CMB Hemispherical Power Asymmetry & its relation with Noncommutative Geometry

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Part 1 — Rudimentary Ideas

Part 2 — The Connection

Cosmic Microwave Radiation

- CMB is a field over sphere, spherical harmonic decomposition can be done

$$\Delta T(\hat{n}) = \sum a_{lm} Y_{lm}(\hat{n})$$

- Satisfies statistical isotropy aka Cosmological Principle

$$\langle \Delta T(\hat{m}) \Delta T(\hat{n}) \rangle = f(\hat{m} \cdot \hat{n}) \Leftrightarrow \langle a_{lm} a^{\star}_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l$$

- Two point correlation depends upon angle between points of observation and not location (or does it?)

Hemispherical Power Asymmetry

- Analysis of 2003 WMAP data revealed extra correlations (Eriksen et.al. 2004)
- Coined as Hemispherical Power Asymmetry
- Parametrisation(Gordon et.al. 2005, Prunet et.al. 2005, Bennett et.al. 2011)

$$\Delta T\left(\hat{n}\right) = \Delta T_{\mathrm{iso}}\left(\hat{n}\right)\left(1 + A\hat{\lambda}\cdot\hat{n}\right)$$

- Effect is absent at high l values (Donoghue 2005)
- Thus this is a large scale anisotropy

Modelling — The idea

R. Kothari et.al. Imprint of Inhomogeneous and Anisotropic Primordial Power Spectrum on CMB Polarisation, MNRAS **460**, 1577-1587

- Isotropic and homogeneous power spectra leads to isotropy and homogeneity
- Dipole modulation might be related to an early phase of inhomogeneous and/ or isotropic phase
- These modes generated during early phases of inflation may later re-enter the horizon and cause anisotropy
- Thus by modifying primordial power spectra HPA can be explained

The Algorithm

Before going any further, I would like to discuss the algorithm to calculate final correlations starting from two point density correlations

Model Real space Correlations
$\langle \delta(\mathbf{x})\, \delta(\mathbf{y}) \rangle$

Take Fourier transform

Fourier space two Point Correlations
$\left\langle \tilde{\delta}(\mathbf{k})\, \tilde{\delta}^{\star}(\mathbf{k}') \right\rangle$

Spherical harmonic Correlations
$\langle a_{lm} a^{\star}_{l'm'} \rangle =$

$$\iint \mathscr{G}(l, l', \mathbf{k}, \mathbf{k}') \left\langle \tilde{\delta}(\mathbf{k})\, \tilde{\delta}^{\star}(\mathbf{k}') \right\rangle d(\mathbf{k}, \mathbf{k}')$$

transfer function and geometric quantities

two point density correlations

Integration measure

Calculate Involved Integrals Using CAMB

Obtain required correlations

Example

- For the anisotropic model real space density correlations

$$\langle \delta(\mathbf{x})\,\delta(\mathbf{y})\rangle = f_1(R) + B_i R_i f_2(R)\,,\; B_i \in \mathbb{R},\; \mathbf{R} = \mathbf{x} - \mathbf{y}$$

- Then the two point density correlations in Fourier space

$$\left\langle \tilde{\delta}(\mathbf{k})\,\tilde{\delta}^{\star}(\mathbf{k}')\right\rangle = \left[P_{\text{iso}}(k) - i\hat{k}\cdot\hat{\lambda}\,g(k)\right]\delta\left(\mathbf{k}-\mathbf{k}'\right)$$

- Harmonic coefficients correlations

- Evaluated with the help of CAMB

$$\langle a_{lm} a^{\star}_{l'm'}\rangle_{\text{aniso}} = \delta_{mm'}\,\delta_{l,l+1}\,(4\pi T_0)^2\sqrt{\frac{(l+1)^2 - m^2}{(2l+1)(2l+3)}}\boxed{\int_0^{\infty} k^2 dk\,\Delta(l,k)\,\Delta\left(l',k\right)g(k)}$$

Evaluated using CAMB

Part 1 — Rudimentary Ideas

Part 2 — The Connection

Derivation of Anisotropic Power Spectrum

- Direction dependence introduces anisotropy so we want a correlation that depends upon direction

- The simplest of such correlations is (from slide 7)

$$\langle \delta(\mathbf{x})\,\delta(\mathbf{y})\rangle = f_1(R) + B_i R_i f_2(R)\,,\ B_i \in \mathbb{R},\ \mathbf{R} = \mathbf{x} - \mathbf{y}$$

- I'll outline a proof that such a form isn't possible in commutative field theory

- Noncommutative spacetime gives us a way out

The Proof

Proposition: *Anisotropic form of the power spectrum*

$$F(\mathbf{R},\mathbf{X}) = \langle \delta(\mathbf{x})\,\delta(\mathbf{y})\rangle = f_1(R) + B_i R_i f_2(R)\,,\ B_i \in \mathbb{R},\ \mathbf{X} = (\mathbf{x}+\mathbf{y})/2$$

isn't possible in commutative spacetimes.

Proof: Let us assume this to be true so that $f_2(R) \neq 0$. In commutative regime fields commute, so that

$$\langle \delta(\mathbf{x})\,\delta(\mathbf{y})\rangle = \langle \delta(\mathbf{y})\,\delta(\mathbf{x})\rangle \Rightarrow F(\mathbf{R},\mathbf{X}) = F(-\mathbf{R},\mathbf{X})$$

thus the above condition implies

$$f_1(|\mathbf{x}-\mathbf{y}|) + B_i(x_i - y_i) f_2(|\mathbf{x}-\mathbf{y}|) = f_1(|\mathbf{y}-\mathbf{x}|) + B_i(y_i - x_i) f_2(|\mathbf{y}-\mathbf{x}|)$$

or in other words

$$f_2(|\mathbf{x}-\mathbf{y}|) = f_2(R) = 0$$

which is a contradiction. Hence this form isn't possible in commutative regime.

Generalised Moyal Product

Rahul Kothari, Pranati Rath & Pankaj Jain, Cosmological Power Spectrum in Non-commutative Spacetime, Physical Review D, **94**, 063531

- The power spectrum is defined to be the Fourier transform of

$$\langle 0 | \phi(\mathbf{x},t) \star \phi(\mathbf{y},t) | 0 \rangle$$

- The 'simple' product between the fields must be changed to Moyal star product

$$\Omega^{j_1,j_2,\ldots,j_m}_{i_1,i_2,\ldots,i_m} \star \Omega^{l_1,l_2,\ldots,l_n}_{k_1,k_2,\ldots,k_n} = \left[1 + \frac{i}{2}\Theta^{\mu\nu}\sum_{p=1}^{m}\sum_{q=1}^{n} F_{j_p l_q}\frac{\partial}{\partial x^{\mu}_{j_p}}\frac{\partial}{\partial x^{\nu}_{l_q}} + \ldots\right]\Omega^{j_1,j_2,\ldots,j_m}_{i_1,i_2,\ldots,i_m}\Omega^{l_1,l_2,\ldots,l_n}_{k_1,k_2,\ldots,k_n}$$

- Here

$$\Omega^{j_1,j_2,\ldots,j_m}_{i_1,i_2,\ldots,i_m} = f_{i_1}(x_{j_1}) \star \left(f_{i_2}(x_{j_2}) \star \ldots \star \left(f_{i_{m-1}}\left(x_{j_{m-1}}\right) \star f_{i_m}(x_{j_m})\right)\right)$$

- This is general analysis, functions f are to be taken as scalar fields later and F is the form factor

Generalised Moyal Product Properties & Features

- Calculation was done at the linear order
- Definition having different spacetime points was absent in the literature, we gave a recursive definition approach
- When all spacetime points become same this becomes standard star product
- Generalised product is still — (a) Associativity & (b) cyclic
- Our definition can be used to prove associativity of the standard star product

Can tell about associativity of real field

Results & Conclusion

- After all calculations are done the following correction to the standard power spectrum is obtained

$$i\frac{b\Theta^{0i}H^3}{k^5}\left(15k_i\left(\vec{\alpha}\cdot\hat{k}\right)^2\boxed{-3k_i\left|\vec{\alpha}\right|^2}-6k\alpha_i\vec{\alpha}\cdot\hat{k}\right)$$

required form

- Thus although other correlations are present, it has been shown that it is possible to have such a form using noncommutative geometry

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Thank You

References

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- Prunet et.al. 2005, PRD 71, 083508
- Bennett et.al. 2011, ApJS 192, 17
- Donoghue 2005, PRD 71, 043002