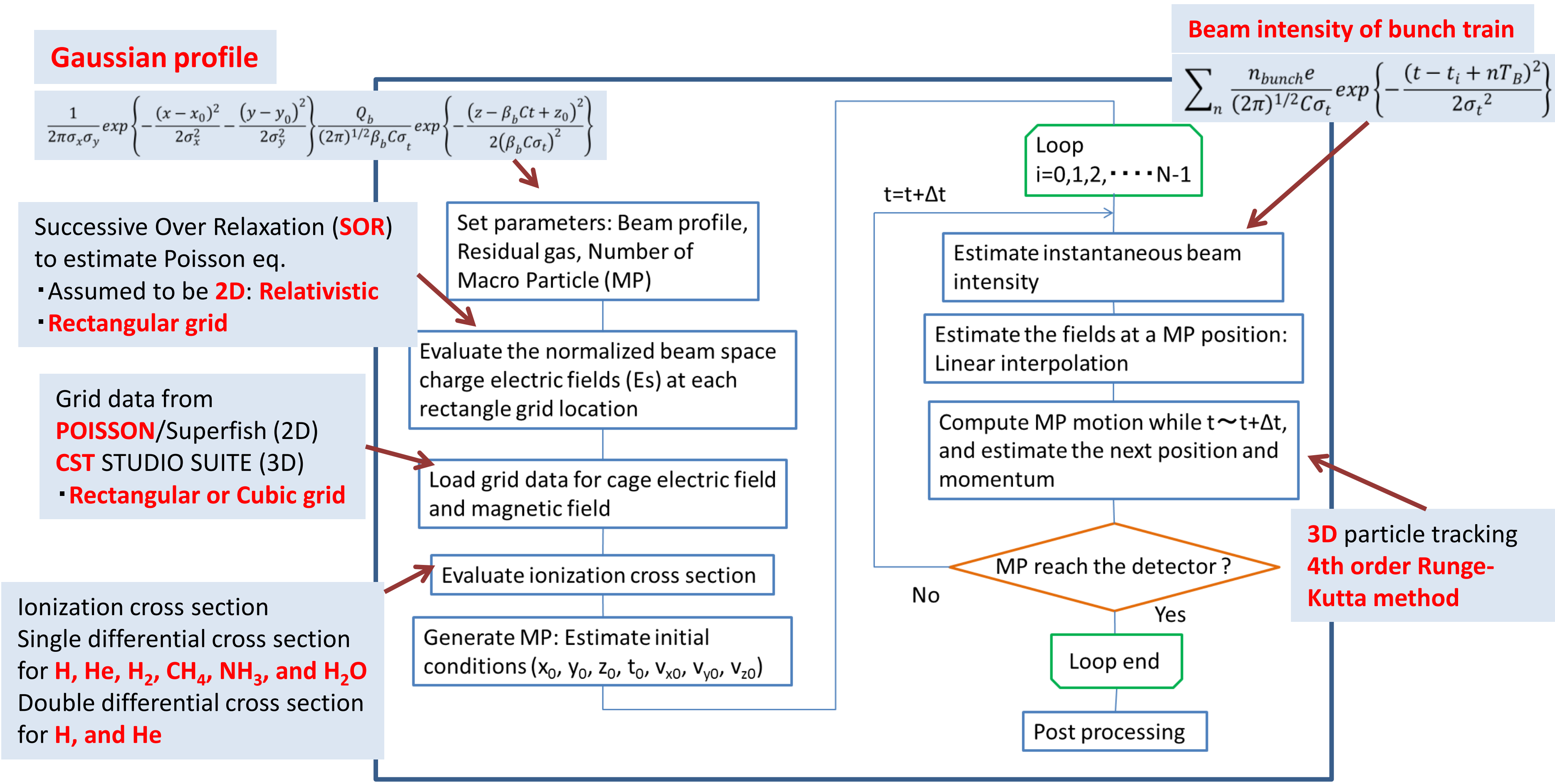
Ec(x,y,z): Cage E field

Es(x,y): Space charge E field
Assum.: Lolentz β=1 -> Es(z)=0, Bs=0

B(x,y,z): Guide B field



Python based 3D particle tracking code
Profile simulator for IPM design

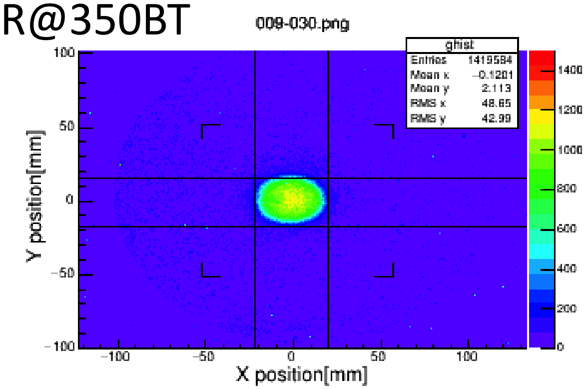


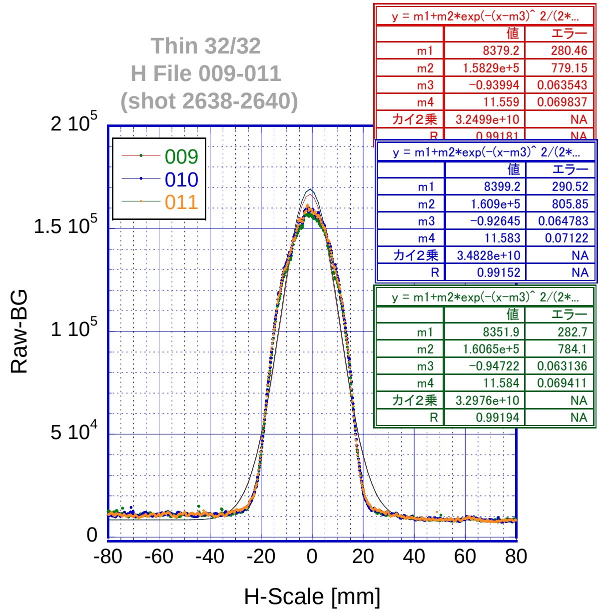
Flow chart of the simulator

But in a real case, the profile could be far from the Gaussian-shape: J-PARC MR

Transversal profile

OTR@350BT





y = m1+m2*exp(-(x-m3)^ 2/(2*...

	値	エラー
m1	8379.2	280.46
m2	1.5829e+5	779.15
m3	−0.93994	0.063543
m4	11.559	0.069837
カイ2乗	3.2499e+10	NA
R	0.99181	NA

y = m1+m2*exp(-(x-m3)^ 2/(2*...

	値	エラー
m1	8399.2	290.52
m2	1.609e+5	805.85
m3	−0.92645	0.064783
m4	11.583	0.07122
カイ2乗	3.4828e+10	NA
R	0.99152	NA

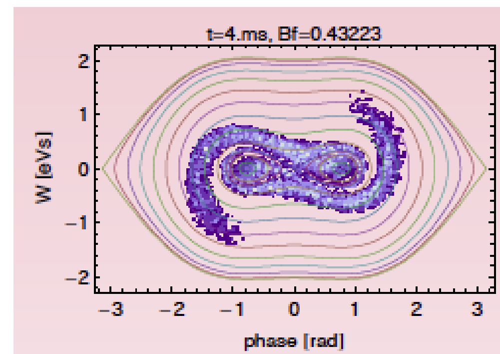
y = m1+m2*exp(-(x-m3)^ 2/(2*...

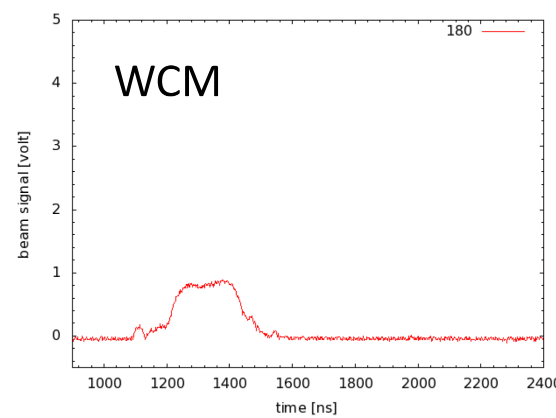
	値	エラー
m1	8351.9	282.7
m2	1.6065e+5	784.1
m3	−0.94722	0.063136
m4	11.584	0.069411
カイ2乗	3.2976e+10	NA
R	0.99194	NA

Longitudinal profile

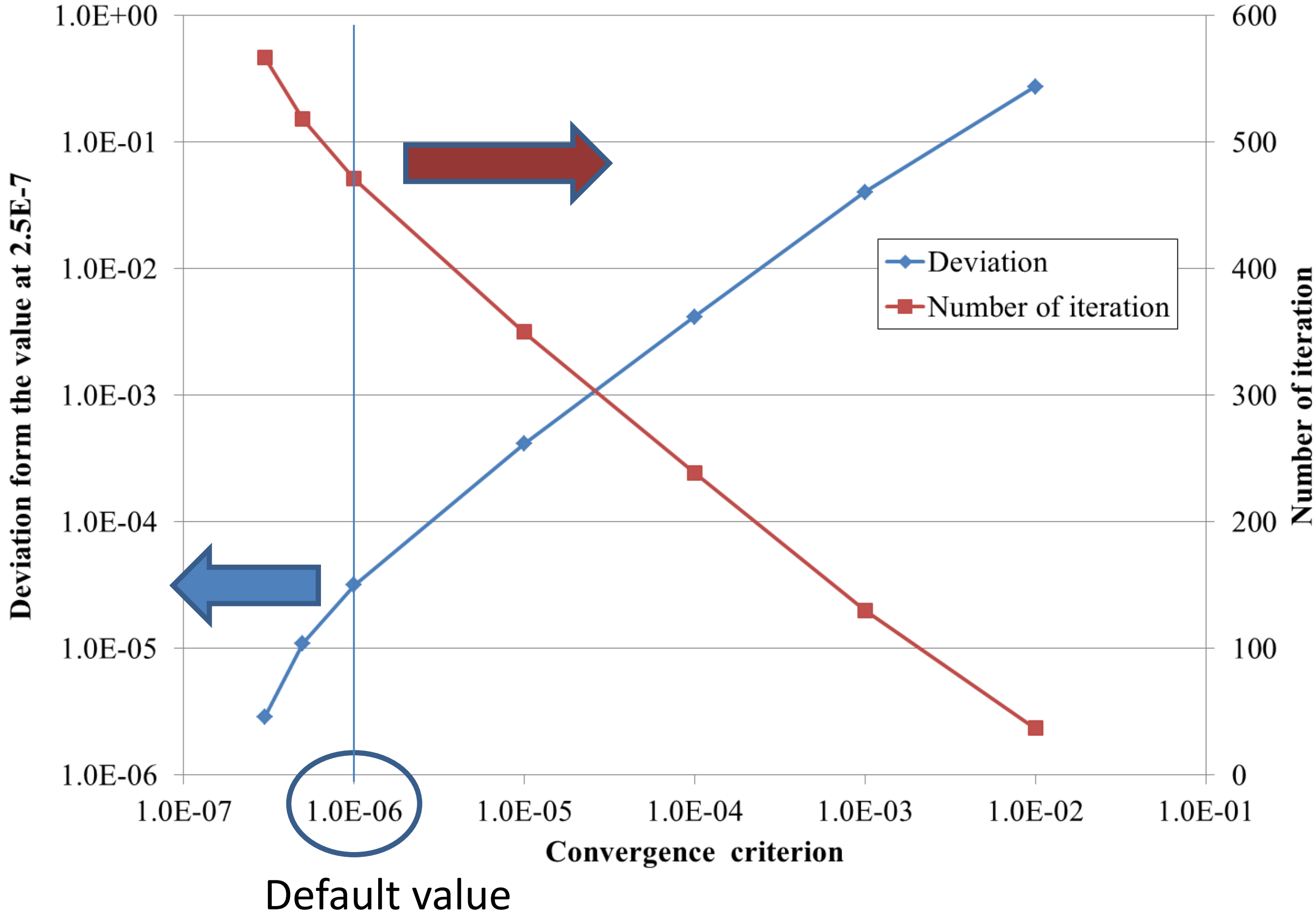
Space charge dilution by the 2[nd] RF harmonics

Simulation
(100 kV, 70 kV)





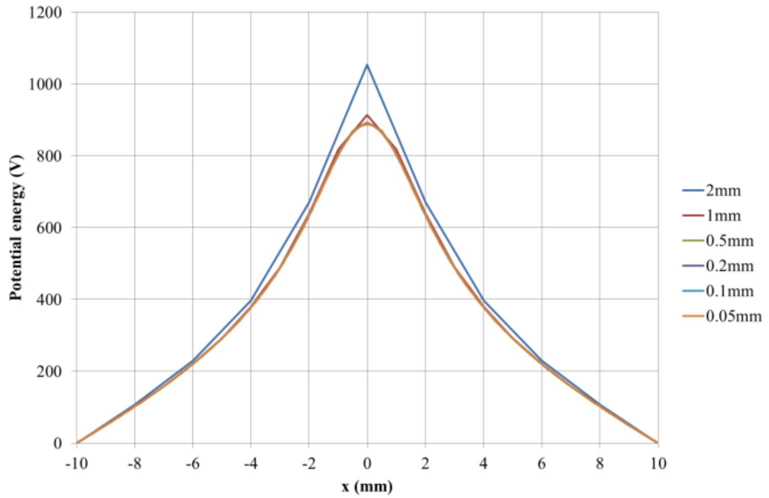
SOR method
Beam Potential Convergence at the Grids

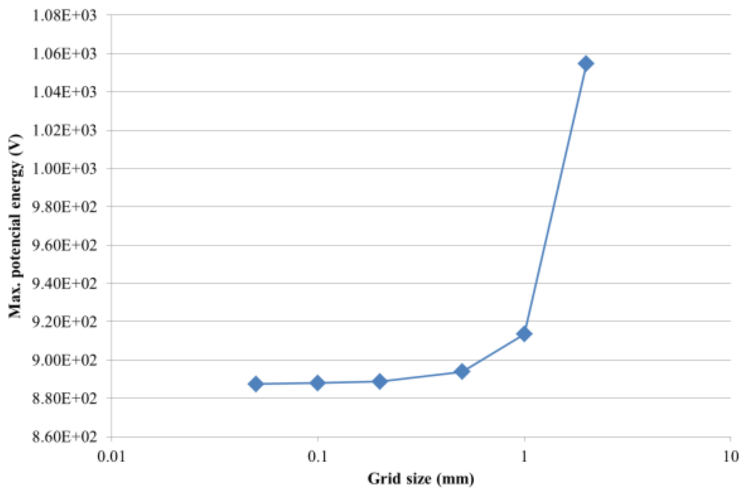


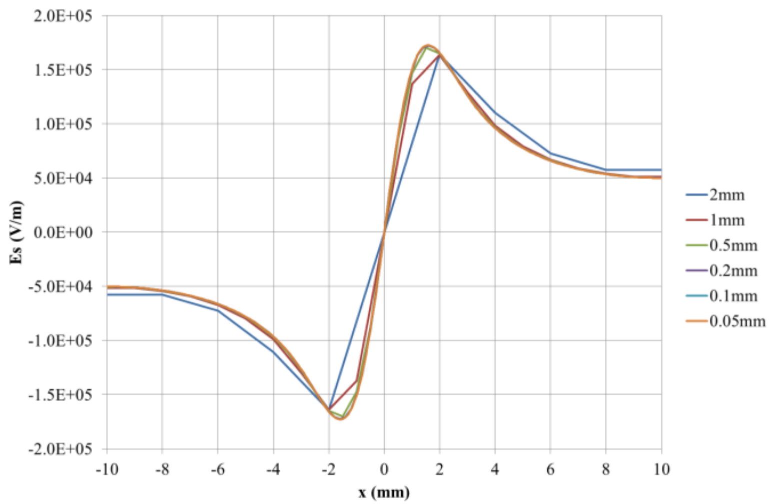
About 470 iterations are needed to obtain a potential value at each grid point

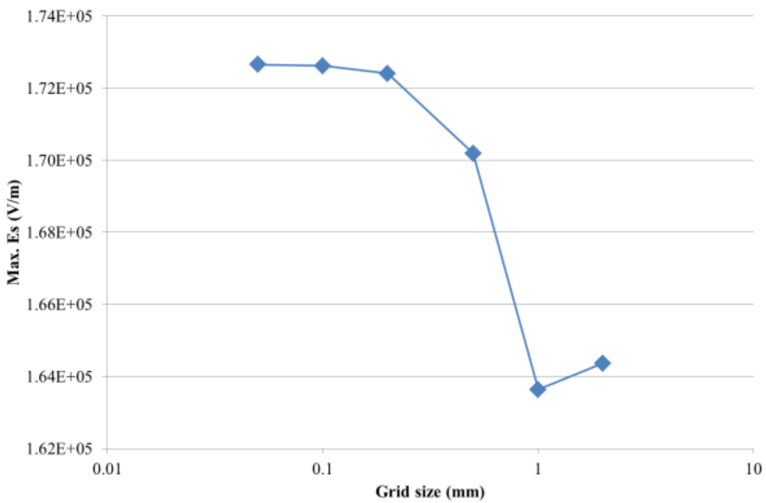
Es calc.: Grid size selection

Beam size: σx=σy=1mm



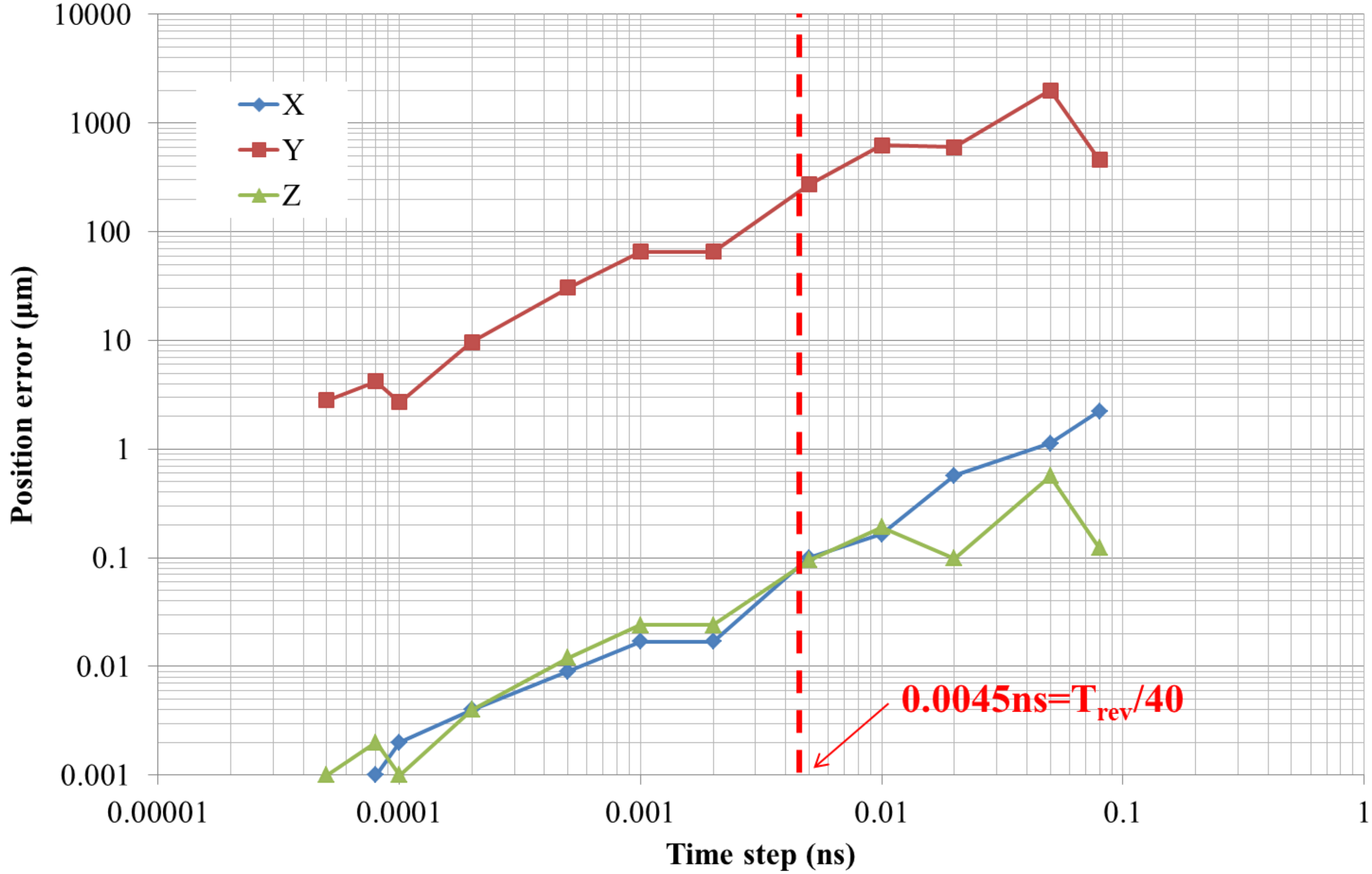






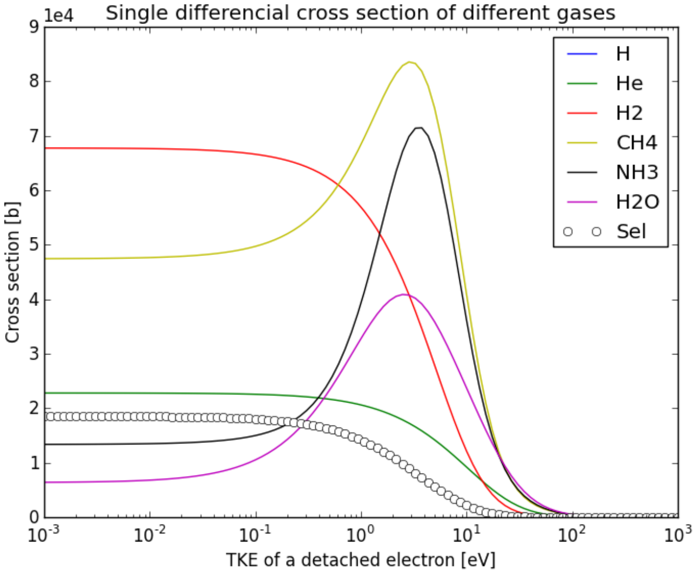
Tracking error vs. time step

4th dorder Runge-Kutta



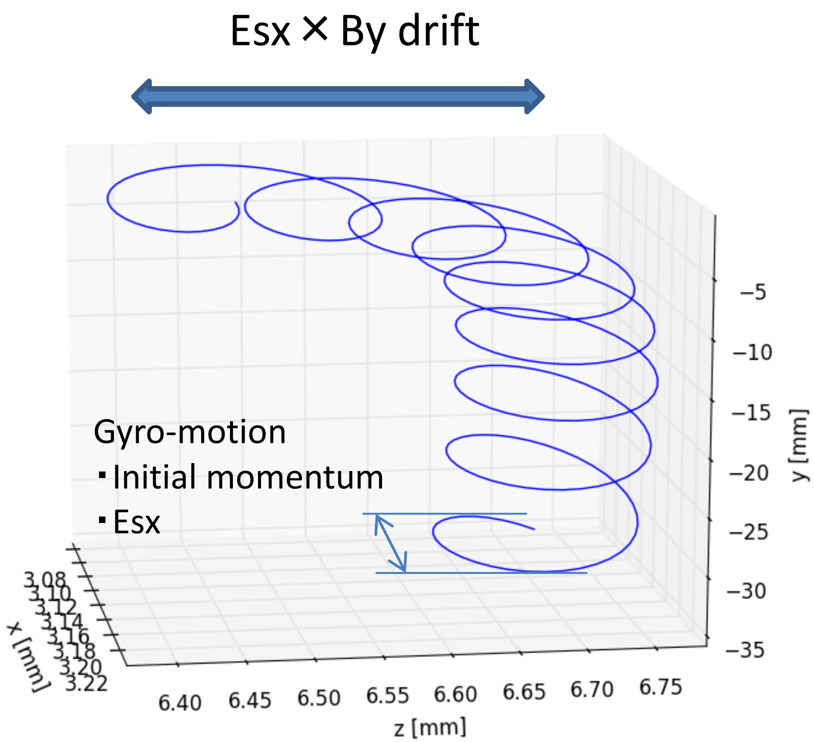
Initial conditions of a detached electron

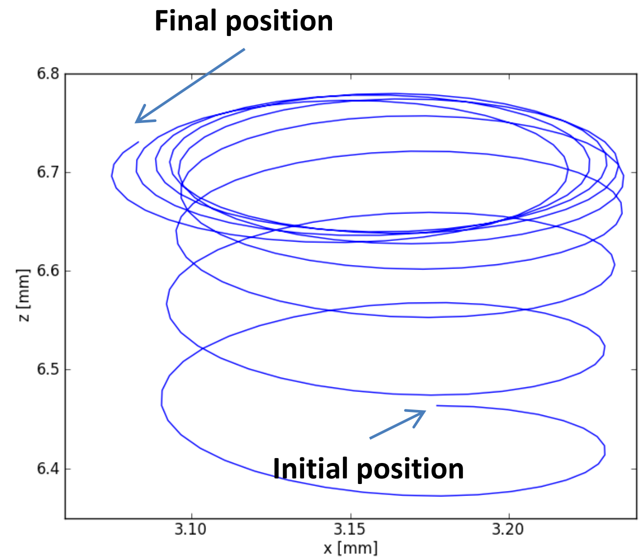
Single differential cross section (H, He, H2, CH4, NH3, H2O)
Or
Double differential cross section (H, He)
are used for initial TKE of detached electron

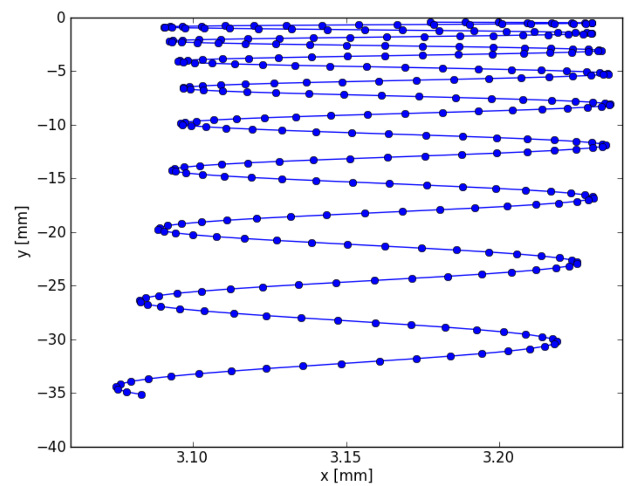


- Ref.
  - "Differential cross section for ionization of helium,neon,and argon by high-velocity ions"
    - J.H.Miller et. al., Phys. rev. A, 27 (1337), 1983.
  - "Analytic representation of secondary-electron spectra"
    - J. Chem. Phys. 87 (12), 15 Dec. 1987
  - "Electron production in proton collisions with atoms and molecules: energy distribution"
    - M.E. Rudd et. al., Rev. Mod. Phys., 64, No2, 441 (1992)

An example of tracking

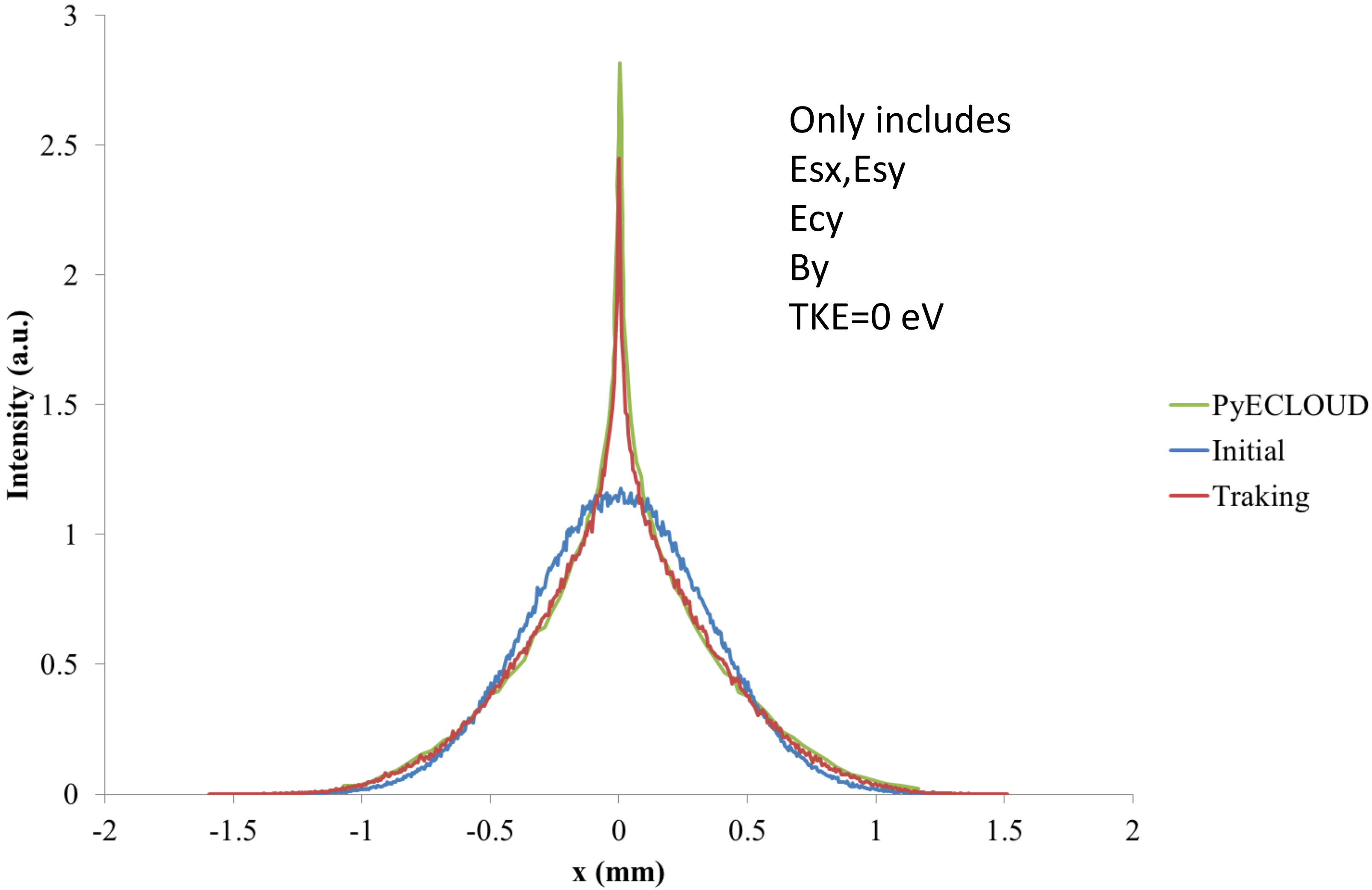






Cross check with PyECLOUD

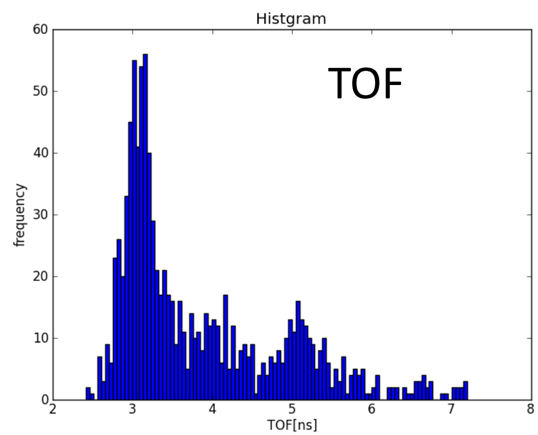
(Cross check with ESS code are now on going)

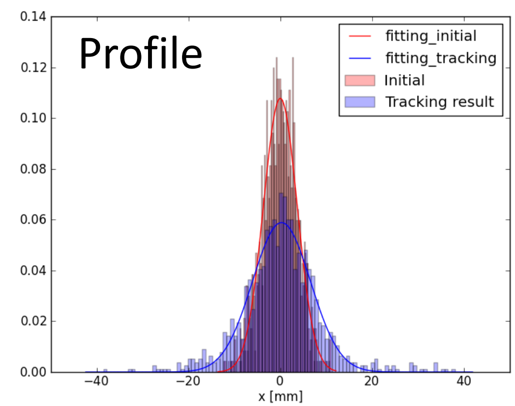


Calculation time using my 5 years old laptop PC

▪Intel Core i5 2.5GHz(4 CPUs): 64bit
▪8.2GB memory

- SOR: Space charge E field -> **1079 s (18min)**
  - Grid size: 70 × 70
  - Conversion criteria: 1E-6
- Tracking -> **124s (2min) / 1000 macroparticles**
  - Time step: 0.005 ns
  - 600step/3ns





Demonstration

How to use the code

In the near future upgrade

- Es calcu.
  - Non-Gaussian beam profile: SOR method
  - Analytical solution for Gaussian beam (Same as ESS code)
  - Non relativistic 3D Gaussian beam: Taking into account, Esz, Bs(x,y,z) fields