ESS IPM Model

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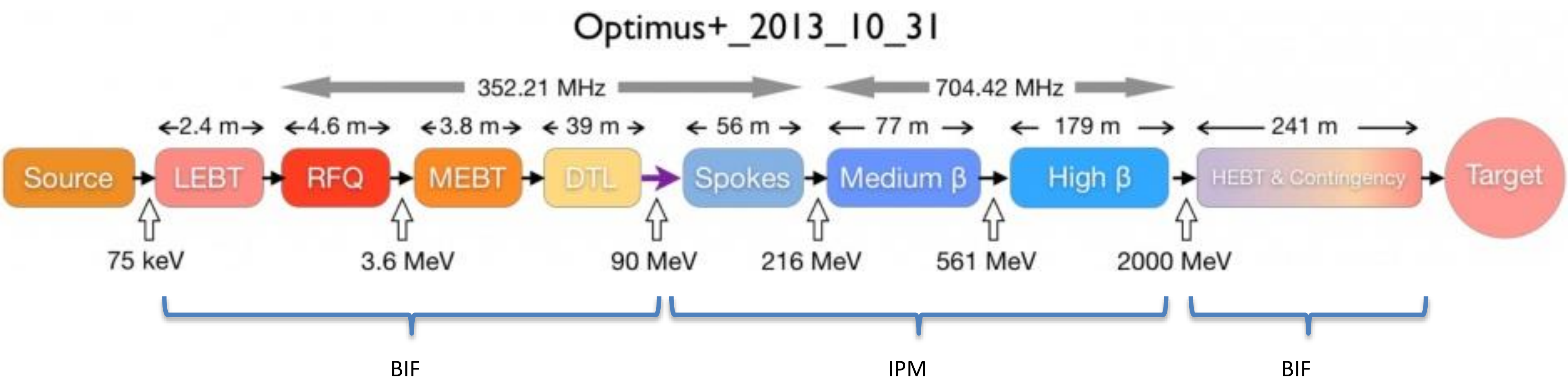
ESS Linac and NPMs



NPM:
- Transverse profile measurement up to full power
  - ➢ Tuning and operation diagnostic

Baseline:
- BIF: 1st choice, everywhere it is possible
- IPM: where BIF cannot deliver expected performance

Commissioning tool: profile and measured size for **matching** beam and lattice parameters to the model predicted **nominal values**, emittance measurement and measure emittance growth

Operation: monitor beam nominal parameters

Performance:
Profile measurement per pulse (14Hz)
Intra-pulse measurement bonus (highly desired)

The Model

Equation of Motion

$$
\begin{cases}
\frac{d}{dt}(mv_x) = F_x = Q_0\gamma_b(1-\beta_b v_z/c)\bar{E}_x \\
\frac{d}{dt}(mv_y) = F_y = Q_0\left(E_{y,IPM} + \gamma_b(1-\beta_b v_z/c)\bar{E}_y\right) \\
\frac{d}{dt}(mv_z) = F_z = Q0\left(\bar{E}_z + \beta_b\left(\gamma\bar{E}_x\frac{u_x}{c} + \gamma\bar{E}_y\frac{u_y}{c}\right)\right)
\end{cases}
$$

Bunch E Field
In rest frame

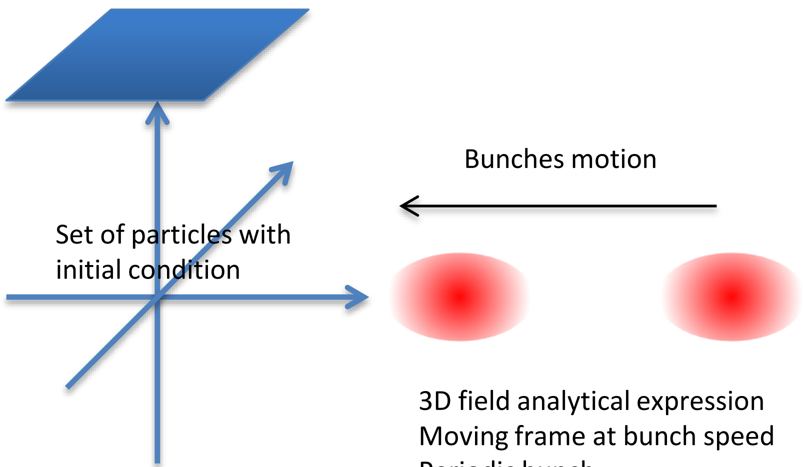
$$
\begin{cases}
\bar{E}_x = \frac{Q_b}{4\pi\epsilon_0}\frac{1}{\sqrt{\pi}}\int_0^\infty dq\, 2\,x\,\frac{e^{-\frac{x^2}{q_x}-\frac{y^2}{q_y}-\frac{z^2}{q_z}}}{(q_x^3 q_y q_z)^{\frac{1}{2}}} \\
\bar{E}_y = \frac{Q_b}{4\pi\epsilon_0}\frac{1}{\sqrt{\pi}}\int_0^\infty dq\, 2\,y\,\frac{e^{-\frac{x^2}{q_x}-\frac{y^2}{q_y}-\frac{z^2}{q_z}}}{(q_x q_y^3 q_z)^{\frac{1}{2}}} \\
\bar{E}_z = \frac{Q_b}{4\pi\epsilon_0}\frac{1}{\sqrt{\pi}}\int_0^\infty dq\, 2\,z\,\frac{e^{-\frac{x^2}{q_x}-\frac{y^2}{q_y}-\frac{z^2}{q_z}}}{(q_x q_y q_z^3)^{\frac{1}{2}}}
\end{cases}
$$

Bunch E Field
In lab frame

$$\bar{x} = x,\ \bar{y} = y,\ \bar{z} = \gamma_b z,\ \sigma_{\bar{z}} = \gamma_b\,\sigma_z$$

$$
\vec{E}(x,y,z) = \begin{cases}
\gamma_b\,\bar{E}_x(\bar{x},\bar{y},\bar{z}) \\
\gamma_b\,\bar{E}_y(\bar{x},\bar{y},\bar{z}) \\
\bar{E}_z(\bar{x},\bar{y},\bar{z})
\end{cases}
$$

Screen for stopping calculation



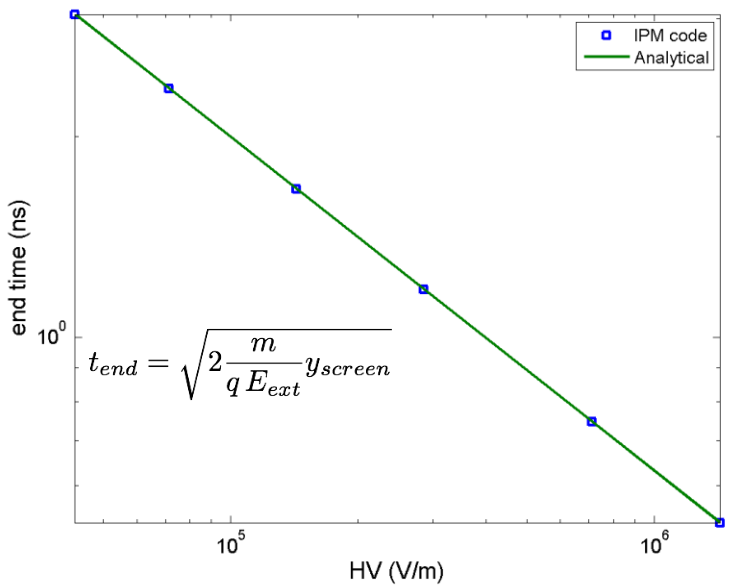
3D field analytical expression
Moving frame at bunch speed
Periodic bunch

Equation of motion solve
with Runge-Kutta non-
linear solver

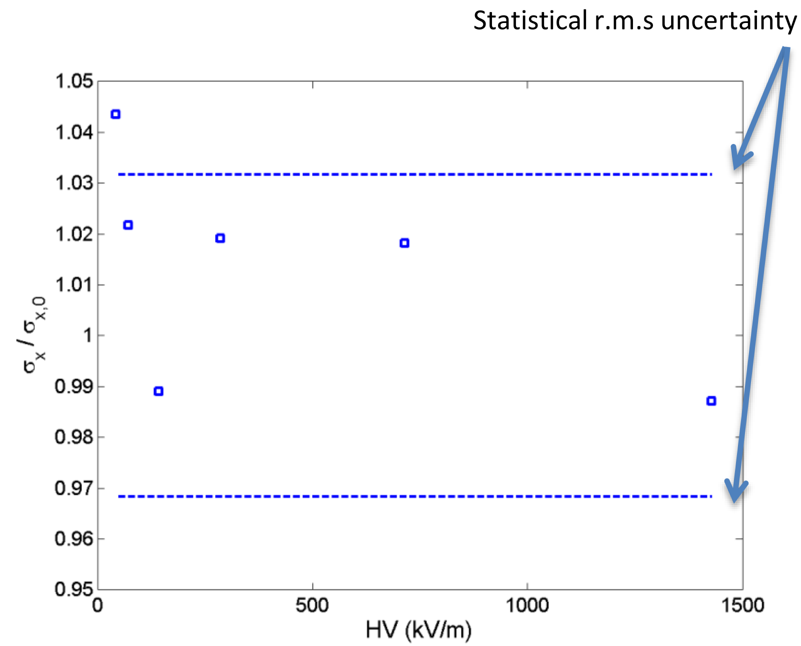
Checking the code

- No charge in the beam: checking the Vertical Force applied, good behaviour of the code with simple initial conditions
  - Time of arrival
  - Transverse size calculation (r.m.s value of the projected distribution)
  - Transverse size result (within statistical uncertainty)

- With charge in the beam, verify the forces induced by the beam
  - $Q_0 Q_b < 0$: focussing
  - $Q_0 Q_b > 0$: defocussing



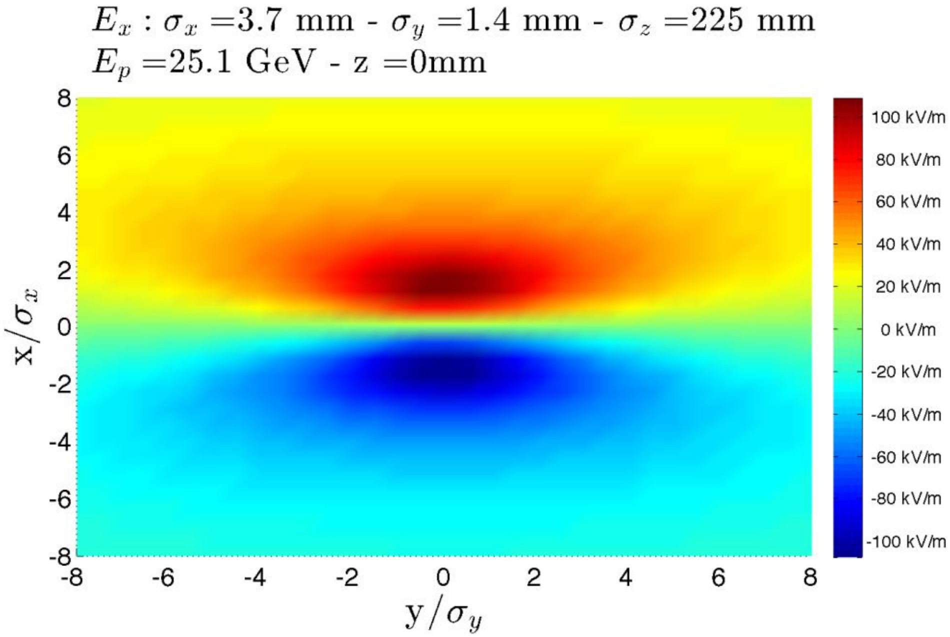
CERN M

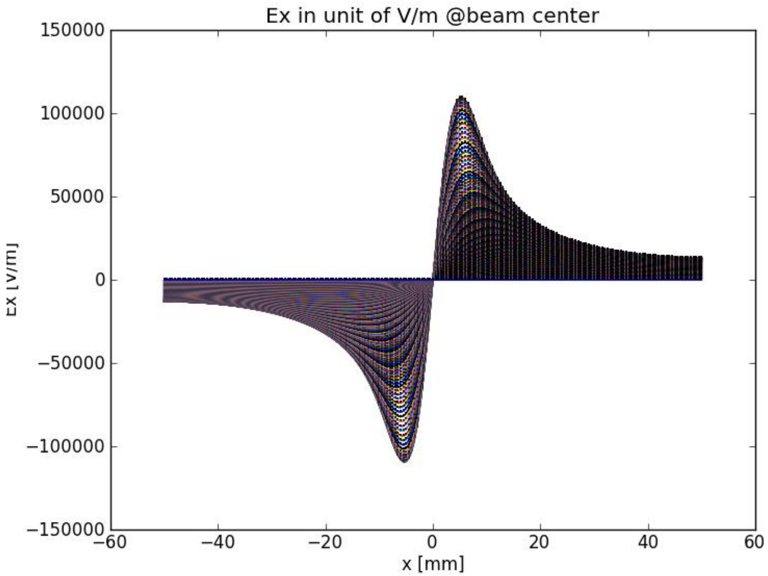


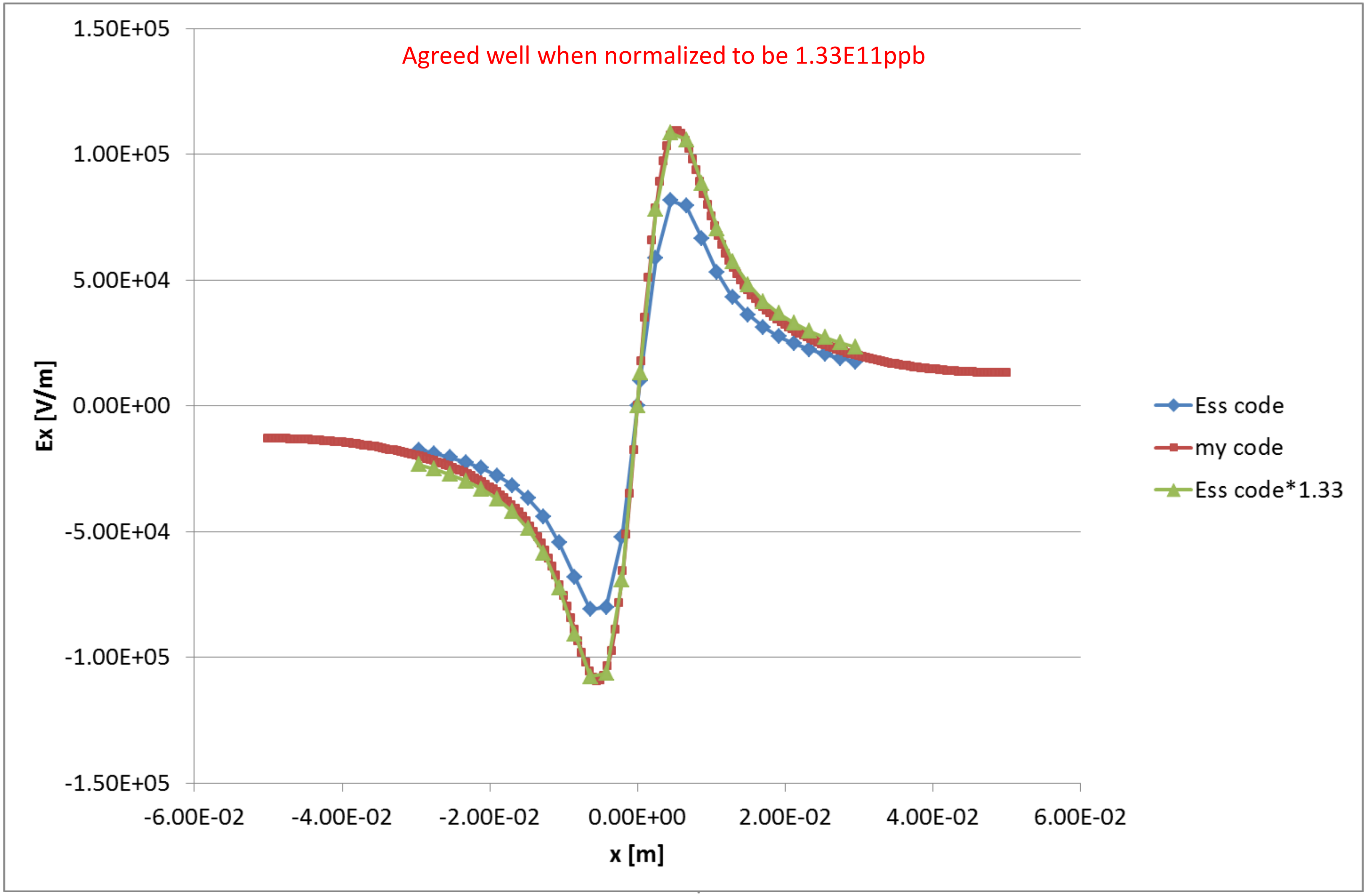
E field benchmarking

ESS code
E_p=26GeV
σ_x=3.7mm
σ_y=1.4mm
σ_s=0.75ns
Nb=1.0E11 ppb

J-Park code
E_p=100GeV -> only used for ionization process
σ_x=3.7mm
σ_y=1.4mm
σ_s=0.75ns
Nb=1.33E11 ppb
※set as Dirichlet Boundary on the lines,
x=±50mm, y=±35mm.






Agreed well when normalized to be 1.33E11ppb
1.50E+05
1.00E+05
5.00E+04
0.00E+00
-5.00E+04
-1.00E+05
-1.50E+05
Ex [V/m]
-6.00E-02
-4.00E-02
-2.00E-02
0.00E+00
2.00E-02
4.00E-02
6.00E-02
x [m]
Ess code
my code
Ess code*1.33

PS Parameters for Benchmarking

Beam Conditions:

- Beam position: (0, 0)
- Beam profile: 3D Gaussian distribution
  - σ_x: 3.7mm σ_y: 1.4mm σ_t: 3/4ns
  - Beam intensity: 1.33×10^{11} ppb
  - Bunch space: 25ns
  - Tracked particle: electron
  - Initial momentum: 0eV/C
  - Dirichlet boundary condition: equipotential boundary for space charge E-field estimate
- X: -50, 50mm
- Y: -35, 35mm
- Cage field: HV/70mm×(0, 1, 0), where HV=3, 5, 10, 20, 50, 100kV
- Magnetic field: 0T

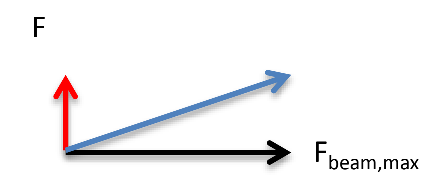
Quick calculation checks

EUROPEAN SPALLATION SOURCE

- Formulae to anticipate space charge effect *: $N = \frac{N_b}{\beta\,\sigma_x^2\,\sigma_y\,\sigma_z}$
  - N = 3.08

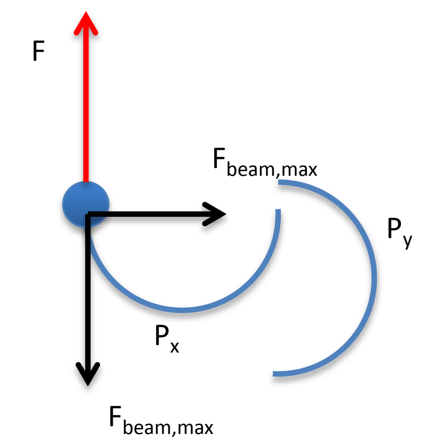
* K. Satou, J-Park

- Time of flight with no beam:
  - y_{screen} = 35mm; F=3kV/70mm=42.8kV/m; t_{end} = 2.38ns;
- Force from the bunch $F_{beam,max}$= 100kV/m : $F_{beam,max}$ / F > 2
  - First kick at max: 66 degrees: strong effect from the beam forces!

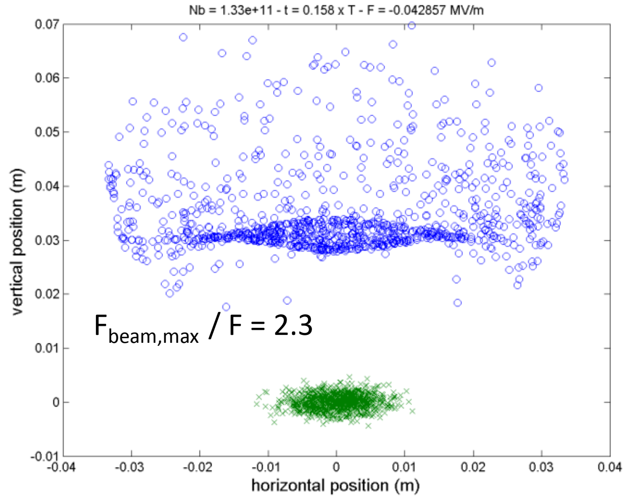


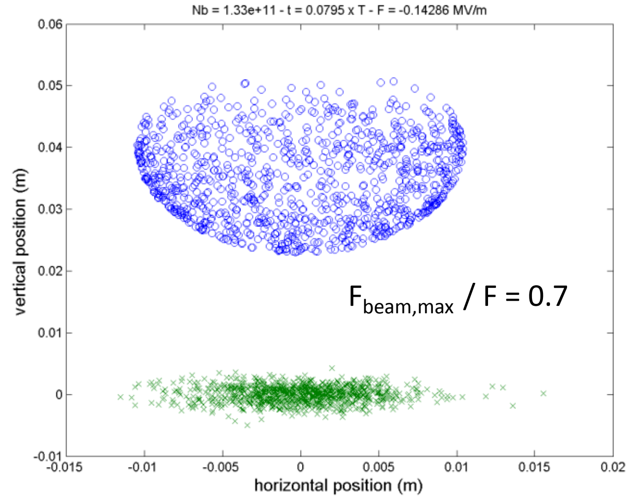
- Expected displacement? $\Delta x = \frac{Q_0}{2\,m}\,E_x\,t^2$
  - Overestimation: First case: Δx = 130mm

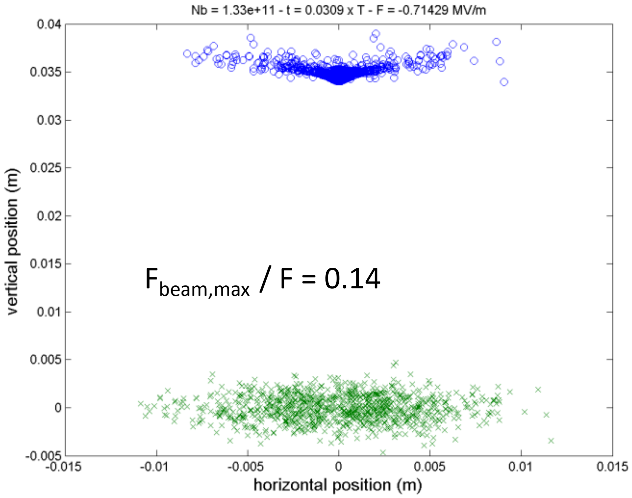
- Px, Py, potential, attractive when $Q_0\,Q_b > 0$
- Curved path: permit escape from trap

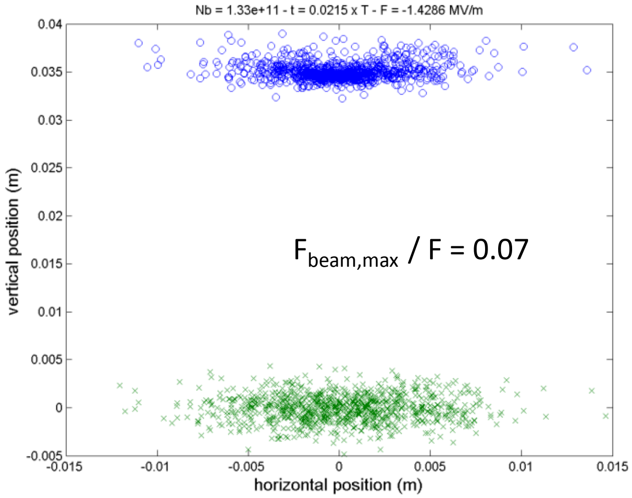


End results for the first benchmark test





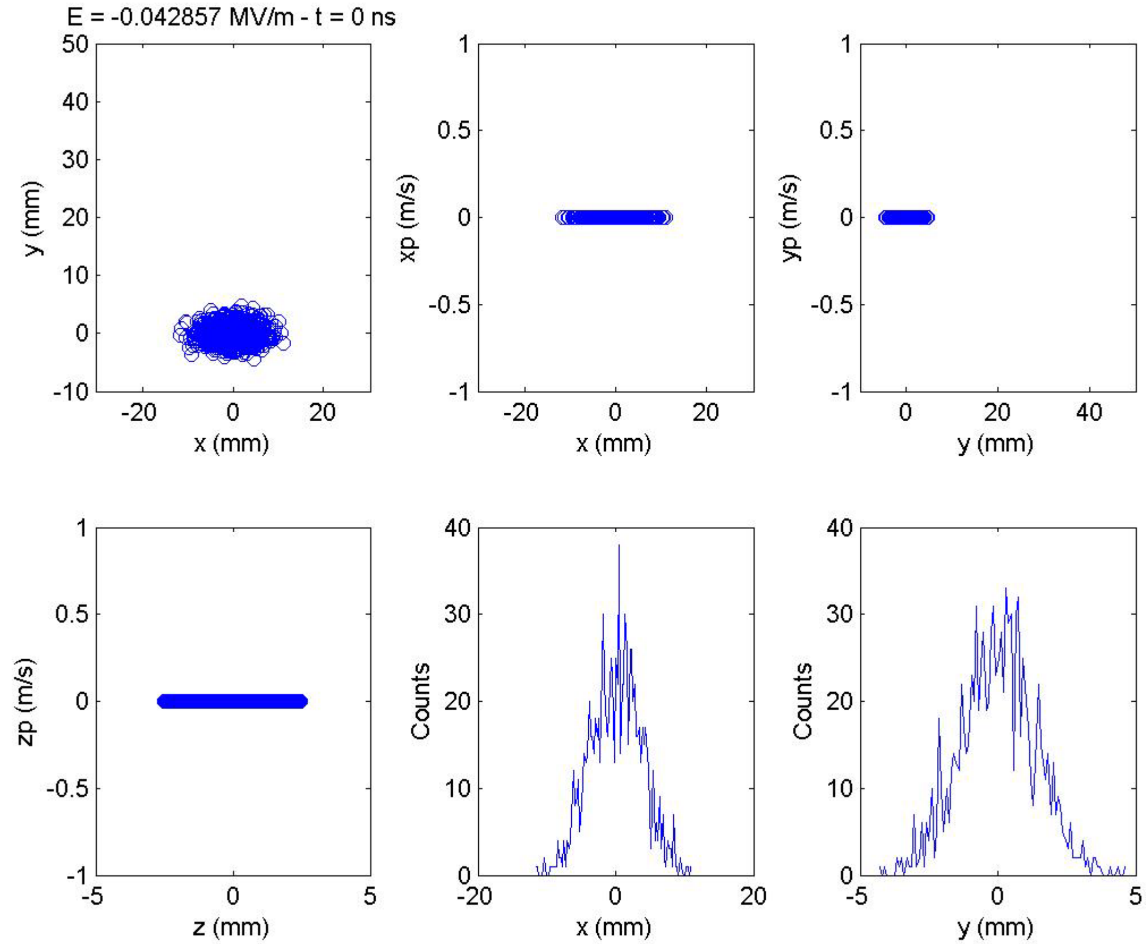




Case: F=3kV/70mm

End time: 3.87 ns

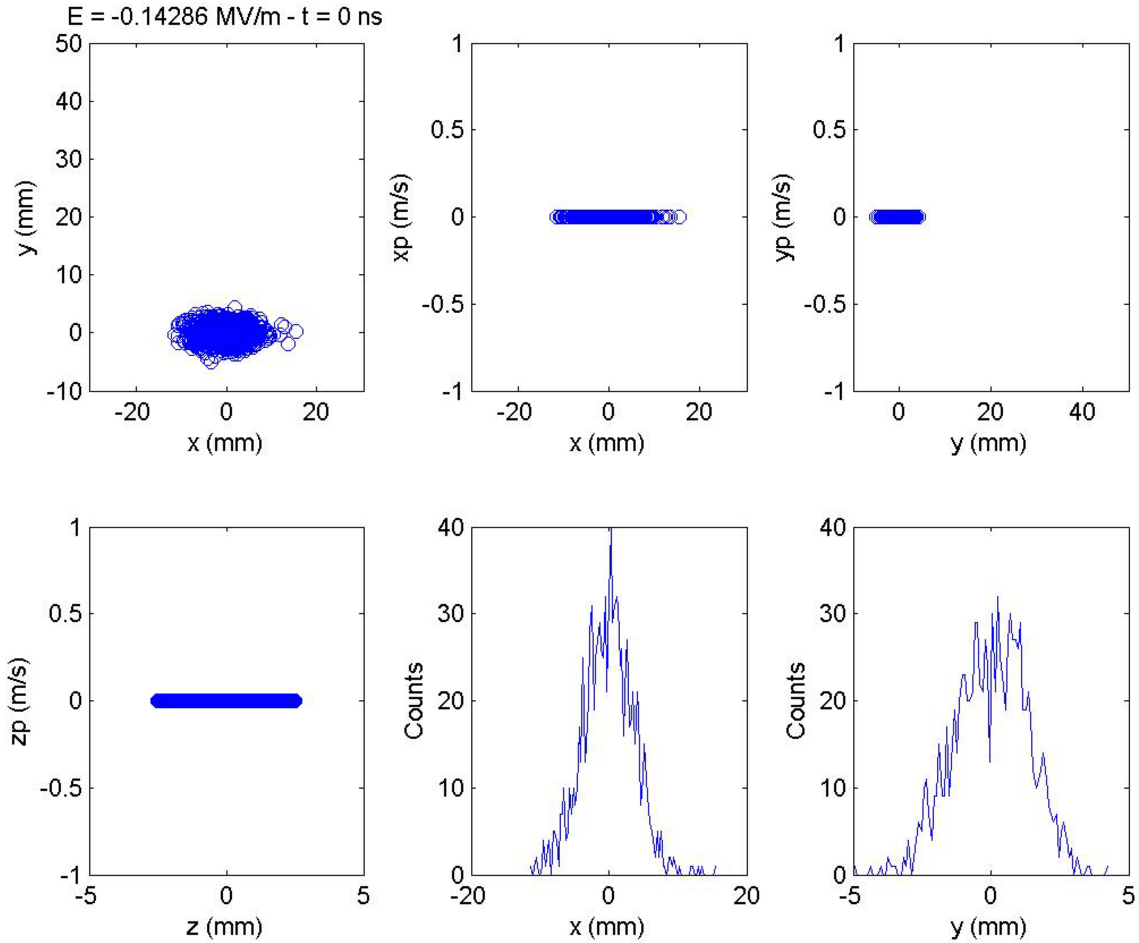
Expected: 3.05 ns



Case: F=10kV/70mm

End time: 1.85 ns

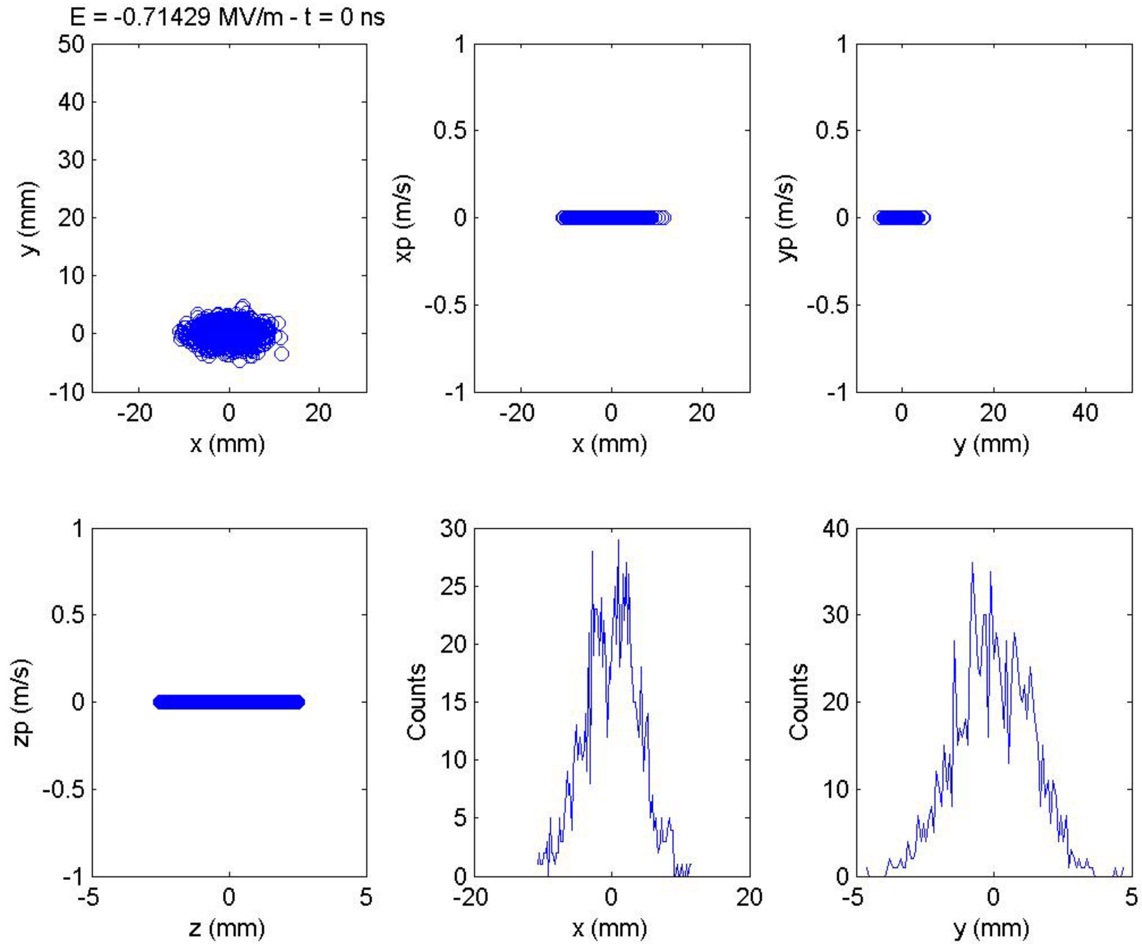
Expected: 1.67 ns



Case: F=50kV/70mm

End time: 0.749 ns

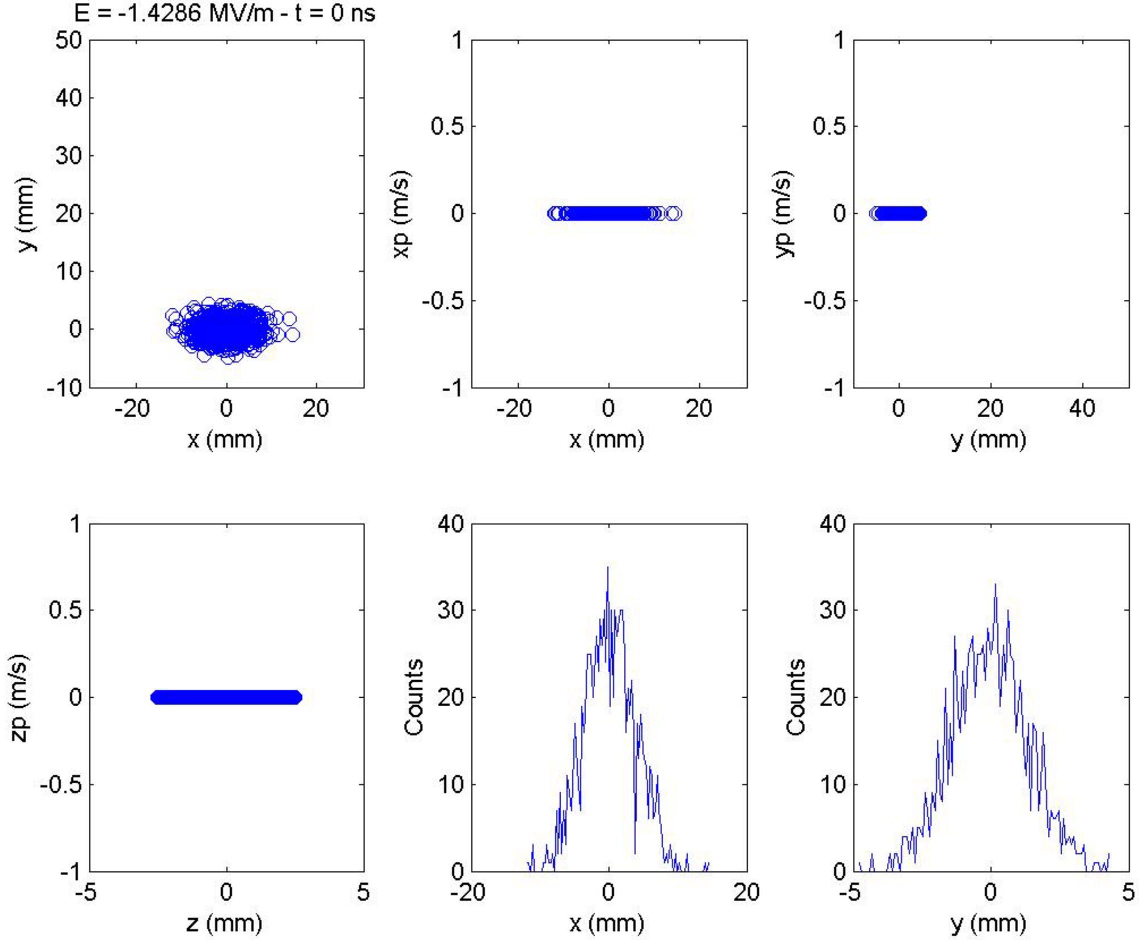
Expected: 0.747 ns



Case: F=100kV/70mm

End time: 0.53 ns

Expected: 0.528 ns



Concluding remarks

- ✓ ESS IPM code has been written. This is a matlab code, and benefit from efficient and debugged routine to solve the equation of motion. The code uses an analytical expression for the generation of the bunch generated E, and B fields, with real physical bunch parameters, based on a 3D Gaussian distribution, moving along its axis at relativistic speed

- ✓ Physical principle and results from the code agrees

- ✓ The PS case studied for benchmarking shows strong effects, which could reveal presence of a bug in the code, yet to be discovered

- ✓ Debugging is in progress …

Specials thanks to K. Satou, J. Marroncle, J. Storey, B. Dehning