ALL-SCALE cosmological perturbations and SCREENING OF GRAVITY in inhomogeneous Universe

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Outline

Introduction

(concordance cosmology, perturbation theory)

Discrete picture of (scalar and vector) cosmological perturbations (at all sub- and super-horizon scales)

(weak gravitational field limit, point-like masses)

Menu of properties, benefits, and bonuses

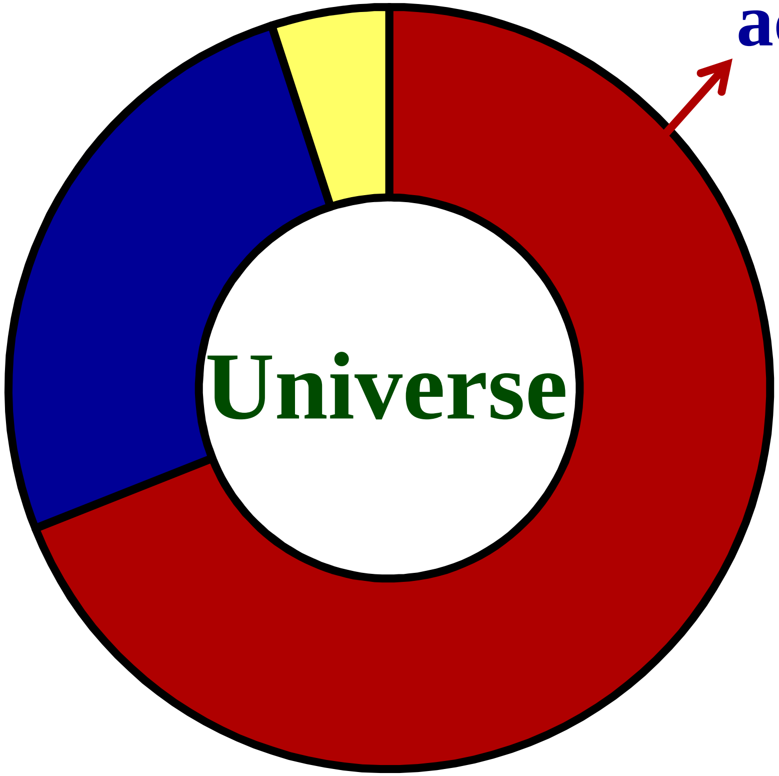
- Minkowski background limit
- Newtonian approximation and homogeneity scale
- Yukawa interaction and zero average values
- Transformation of spatial coordinates
- Nonzero spatial curvature and screening of gravity

Conclusion + fun

Introduction

Concordance cosmology: Λ C(old) D(ark) M(atter) model

acceleration of global expansion



- **~ 69% (Λ)**
- **~ 26% (CDM)**
- **~ 5% (SM) (baryons, photons)**

Planck 2015 results. XIII. Cosmological parameters **arXiv:1502.01589**

Cosmological principle

on large enough scales the Universe is treated as being homogeneous and isotropic → F(riedmann)-L(emaître)-R(obertson)-W(alker) background metric

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observed separate galaxies, their groups and clusters → on sufficiently small scales the Universe is highly inhomogeneous

↑ (structure formation → observed galaxies) ↓ (inhomogeneous → perturbation theory)

structure formation from primordial fluctuations at earliest evolution stages ← perturbation theory

Two main distinct approaches to structure growth investigation

relativistic perturbation theory	N-body simulations generally based on Newtonian cosmological approximation

Keywords

early Universe; linearity; large scales	late Universe; nonlinearity; small scales
fails in describing nonlinear dynamics at small distances	do not take into account relativistic effects becoming non-negligible at large distances

The acute problem: construction of a self-consistent unified scheme, which would be valid for arbitrary (sub- & super-horizon) scales and incorporate linear & nonlinear effects.

very promising in precision cosmology era

Weak gravitational field limit

Deviations of the metric coefficients from their background (average) values are considered as 1^{st} order quantities, while the 2^{nd} order is completely disregarded.

A couple of previous attempts to develop a unified perturbation theory

I. Generalization of nonrelativistic post-Minkowski formalism to the cosmological case in the form of relativistic post-Friedmann formalism, which would be valid on all scales and include the full nonlinearity of Newtonian gravity at small distances:

expansion of the metric in powers of the parameter 1/c (the inverse speed of light)

I. Milillo, D. Bertacca, M. Bruni and A. Maselli, Phys. Rev. D 92, 023519 (2015) arXiv:1502.02985

II. Formalism for relativistic N-body simulations:

different orders of smallness given to the metric corrections and their spatial derivatives ("dictionary")

J. Adamek, D. Daverio, R. Durrer and M. Kunz,
Phys. Rev. D 88, 103527 (2013) arXiv:1308.6524

S.R. Green and R.M. Wald, Phys. Rev. D 85, 063512 (2012)
arXiv:1111.2997

Discrete cosmology: presenting nonrelativistic matter as separate point-like massive particles

Advantages of the unified scheme developed here

1) no any supplementary approximations or extra assumptions in addition to the weak field limit;

2) spatial and temporal derivatives are treated on an equal footing, no “dictionaries”;

3) no expansion into series with respect to the ratio 1/c;

LET’S GO !!!

4) no artificial mixing of first- and second-order contributions to the metric;

5) sub- or super-horizon regions are not singled out

Discrete picture of (scalar and vector) cosmological perturbations

Unperturbed FLRW metric describing (homogeneous and isotropic on the average) Universe:

$$ds^2 = a^2\left(d\eta^2 - \delta_{\alpha\beta}dx^\alpha dx^\beta\right) \qquad \alpha,\beta = 1,2,3$$

scale factor (a)

conformal time (η)

comoving coordinates (x^α, x^β)

spatial curvature is zero (generalization to non-flat spatial geometry is simple)

Friedmann Eqs. in the framework of the pure ΛCDM model (with a negligible radiation contribution):

$$\frac{3\tilde{H}^2}{a^2} = \kappa\overline{\varepsilon} + \Lambda$$

energy density of nonrelativistic pressureless matter

$$\frac{2\tilde{H}' + \tilde{H}^2}{a^2} = \Lambda$$

cosmological constant

$$\tilde{H} \equiv \frac{a'}{a} \qquad \kappa \equiv 8\pi G_N / c^4$$

Newtonian gravitational constant

overline: average value ;

prime: derivative with respect to η

Perturbed metric describing (inhomogeneous and anisotropic) Universe:

$$ds^2 = a^2\left[\left(1+2\Phi\right)d\eta^2 + 2B_\alpha dx^\alpha d\eta - \left(1-2\Phi\right)\delta_{\alpha\beta}dx^\alpha dx^\beta\right]$$

function $\Phi\left(\eta,\mathbf{r}\right)$ **and spatial vector** $\mathbf{B}\left(\eta,\mathbf{r}\right)\equiv\left(B_1,B_2,B_3\right)$**:**
scalar and vector perturbations, respectively

$$\nabla\mathbf{B} = \delta^{\alpha\beta}\frac{\partial B_\alpha}{\partial x^\beta} = 0$$

tensor perturbations are not taken into account

Einstein Eqs.: $G_i^k = \kappa T_i^k + \Lambda\delta_i^k \qquad i,k = 0,1,2,3$

mixed components of Einstein and matter energy-momentum tensors

$$G_0^0 = \kappa T_0^0 + \Lambda \quad \Rightarrow \quad \Delta\Phi - 3\tilde{H}\left(\Phi' + \tilde{H}\Phi\right) = \frac{1}{2}\kappa a^2 \delta T_0^0$$

$$G_\alpha^0 = \kappa T_\alpha^0 \quad \Rightarrow \quad \frac{1}{4}\Delta B_\alpha + \frac{\partial}{\partial x^\alpha}\left(\Phi' + \tilde{H}\Phi\right) = \frac{1}{2}\kappa a^2 \delta T_\alpha^0$$

$$G_\alpha^\beta = \kappa T_\alpha^\beta + \Lambda \delta_\alpha^\beta \quad \Rightarrow \quad \Phi'' + 3\tilde{H}\Phi' + \left(2\tilde{H}' + \tilde{H}^2\right)\Phi = 0$$

$$\left(\frac{\partial B_\alpha}{\partial x^\beta} + \frac{\partial B_\beta}{\partial x^\alpha}\right)' + 2\tilde{H}\left(\frac{\partial B_\alpha}{\partial x^\beta} + \frac{\partial B_\beta}{\partial x^\alpha}\right) = 0$$

$$\Delta \equiv \delta^{\alpha\beta}\frac{\partial^2}{\partial x^\alpha \partial x^\beta}$$

$$T_i^k = \bar{T}_i^k + \delta T_i^k \qquad \bar{T}_0^0 = \bar{\varepsilon}$$

Laplace operator in comoving coordinates

only nonzero average mixed component

in the spirit of the particle-particle method of N-body simulations

gravitating masses

$$T^{ik} = \sum_n \frac{m_n c^2}{\sqrt{-g}} \frac{dx_n^i}{d\eta} \frac{dx_n^k}{d\eta} \frac{d\eta}{ds_n} \delta(\mathbf{r} - \mathbf{r}_n)$$

$$g \equiv \det(g_{ik})$$

comoving radius-vectors

4-velocities

$$u_n^i \equiv dx_n^i / ds_n$$

$$\rho = \sum_n m_n \delta(\mathbf{r} - \mathbf{r}_n) = \sum_n \rho_n \qquad \rho_n \equiv m_n \delta(\mathbf{r} - \mathbf{r}_n)$$

rest mass density

comoving peculiar velocities

$$\tilde{v}_n^\alpha \equiv dx_n^\alpha / d\eta$$

$\tilde{v}_n^{\alpha}$ **import the 1st order of smallness in rhs of linearized Einstein Eqs.**

$$\delta\rho \equiv \rho - \bar{\rho}$$

$$\delta T_0^0 \equiv T_0^0 - \bar{T}_0^0 = \frac{c^2}{a^3}\delta\rho + \frac{3\bar{\rho}c^2}{a^3}\Phi$$

$$\delta T_\alpha^0 = -\frac{c^2}{a^3}\sum_n m_n \delta\left(\mathbf{r}-\mathbf{r}_n\right)\tilde{v}_n^{\alpha} + \frac{\bar{\rho}c^2}{a^3}B_\alpha = -\frac{c^2}{a^3}\sum_n \rho_n \tilde{v}_n^{\alpha} + \frac{\bar{\rho}c^2}{a^3}B_\alpha$$

$$\delta T_\alpha^\beta = 0$$

replacements:

$$\rho\Phi \to \bar{\rho}\Phi, \quad \rho\mathbf{B} \to \bar{\rho}\mathbf{B}$$

1st order

$$\Delta\Phi - 3\tilde{H}\left(\Phi' + \tilde{H}\Phi\right) - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi = \frac{\kappa c^2}{2a}\delta\rho$$

$$\frac{1}{4}\Delta\mathbf{B} + \nabla\left(\Phi' + \tilde{H}\Phi\right) - \frac{\kappa\bar{\rho}c^2}{2a}\mathbf{B} = -\frac{\kappa c^2}{2a}\sum_n m_n\delta\left(\mathbf{r}-\mathbf{r}_n\right)\tilde{\mathbf{v}}_n = -\frac{\kappa c^2}{2a}\sum_n \rho_n \tilde{\mathbf{v}}_n$$

Continuity Eq.: $$\rho_n' + \nabla\left(\rho_n \tilde{\mathbf{v}}_n\right) = 0$$

$$\tilde{\mathbf{v}}_n(\eta) \equiv d\mathbf{r}_n / d\eta \equiv \left(\tilde{v}_n^1, \tilde{v}_n^2, \tilde{v}_n^3\right)$$

$$\sum_n \rho_n \tilde{\mathbf{v}}_n = \underbrace{\nabla\Xi}_{\textbf{grad}} + \underbrace{\left(\sum_n \rho_n \tilde{\mathbf{v}}_n - \nabla\Xi\right)}_{\textbf{curl}}$$

$$\Xi = \frac{1}{4\pi} \sum_n m_n \frac{\left(\mathbf{r} - \mathbf{r}_n\right)\tilde{\mathbf{v}}_n}{\left|\mathbf{r} - \mathbf{r}_n\right|^3}$$

$$\Phi' + \tilde{H}\Phi = -\frac{\kappa c^2}{2a}\Xi$$

Fourier transform:

$$\hat{\Xi}(\eta, \mathbf{k}) \equiv \int \Xi(\eta, \mathbf{r}) \exp(-i\mathbf{k}\mathbf{r})\, d\mathbf{r} = -\frac{i}{k^2} \sum_n m_n \left(\mathbf{k}\tilde{\mathbf{v}}_n\right) \exp\left(-i\mathbf{k}\mathbf{r}_n\right) \qquad k \equiv |\mathbf{k}|$$

$$\hat{\rho}_n(\eta, \mathbf{k}) \equiv \int \rho_n(\eta, \mathbf{r}) \exp(-i\mathbf{k}\mathbf{r})\, d\mathbf{r} = m_n \int \delta\left(\mathbf{r} - \mathbf{r}_n\right) \exp(-i\mathbf{k}\mathbf{r})\, d\mathbf{r} = m_n \exp\left(-i\mathbf{k}\mathbf{r}_n\right)$$

$$\frac{1}{4}\Delta\mathbf{B}-\frac{\kappa\bar{\rho}c^{2}}{2a}\mathbf{B}=-\frac{\kappa c^{2}}{2a}\left(\sum_{n}\rho_{n}\tilde{\mathbf{v}}_{n}-\nabla\Xi\right)$$

$$-\frac{k^{2}}{4}\hat{\mathbf{B}}-\frac{\kappa\bar{\rho}c^{2}}{2a}\hat{\mathbf{B}}=-\frac{\kappa c^{2}}{2a}\left(\sum_{n}\hat{\rho}_{n}\tilde{\mathbf{v}}_{n}-i\mathbf{k}\hat{\Xi}\right)$$

$$\hat{\mathbf{B}}=\frac{2\kappa c^{2}}{a}\left(k^{2}+\frac{2\kappa\bar{\rho}c^{2}}{a}\right)^{-1}\sum_{n}m_{n}\exp\left(-i\mathbf{k}\mathbf{r}_{n}\right)\left(\tilde{\mathbf{v}}_{n}-\frac{\left(\mathbf{k}\tilde{\mathbf{v}}_{n}\right)}{k^{2}}\mathbf{k}\right)$$

$$\boxed{\begin{aligned}\mathbf{B}=\frac{\kappa c^{2}}{8\pi a}\sum_{n}\Bigg[&\frac{m_{n}\tilde{\mathbf{v}}_{n}}{\left|\mathbf{r}-\mathbf{r}_{n}\right|}\cdot\frac{\left(3+2\sqrt{3}q_{n}+4q_{n}^{2}\right)\exp\left(-2q_{n}/\sqrt{3}\right)-3}{q_{n}^{2}}\\&+\frac{m_{n}\left[\tilde{\mathbf{v}}_{n}\left(\mathbf{r}-\mathbf{r}_{n}\right)\right]}{\left|\mathbf{r}-\mathbf{r}_{n}\right|^{3}}\left(\mathbf{r}-\mathbf{r}_{n}\right)\cdot\frac{9-\left(9+6\sqrt{3}q_{n}+4q_{n}^{2}\right)\exp\left(-2q_{n}/\sqrt{3}\right)}{q_{n}^{2}}\Bigg]\end{aligned}}$$

$$\Delta\Phi - \frac{3\kappa\bar{\rho}c^2}{2a}\Phi = \frac{\kappa c^2}{2a}\delta\rho - \frac{3\kappa c^2\tilde{H}}{2a}\boldsymbol{\Xi}$$

$$-k^2\hat{\Phi} - \frac{3\kappa\bar{\rho}c^2}{2a}\hat{\Phi} = \frac{\kappa c^2}{2a}\sum_n \hat{\rho}_n - \frac{\kappa\bar{\rho}c^2}{2a}(2\pi)^3\delta(\mathbf{k}) - \frac{3\kappa c^2\tilde{H}}{2a}\hat{\boldsymbol{\Xi}}$$

$$\hat{\Phi} = -\frac{\kappa c^2}{2a}\left(k^2 + \frac{3\kappa\bar{\rho}c^2}{2a}\right)^{-1}\left[\sum_n m_n \exp(-i\mathbf{k}\mathbf{r}_n)\left(1 + 3i\tilde{H}\frac{(\mathbf{k}\tilde{\mathbf{v}}_n)}{k^2}\right) - \bar{\rho}(2\pi)^3\delta(\mathbf{k})\right]$$

$$\boxed{\begin{aligned}\Phi = {} & \frac{1}{3} - \frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\mathbf{r}-\mathbf{r}_n|}\exp(-q_n) \\ & + \frac{3\kappa c^2}{8\pi a}\tilde{H}\sum_n \frac{m_n\left[\tilde{\mathbf{v}}_n(\mathbf{r}-\mathbf{r}_n)\right]}{|\mathbf{r}-\mathbf{r}_n|}\cdot\frac{1-(1+q_n)\exp(-q_n)}{q_n^2}\end{aligned}}$$

$$\mathbf{q}_n(\eta,\mathbf{r}) \equiv \sqrt{\frac{3\kappa\bar{\rho}c^2}{2a}}(\mathbf{r}-\mathbf{r}_n) \qquad q_n \equiv |\mathbf{q}_n|$$

Thus, explicit expressions for 1st order vector and scalar cosmological perturbations are determined.

satisfied

???

satisfied

???

??? $\longrightarrow$ $\hat{\mathbf{B}}' + 2\tilde{H}\hat{\mathbf{B}} = 0$???

Incidentally, $\nabla \mathbf{B} = 0 \rightarrow \mathbf{k}\hat{\mathbf{B}} = 0$ satisfied

Equations of motion

Spacetime interval for the n-th particle

$$ds_n = a\left[1+2\Phi+2B_\alpha \tilde{v}_n^\alpha-(1-2\Phi)\,\delta_{\alpha\beta}\tilde{v}_n^\alpha \tilde{v}_n^\beta\right]^{1/2} d\eta$$

$$\left[a\left(\mathbf{B}\big|_{\mathbf{r}=\mathbf{r}_n}-\tilde{\mathbf{v}}_n\right)\right]' = a\nabla\Phi\big|_{\mathbf{r}=\mathbf{r}_n} \qquad \big|\rho_n \qquad \Big|\sum_n$$

$$\rho(a\mathbf{B})' - \sum_n \rho_n (a\tilde{\mathbf{v}}_n)' = a\rho\nabla\Phi \qquad \sum_n \rho_n (a\tilde{\mathbf{v}}_n)' = -a\bar{\rho}\nabla\Phi + \bar{\rho}(a\mathbf{B})'$$

$$\sum_n \hat{\rho}_n (a\tilde{\mathbf{v}}_n)' = \sum_n m_n \exp(-i\mathbf{k}\mathbf{r}_n)(a\tilde{\mathbf{v}}_n)' = -a\bar{\rho}\cdot i\mathbf{k}\hat{\Phi} + \bar{\rho}\left(a\hat{\mathbf{B}}\right)'$$

all ??? satisfied

$$\boxed{(a\tilde{\mathbf{v}}_n)' = -a\left(\nabla\Phi\big|_{\mathbf{r}=\mathbf{r}_n} + \tilde{H}\,\mathbf{B}\big|_{\mathbf{r}=\mathbf{r}_n}\right)}$$

Menu of properties, benefits, and bonuses

Minkowski background limit

$$a \to \text{const} \quad \Rightarrow \quad \tilde{H} \to 0 \qquad\qquad \bar{\rho} \to 0 \quad \Rightarrow \quad q_n \to 0$$

$$\Phi \to -\frac{\kappa c^2}{8\pi a}\sum_n \frac{m_n}{|\mathbf{r}-\mathbf{r}_n|} = -\frac{G_N}{c^2}\sum_n \frac{m_n}{|\mathbf{R}-\mathbf{R}_n|}$$

$$\mathbf{R} = a\mathbf{r}$$

$$\mathbf{R}_n = a\mathbf{r}_n$$

physical radius-vectors

The constant 1/3 has been dropped since it originates exclusively from the terms containing $\bar{\rho}$.

$$\mathbf{B} \to \frac{\kappa c^2}{4\pi a}\sum_n \left[\frac{m_n \tilde{\mathbf{v}}_n}{|\mathbf{r}-\mathbf{r}_n|} + \frac{m_n \left[\tilde{\mathbf{v}}_n (\mathbf{r}-\mathbf{r}_n) \right]}{|\mathbf{r}-\mathbf{r}_n|^3} (\mathbf{r}-\mathbf{r}_n) \right]$$

$$= \frac{G_N}{2c^2}\sum_n \frac{m_n}{|\mathbf{R}-\mathbf{R}_n|} \left[4\tilde{\mathbf{v}}_n + \frac{4\left[\tilde{\mathbf{v}}_n (\mathbf{R}-\mathbf{R}_n) \right]}{|\mathbf{R}-\mathbf{R}_n|} \frac{\mathbf{R}-\mathbf{R}_n}{|\mathbf{R}-\mathbf{R}_n|} \right]$$

$$\tilde{\mathbf{v}}_n \equiv \frac{d\mathbf{r}_n}{d\eta}, \quad \mathbf{v}_n \equiv \frac{d\mathbf{r}_n}{dt}$$

$$cdt = ad\eta \quad \Rightarrow \quad \tilde{\mathbf{v}}_n = \frac{a\mathbf{v}_n}{c}$$

synchronous time

The sum of these integers

4 + 4 = 8

is the same for the other appropriate choices of gauge conditions as well.

complete agreement with textbooks

Newtonian approximation
Homogeneity scale

$$\tilde{\mathbf{v}}_n \to 0$$

$$q_n \ll 1$$

(peculiar motion as a gravitational field source is completely ignored)

$$\Phi \to -\frac{G_N}{c^2}\sum_n \frac{m_n}{\left|\mathbf{R}-\mathbf{R}_n\right|} \qquad \mathbf{B} \to 0$$

The constant 1/3 has been dropped for the other reason: only the gravitational potential gradient enters into Eqs. of motion describing dynamics of the considered system of gravitating masses.

$$\ddot{\mathbf{R}}_j - \frac{\ddot{a}}{a}\mathbf{R}_j = -G_N \sum_{n \neq j} \frac{m_n \left(\mathbf{R}_j - \mathbf{R}_n\right)}{\left|\mathbf{R}_j - \mathbf{R}_n\right|^3}$$

dot: **derivative with respect to** t

complete agreement with Eqs. for N-body simulations

What are the applicability bounds for the inequality?

$$q_n \ll 1 \quad \Leftrightarrow \quad \left|\mathbf{R} - \mathbf{R}_n\right| \ll \lambda$$

$$H \equiv \frac{\dot{a}}{a} = \frac{c\tilde{H}}{a}$$

$$\boxed{\lambda \equiv \sqrt{\frac{2a^3}{3\kappa\bar{\varphi}c^2}} = \sqrt{\frac{2c^2}{9H_0^2\Omega_M}\left(\frac{a}{a_0}\right)^3}}$$

$$\Omega_M \equiv \frac{\kappa\bar{\varphi}c^4}{3H_0^2 a_0^3}$$

Hubble parameter

$$H_0 \approx 68 \text{ km/s/Mpc}$$

$$\Omega_M \approx 0.31$$

current values

today $\boxed{\lambda_0 \approx 3700\ \mathrm{Mpc} = 3.7\ \mathrm{Gpc} \approx 12\ \mathrm{Gly}}$

This Yukawa interaction range and dimensions of the known largest cosmic structures are of the same order !

Hercules-Corona Borealis Great Wall ~ 2-3 Gpc

I. Horvath, J. Hakkila and Z. Bagoly, A&A 561, L12 (2014); arXiv:1401.0533

Giant Gamma Ray Burst Ring ~ 1.7 Gpc

L.G. Balazs, Z. Bagoly, J.E. Hakkila, I. Horvath, J. Kobori, I. Racz, L.V. Toth, Mon. Not. R. Astron. Soc. 452, 2236 (2015); arXiv:1507.00675

Huge Large Quasar Group ~ 1.2 Gpc

R.G. Clowes, K.A. Harris, S. Raghunathan, L.E. Campusano, I.K. Soechting, M.J. Graham, Mon. Not. R. Astron. Soc. 429, 2910 (2013); arXiv:1211.6256

Formidable challenge: dimensions of the largest cosmic structures essentially exceed the scale of homogeneity ~ 370 Mpc.

J.K. Yadav, J.S. Bagla and N. Khandai, Mon. Not. Roy. Astron. Soc. 405, 2009 (2010) arXiv:1001.0617

obvious hint at a resolution opportunity: to associate the scale of homogeneity with λ (~ 3.7 Gpc today) instead of ~ 370 Mpc

$$\lambda \sim a^{3/2}$$

Cosmological principle (Universe is homogeneous and isotropic when viewed at a sufficiently large scale) is saved and reinstated when this typical averaging scale is greater than λ.

$$a\downarrow \ \Rightarrow \ \lambda\downarrow$$

What are the applicability bounds for peculiar motion ignoring?

$$\left|\frac{\dfrac{3\kappa c^2}{8\pi a}\tilde{H}\cdot\dfrac{m_n\left[\tilde{\mathbf{v}}_n\left(\mathbf{r}-\mathbf{r}_n\right)\right]}{\left|\mathbf{r}-\mathbf{r}_n\right|}\cdot\dfrac{1}{2}}{-\dfrac{\kappa c^2}{8\pi a}\cdot\dfrac{m_n}{\left|\mathbf{r}-\mathbf{r}_n\right|}}\right|\sim ???$$

ratio of velocity -dependent and -independent terms in Φ

For a single gravitating mass m_1 momentarily located at the origin of coordinates with the velocity collinear to $\mathbf{r}$:

$$\frac{\dfrac{3\kappa c^2}{8\pi a}\tilde{H}\cdot m_1\tilde{v}_1\cdot\dfrac{1}{2}}{\dfrac{\kappa c^2}{8\pi a}\cdot\dfrac{m_1}{r}}=\frac{3}{2}\tilde{H}\tilde{v}_1 r=\frac{3}{2}\frac{Hav_1R}{c^2}$$

$$v_1\equiv\left|\mathbf{v}_1\right|=c\tilde{v}_1/a$$

$$R\equiv\left|\mathbf{R}\right|=ar$$

$$q_1\ll 1$$

$$3H \cdot \underbrace{av_1} \cdot R/\left(2c^2\right) \ll (1 \div 2) \times 10^{-3}$$

same estimate for a ratio of derivatives

absolute value of the particle's physical peculiar velocity $\sim (250 \div 500)\,\mathrm{km/s}$

$$\lambda \neq \underbrace{c/H}$$

Hubble radius

if $q \equiv -\ddot{a}/\left(aH^2\right) \neq -2/3$ $\downarrow$

deceleration parameter

$a/a_0 \neq 1.16$ **(future)**

$$\boxed{\frac{1}{\lambda^2} = \frac{3H^2}{c^2}(1+q) = -\frac{3\dot{H}}{c^2}}$$

$$\boxed{\lambda = \frac{1}{\sqrt{3(1+q)}}\frac{c}{H}}$$

Yukawa interaction
Zero average values

$$\Phi = \frac{1}{3} + \sum_n \phi_n + \text{velocity-dependent part}$$

manifestation of the superposition principle

$$\phi_n = -\frac{\kappa c^2}{8\pi a}\frac{m_n}{|\mathbf{r}-\mathbf{r}_n|}\exp(-q_n) = -\frac{G_N m_n}{c^2|\mathbf{R}-\mathbf{R}_n|}\exp\left(-\frac{|\mathbf{R}-\mathbf{R}_n|}{\lambda}\right)$$

↓

Yukawa potentials coming from each single particle, with the same interaction radius λ

Computation of a sum in Newtonian approximation

P.J.E. Peebles, The large-scale structure of the Universe, Princeton University Press, Princeton (1980).

$$\Phi \sim \int d\mathbf{r}' \frac{\rho|_{\mathbf{r}=\mathbf{r}'} - \bar{\rho}}{|\mathbf{r}-\mathbf{r}'|} \qquad \textbf{(8.1)}$$

$$-\nabla\Phi \sim \int d\mathbf{r}' \frac{\rho|_{\mathbf{r}=\mathbf{r}'}}{|\mathbf{r}-\mathbf{r}'|^3}(\mathbf{r}-\mathbf{r}') \qquad \textbf{(8.3)}$$

$$-\nabla\Phi \sim \underbrace{\sum_n \frac{m_n}{|\mathbf{r}-\mathbf{r}_n|^3}(\mathbf{r}-\mathbf{r}_n)} \qquad \textbf{(8.5)}$$

not well-defined, depends on the order of adding terms;

addition in the order of increasing distances $|\mathbf{r}-\mathbf{r}_n|$ and a spatially homogeneous and isotropic random process with the correlation length $\ll c/H$ for the distribution of particles are required for convergence of such a sum

Summing up the Yukawa potentials

convergent in all points except the positions of the gravitating masses

no famous Neumann-Seeliger gravitational paradox

$$\sum_n \phi_n$$

no obstacles in the way of computation, the order of adding terms corresponding to different particles is arbitrary and does not depend on their locations

particles' distribution may be nonrandom and anisotropic (e.g., the lattice Universe)

$$\bar{\phi}_n \equiv \frac{1}{V}\int_V d\mathbf{r}\,\phi_n = -\frac{\kappa c^2}{8\pi a}\frac{m_n}{V}\int_V \frac{d\mathbf{r}}{|\mathbf{r}-\mathbf{r}_n|}\exp\left(-\frac{a|\mathbf{r}-\mathbf{r}_n|}{\lambda}\right) = -\frac{\kappa c^2}{8\pi a}\frac{m_n}{V}\frac{4\pi\lambda^2}{a^2} = -\frac{m_n}{V}\frac{1}{3\bar{\rho}}$$

$\downarrow\downarrow$

comoving averaging volume, tending to infinity

$$\frac{1}{V}\sum_n m_n \equiv \bar{\rho} \qquad \sum_n \bar{\phi}_n = -\frac{1}{3\bar{\rho}}\cdot\frac{1}{V}\sum_n m_n = -\frac{1}{3}$$

$$\bar{\Phi} = \frac{1}{3} + \underbrace{\sum_n \bar{\phi}_n}_{-1/3} + \underbrace{\overline{\text{velocity-dependent part}}}_{0} = 0$$

$$\bar{\mathbf{B}} = 0$$

$$\overline{\delta T}{}^0_{\ 0} = 0$$

$$\overline{\delta T}{}^0_{\ \alpha} = 0$$

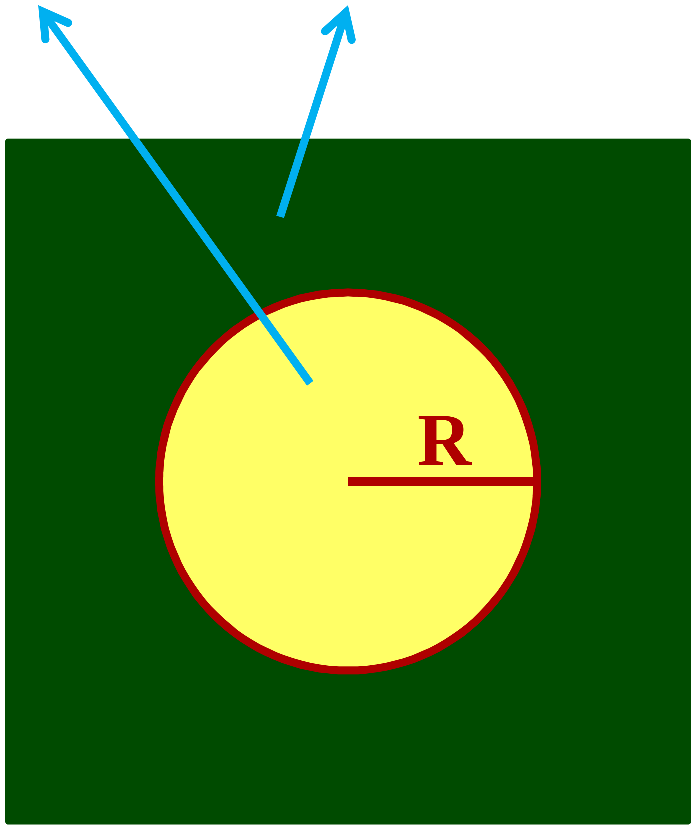
no first-order backreaction effects

In addition, in the limiting case of the homogeneous mass distribution $\Phi = 0$ at any point. For example, on the surface of a sphere of the physical radius R the contributions from its inner and outer regions combined with 1/3 give 0.

Then Eq. of motion of a test cosmic body reads:

$$\ddot{\mathbf{R}} = \frac{\ddot{a}}{a}\mathbf{R}$$

($\ddot{\mathbf{R}}$ is reasonably connected with the acceleration of the global Universe expansion)



Proof:

$$\ddot{\mathbf{R}} - \frac{\ddot{a}}{a}\mathbf{R} = -c^2 \frac{\partial \Phi}{\partial \mathbf{R}} \qquad \frac{\partial \Phi}{\partial \mathbf{R}} = \frac{\partial}{\partial \mathbf{R}}\left(\sum_n \phi_n\right)$$

M. Eingorn and A. Zhuk, Class. Quant. Grav. 27, 055002 (2010) **arXiv:0910.3507**

radial acceleration of a test body within a uniformly filled spherical shell of the inner and outer radii $R_{1,2}$:

$$-c^2 \frac{\partial \Phi}{\partial R} = -\frac{4\pi G_N \bar{\rho} \lambda^3}{a^3 R^2}\left\{h(R)\left(1+\frac{R_2}{\lambda}\right)\exp\left(-\frac{R_2}{\lambda}\right) - h(R_1)\left(1+\frac{R}{\lambda}\right)\exp\left(-\frac{R}{\lambda}\right)\right\}$$

$$h(R) \equiv \frac{R}{\lambda}\cosh\left(\frac{R}{\lambda}\right) - \sinh\left(\frac{R}{\lambda}\right)$$

Homogeneous Universe corresponds to simultaneous limits $R_1 \to 0, \quad R_2 \to +\infty \quad \longrightarrow \quad -c^2 \frac{\partial \Phi}{\partial R} = 0, \quad \ddot{\mathbf{R}} - \frac{\ddot{a}}{a}\mathbf{R} = 0$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N \bar{\rho}}{3a^3} + \frac{\Lambda c^2}{3} \quad \longrightarrow \quad \ddot{\mathbf{R}} = \left(-\frac{4\pi G_N \bar{\rho}}{3a^3} + \frac{\Lambda c^2}{3} \right) \mathbf{R}$$

If a particle is so far from its closest neighbors that their fields are negligible at its location, then such a particle obeys this equation of motion approaching asymptotically the Hubble flow $\dot{\mathbf{R}} = H\mathbf{R}$.

On the contrary, in the framework of Newtonian cosmological approximation the outer region does not contribute to the radial acceleration while the inner region does:

$$-c^2 \frac{\partial \Phi}{\partial R} = -\frac{4\pi G_N \bar{\rho}}{3a^3} R \quad \longrightarrow \quad \ddot{\mathbf{R}} = \left(-\frac{8\pi G_N \bar{\rho}}{3a^3} + \frac{\Lambda c^2}{3} \right) \mathbf{R}$$

Transformation of spatial coordinates

Poisson/longitudinal/conformal-Newtonian gauge:

$$ds^2 = a^2\left[\left(1+2\Phi\right)d\eta^2 - \left(1-2\Phi\right)\delta_{\alpha\beta}dx^\alpha dx^\beta\right]$$

gauge-invariant Bardeen potential

vector and tensor perturbations are not taken into account

Transformation of coordinates:

$$\eta = \tau + A, \quad x^\alpha = \xi^\alpha + \frac{\partial L}{\partial \xi^\alpha}$$

(first-order) functions of the new conformal time and comoving coordinates

$$ds^2 = a^2 \left[\begin{array}{l} \left(1+2\Phi+2A'+2\tilde{H}A\right)d\tau^2 + 2\left(\frac{\partial A}{\partial \xi^\alpha} - \frac{\partial L'}{\partial \xi^\alpha}\right) d\tau d\xi^\alpha \\ -\left(\left(1-2\Phi+2\tilde{H}A\right)\delta_{\alpha\beta} + 2\frac{\partial^2 L}{\partial \xi^\alpha \partial \xi^\beta}\right) d\xi^\alpha d\xi^\beta \end{array} \right]$$

prime: **derivative with respect to** τ

$A = 0 \longrightarrow$ **coincidence of fluctuations of mixed energy-momentum tensor components with corresponding gauge-invariant ones**

$$\delta T_0^0 = \frac{c^2}{a^3}\delta\rho_\xi + \frac{\bar{\rho}c^2}{a^3}\left(3\Phi - \Delta_\xi L\right) \qquad \Delta_\xi \equiv \delta^{\alpha\beta}\frac{\partial^2}{\partial \xi^\alpha \partial \xi^\beta}$$

N-body gauge idea:

C. Fidler, C. Rampf, T. Tram, R. Crittenden, K. Koyama and D. Wands, Phys. Rev. D 92, 123517 (2015) arXiv:1505.04756

$$\Delta_\xi L = 3\Phi$$

$$\delta T_0^0 = \frac{c^2}{a^3}\delta\rho_\xi \quad \overset{???}{\longleftrightarrow} \quad \delta T_0^0 = \frac{c^2}{a^3}\delta\rho + \frac{3\bar{\rho}c^2}{a^3}\Phi$$

$$\delta\rho_\xi = \rho_\xi - \bar{\rho} \qquad \rho_\xi = \sum_n m_n \delta\left(\vec{\xi} - \vec{\xi}_n\right)$$

$$\rho = \frac{\rho_\xi}{\det\left(\partial x^\alpha / \partial \xi^\beta\right)} = \frac{\rho_\xi}{1 + \Delta_\xi L} \qquad \delta\rho_\xi = \delta\rho + \bar{\rho}\Delta_\xi L = \delta\rho + 3\bar{\rho}\Phi$$

Initial displacement of particles idea:

$$x^\alpha = \xi^\alpha + \delta x_{\text{in}}^\alpha$$

N.E. Chisari and M. Zaldarriaga, Phys. Rev. D 83, 123505 (2011) arXiv:1101.3555

$$\frac{\partial}{\partial \xi^\alpha}\left(\delta x^\alpha_{\mathrm{in}}\right) = 3\zeta_{\mathrm{in}} \longrightarrow$$

initial value of comoving curvature, or curvature perturbation variable

$\Delta_\xi L$ **is replaced by** $3\zeta_{\mathrm{in}}$

$$\zeta = \frac{2a\tilde{H}\left(\Phi' + \tilde{H}\Phi\right)}{\kappa\bar{\rho}c^2} + \Phi$$

$$\Delta\Phi - 3\tilde{H}\left(\Phi' + \tilde{H}\Phi\right) = \frac{\kappa c^2}{2a}\left[\delta\rho_\xi + 3\bar{\rho}\left(\Phi - \zeta_{\mathrm{in}}\right)\right]$$

The introduced comoving curvature does not evolve at large enough scales: $\zeta_{\mathrm{in}} \approx \zeta \longrightarrow$

$$\Delta\Phi = \frac{\kappa c^2}{2a}\delta\rho_\xi \quad \overset{???}{\longleftrightarrow} \quad \Delta\Phi - 3\tilde{H}\left(\Phi' + \tilde{H}\Phi\right) = \frac{\kappa c^2}{2a}\left(\delta\rho + 3\bar{\rho}\Phi\right)$$

$$\delta\rho_\xi = \delta\rho + 3\bar{\rho}\zeta_{\mathrm{in}}$$

Nonzero spatial curvature
Screening of gravity

$$\Delta\Phi + \left(3K - \frac{3\kappa\bar{\rho}c^2}{2a}\right)\Phi = \frac{\kappa c^2}{2a}\delta\rho$$

velocities' contributions dropped

+ 1 for the spherical (closed) space
– 1 for the hyperbolic (open) space

Solutions are smooth at any point except particles' positions (where Newtonian limits are reached) and characterized by zero average values as before.

$$\lambda = \frac{1}{\sqrt{3(1+q)}} \frac{c}{H}$$

not only for the curved space,
but also in the presence of
an arbitrary number
of additional Universe components
in the form of barotropic perfect fluids

at the radiation-dominated stage
of the Universe evolution $\lambda \sim a^2$

Since λ may be associated with the homogeneity scale, asymptotic behaviour $\lambda \to 0$ **when** $a \to 0$ **supports the idea of the homogeneous Big Bang.**

irresistible temptation of associating the Yukawa interaction range λ with the graviton Compton wavelength $h/(m_g c)$

Planck constant **graviton mass**

$$\hbar \equiv h/(2\pi)$$

$$\lambda = \hbar/(m_g c)$$

(today)

$$\frac{1}{\lambda^2} = \frac{m_g^2 c^2}{\hbar^2} = \frac{2\Lambda}{3}$$

$$m_g = \hbar/(\lambda c) \approx 1.7\times 10^{-33}\,\mathrm{eV}$$

$$9\Omega_M = 4\Omega_\Lambda$$

$$\Omega_\Lambda \equiv \frac{\Lambda c^2}{3H_0^2}$$

$$\Omega_M + \Omega_\Lambda = 1$$

$$\Omega_M = 4/13 \approx 0.31,\quad \Omega_\Lambda = 9/13 \approx 0.69$$

Conclusion

I

first-order scalar and vector cosmological perturbations, produced by inhomogeneities in the discrete form of a system of separate point-like gravitating masses, are derived without any extra approximations in addition to the weak gravitational field limit (no c^{-1} series expansion, no “dictionaries”);

II

obtained metric corrections are valid at all (sub-horizon and super-horizon) scales and converge in all points except locations of sources (where Newtonian limits are reached), and their average values are zero (no first-order backreaction effects);

III

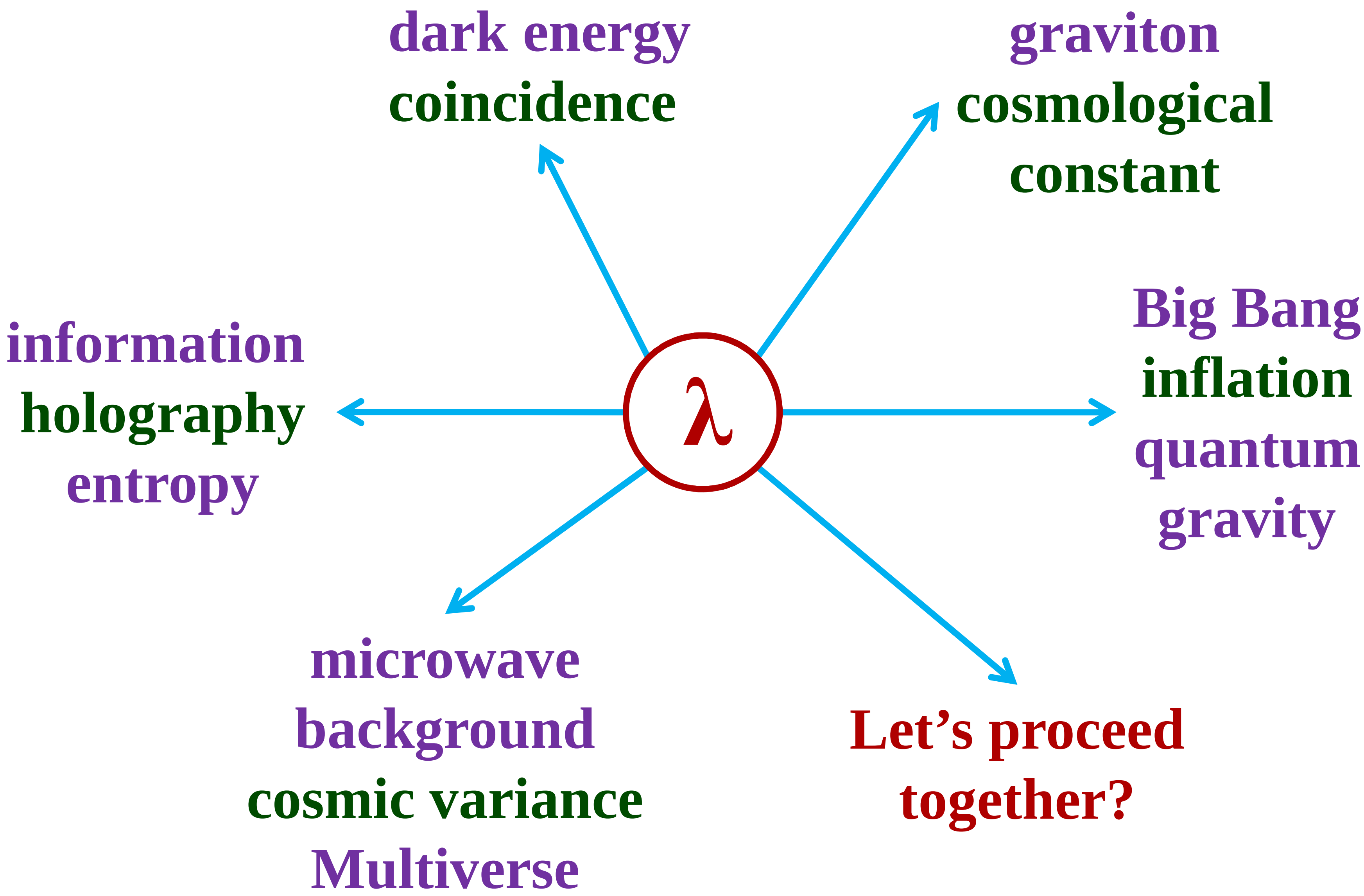
the Minkowski background limit and Newtonian cosmological approximation are particular cases;

IV

the velocity-independent part of the scalar perturbation contains a sum of Yukawa potentials with the same finite time-dependent Yukawa interaction range, which may be connected with the scale of homogeneity, thereby explaining existence of the largest cosmic structures;

V

the general Yukawa range definition is given for various extensions of the concordance model (nonzero spatial curvature, additional perfect fluids).


dark energy
coincidence
graviton
cosmological
constant
information
holography
entropy
λ
Big Bang
inflation
quantum
gravity
microwave
background
cosmic variance
Multiverse
Let’s proceed
together?

FUN

What is the length of the Sydney Harbour Bridge?

A. 1149 m	B. 21 cm
C. 0.7 AU	D. 3.7 Gpc

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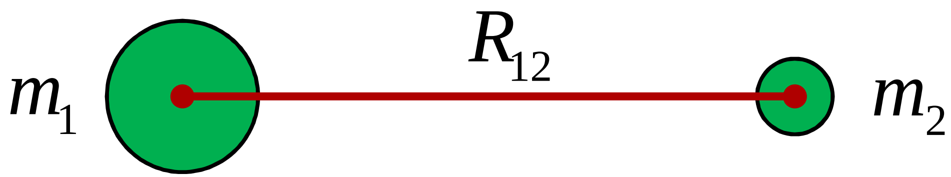


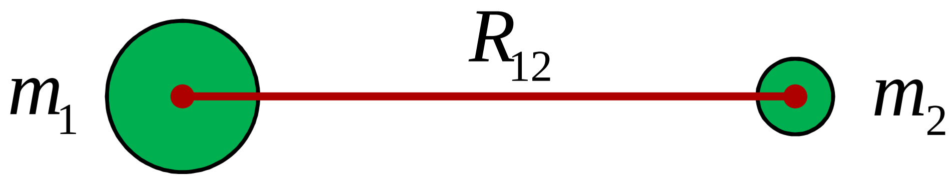
Whose law is the most appropriate for description of universal gravitation???

A. Newton

B. Einstein

C. Yukawa

D. Coulomb



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THANK YOU FOR ATTENTION !