ALL-SCALE cosmological perturbations and SCREENING OF GRAVITY in inhomogeneous Universe

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Outline

Introduction
(concordance cosmology, perturbation theory)

Discrete picture of (scalar and vector) cosmological perturbations (at all sub- and super-horizon scales)
(weak gravitational field limit, point-like masses)

Menu of properties, benefits, and bonuses
- Minkowski background limit
- Newtonian approximation and homogeneity scale
- Yukawa interaction and zero average values
- Transformation of spatial coordinates
- Nonzero spatial curvature and screening of gravity

Conclusion + fun
Introduction

Concordance cosmology: $\Lambda$ C(old) D(ark) M(atter) model

acceleration of global expansion

- $\sim 69\% \ (\Lambda)$
- $\sim 26\% \ (CDM)$
- $\sim 5\% \ (SM)$

(baryons, photons)

Planck 2015 results. XIII. Cosmological parameters  arXiv:1502.01589
Cosmological principle

on large enough scales the Universe is treated as being homogeneous and isotropic

Friedmann-Lemaître-Robertson-Walker background metric

observed separate galaxies, their groups and clusters

structure formation from primordial fluctuations at earliest evolution stages

on sufficiently small scales the Universe is highly inhomogeneous

perturbation theory
### Two main distinct approaches to structure growth investigation

<table>
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<th>relativistic perturbation theory</th>
<th>N-body simulations generally based on Newtonian cosmological approximation</th>
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#### Keywords

<table>
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<tr>
<th>early Universe; linearity; large scales</th>
<th>late Universe; nonlinearity; small scales</th>
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<td>fails in describing nonlinear dynamics at small distances</td>
<td>do not take into account relativistic effects becoming non-negligible at large distances</td>
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The acute problem: construction of a self-consistent unified scheme, which would be valid for arbitrary (sub- & super-horizon) scales and incorporate linear & nonlinear effects.

very promising in precision cosmology era

Weak gravitational field limit

Deviations of the metric coefficients from their background (average) values are considered as 1\textsuperscript{st} order quantities, while the 2\textsuperscript{nd} order is completely disregarded.
A couple of previous attempts to develop a unified perturbation theory

I. Generalization of nonrelativistic post-Minkowski formalism to the cosmological case in the form of relativistic post-Friedmann formalism, which would be valid on all scales and include the full nonlinearity of Newtonian gravity at small distances:

expansion of the metric in powers of the parameter $1/c$ (the inverse speed of light)

II. Formalism for relativistic N-body simulations:

**different orders of smallness given to the metric corrections and their spatial derivatives ("dictionary")**


Discrete cosmology: presenting nonrelativistic matter as separate point-like massive particles
Advantages of the unified scheme developed here

1) no any supplementary approximations or extra assumptions in addition to the weak field limit;

2) spatial and temporal derivatives are treated on an equal footing, no “dictionaries”;

3) no expansion into series with respect to the ratio 1/c;

4) no artificial mixing of first- and second-order contributions to the metric;

5) sub- or super-horizon regions are not singled out
Discrete picture of (scalar and vector) cosmological perturbations

Unperturbed FLRW metric describing (homogeneous and isotropic on the average) Universe:

\[ ds^2 = a^2 \left( d\eta^2 - \delta_{\alpha\beta} dx^\alpha dx^\beta \right) \]

- scale factor
- comoving coordinates
- spatial curvature is zero (generalization to non-flat spatial geometry is simple)
Friedmann Eqs. in the framework of the pure $\Lambda$CDM model (with a negligible radiation contribution):

\[ \frac{3\tilde{H}^2}{a^2} = \kappa \bar{\varepsilon} + \Lambda \]

energy density of nonrelativistic pressureless matter

\[ \frac{2\tilde{H}' + \tilde{H}^2}{a^2} = \Lambda \]

cosmological constant

\[ \tilde{H} \equiv \frac{a'}{a} \]

Newtonian gravitational constant

\[ \kappa \equiv 8\pi G_N / c^4 \]
Perturbed metric describing (inhomogeneous and anisotropic) Universe:

\[ ds^2 = a^2 \left[ (1 + 2\Phi) d\eta^2 + 2B_\alpha dx^\alpha d\eta - (1 - 2\Phi) \delta_{\alpha\beta} dx^\alpha dx^\beta \right] \]

function \( \Phi(\eta, r) \) and spatial vector \( \mathbf{B}(\eta, r) \equiv (B_1, B_2, B_3) \): scalar and vector perturbations, respectively

\[ \nabla \mathbf{B} = \delta^{\alpha\beta} \frac{\partial B_\alpha}{\partial x^\beta} = 0 \]

tensor perturbations are not taken into account

Einstein Eqs.:

\[ G_i^k = \kappa T_i^k + \Lambda \delta_i^k \]

mixed components of Einstein and matter energy-momentum tensors
\[ G_0^0 = \kappa T_0^0 + \Lambda \Rightarrow \Delta \Phi - 3\tilde{H}\left(\Phi' + \tilde{H}\Phi\right) = \frac{1}{2}\kappa a^2 \delta T_0^0 \]

\[ G_\alpha^0 = \kappa T_\alpha^0 \Rightarrow \frac{1}{4} \Delta B_\alpha + \frac{\partial}{\partial x^\alpha}\left(\Phi' + \tilde{H}\Phi\right) = \frac{1}{2}\kappa a^2 \delta T_\alpha^0 \]

\[ G_\alpha^\beta = \kappa T_\alpha^\beta + \Lambda \delta_\alpha^\beta \Rightarrow \Phi'' + 3\tilde{H}\Phi' + \left(2\tilde{H}' + \tilde{H}^2\right)\Phi = 0 \]

\[ \left(\frac{\partial B_\alpha}{\partial x^\beta} + \frac{\partial B_\beta}{\partial x^\alpha}\right)' + 2\tilde{H}\left(\frac{\partial B_\alpha}{\partial x^\beta} + \frac{\partial B_\beta}{\partial x^\alpha}\right) = 0 \]

\[ \Delta \equiv \delta^{\alpha\beta} \frac{\partial^2}{\partial x^\alpha \partial x^\beta} \]

\[ T_i^k = \overline{T_i^k} + \delta T_i^k \]

\[ \overline{T_0^0} = \overline{\epsilon} \]

Laplace operator in comoving coordinates

only nonzero average mixed component
gravitating masses

\[ T^{ik} = \sum_n m_n c^2 \frac{dx^i_n}{\sqrt{-g}} \frac{dx^k_n}{d\eta} \frac{d\eta}{ds_n} \delta(r - r_n) \]

\[ g = \det(g_{ik}) \]

comoving radius-vectors

\[ \rho = \sum_n m_n \delta(r - r_n) = \sum_n \rho_n \]

rest mass density

\[ \tilde{v}^{\alpha}_n = dx^{\alpha}_n / d\eta \]

4-velocities

\[ u^i_n = dx^i_n / ds_n \]

comoving peculiar velocities

in the spirit of the particle-particle method of N-body simulations
\[ \tilde{v}_n^\alpha \] import the 1st order of smallness in rhs of linearized Einstein Eqs.

\[ \delta T^0_0 \equiv T^0_0 - \bar{T}^0_0 = \frac{c^2}{a} \delta \rho + \frac{3 \bar{\rho} c^2}{a^3} \Phi \]

\[ \delta T^0_\alpha = -\frac{c^2}{a^3} \sum_n m_n \delta \left( \mathbf{r} - \mathbf{r}_n \right) \tilde{v}_n^\alpha + \frac{\bar{\rho} c^2}{a^3} B_\alpha = -\frac{c^2}{a^3} \sum_n \rho_n \tilde{v}_n^\alpha + \frac{\bar{\rho} c^2}{a^3} B_\alpha \]

\[ \delta T^\beta_\alpha = 0 \]

replacements:
\[ \rho \Phi \rightarrow \bar{\rho} \Phi, \quad \rho \mathbf{B} \rightarrow \bar{\rho} \mathbf{B} \]

1st order

\[ \Delta \Phi - 3 \bar{H} \left( \Phi' + \bar{H} \Phi \right) = -\frac{3 \kappa \bar{\rho} c^2}{2a} \Phi = \frac{\kappa c^2}{2a} \delta \rho \]

\[ \frac{1}{4} \Delta \mathbf{B} + \nabla \left( \Phi' + \bar{H} \Phi \right) = -\frac{\kappa \bar{\rho} c^2}{2a} \mathbf{B} = -\frac{\kappa c^2}{2a} \sum_n m_n \delta \left( \mathbf{r} - \mathbf{r}_n \right) \tilde{v}_n = -\frac{\kappa c^2}{2a} \sum_n \rho_n \tilde{v}_n \]
Continuity Eq.: \[ \rho_n' + \nabla (\rho_n \tilde{v}_n) = 0 \]
\[ \tilde{v}_n (\eta) = d\mathbf{r}_n / d\eta \equiv (\tilde{v}_n^1, \tilde{v}_n^2, \tilde{v}_n^3) \]

\[ \sum_n \rho_n \tilde{v}_n = \nabla \Xi + \left( \sum_n \rho_n \tilde{v}_n - \nabla \Xi \right) \]
\[ \Xi = \frac{1}{4\pi} \sum_n m_n \frac{(\mathbf{r} - \mathbf{r}_n) \tilde{v}_n}{|\mathbf{r} - \mathbf{r}_n|^3} \]

\[ \Phi' + \tilde{H} \Phi = -\frac{\kappa c^2}{2a} \Xi \]

Fourier transform:
\[ \hat{\Xi} (\eta, \mathbf{k}) \equiv \int \Xi (\eta, \mathbf{r}) \exp (-i\mathbf{k} \cdot \mathbf{r}) d\mathbf{r} = -\frac{i}{k^2} \sum_n m_n (\mathbf{k} \tilde{v}_n) \exp (-i\mathbf{k} \cdot \mathbf{r}) \quad k \equiv |\mathbf{k}| \]
\[ \hat{\rho}_n (\eta, \mathbf{k}) \equiv \int \rho_n (\eta, \mathbf{r}) \exp (-i\mathbf{k} \cdot \mathbf{r}) d\mathbf{r} = m_n \int \delta (\mathbf{r} - \mathbf{r}_n) \exp (-i\mathbf{k} \cdot \mathbf{r}) d\mathbf{r} = m_n \exp (-i\mathbf{k} \cdot \mathbf{r}_n) \]
\[
\frac{1}{4} \Delta B - \frac{\kappa \rho c^2}{2a} B = -\frac{\kappa c^2}{2a} \left( \sum_n \rho_n \tilde{v}_n - \nabla \Xi \right)
\]

\[
-\frac{k^2}{4} \hat{B} - \frac{\kappa \rho c^2}{2a} \hat{B} = -\frac{\kappa c^2}{2a} \left( \sum_n \hat{\rho}_n \tilde{v}_n - i\mathbf{k} \hat{\Xi} \right)
\]

\[
\hat{B} = \frac{2\kappa c^2}{a} \left( k^2 + \frac{2\kappa \rho c^2}{a} \right)^{-1} \sum_n m_n \exp(-i\mathbf{k} \mathbf{r}_n) \left( \tilde{v}_n - \frac{(\mathbf{k} \tilde{v}_n)}{k^2} \mathbf{k} \right)
\]

\[
\mathbf{B} = \frac{\kappa c^2}{8\pi a} \sum_n \left[ \frac{m_n \tilde{v}_n}{|\mathbf{r} - \mathbf{r}_n|} \cdot \frac{3 + 2\sqrt{3} q_n + 4 q_n^2}{q_n^2} \exp\left(-2q_n / \sqrt{3}\right) - 3 \right.
\]

\[
+ \left. \frac{m_n \left[ \tilde{v}_n (\mathbf{r} - \mathbf{r}_n) \right]}{|\mathbf{r} - \mathbf{r}_n|^3} (\mathbf{r} - \mathbf{r}_n) \cdot \frac{9 - \left(9 + 6\sqrt{3} q_n + 4 q_n^2\right) \exp\left(-2q_n / \sqrt{3}\right)}{q_n^2} \right]
\]
\[ \Delta \Phi - \frac{3 \kappa \rho c^2}{2a} \Phi = \frac{\kappa c^2}{2a} \partial \rho - \frac{3 \kappa c^2 \tilde{H}}{2a} \Xi \]

\[ -k^2 \hat{\Phi} - \frac{3 \kappa \rho c^2}{2a} \hat{\Phi} = \frac{\kappa c^2}{2a} \sum_n \hat{\rho}_n - \frac{\kappa \rho c^2}{2a} (2\pi)^3 \delta(k) - \frac{3 \kappa c^2 \tilde{H}}{2a} \hat{\Xi} \]

\[ \hat{\Phi} = -\frac{\kappa c^2}{2a} \left( k^2 + \frac{3 \kappa \rho c^2}{2a} \right)^{-1} \left[ \sum_n m_n \exp(-ikr_n) \left( 1 + 3i\tilde{H} \frac{(k\tilde{v}_n)}{k^2} \right) - \rho (2\pi)^3 \delta(k) \right] \]

\[ \Phi = \frac{1}{3} - \frac{\kappa c^2}{8\pi a} \sum_n \frac{m_n}{|r - r_n|} \exp(-q_n) \]

\[ + \frac{3 \kappa c^2}{8\pi a} \tilde{H} \sum_n m_n \left[ \tilde{v}_n (r - r_n) \right] \frac{1 - (1 + q_n) \exp(-q_n)}{q_n^2} \]

\[ q_n (\eta, r) \equiv \sqrt{\frac{3 \kappa \rho c^2}{2a}} (r - r_n) \quad q_n \equiv |q_n| \]
Thus, explicit expressions for 1st order vector and scalar cosmological perturbations are determined.

\[ B' + 2 \tilde{H} B = 0 \]

Incidentally, \( \nabla B = 0 \rightarrow kB = 0 \) is satisfied.
Equations of motion

Spacetime interval for the \( n \)-th particle

\[
ds_n = a \left[ 1 + 2\Phi + 2B_\alpha \tilde{v}_n^\alpha - (1 - 2\Phi) \delta_{\alpha\beta} \tilde{v}_n^\alpha \tilde{v}_n^\beta \right]^{1/2} \, d\eta
\]

\[
\left[ a \left( B|_{r=r_n} - \tilde{v}_n \right) \right]' = a \nabla \Phi|_{r=r_n}
\]

\[
\rho(aB)' - \sum_n \rho_n(a\tilde{v}_n)' = a \rho \nabla \Phi
\]

\[
\sum_n \rho_n(a\tilde{v}_n)' = -a \bar{\rho} \nabla \Phi + \bar{\rho}(aB)'
\]

\[
\sum_n \hat{\rho}_n(a\tilde{v}_n)' = \sum_n m_n \exp(-ikr_n)(a\tilde{v}_n)' = -a \bar{\rho} \cdot ik \Phi + \bar{\rho}(a\hat{B})'
\]

all \( \Box \Box \Box \) satisfied

\[
(a\tilde{v}_n)' = -a \left( \nabla \Phi|_{r=r_n} + \tilde{H} B|_{r=r_n} \right)
\]
Menu of properties, benefits, and bonuses

Minkowski background limit

\[ a \to \text{const} \implies \tilde{H} \to 0 \quad \bar{\rho} \to 0 \implies q_n \to 0 \]

\[ \Phi \to -\frac{\kappa c^2}{8\pi a} \sum_n \frac{m_n}{|\mathbf{r} - \mathbf{r}_n|} = -\frac{G_N}{c^2} \sum_n \frac{m_n}{|\mathbf{R} - \mathbf{R}_n|} \]

The constant $1/3$ has been dropped since it originates exclusively from the terms containing $\bar{\rho}$.

\[ \mathbf{R} = a\mathbf{r} \]

\[ \mathbf{R}_n = a\mathbf{r}_n \]

physical radius-vectors
\[
B \rightarrow \frac{\kappa c^2}{4\pi a} \sum_n \left[ \frac{m_n \tilde{v}_n}{|r - r_n|} + \frac{m_n [\tilde{v}_n (r - r_n)]}{|r - r_n|^3} (r - r_n) \right]
\]

\[
= \frac{G_N}{2c^2} \sum_n \frac{m_n}{|R - R_n|} \left[ 4\tilde{v}_n + 4 \frac{[\tilde{v}_n (R - R_n)]}{|R - R_n|} \frac{R - R_n}{|R - R_n|} \right]
\]

\[
\tilde{v}_n \equiv \frac{dr_n}{d\eta}, \quad v_n \equiv \frac{dr_n}{dt}
\]

\[
cdt = ad\eta \quad \Rightarrow \quad \tilde{v}_n = \frac{av_n}{c}
\]

The sum of these integers \(4 + 4 = 8\)
is the same for the other appropriate choices of gauge conditions as well.

complete agreement with textbooks
Newtonian approximation
Homogeneity scale

\( \tilde{v}_n \to 0 \) (peculiar motion as a gravitational field source is completely ignored)
\( q_n \ll 1 \)

\[ \Phi \to -\frac{G_N}{c^2} \sum_n \frac{m_n}{|\mathbf{R} - \mathbf{R}_n|} \quad \mathbf{B} \to 0 \]

The constant \( \frac{1}{3} \) has been dropped for the other reason: only the gravitational potential gradient enters into Eqs. of motion describing dynamics of the considered system of gravitating masses.
\[
\ddot{R}_j - \frac{\ddot{a}}{a} R_j = -G_N \sum_{n \neq j} \frac{m_n (R_j - R_n)}{\left| R_j - R_n \right|^3}
\]
dot: derivative with respect to \( t \)

complete agreement with Eqs. for N-body simulations

What are the applicability bounds for the inequality?

\[ q_n \ll 1 \iff |R - R_n| \ll \lambda \]

\[
H \equiv \frac{\dot{a}}{a} = \frac{cH}{a}
\]
Hubble parameter

\[
\lambda \equiv \sqrt{\frac{2a^3}{3k\bar{\rho}c^2}} = \sqrt{\frac{2c^2}{9H_0^2 \Omega_M} \left( \frac{a}{a_0} \right)^3}
\]

\[ \Omega_M \equiv \frac{k\bar{\rho}c^4}{3H_0^2 a_0^3} \]
current values

\[
H_0 \approx 68 \text{ km/s/Mpc}
\]
\[ \Omega_M \approx 0.31 \]
This Yukawa interaction range and dimensions of the known largest cosmic structures are of the same order!

Hercules-Corona Borealis Great Wall \( \sim 2\text{-}3\ \text{Gpc} \)

Giant Gamma Ray Burst Ring \( \sim 1.7\ \text{Gpc} \)

Huge Large Quasar Group \( \sim 1.2\ \text{Gpc} \)
Obvious hint at a resolution opportunity: to associate the scale of homogeneity with $\lambda$ ($\sim 3.7$ Gpc today) instead of $\sim 370$ Mpc.

Formidable challenge: dimensions of the largest cosmic structures essentially exceed the scale of homogeneity $\sim 370$ Mpc.


Cosmological principle (Universe is homogeneous and isotropic when viewed at a sufficiently large scale) is saved and reinstated when this typical averaging scale is greater than $\lambda$.

$\lambda \sim a^{3/2}$

$a \downarrow \Rightarrow \lambda \downarrow$
What are the applicability bounds for peculiar motion ignoring?

\[
\frac{3\kappa c^2}{8\pi a} \tilde{H} \cdot \frac{m_n}{|r-r_n|} \left[ \frac{1}{2} \frac{\tilde{v}_n(r-r_n)}{|r-r_n|} \right] \sim ???
\]

ratio of velocity-dependent and-independent terms in \( \Phi \)

For a single gravitating mass \( m_1 \) momentarily located at the origin of coordinates with the velocity collinear to \( r \):

\[
\frac{3\kappa c^2}{8\pi a} \tilde{H} \cdot m_1 \tilde{v}_1 \cdot \frac{1}{2} = \frac{3}{2} \tilde{H} \tilde{v}_1 r = \frac{3}{2} \frac{Hav_1 R}{c^2}
\]

\[
v_1 \equiv |\mathbf{v}_1| = c\tilde{v}_1/a
\]

\[
R \equiv |\mathbf{R}| = ar
\]

\[
q_1 \ll 1
\]
3H \cdot a v_1 \cdot R/(2c^2) \ll (1 \div 2) \times 10^{-3}

absolute value of the particle’s physical peculiar velocity \sim (250 \div 500) \text{ km/s}

\lambda \neq c/H \quad \text{if} \quad q \equiv -\ddot{a}/(aH^2) \neq -2/3

Hubble radius

deceleration parameter

\[ \frac{1}{\lambda^2} = \frac{3H^2}{c^2}(1+q) = -\frac{3\dot{H}}{c^2} \]

\[ \lambda = \frac{1}{\sqrt{3(1+q)}} \frac{c}{H} \]
Yukawa interaction
Zero average values

\[ \Phi = \frac{1}{3} + \sum_n \phi_n + \text{velocity-dependent part} \]

manifestation of the superposition principle

\[ \phi_n = -\frac{k c^2}{8 \pi a} \frac{m_n}{|r - r_n|} \exp(-q_n) = -\frac{G_N m_n}{c^2 |R - R_n|} \exp \left( -\frac{|R - R_n|}{\lambda} \right) \]

Yukawa potentials coming from each single particle, with the same interaction radius \( \lambda \)
Computation of a sum in Newtonian approximation


\[ \Phi \sim \int dr' \frac{\rho|_{r=r'} - \bar{\rho}}{|r-r'|} \]  
\[ (8.1) \]

\[ -\nabla \Phi \sim \int dr' \frac{\rho|_{r=r'}}{|r-r'|^3} (r-r') \]  
\[ (8.3) \]

\[ -\nabla \Phi \sim \sum_n \frac{m_n}{|r-r_n|^3} (r-r_n) \]  
\[ (8.5) \]

not well-defined, depends on the order of adding terms; addition in the order of increasing distances \(|r-r_n|\) and a spatially homogeneous and isotropic random process with the correlation length \(\ll c/H\) for the distribution of particles are required for convergence of such a sum
Summing up the Yukawa potentials

convergent in all points except the positions of the gravitating masses

no obstacles in the way of computation, the order of adding terms corresponding to different particles is arbitrary and does not depend on their locations

no famous Neumann-Seeliger gravitational paradox

particles’ distribution may be nonrandom and anisotropic (e.g., the lattice Universe)
\[
\bar{\phi}_n \equiv \frac{1}{V} \int d\mathbf{r} \phi_n = -\frac{\kappa c^2}{8\pi a} \frac{m_n}{V} \int d\mathbf{r} \exp \left( - \frac{a |\mathbf{r} - \mathbf{r}_n|}{\lambda} \right) = -\frac{\kappa c^2}{8\pi a} \frac{m_n}{V} \frac{4\pi \lambda^2}{a^2} = -\frac{m_n}{V} \frac{1}{3\bar{\rho}}
\]

**comoving averaging volume, tending to infinity**

\[
\frac{1}{V} \sum_n m_n \equiv \bar{\rho}
\]

\[
\sum_n \bar{\phi}_n = -\frac{1}{3\bar{\rho}} \cdot \frac{1}{V} \sum_n m_n = -\frac{1}{3}
\]

\[
\bar{\Phi} = \frac{1}{3} + \sum_n \bar{\phi}_n + \text{velocity-dependent part} = 0
\]

\[
\bar{\delta T}_0 = 0
\]

\[
\bar{\delta T}_\alpha = 0
\]

**no first-order backreaction effects**
In addition, in the limiting case of the homogeneous mass distribution \( \Phi = 0 \) at any point. For example, on the surface of a sphere of the physical radius \( R \) the contributions from its inner and outer regions combined with \( 1/3 \) give 0.

Then Eq. of motion of a test cosmic body reads:

\[
\ddot{R} = \frac{\ddot{a}}{a} R
\]

(\( \ddot{R} \) is reasonably connected with the acceleration of the global Universe expansion)
Proof: \[
\ddot{R} - \frac{\dddot{a}}{a} R = -c^2 \frac{\partial \Phi}{\partial R} \quad \frac{\partial \Phi}{\partial R} = \frac{\partial}{\partial R} \left( \sum_n \phi_n \right)
\]


radial acceleration of a test body within a uniformly filled spherical shell of the inner and outer radii \( R_{1,2} \):

\[-c^2 \frac{\partial \Phi}{\partial R} = - \frac{4 \pi G_N \overline{\rho} \lambda^3}{a^3 R^2} \left\{ h(R) \left( 1 + \frac{R_2}{\lambda} \right) \exp \left( - \frac{R_2}{\lambda} \right) - h(R_1) \left( 1 + \frac{R}{\lambda} \right) \exp \left( - \frac{R}{\lambda} \right) \right\} \]

\[h(R) \equiv \frac{R}{\lambda} \cosh \left( \frac{R}{\lambda} \right) - \sinh \left( \frac{R}{\lambda} \right)\]

**Homogeneous Universe corresponds to simultaneous limits** \( R_1 \to 0, \quad R_2 \to +\infty \)

\[-c^2 \frac{\partial \Phi}{\partial R} = 0, \quad \ddot{R} - \frac{\dddot{a}}{a} R = 0\]
If a particle is so far from its closest neighbors that their fields are negligible at its location, then such a particle obeys this equation of motion approaching asymptotically the Hubble flow $\dot{R} = HR$.

On the contrary, in the framework of Newtonian cosmological approximation the outer region does not contribute to the radial acceleration while the inner region does:

$$-c^2 \frac{\partial \Phi}{\partial R} = -\frac{4\pi G_N \bar{\rho}}{3a^3} R \quad \Rightarrow \quad \ddot{R} = \left(-\frac{8\pi G_N \bar{\rho}}{3a^3} + \frac{\Lambda c^2}{3}\right) R$$
Transformation of spatial coordinates

Poisson/longitudinal/conformal-Newtonian gauge:

\[ ds^2 = a^2 \left[ (1 + 2\Phi) d\eta^2 - (1 - 2\Phi) \delta_{\alpha\beta} dx^\alpha dx^\beta \right] \]

gauge-invariant

Bardeen potential

vector and tensor perturbations are not taken into account

Transformation of coordinates:

\[ \eta = \tau + A, \quad x^\alpha = \xi^\alpha + \frac{\partial L}{\partial \xi^\alpha} \]

(first-order) functions of the new conformal time and comoving coordinates
\begin{align*}
\text{prime:} & \quad \text{derivative with respect to } \tau \\
A = 0 & \quad \text{coincidence of fluctuations of mixed energy-momentum tensor components with corresponding gauge-invariant ones}
\end{align*}
\begin{align*}
\delta T_0^0 &= \frac{c^2}{a^3} \delta \rho_\xi + \frac{\bar{\rho} c^2}{a^3} \left( 3 \Phi - \Delta_\xi L \right) \\
\Delta_\xi &\equiv \delta^{\alpha\beta} \frac{\partial^2}{\partial \xi^\alpha \partial \xi^\beta}
\end{align*}
**N-body gauge idea:**

\[ \Delta_\xi L = 3\Phi \]

\[ \delta T^0_0 = \frac{c^2}{a^3} \delta \rho_\xi \]

\[ \delta \rho_\xi = \rho_\xi - \bar{\rho} \]

\[ \rho = \frac{\rho_\xi}{\det \left( \frac{\partial x^\alpha}{\partial \xi^\beta} \right)} = \frac{\rho_\xi}{1 + \Delta_\xi L} \]

\[ \delta \rho_\xi = \delta \rho + \bar{\rho} \Delta_\xi L = \delta \rho + 3 \bar{\rho} \Phi \]

**Initial displacement of particles idea:**

\[ x^\alpha = \xi^\alpha + \delta x^\alpha_{in} \]

N.E. Chisari and M. Zaldarriaga, Phys. Rev. D 83, 123505 (2011) \hspace{1cm} arXiv:1101.3555
\[
\frac{\partial}{\partial \xi^\alpha} (\delta x^\alpha) = 3 \zeta_{\text{in}} \quad \rightarrow \quad \text{initial value of comoving curvature, or curvature perturbation variable}
\]

\[
\Delta \xi L \quad \text{is replaced by} \quad 3 \zeta_{\text{in}}
\]

\[
\zeta = \frac{2a \tilde{H} (\Phi' + \tilde{H} \Phi)}{\kappa \bar{\rho} c^2} + \Phi
\]

\[
\Delta \Phi - 3 \tilde{H} (\Phi' + \tilde{H} \Phi) = \frac{\kappa c^2}{2a} \left[ \delta \rho_\xi + 3 \bar{\rho} (\Phi - \zeta_{\text{in}}) \right]
\]

The introduced comoving curvature does not evolve at large enough scales: \( \zeta_{\text{in}} \approx \zeta \quad \rightarrow \quad \)

\[
\Delta \Phi = \frac{\kappa c^2}{2a} \delta \rho_\xi \quad \leftarrow ?? \quad \Delta \Phi - 3 \tilde{H} (\Phi' + \tilde{H} \Phi) = \frac{\kappa c^2}{2a} (\delta \rho + 3 \bar{\rho} \Phi)
\]

\[
\delta \rho_\xi = \delta \rho + 3 \bar{\rho} \zeta_{\text{in}}
\]
Nonzero spatial curvature
Screening of gravity

\[
\Delta \Phi + \left( 3K - \frac{3\kappa \bar{\rho}c^2}{2a} \right) \Phi = \frac{\kappa c^2}{2a} \delta \rho
\]

velocities’ contributions dropped

+ 1 for the spherical (closed) space
– 1 for the hyperbolic (open) space

Solutions are smooth at any point except particles’ positions (where Newtonian limits are reached) and characterized by zero average values as before.
\[ \lambda = \frac{1}{\sqrt{3(1+q)}} \frac{c}{H} \]

not only for the curved space, but also in the presence of an arbitrary number of additional Universe components in the form of barotropic perfect fluids

at the radiation-dominated stage of the Universe evolution \( \lambda \sim a^2 \)

Since \( \lambda \) may be associated with the homogeneity scale, asymptotic behaviour \( \lambda \to 0 \) when \( a \to 0 \) supports the idea of the homogeneous Big Bang.
irresistible temptation of associating the Yukawa interaction range $\lambda$ with the graviton Compton wavelength $\frac{\hbar}{(m_g c)}$

Planck constant $\hbar$ 
graviton mass $m_g$ 

\[
\frac{1}{\lambda^2} = \frac{m_g^2 c^2}{\hbar^2} = \frac{2\Lambda}{3}
\]

$9\Omega_M = 4\Omega_\Lambda$

$\Omega_M = \frac{4}{13} \approx 0.31, \quad \Omega_\Lambda = \frac{9}{13} \approx 0.69$

$\hbar \equiv \hbar/(2\pi)$

$\lambda = \frac{\hbar}{(m_g c)}$

(today)

$m_g = \frac{\hbar}{(\lambda c)} \approx 1.7 \times 10^{-33}$ eV

$\Omega_\Lambda = \frac{\Lambda c^2}{3H_0^2}$

$\Omega_M + \Omega_\Lambda = 1$
Conclusion

first-order scalar and vector cosmological perturbations, produced by inhomogeneities in the discrete form of a system of separate point-like gravitating masses, are derived without any extra approximations in addition to the weak gravitational field limit (no $c^{-1}$ series expansion, no “dictionaries”);
II

Obtained metric corrections are valid at all (sub-horizon and super-horizon) scales and converge in all points except locations of sources (where Newtonian limits are reached), and their average values are zero (no first-order backreaction effects);

III

The Minkowski background limit and Newtonian cosmological approximation are particular cases;
the velocity-independent part of the scalar perturbation contains a sum of Yukawa potentials with the same finite time-dependent Yukawa interaction range, which may be connected with the scale of homogeneity, thereby explaining existence of the largest cosmic structures;

the general Yukawa range definition is given for various extensions of the concordance model (nonzero spatial curvature, additional perfect fluids).
Let’s proceed together?
What is the length of the Sydney Harbour Bridge?

A. 1149 m
B. 21 cm
C. 0.7 AU
D. 3.7 Gpc
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A. 1149 m
B. 21 cm
C. 0.7 AU
D. 3.7 Gpc
Whose law is the most appropriate for description of universal gravitation???

A. Newton  B. Einstein  
C. Yukawa  D. Coulomb
Whose law is the most appropriate for description of universal gravitation???

A. Newton  B. Einstein  C. Yukawa  D. Coulomb
THANK YOU FOR ATTENTION!

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