

# Magnetic fields at cosmological recombination

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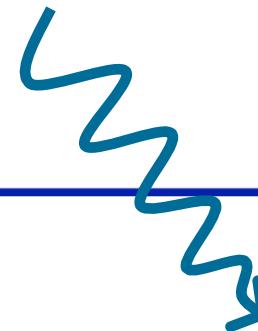


## Abstract

Standard  $\Lambda$ CDM cosmology

~~The 1<sup>st</sup> order vector mode~~

**The 2<sup>nd</sup> order vector mode!!!**



Magnetic fields

# 1 Introduction

## Observations

Magnetic fields are observed ubiquitously even on large scales:

- Galaxy, cluster scales  $O(10^{-6})$  Gauss [P.Blas et al. \[astro-ph/9812487\], ...](#)
- CMB scales  $< O(10^{-9})$  Gauss :upper bound [Planck XIX \[1502.01594\], ...](#)
- Intergalactic scales (or voids?)  $O(10^{-17} \sim -15)$  Gauss

[A.Neronov and L.Vovk \[1006.3504\]](#)  
[K.Takahashi et al. \[1303.3069\]](#)

## Scenario

Cosmological magnetic fields = **seed fields** + dynamo mechanism

## Goal

The origin of seed fields from **the standard cosmology**

## 1.1 (Previous) Mechanisms

### Modification of the electromagnetic sector in the inflationary era

$$\text{ex)} \quad -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \rightarrow -\frac{1}{4}f^2(\phi)F^{\mu\nu}F_{\mu\nu}$$

→ Back reaction of EM fields spoils accelerated expansion.

### Cosmological phase transitions

→ The coherent scale of magnetic fields corresponds to the horizon scale at the phase transition time.

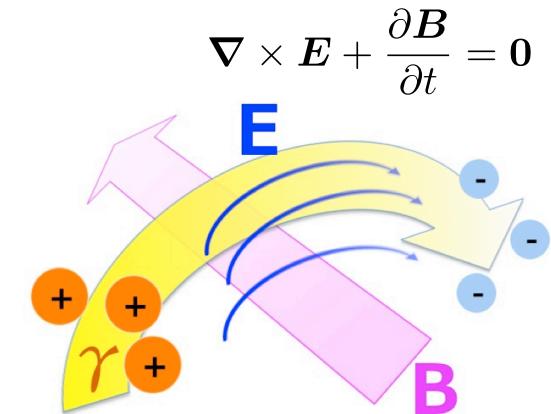
These scenarios do not work.

## 1.2 Harrison mechanism

Primordial plasma in the early universe

→ Thomson scattering induces the relative velocity between protons and electrons.

→ The rotational currents induce magnetic fields.



Based on ...

- Euler equations for electrons, protons, and photons
- Maxwell equation

$$\frac{dB^i}{dt} = \frac{4\sigma_T \rho_\gamma^{(0)} a}{3e} \epsilon^{ijk} \left[ \frac{1}{2} \delta v_{\gamma b j, k}^{(2)} - \delta_{\gamma, j}^{(1)} \delta v_{\gamma b k}^{(1)} - \frac{3}{4} \left( v_{el}^{(1)} \Pi_{\gamma j}^{(1)l} \right)_{,k} \right]$$

2<sup>nd</sup> order Slip term       $\delta v_{\gamma b j} \equiv v_{\gamma j} - v_{b j}$   
 Slip term      Anisotropic stress

All terms are the 2<sup>nd</sup> order effects.

→ 2<sup>nd</sup> order cosmological perturbation theory

## 1.3 Harrison mechanism

1<sup>st</sup> order case:

$$\frac{dB^i}{dt} = \frac{4\sigma_T \rho_\gamma^{(0)} a}{3e} \epsilon^{ijk} \delta v_{\gamma b j, k}^{(1)}$$

In the standard cosmology,

$$\epsilon^{ijk} \delta v_{\gamma b j, k}^{(1)} \propto \epsilon^{ijk} \hat{k}_j \hat{k}_k \rightarrow 0$$

※ Beyond the standard cosmology, for example,

- Cosmic defects
- Modified gravity with vector fields
- Primordial neutrino vorticity

$$\epsilon^{ijk} \delta v_{\gamma b j, k}^{(1)} \neq 0$$

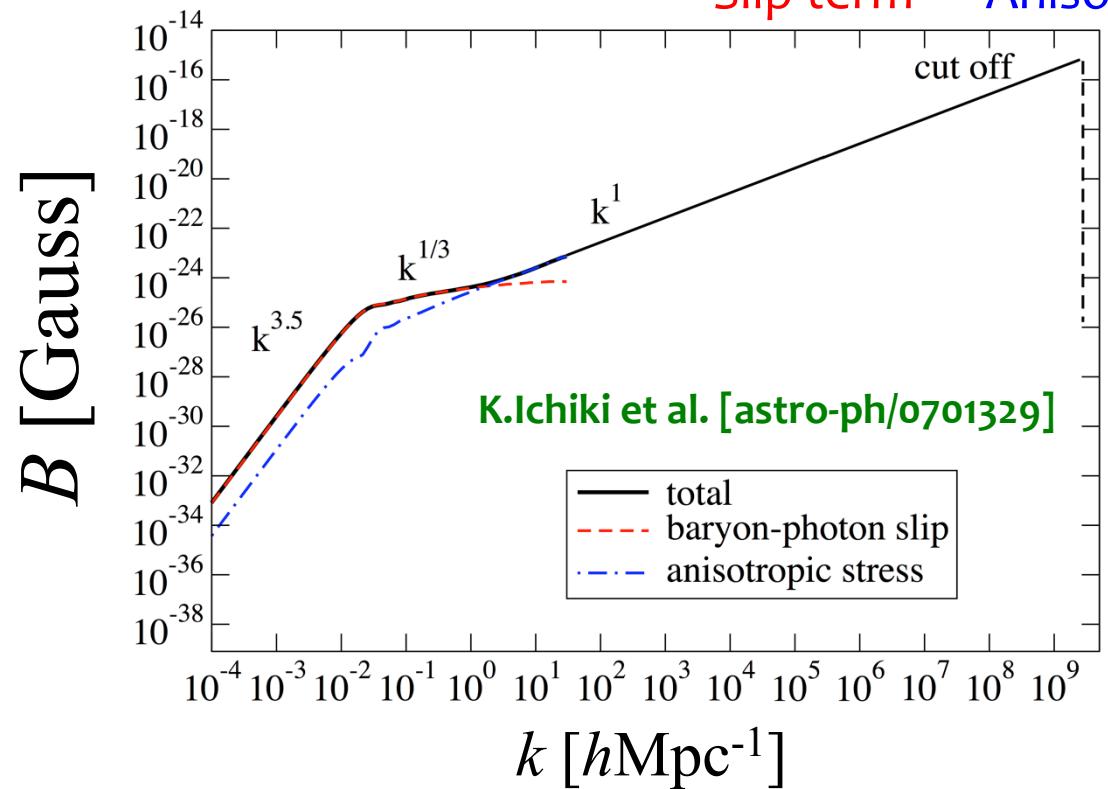
**The amplitude of magnetic fields depends on model parameters.**

## 1.4 Previous studies

2<sup>nd</sup> order Slip term

$$\frac{dB^i}{dt} = \frac{4\sigma_T \rho_\gamma^{(0)} a}{3e} \epsilon^{ijk} \left[ \frac{1}{2} \delta v_{\gamma b i}^{(2)} - \delta_{\gamma j}^{(1)} \delta v_{\gamma b k}^{(1)} - \frac{3}{4} \left( v_{el}^{(1)} \Pi_{\gamma j}^{(1)l} \right)_{,k} \right]$$

Slip term Anisotropic stress



→ The 2<sup>nd</sup> order perturbation theory

## 2. 2<sup>nd</sup> order perturbation theory

### 1<sup>st</sup> order

OK

#### Scalar mode

- Density fluctuations
- CMB fluctuation

Decaying mode

#### Vector mode

- Magnetic fields
- CMB B-mode

Planck?

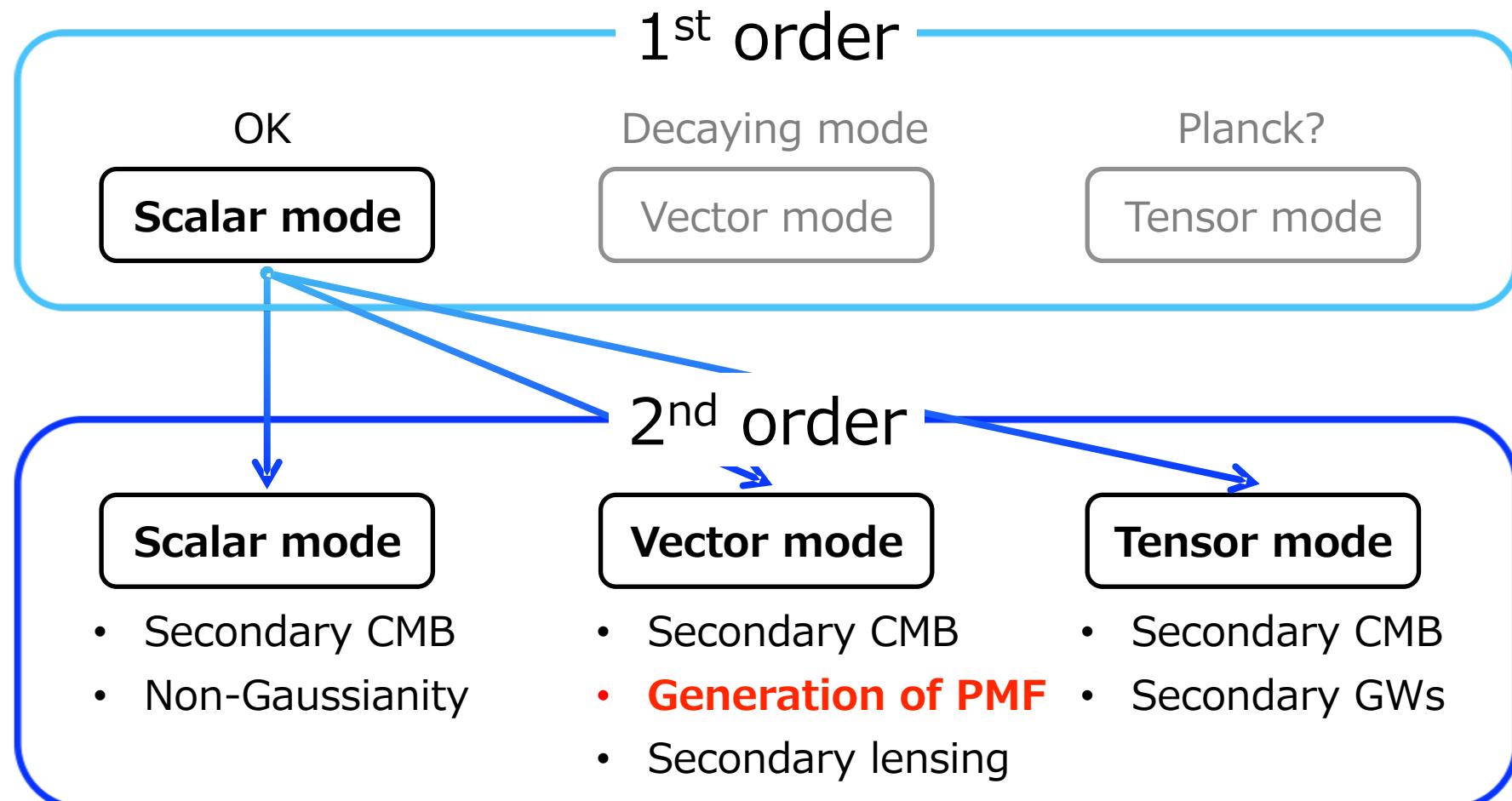
#### Tensor mode

- PGWs
- CMB B-mode

In the 1<sup>st</sup> order theory, each mode evolves independently and ...

1. current observations show in good agreement with the 1<sup>st</sup> order scalar mode.
2. the vector mode is neglected.
3. the tensor mode is provided an upper limit.

## 2.1 2<sup>nd</sup> order perturbation theory



## 2.2 Einstein-Boltzmann system

**Gravity: Einstein eq.**

Poisson gauge:  $ds^2 = a^2 \left[ -e^{2\Psi} d\eta^2 + 2\omega_i d\eta dx^i + (e^{-2\Phi} \delta_{ij} + \chi_{ij}) dx^i dx^j \right]$

$\Psi, \Phi$  (scalar)    $\omega_i$  (vector)    $\chi_{ij}$  (tensor)

$$\begin{aligned} \Psi &= \Psi^{(1)} + \frac{1}{2}\Psi^{(2)} + \dots & \omega_i &= \cancel{\omega_i^{(1)}} + \frac{1}{2}\omega_i^{(2)} + \dots & G_{\mu\nu}^{(1)} &= 8\pi G T_{\mu\nu}^{(1)} \\ \Phi &= \Phi^{(1)} + \frac{1}{2}\Phi^{(2)} + \dots & \chi_{ij} &= \cancel{\chi_{ij}^{(1)}} + \frac{1}{2}\chi_{ij}^{(2)} + \dots & G_{\mu\nu}^{(2)} &= 8\pi G T_{\mu\nu}^{(2)} \end{aligned}$$

**Relativistic fluids: Boltzmann eq.**

$$\Delta_{\ell,m \text{ rel}} = \Delta_{\ell,m \text{ rel}}^{(1)} + \frac{1}{2}\Delta_{\ell,m \text{ rel}}^{(2)} + \dots$$

$$\frac{\partial \Delta_{\ell,m}^{(1,2)}}{\partial \eta} + k \left[ \frac{c_{\ell+1,m}}{2\ell+3} \Delta_{\ell+1,m}^{(1,2)} - \frac{c_{\ell,m}}{2\ell-1} \Delta_{\ell-1,m}^{(1,2)} \right] - \dot{\tau}_c \Delta_{\ell,m}^{(1,2)} = S_{\ell,m}^{(1,2)}$$

$m = 0$  (scalar),  $m = \pm 1$  (vector),  $m = \pm 2$  (tensor)

**Non-relativistic fluids: Euler eq.**

$$\delta_{\text{non-rel}} = \delta_{\text{non-rel}}^{(1)} + \frac{1}{2}\delta_{\text{non-rel}}^{(2)} + \dots$$

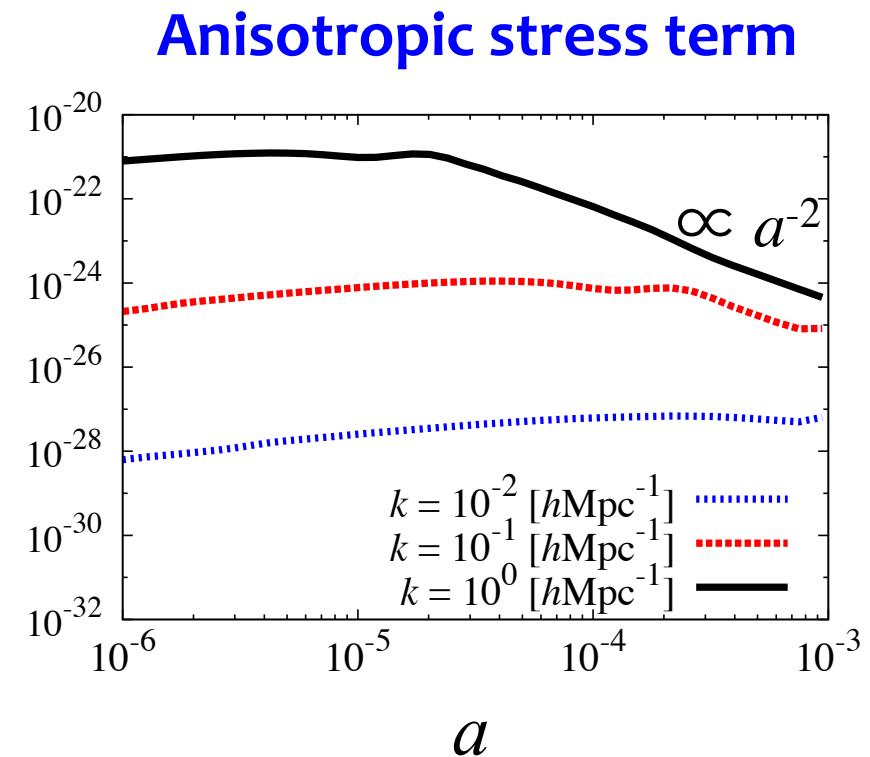
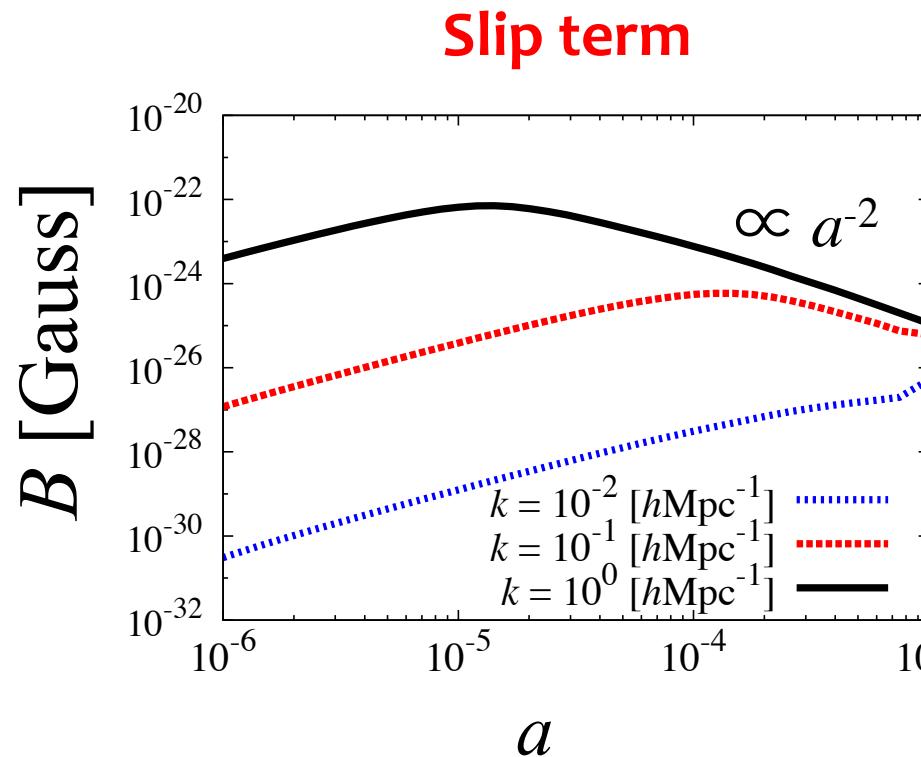
$$v_{\text{non-rel}} = v_{\text{non-rel}}^{(1)} + \frac{1}{2}v_{\text{non-rel}}^{(2)} + \dots$$

→ **The 2<sup>nd</sup> order relative velocity:  $\delta v_{\gamma b}$**

### 3. Results

$$\frac{dB^i}{dt} = \frac{4\sigma_T \rho_\gamma^{(0)} a}{3e} \epsilon^{ijk} \left[ \frac{1}{2} \delta v_{\gamma b j, k}^{(2)} - \delta_{\gamma, j}^{(1)} \delta v_{\gamma b k}^{(1)} - \frac{3}{4} \left( v_{\text{el}}^{(1)} \Pi_{\gamma j}^{(1)l} \right)_{,k} \right]$$

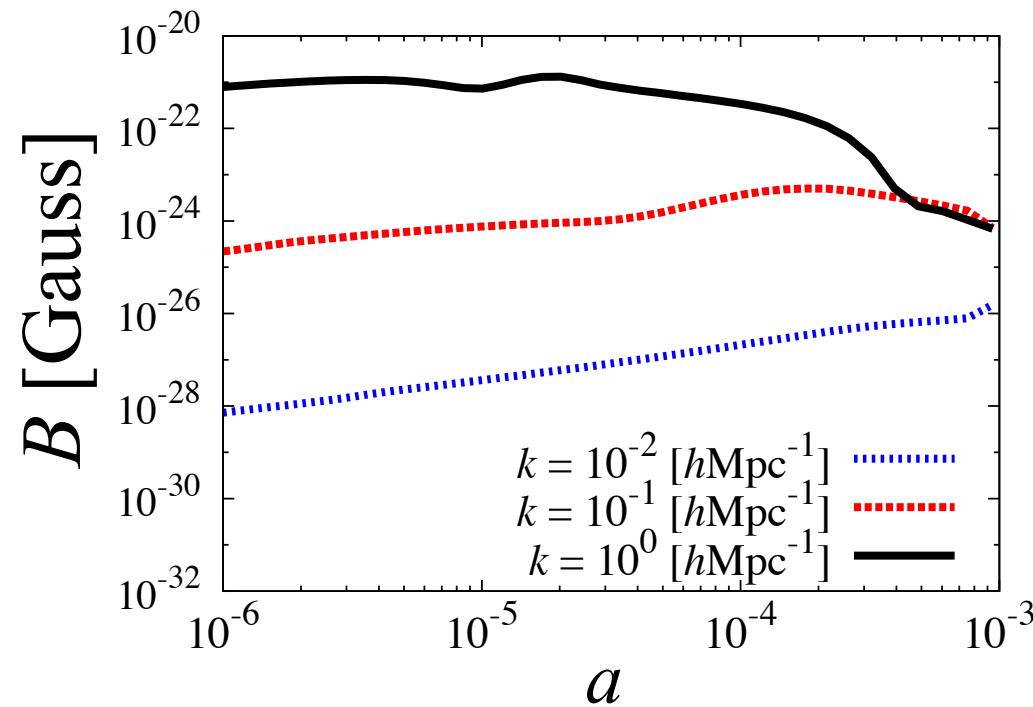
Time evolutions of magnetic fields:



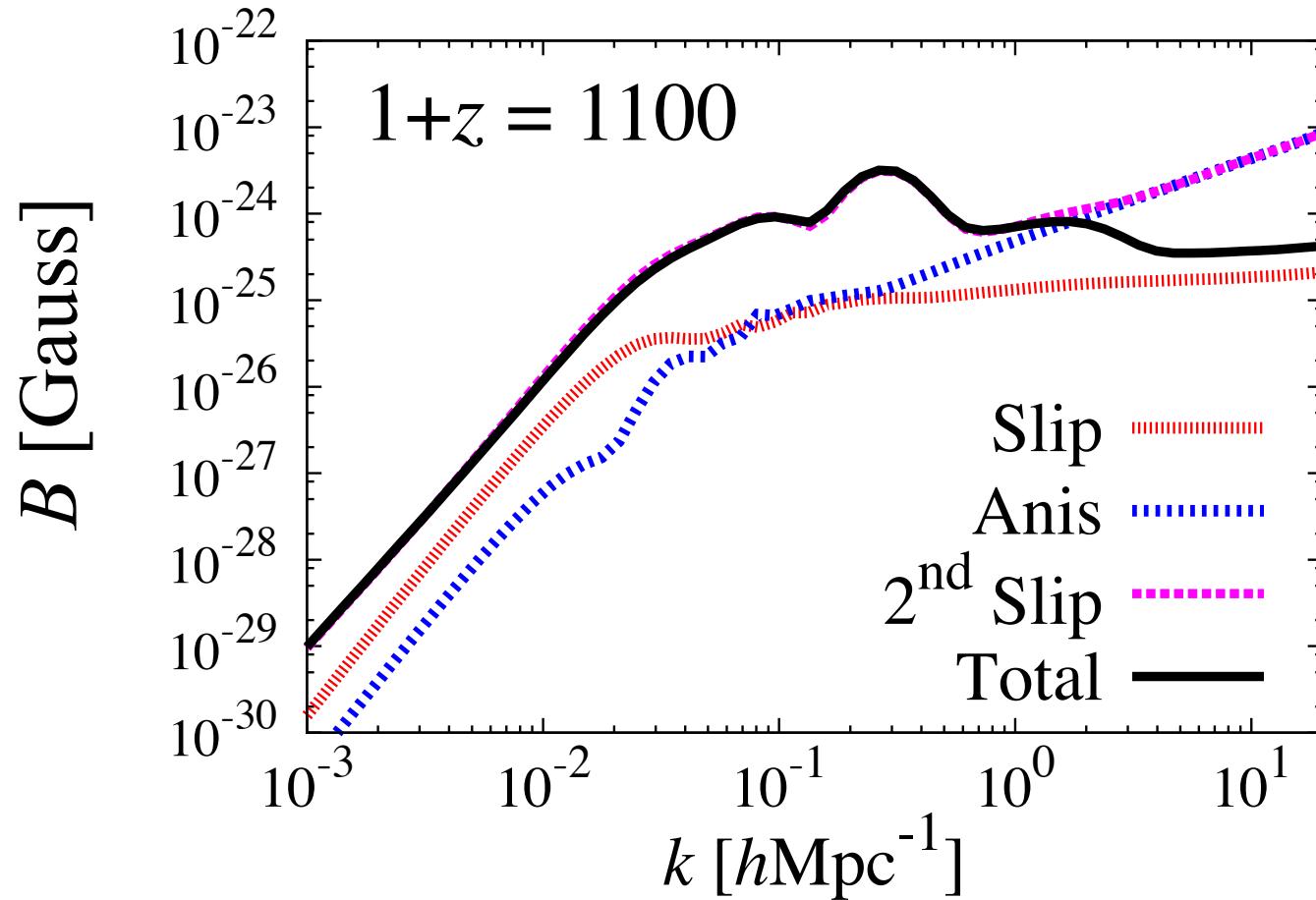
### 3.1 Results

$$\frac{dB^i}{dt} = \frac{4\sigma_T \rho_\gamma^{(0)} a}{3e} \epsilon^{ijk} \left[ \frac{1}{2} \delta v_{\gamma b j, k}^{(2)} - \delta_{\gamma, j}^{(1)} \delta v_{\gamma b k}^{(1)} - \frac{3}{4} \left( v_{\text{el}}^{(1)} \Pi_{\gamma j}^{(1)l} \right)_{,k} \right]$$

Time evolutions of magnetic fields: **2<sup>nd</sup> order Slip term**

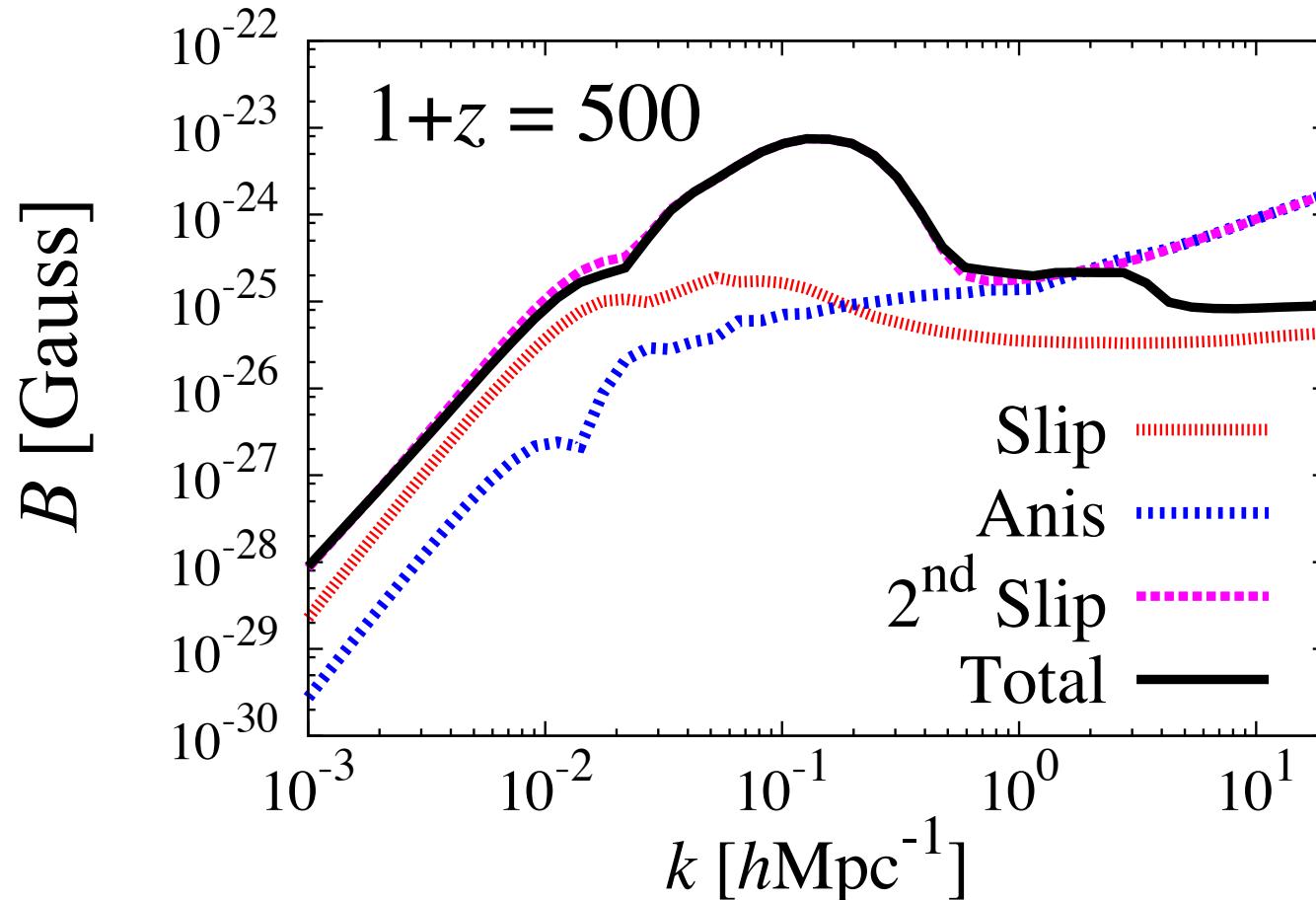


### 3.2 Total magnetic fields “at” the recombination



- ✓ Non-trivial cancellation
- ✓ Bump at  $k \sim 0.5 h\text{Mpc}^{-1}$

### 3.3 Total magnetic fields “after” the recombination



- ✓ At the recombination, the bump is amplified.
- ✓ The amplitude of resultant magnetic fields is about  $10^{-23}$  Gauss

## 3.4 Helical or Non-helical?

Spectrum of magnetic fields:

$$\langle B_i(\mathbf{k})B_j(\mathbf{k}') \rangle = (2\pi)^3 \delta_{\text{D}}^3(\mathbf{k} - \mathbf{k}') \left[ (\delta_{ij} - \hat{k}_i \hat{k}_j) \frac{P_{\text{B}}(k)}{2} + i\epsilon_{ijk} \hat{k}^k \frac{P_{\text{H}}(k)}{2} \right]$$

Non-helical part    Helical part

Helical part → A signature of the parity violation

The Harrison mechanism is based on  
**Standard Maxwell theory and General relativity.**

The Harrison mechanism provides only **non-helical** magnetic fields.

## 4. Summary

- ✓ Resultant 2<sup>nd</sup> order magnetic field has an amplitude about  $10^{-23}$  Gauss at cosmological recombination.
- ✓ Non-trivial cancellation between **anisotropic stress** and **2<sup>nd</sup> order slip** terms on sub-horizon scales.
- ✓ The magnetic spectrum has a bump at  $k \sim 0.5 h\text{Mpc}^{-1}$ .

**No model parameters!**

- ✓ The dynamo mechanism should be understood.